CS215 2026-2027

CIDS Dept

Signals and Information Theory

Communications and intelligent digital systems Engineering

2nd Stage | first semester



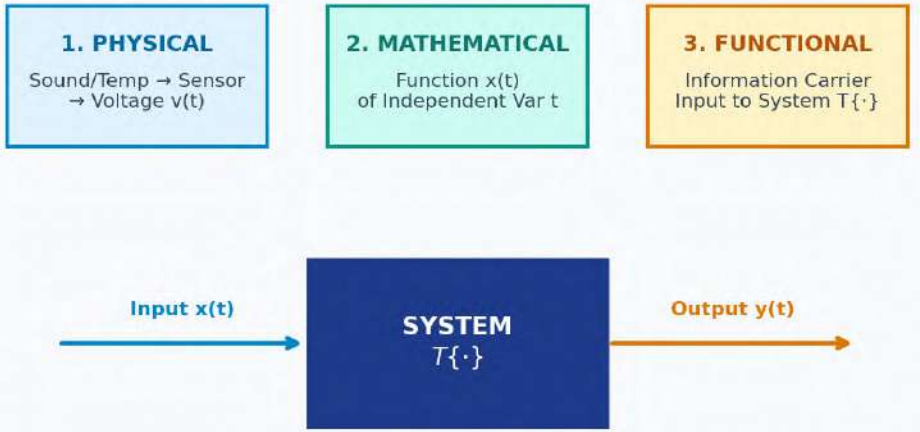
Introduction to Signals

Concepts, Perspectives, Classifications, Equations, and Visual Examples

Dr. Omar Mustafa Ali

1. What is a Signal? Understanding Signals from Three Essential Perspectives

- **Physical Perspective:** Sensors/transducers convert real-world physical phenomena (sound, temperature, heart rate) into electrical voltage v(t) or current i(t).
- **Mathematical Perspective:** A signal is modeled as a function of one or more independent variables, typically continuous time t in seconds: x(t) or discrete index n as x[n].
- **Functional Perspective:** In engineering, a signal is an information carrier passed through a system T{·} to produce an output signal y(t).
- **Key Intuition:** A signal is simply data carrying information about the state or behavior of a physical system.



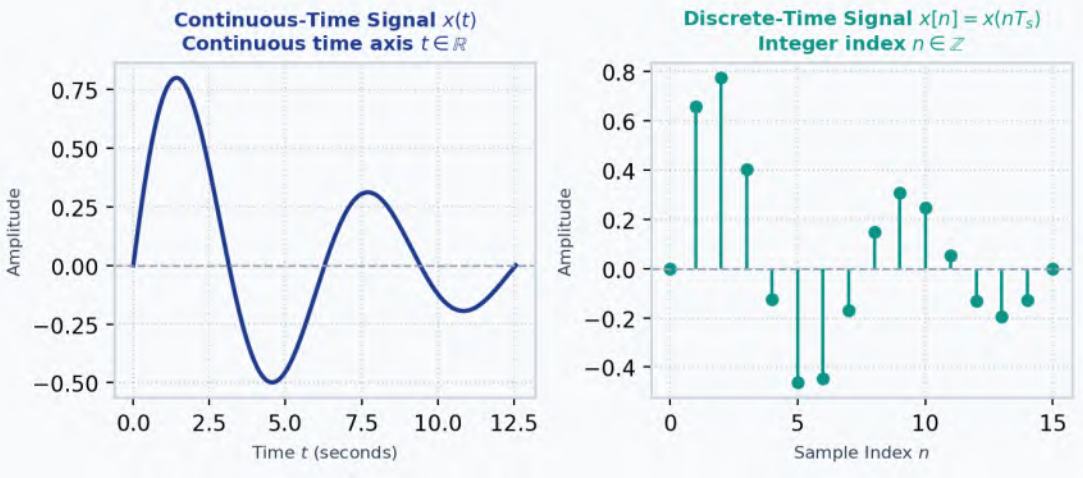
FUNDAMENTAL MATHEMATICAL SIGNAL NOTATION

Continuous-Time: x(t) | Discrete-Time: x[n]

Where x represents amplitude (volts, pressure) and t (seconds) or n (sample index) represents the independent time axis.

2. Signal Classification: Continuous-Time (CT) vs. Discrete-Time (DT) Signals

- **Continuous-Time (CT) Signals:** Defined for a continuous continuum of time values t in (-inf, +inf). Denoted with parentheses x(t).
- **Discrete-Time (DT) Signals:** Defined ONLY at discrete integer time instants n in Z. Denoted with square brackets x[n].
- **Sampling Relationship:** DT signals are produced by sampling a CT signal every Ts seconds: x[n] = x(n*Ts), where Ts is the sampling period.
- **Real-World Examples:** Analog speech recorded by a microphone (CT) vs. digital audio samples (DT).



