# Probability and statistics

University of Mosul
College of Engineering
Department of Communications and Intelligent Digital Systems

***Probability and statistics 1st Class (2025-2026)***

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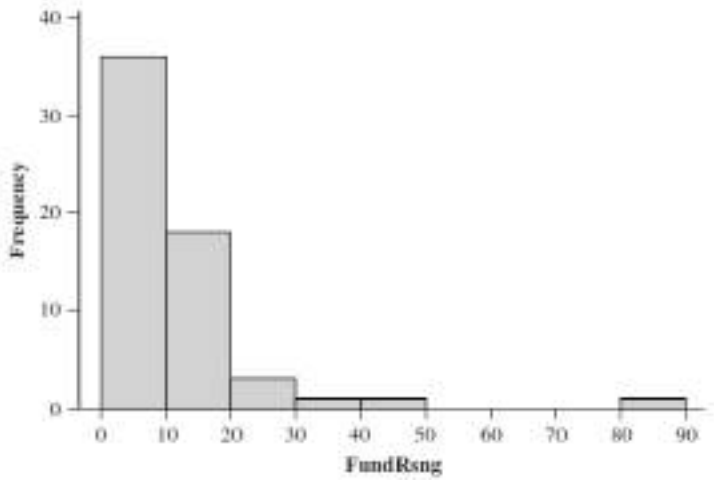
# Introduction of Statistics and probability

- Statistics is a branch of scientific inquiry that provides methods for organizing and summarizing data, and for using information in the data to draw various conclusions.

- Simply put, it is a collection of scientific methods for collecting, displaying, analyzing, and drawing conclusions from data.

Example: Here is data on fundraising expenses as a percentage of total expenditures for a random sample of 60 charities:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6.1 | 12.6 | 34.7 | 1.6 | 18.8 | 2.2 | 3.0 | 2.2 | 5.6 | 3.8 |
| 2.2 | 3.1 | 1.3 | 1.1 | 14.1 | 4.0 | 21.0 | 6.1 | 1.3 | 20.4 |
| 7.5 | 3.9 | 10.1 | 8.1 | 19.5 | 5.2 | 12.0 | 15.8 | 10.4 | 5.2 |
| 6.4 | 10.8 | 83.1 | 3.6 | 6.2 | 6.3 | 16.3 | 12.7 | 1.3 | 0.8 |
| 8.8 | 5.1 | 3.7 | 26.3 | 6.0 | 48.0 | 8.2 | 11.7 | 7.2 | 3.9 |
| 15.3 | 16.6 | 8.8 | 12.0 | 4.7 | 14.7 | 6.4 | 17.0 | 2.5 | 16.2 |



# Introduction of Statistics and probability

## Types of Statistics

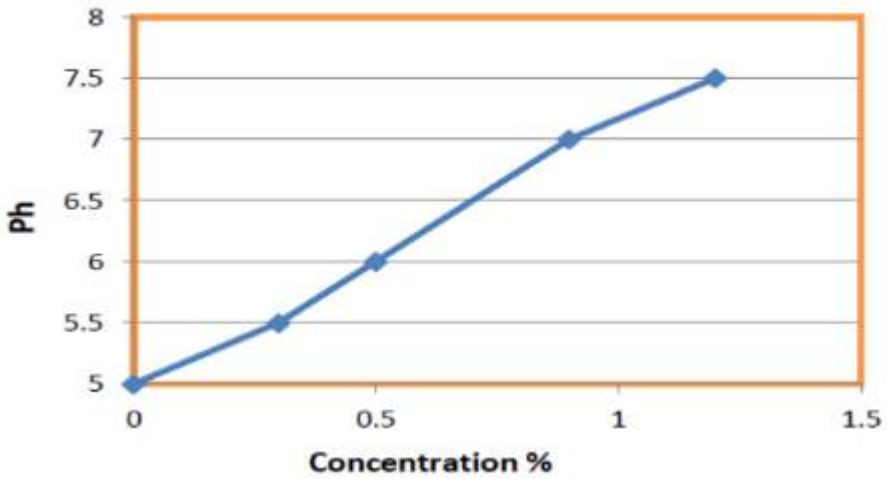
Statistics is mainly divided into two types (**Descriptive, Inferential)**:

### A. Descriptive Statistics:

Descriptive statistics are used to organize and summarize data using numbers and graphs. You are simply describing what the data shows.

- **Summary & Description:** Displaying data using histograms, box plots, scatter plots, etc.
- **Summarize:** Calculating mean, variance, correlation coefficients, etc.

For example, Figure 1 shows that Ph increase as the concentration increase.



# Introduction of Statistics and probability

**B. Inferential Statistics (الإحصاء الاستدلالي):**

Inferential statistics are used to make inferences using a sample data taken from a population or draw a conclusion about the population.

- It takes data from a random sample and makes conclusions about the larger population.
- Given a sample, whose properties are observed, we generalize from this sample about the population itself.

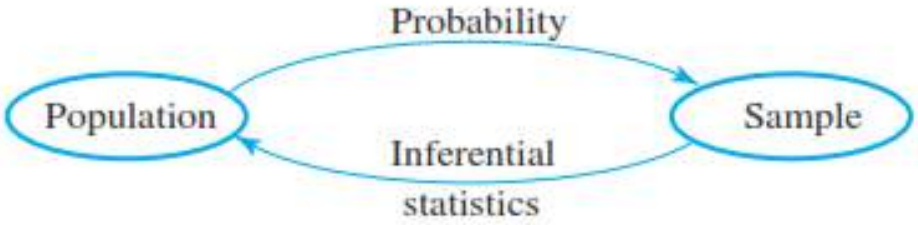
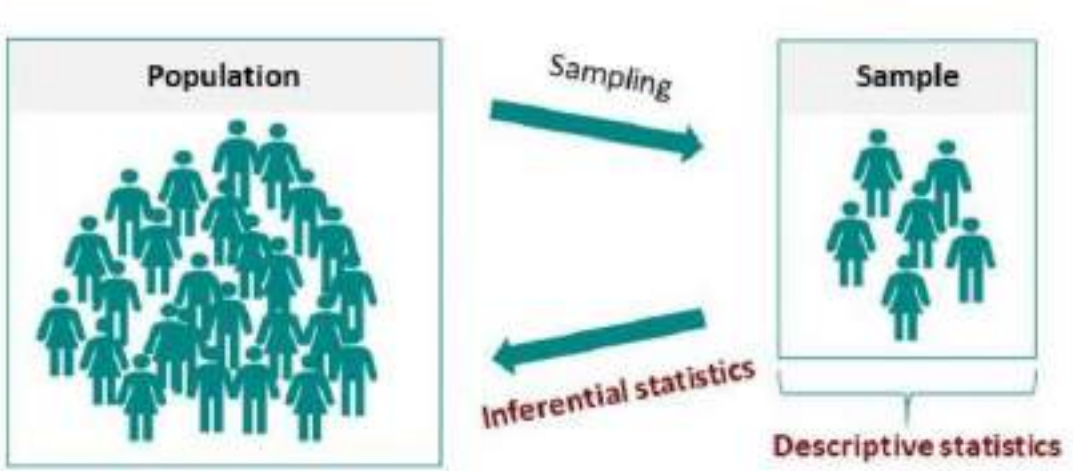



Figure 2: The relationship between probability and inferential statistics



# The advantages of Statistics

## The advantages of Statistics

1.Making sense of data.

2.Identifying patterns and trends

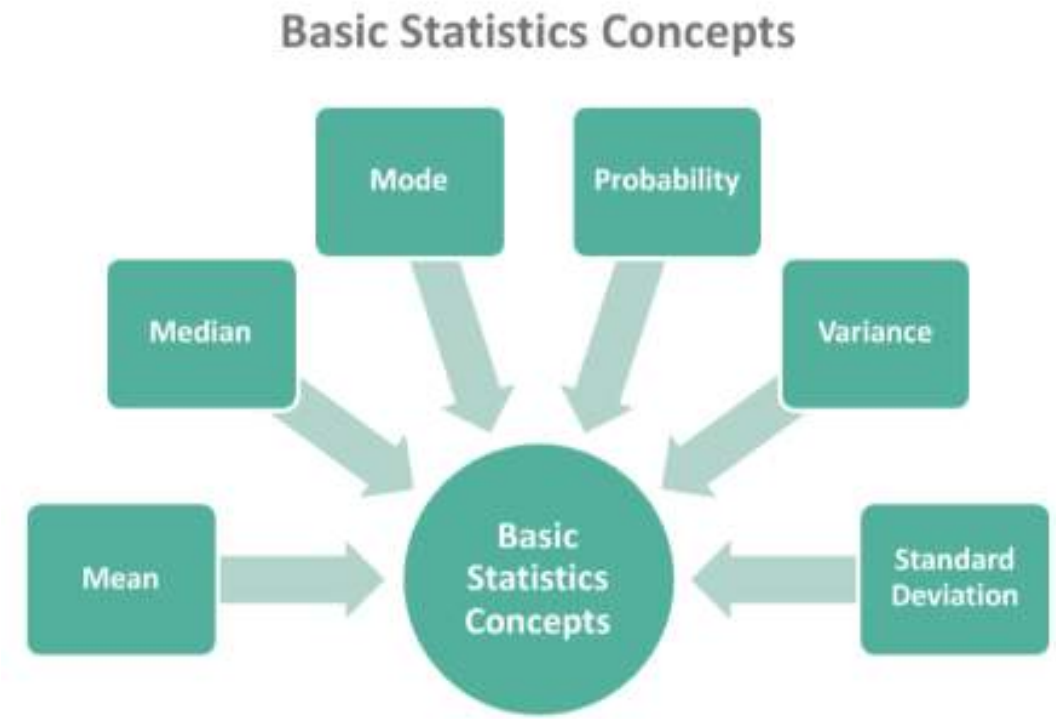
3.Making predictions

4.Making decisions

5.Communicating information

## Basic concepts of statistics

Common concepts include population, sample, measures of Central tendency, variance, and standard deviation, and analysis of variance (ANOVA).



# The advantages of Statistics

**Population** refers to the members of the group that study takes.

**Population:** A population of participants or objects consists of all those participants or objects that are relevant in a particular study.

**Sample** is a part of the population taken for analysis, while the **parameter** is the numerical measure that describes the characteristics of a population set.