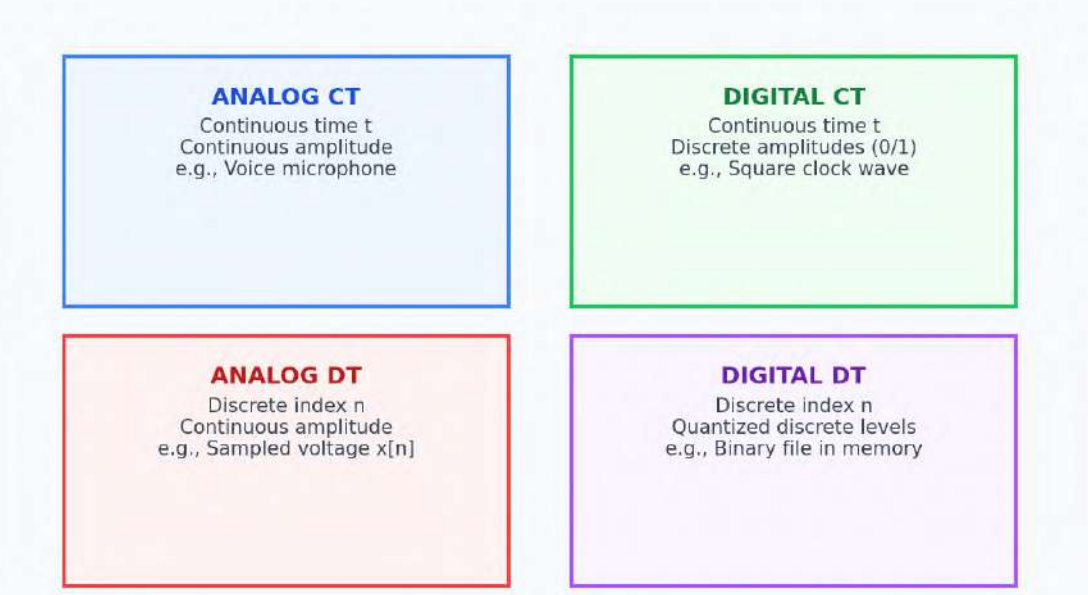
SAMPLING EQUATION & INDEXING MODEL

$$x[n] = x(t) \text{ at } \{t = n\,T_s\} = x(n\,T_s),\ n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

Sampling frequency $f_s = \frac{1}{T_S}$ Hz dictates how many samples per second are captured from the continuous signal x(t).

3. Signal Classification: Analog vs. Digital Signals

- **Amplitude vs. Time Domains:** CT vs. DT describes the horizontal time axis. Analog vs. Digital describes the vertical amplitude axis!.
- **Analog Signals:** Signal amplitude can take on an infinite continuous range of real values (e.g., any voltage between -5V and +5V).
- **Digital Signals:** Signal amplitude is restricted to a finite set of discrete quantized levels (e.g., binary states 0 and 1).
- **A/D Conversion Process:** An Analog-to-Digital Converter (ADC) performs Sampling in time, followed by Quantization in amplitude.