**Sample:** A sample is any subset of the population of individuals or things under study.

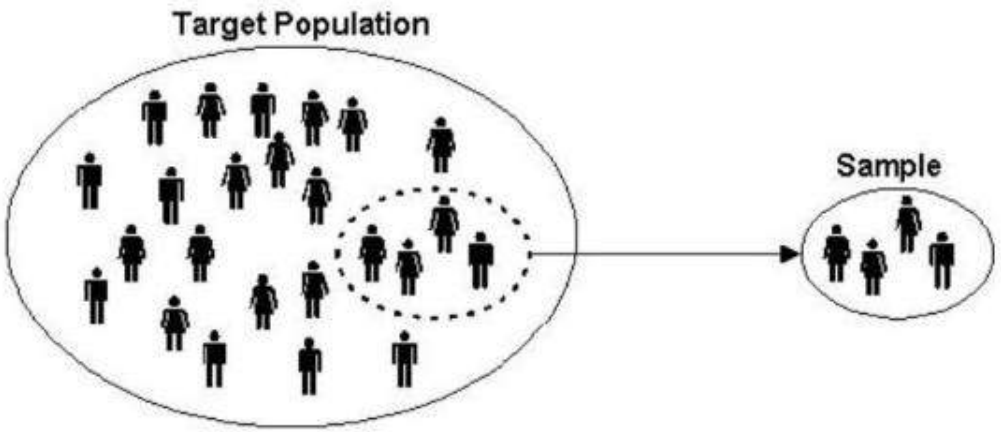
The context depends on the process that generated the data collecting a subset (sample) of observations from a larger set (population) of observations of interest. The idea of taking a sample from a population is central to understanding statistics and the heart of most statistical procedures.

Engineering Example (Telecommunications)

**Population**: All routers produced by a specific company in 2023 (let's assume it is 1,000,000 devices).

**Parameter:** The exact average lifespan of all these 1,000,000 devices.

(In contrast, if we select only 100 devices for testing, the average lifespan of these 100 devices is called a "**Statistic**" and not a parameter.)



# The type of samples

**Type of samples:**

1- Simple random sample
2- Stratified random sample
3- Regular random sample

**1- Simple random sample**: It is the smallest and simplest type of samples taken in a random manner and the number of its individuals should not be less than (30) individuals or items

**2- Stratified random sample** (العينة العشوائية الطبقية)**:** It is taken from population if it is in the form of stratified or if the goal is to study stratified the population.

**3- Regular random sample:** It is taken randomly, but at regular intervals or from regular places

# The variables

**Variables** are some characteristics of the individuals within the population. Variables can take on various types of values, some of them are numbers and some are categories.

It is a characteristic or attribute that can assume different values. Like water temperature, turbidity, building height.

Variables can be classified into two groups: Categorical (فئوية) (or qualitative (نوعية)) and Quantitative (كمية).

**1. Categorical or qualitative variables** are variables that can be placed into distinct categories, according to these variables cannot be measured, but it is counted. For some characteristic or attribute. example, the **color** of eye, the **taste** of water and the **gender** (male or female).

Categorical or qualitative allow listing individual's characteristics into categories. Categorical data represent characteristics such as a person's gender, marital status, and hometown. Categorical data can take on numerical values (such as "1" indicating male and "2" indicating female), but those numbers don't have mathematical meaning. You couldn't add them together .The data collected for a categorical variable are qualitative data. They may be further described as either ordinal or nominal:

# The variables

**1- Nominal variables** (المتغيرات الاسمية): Elements of a sample or a population can be classified using a nominal variable. The value of the variable places an element in a certain class or category. Examples of such variables are:

- Gender (male/female).
- Nationality (الجنسية).
- Religion (الديانة).
- Whether or not one owns a car (yes/no).

**2- Ordinal variables** (المتغيرات الترتيبية) Ordinal variable implies a logical order between the elements of a sample. Typical examples of ordinal variables can be found in all kinds of surveys. There, respondents are typically asked whether they consider the quality of a product or service as “1: very good”, “2: good”, “3: moderate”, “4: bad”, or “5: very bad”.

In other surveys, the respondents are asked if they “1: strongly disagree”, “2: rather disagree”, “3: neither agree nor disagree”, “4: rather agree”, or “5: strongly agree” with a particular statement.