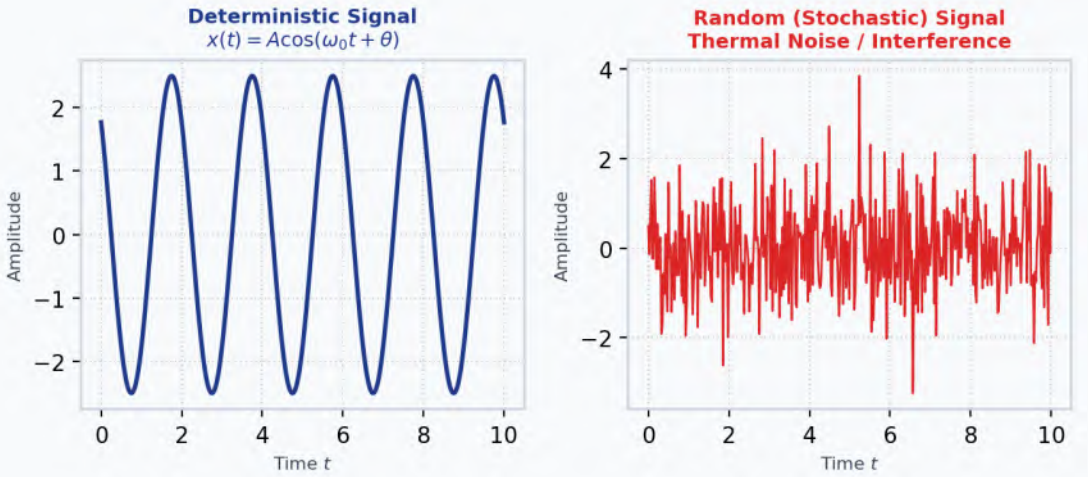
QUANTIZATION STEP SIZE IN DIGITAL SYSTEMS

$$\Delta = (V_{Max} - V_{Min}) / (2^b - 1)$$

For a b-bit digital system, the total voltage range is divided into 2^b discrete amplitude levels, defining the quantization resolution Δ.

4. Signal Classification: Deterministic vs. Random (Stochastic) Signals

- **Deterministic Signals:** Values are completely specified at every time instant by an exact mathematical formula.
- **Random (Stochastic) Signals:** Future values cannot be predicted precisely in advance due to randomness; must be modeled using probability.
- **Analytical Methods:** Deterministic signals use exact time calculus and Fourier transforms; Random signals use mean, variance, and power spectral density.
- **Practical Examples:** Laboratory function generator wave x(t) = A cos(w_ot) (Deterministic) vs. Thermal noise in a resistor or radio interference (Random).



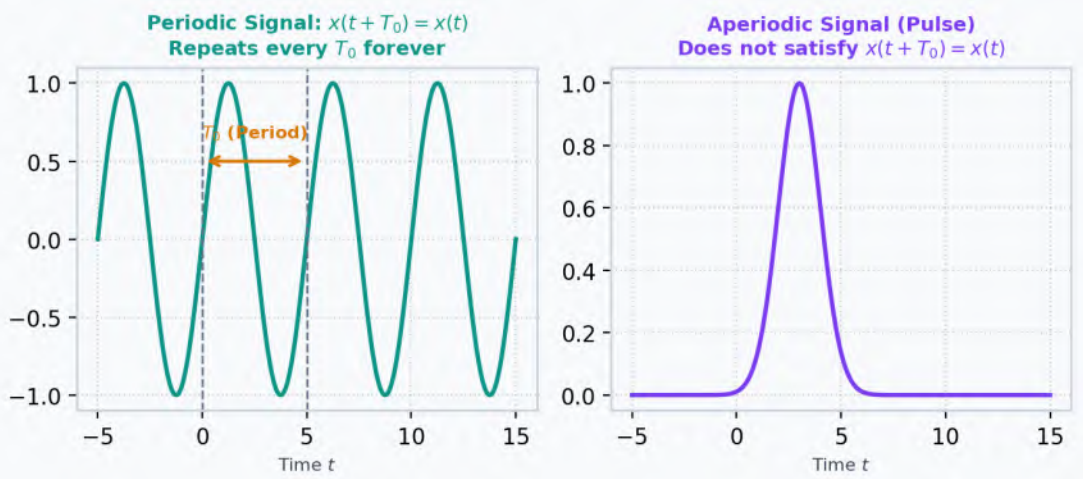
DETERMINISTIC MODEL VS. RANDOM PROBABILITY MODEL

Deterministic: x(t) = A cos(ωo t + θ) | Random: p(x) = (1 / √(2πσ²)) * exp(-(x - μ)² / (2σ²))

Deterministic signals have fixed parameter values; Random signals are characterized by probability density functions p(x).

5. Signal Classification: Periodic vs. Aperiodic Signals

- **Periodic Definition:** A signal x(t) is periodic if it repeats its exact waveform pattern at regular time intervals T_0 forever.
- **Fundamental Period (T_0):** The smallest positive value $T_0 > 0$ that satisfies $x(t + T_0) = x(t)$ for all t in (-inf, +inf).
- **Fundamental Frequency:** $f_0 = 1 / T_0$ Hz (cycles per second), or angular frequency $w_0 = 2*\pi / T_0$ rad/s.
- **Aperiodic Signals:** Any signal that does not repeat periodically (e.g., single rectangular pulses, lightning flashes, speech words).



FUNDAMENTAL PERIODICITY CONDITION

$$x(t + T_0) = x(t) \text{ for all } t \quad , \quad f_0 = 1 / T_0 \quad , \quad \omega_0 = 2\pi f_0 = 2\pi / T_0$$

Notice that periodic signals must theoretically exist for all time from t = -inf to +inf (everlasting signals).