ما الفرق بين المتغيرات الاسمية والترتيبية؟

# The variables

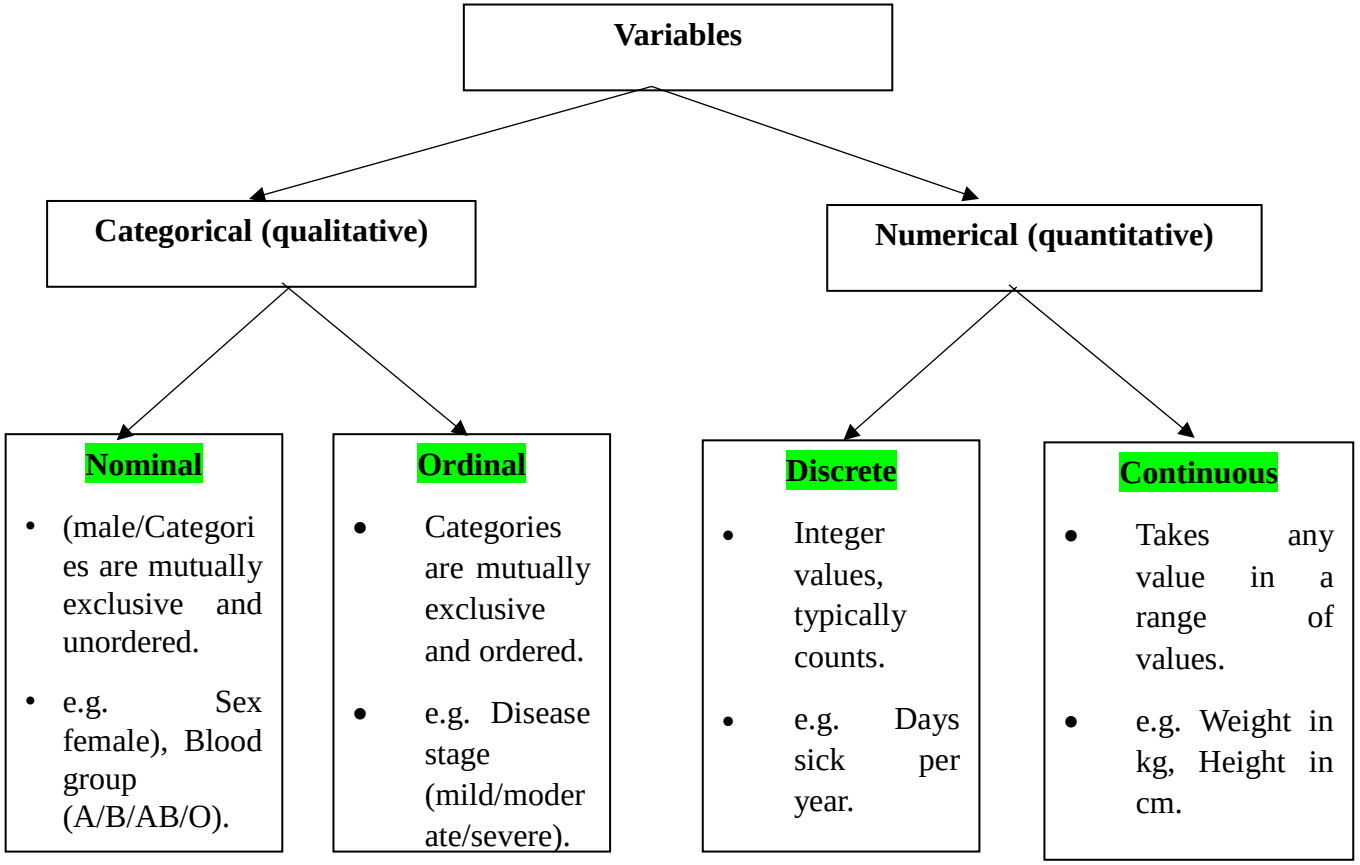
**2. Quantitative variables** A variable that is measured on a quantitative scale can be expressed as a fixed number of measurement units. Examples are **length**, **area**, **volume**, **weight**, **duration**, **price**, income, **waiting time**, and so on. It can be divided into:

**1- Discrete variables** A Discrete variable is a quantitative variable that will assume a finite, or a countable, set of values. A discrete variable cannot take on every possible value in an interval on the real line. Each value can be plotted as a separate point on the real line, with space between any two consecutive points. Examples of discrete variables are **the number of passengers on a flight**, **the number of children in a family**, **the number of students in a college**.

**2- Continuous variables** A Continuous variable is a quantitative variable that has an uncountable number of values. In other words, a continuous variable can assume any value between any two points on the real line, and thus the possible values of a continuous variable can form an interval on the number line, with no spaces between the points. Examples of continuous variables are **length**, **weight**.

# The variables

The classification chart of variables is shown in Figure.



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# Measures of Central Tendency

The three indicators of central tendency are the mean, median, and mode. One can determine the central value of the given set of data (grouped and ungrouped) using the three measures of central tendency.

Types of Averages:

There are five averages. Among them mean, median and mode are called simple averages and the other two averages geometric mean and harmonic mean are called special averages.

Types of mean:

1- Arithmetic Mean

2- The Geometric Mean

3- The Harmonic Mean

# The mean Arithmetic (المعدل الحسابي)

Arithmetic Mean ($\overline{x}$) the sample mean, or ($\mu_x$) population mean.

The arithmetic mean (average) of a set of numbers is obtained by dividing the sum of numbers of the set by the number of values. If the variable x assumes n values x1, x2, . . . .,xn then the mean, is given by:

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum x}{n}$$

Ungrouped Data (بيانات غير مبوبة)

Example: Calculate the mean for 2, 4, 6, 8, and 10.

$$Mean = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

Example: find the mean of a student's scores on five exams
(80, 82,76,87,84)

$\overline{X}_{mean}$= (80+82+76+87+84)/5=(409)/5 =81.8

# The mean Arithmetic (المعدل الحسابي)

Example: Calculate the mean height of University students

| Height (cm) |
|---|
| 175 |
| 163 |
| 155 |
| 149 |
| 180 |
| 165 |

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{(175 + 163 + 155 + 149 + 180 + 165)}{6} = \frac{987}{6}$$

$$= 164.5 \text{ cm}$$

Example: The lengths of ten students in centimeters are as follows:

**160 161 163 160 162 165 167 164 163 170**

Find the mean length of the students?

Summation of the length = 160 + 161 + . . . . . + 170 = 1635 cm

Mean = Sum of the values/ n = 1635/10 = 163.5 cm

# The mean Arithmetic (المعدل الحسابي)

Direct method : If the observations $x_1, x_2 \ldots x_n$ have frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then the mean is given by :

$$Mean(\bar{X}) =) \frac{(f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \cdots + f_n} = \frac{\sum f_i x_i}{\sum f_i}$$

Grouped data (بيانات مبوبة)

# The mean Arithmetic (المعدل الحسابي)

Example: Calculate the mean for the table?

| Class group (C)(1) | Frequency . (fi)(3) |
|---|---|
| 36-45 | 5 |
| 46-55 | 7 |
| 56-65 | 10 |
| 66-75 | 14 |
| 76-85 | 12 |
| 86-95 | 8 |
| 96-105 | 4 |
| | |

Solution:

Mid-class / Midpoint $X_i$: This is the average of the lower and upper limits of each class.

$f_i X_i$ : This is the product of the frequency and the midpoint for each class.

| Class group (C)(1) | Frequency . (fi)(3) | Mid class (Xi)(2) | fiXi (2)*(3) |
|---|---|---|---|
| 36-45 | 5 | 40.5 | 202.5 |
| 46-55 | 7 | 50.5 | 353.5 |
| 56-65 | 10 | 60.5 | 605 |
| 66-75 | 14 | 70.5 | 987 |
| 76-85 | 12 | 80.5 | 966 |
| 86-95 | 8 | 90.5 | 724 |
| 96-105 | 4 | 100.5 | 402 |
| | 60 | | 4240 |

$$\sum_{i=1}^{n} f_i X_i = 4240$$

$$X_{mean} = \frac{\sum_{i=1}^{n} f_i X_i}{n} = 4240/60 = 70.667$$

# The mean Arithmetic (المعدل الحسابي)

Example: Find the mean weight of 80 students for the grouped data below (all the data in kilogram):

| Classes true limits | Frequency ($f_i$) |
|---|---|
| 63.5–64.5 | 1 |
| 64.5–65.5 | 0 |
| 65.5–66.5 | 2 |
| 66.5–67.5 | 5 |
| 67.5–68.5 | 9 |
| 68.5–69.5 | 22 |
| 69.5–70.5 | 16 |
| 70.5–71.5 | 12 |
| 71.5–72.5 | 8 |
| 72.5–73.5 | 3 |
| 73.5–74.5 | 1 |
| 74.5–75.5 | 1 |

Solution:

$$\bar{y} = \frac{\sum_{i=1}^{k} y_i f_i}{\sum_{i=1}^{k} f_i} = \frac{y_1 f_1 + y_2 f_2 + \ldots\ldots + y_k f_k}{\sum f_i}$$

$$\bar{y} = 5577 / 80 = 69.70$$

| Classes true limits | Frequency ($f_i$) | Class mid-point ($y_i$) | $y f_i$ |
|---|---|---|---|
| 63.5–64.5 | 1 | (63.5+64.5)/2= 64 | 64 |
| 64.5–65.5 | 0 | 65 | 0 |
| 65.5–66.5 | 2 | 66 | 132 |
| 66.5–67.5 | 5 | 67 | 335 |
| 67.5–68.5 | 9 | 68 | 612 |
| 68.5–69.5 | 22 | 69 | 1518 |
| 69.5–70.5 | 16 | 70 | 1120 |
| 70.5–71.5 | 12 | 71 | 852 |
| 71.5–72.5 | 8 | 72 | 576 |
| 72.5–73.5 | 3 | 73 | 219 |
| 73.5–74.5 | 1 | 74 | 74 |
| 74.5–75.5 | 1 | 75 | 75 |
| Total | 80 | | 5577 |

# The mean Arithmetic (المعدل الحسابي)

Example: Given the following frequency distribution, calculate the arithmetic mean

| Marks (x) | 50 | 55 | 60 | 65 | 70 | 75 |
|---|---|---|---|---|---|---|
| No of Students (f) | 2 | 5 | 4 | 4 | 5 | 5 |

**Solution:**

| **Marks (x)** | 50 | 55 | 60 | 65 | 70 | 75 | Total |
|---|---|---|---|---|---|---|---|
| **No of Students (f)** | 2 | 5 | 4 | 4 | 5 | 5 | 25 |
| **fx** | 100 | 275 | 240 | 260 | 350 | 375 | 1600 |

$$Mean(\bar{X}) =) \frac{(f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \cdots + f_n} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{1600}{25} = 64$$

# The mean Arithmetic (المعدل الحسابي)

Example: Given the following frequency distribution, calculate the arithmetic mean

| Marks (x) | 50 | 55 | 60 | 65 | 70 | 75 |
|---|---|---|---|---|---|---|
| No of Students (f) | 2 | 5 | 4 | 4 | 5 | 5 |

Solution:

| x | f | fx |
|---|---|---|
| 50 | 2 | 100 |
| 55 | 5 | 275 |
| **60** | 4 | 240 |
| 65 | 4 | 260 |
| 70 | 5 | 350 |
| 75 | 5 | 375 |
| | 25 | 1600 |

$$Mean(\bar{X}) = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{1600}{25} = 64$$

# The mean Arithmetic (المعدل الحسابي)

Example: Given is the accidents data, find the sample mean

| Number of accidents, $x_i$ | Frequency $f_i$ |
|---|---|
| 0 | 55 |
| 1 | 14 |
| 2 | 5 |
| 3 | 2 |
| 4 | 0 |
| 5 | 2 |
| 6 | 1 |
| 7 | 0 |
| 8 | 1 |

Solution:

| Number of accidents, $x_i$ | Frequency $f_i$ | $f_i x_i$ |
|---|---|---|
| 0 | 55 | 0 |
| 1 | 14 | 14 |
| 2 | 5 | 10 |
| 3 | 2 | 6 |
| 4 | 0 | 0 |
| 5 | 2 | 10 |
| 6 | 1 | 6 |
| 7 | 0 | 0 |
| 8 | 1 | 8 |
| TOTAL | 80 | 54 |

$$\Rightarrow \bar{x} = \frac{54}{80} = 0.675$$

# The mean

## 2. Geometric Mean (المعدل الهندسي)

This is often used for growth rates, percentages, or financial investments (الشؤون المالية)where values are multiplied rather than added.

- **Example:** An investment grows by **5%** (1.05) in the first year and **20%** (1.20) in the second year.
- **Calculation:** Multiply the values and take the root based on the number of values.

$$\text{G. Mean} = \sqrt{1.05 \times 1.20} \approx 1.122$$

*(This means an average annual growth of approximately 12.2%).*

## 3. Harmonic Mean (المعدل التوافقي)

This is used when dealing with rates or ratios, such as speed (distance/time).

- **Example:** You drive to work at **60 km/h** and drive back home on the same route at **40 km/h**. What is your average speed for the whole trip?
- **Calculation:** Use the reciprocal of the values.

$$\text{H. Mean} = \frac{2}{\frac{1}{60} + \frac{1}{40}} = 48\ km/h$$

*(Note: The arithmetic mean would be 50, but the harmonic mean is more accurate here because the distance is constant and the time varies).*

# The median (المتوسط)

The median is the middle value of a distribution i.e., median of a distribution is the value of the variable which divides it into two equal parts. It is the value of the variable such that the number of observations above it is equal to the number of observations below it.

**When odd numbers of values**

$$Median, Md = \left(\frac{N+1}{2}\right)^{th} \quad \textit{(ungrouped data)}$$

If there is an odd number of values in the data set, then the median is the middle value:

$$Median = \frac{N+1}{2}$$

**Example** : Find median for the following data 25, 18, 27, 10, 8, 30, 42, 20, 53

**Solution:**

Arranging the data in the increasing order 8, 10, 18, 20, 25, 27, 30, 42, 53

Here, numbers of observations are odd (N= 9)

Hence, Median, Md = $\left(\frac{N+1}{2}\right)^{th}$ item = $\left(\frac{9+1}{2}\right)^{th}$ item= $(5)^{th}$ item

The middle value is the 5th item i.e., 25 is the median value.

# The Median

When even numbers of values are given:

$$\text{median} = \frac{n}{2} + 1 \,, \frac{n}{2} \qquad \text{..... عدد زوجي (n)}$$

**Ungrouped data**

- If there is an even number of values in the data set, then the median is the mean of the two middle values

Example: Find median for the following data 5, 8, 12, 30, 18, 10, 2, 22

Solution:

Arranging the data in the increasing order 2, 5, 8, 10, 12, 18, 22, 30

Here median is the mean of the middle two items (i.e) mean of (10, 12) i.e., $\left(\frac{10+12}{2}\right) = 11$

# The Median

Example: Find median for the following data 9, 10, 8, 12, 7, 14, 15, 11

Solution: The number of the data (n) = 8 (even), then the mean of the two values of order (n/2 = 8/2 = 4) and ((n/2)+1) = 4 +1 =5 is the mean

Sort the data: 7, 8, 9, **10, 11**, 12, 14, 15

Then the median = (10 + 11)/2 = 10.5

Example: Find median for the following data 9, 10, 8, 12, 7, 14, 15, 11

2.7, 1.8, 0.8, 3.1, **2.6,** 2.7, 1.3, 4.5

Solution: 0.8, 1.3, 1.8, **2.6, 2.7**, 2.7, 3.1, 4.5

The sample median in this case is taken to be the average of 2.6 and 2.7, namely (2.6+ 2.7)/2 = 2.65.

# The Median

$$\overline{Me} = L_1 + \left( \frac{\frac{\sum f_i}{2} - F_i}{f_i} \right) w$$

**Grouped data**

$L_1$ = the lower limit of median class القيمة الوسطية بين الفئات (الفجوة)

$\sum f_i$ = summation of frequencies

$F_i$ = Ascending cumulative frequency at starting of median class التكرار التراكمي للفئة التي تسبق الفئة الوسطية.

$f_i$ = frequency of median class

w = length of median frequency عرض الفئة (عدد الأرقام)

Example: Find the median of the frequency distribution table:

| Classes | Frequency ($f_i$) |
|---|---|
| 50–53 | 2 |
| 54–57 | 6 |
| 58–61 | 7 |
| 62–65 | 10 |
| 66–69 | 12 |
| 70–73 | 8 |
| 74–77 | 5 |
| 78–81 | 4 |
| 82–83 | 2 |
| $\sum$ | 56 |

# The Median

Example: Find the median of the frequency distribution table:

| Classes | Frequency ($f_i$) |
|---|---|
| 50–53 | 2 |
| 54–57 | 6 |
| 58–61 | 7 |
| 62–65 | 10 |
| 66–69 | 12 |
| 70–73 | 8 |
| 74–77 | 5 |
| 78–81 | 4 |
| 82–83 | 2 |
| $\Sigma$ | 56 |

$$\overline{Me} = L_1 + \left( \frac{\frac{\sum f_i}{2} - F_i}{f_i} \right) w$$

Solution:

$\sum f_i = 56$

Median order $= (\sum f_i)/2 = 56/2 = 28$

Therefore, the median is the class of ascending cumulative frequency of 37 or in the class 66-69

$L_1 = 66 - 0.5 = 65.5$

$f_i = 12$

$w = 69 - 66 + 1 = 4$

$$\overline{Me} = 65.5 + \left( \frac{\frac{56}{2} - 25}{12} \right) * 4 = 66.5$$

| Classes | Frequency ($f_i$) | Ascending cumulative frequency |
|---|---|---|
| 50–53 | 2 | 2 |
| 54–57 | 6 | 8 |
| 58–61 | 7 | 15 |
| 62–65 | 10 | 25 |
| 66–69 | 12 | 37 |
| 70–73 | 8 | 45 |
| 74–77 | 5 | 50 |
| 78–81 | 4 | 54 |
| 82–83 | 2 | 56 |
| $\Sigma$ | 56 | |

# The Median

Example: Find the median of the frequency distribution table:

| Class group (C)(1) | Frequency . (fi)(3) |
|---|---|
| 36-45 | 5 |
| 46-55 | 7 |
| 56-65 | 10 |
| 66-75 | 14 |
| 76-85 | 12 |
| 86-95 | 8 |
| 96-105 | 4 |
| | |

Solution:

$$\overline{Me} = L_1 + \left( \frac{\frac{\sum f_i}{2} - F_i}{f_i} \right) w$$

$$\bar{X}_{median} = 65.5 + \left[ \frac{\frac{60}{2} - 22}{14} \right] \times 10$$

| Class group (C)(1) | Mid class (Xi)(2) | Frequency . (fi)(3) | Cumulative Frequency(4) |
|---|---|---|---|
| 36-45 | 40.5 | 5 | 5 |
| 46-55 | 50.5 | 7 | 12 |
| 56-65 | 60.5 | 10 | 22 |
| 66-75 | 70.5 | 14 | 36 |
| 76-85 | 80.5 | 12 | 48 |
| 86-95 | 90.5 | 8 | 56 |
| 96-105 | 100.5 | 4 | 60 |
| | | | |

# The Median

Example: Find the median of the frequency distribution table:

| Class | Frequency ($f_i$) |
|---|---|
| 60 – 62 | 5 |
| 63 – 65 | 18 |
| 66 – 68 | 42 |
| 69 – 71 | 27 |
| 72 – 74 | 8 |

Solution:

$$\overline{Me} = L_1 + \left( \frac{\frac{\sum f_i}{2} - F_i}{f_i} \right) w$$

| Class | Frequency ($f_i$) | ascending |
|---|---|---|
| 60 – 62 | 5 | 5 |
| 63 – 65 | 18 | 23 |
| 66 – 68 | 42 | 65 |
| 69 – 71 | 27 | 92 |
| 72 – 74 | 8 | 100 |
| | $\sum$ fi = 100 | |

$$\overline{Me} = 65.5 + \left[ \frac{100/2 - 23}{42} \right] * 3 = 67.43$$

# The Mode (المنوال)

The mode or modal value of a distribution is that value of the variable for which the frequency is the maximum. It refers to that value in a distribution which occurs most frequently. It shows the center of concentration of the frequency in around a given value. Therefore, where the purpose is to know the point of the highest concentration it is preferred. It is, thus, a positional measure.

## Ungrouped data:

**a.** One mode.

Example 1: 2, 7, 10, 15, 10, 17, 8, 10, 2

Mode = $M_0$ = 10

Example 2: What is the mode of the following data? (4, 5, 7, 4, 6, 5, 5, 2, 5)

As 5 has the higher frequency, then the mode = 5.

**b.** In some cases the mode may be absent while in some cases there may be more than one mode.

Example 3: 2, 5, 6, 2, 4, 5, 1

Since 2 and 5 have the same frequency and higher than other values, then the mode = 2 and 5.

**c.** 2, 5, 6, 3, 4, 9

Since all the data have the same frequency 1 than all the is the mode.

# The Mode (المنوال)

For example:

- The mode of the set 2, 2, 5, 7, 9, 9, 9, 10, 10, 11, 12,18 = 9.

And the set 3, 5, 8, 10, 12, 15,16 has ***no mode***.

- The mode here 2, 3, 4, 4, 4, 5, 5, 7, 7, 7, 9 is 4 and 7, and is called ***bimodal.***
- A distribution having only one mode is called ***unimodal***.

# The Mode (المنوال)

**Grouped data:**

$$\overline{Mo} = L_1 + \frac{d_1}{d_1 + d_2} \cdot w$$

$L_1$ = the lower limit of the mode class
$d_1$ = the difference between the frequency of the mode class and the previous class
$d_2$ = the difference between the frequency of the mode class and the following class
$w$ = the length of mode class

Example: Find the mode of the following frequency distribution.

| Classes | Frequency (fi) |
|---|---|
| 58-59 | 5 |
| 60-61 | 12 |
| 62-63 | 25 |
| 64-65 | 40 |
| 66-67 | 28 |
| 68-69 | 16 |
| total | 126 |

Mode class of higher frequency

w = upper limit – lower limit + 1
= 65 – 64 + 1 = 2

$\overline{Mo} = 64 + \frac{15}{15+12} * 2 = 65.11$

$d_1 = 40 - 25 = 15$
$d_2 = 40 - 28 = 12$

# The Mode (المنوال)

**Grouped data:**

Example: Find the mode of the following table.

| Class group (C)(1) | Frequency . (fi)(3) |
|---|---|
| 36-45 | 5 |
| 46-55 | 7 |
| 56-65 | 10 |
| 66-75 | 14 |
| 76-85 | 12 |
| 86-95 | 8 |
| 96-105 | 4 |
| | |

Solution:

$$\overline{Mo} = L_1 + \frac{d_1}{d_1 + d_2} \cdot w$$

| Class group (C)(1) | Mid class (Xi)(2) | Frequency . (fi)(3) |
|---|---|---|
| 36-45 | 40.5 | 5 |
| 46-55 | 50.5 | 7 |
| 56-65 | 60.5 | 10 |
| 66-75 | 70.5 | 14 |
| 76-85 | 80.5 | 12 |
| 86-95 | 90.5 | 8 |
| 96-105 | 100.5 | 4 |
| | | |

d1=14-10=4

d2=14-12=2

$$\bar{X}_{mode} = 65.5 + \left(\frac{4}{4+2}\right) \times 10$$

=72.16

# The Mode (المنوال)

**Grouped data:**

Example: Find the mode of the following table.

| Class | Frequency ($f_i$) |
|---|---|
| 60 – 62 | 5 |
| 63 – 65 | 18 |
| 66 – 68 | 42 |
| 69 – 71 | 27 |
| 72 – 74 | 8 |

Solution:

$$\overline{Mo} = L_1 + \frac{d_1}{d_1 + d_2} \cdot w$$

The modal class is the one with higher frequency, which is 66-68.

$$\therefore Mo = 65.5 + \left(\frac{24}{24 + 15}\right) * 3$$

$$= 67.35$$

# The Mode (المنوال)

**Grouped data:**

Example: Find the mode of the following table.

| Cl | f |
|---|---|
| 0-50 | 5 |
| 50-100 | 14 |
| 100-150 | 40 |
| 150-200 | 91 |
| 200-250 | 150 |
| 250-300 | 87 |
| 300-350 | 60 |
| 350-400 | 38 |
| 400 and above | 15 |

Solution:

$$\overline{Mo} = L_1 + \frac{d_1}{d_1 + d_2} \cdot w$$

The highest frequency is 150 and corresponding class interval is 200 – 250, which is the modal class.

L = 200

$d_1 = 150 - 91 = 59$

$d_2 = 150 - 87 = 63$

w = 250 – 200 +1 = 51

$$\overline{Mo} = 200 + \frac{59}{59 + 63} \cdot 50 = 224.66$$

# Probability and statistics

University of Mosul
College of Engineering
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***Probability and statistics 1st Class (2025-2026)***

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# Variance and Standard deviation

**The variance** is the mean of the square deviation of the data from the average.

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$s^2$ = Sample variance

$\bar{x}$ = Sample mean

$x_i$ = The observation of given data

n = Number of data values in sample

**Standard deviation** if the square root of the variance.

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

## Coefficient of variation (CV)

is a normalized quantity based on the standard deviation and the mean. It is a dimensionless quantity.

$$CV = \frac{\text{Standard deviation}}{Mean} * 100$$

# Variance and Standard deviation

Example: Find the variance, standard deviation and coefficient of variation for the following data: 4, 6, 3, 5, 8, 10

Mean= (4+6+3+5+8+10)/6 = 36/6 = 6

The variance

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$S^2$ = 34/(6-1) = 6.8

| $X_i$ | $X_i - \bar{X}$ | $(X_i - \bar{X})^2$ |
|---|---|---|
| 4 | -2 | 4 |
| 6 | 0 | 0 |
| 3 | -3 | 9 |
| 5 | -1 | 1 |
| 8 | 2 | 4 |
| 10 | 4 | 16 |
| ∑= 36 | | 34 |

The standard deviation

S = 2.61

$$CV = \frac{\text{Standard deviation}}{Mean} * 100$$

$$C.V = \frac{2.61}{6} * 100 = 43.33\%$$

# Variance and Standard deviation

Example: Calculate Sample Variance from the following data 85, 96, 76, 108, 85, 80, 100, 85, 70, 95

**Solution:**

| $x$ | $x - \bar{x} = x - 88$ | $(x - \bar{x})^2$ |
|---|---|---|
| 85 | -3 | 9 |
| 96 | 8 | 64 |
| 76 | -12 | 144 |
| 108 | 20 | 400 |
| 85 | -3 | 9 |
| 80 | -8 | 64 |
| 100 | 12 | 144 |
| 85 | -3 | 9 |
| 70 | -18 | 324 |
| 95 | 7 | 49 |
| --- | --- | --- |
| $\sum x = 880$ | $\sum (x - \bar{x}) = 0$ | $\sum (x - \bar{x})^2 = 1216$ |

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{85 + 96 + 76 + 108 + 85 + 80 + 100 + 85 + 70 + 95}{10}$$

$$= 88$$

$$S^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$= \frac{1216}{9}$$

$$= 135.1111$$