6. Signal Classification: Energy Signals vs. Power Signals

- **Normalized 1-Ohm Power:** Instantaneous signal power dissipated across a 1-Ohm resistor is

$$p(t) = |x(t)|^2$$

- **Signal Energy (Ex):** Total energy integrated over all time:

$$Ex = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

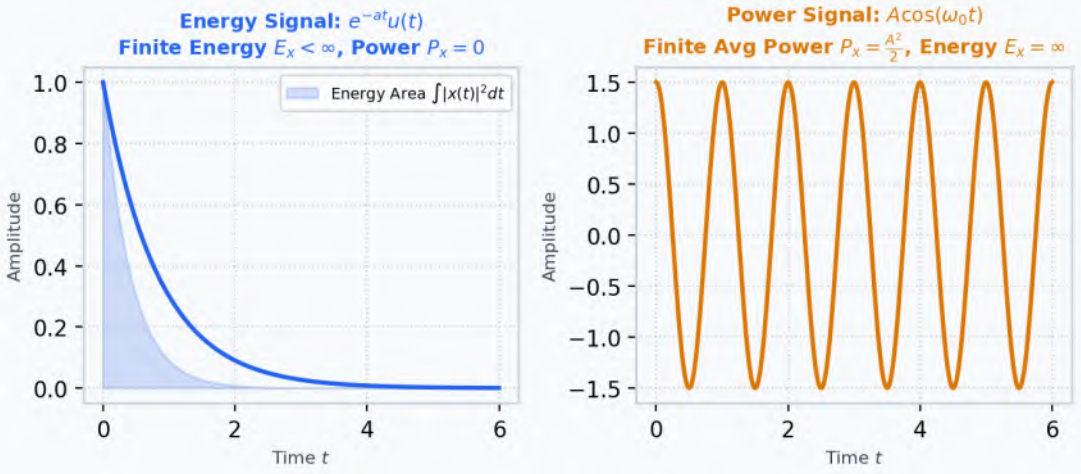
- **Average Power (Px):** Time-averaged energy over infinite duration:

$$Px = \lim_{T\to\infty} \left[(1/T) \int_{-T/2}^{T/2} |x(t)|^2 \, dt \right].$$

- **Classification Rule:**

Energy Signal: 0 < Ex < ∞ and Px = 0 (Pulse signals).

Power Signal: 0 < Px < ∞ and Ex = ∞ (Periodic signals).



CONTINUOUS-TIME ENERGY & POWER INTEGRAL EQUATIONS

Energy Signal: 0 < Ex < ∞ and Px = 0 (Pulse signals). Power Signal: 0 < Px < ∞ and Ex = ∞ (Periodic signals).

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Basic Signal Operations

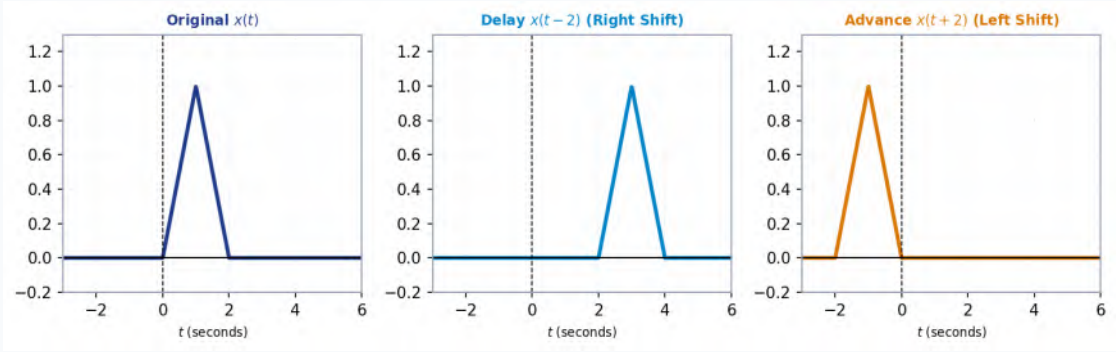
Continuous-Time & Discrete-Time

Time Shifting • Time Scaling • Time Reversal • Combined Transformations • Even/Odd Symmetry

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Continuous-Time Time Shifting: Right vs. Left

- **Shift Right :** x(t - T₀) shifts the entire waveform to the RIGHT by T₀ seconds towards positive time.
- **Shift Left:** x(t + T₀) shifts the waveform to the LEFT towards negative time.



CONTINUOUS TIME-SHIFT EQUATIONS

$x(t - T_0)$ [Right Shift]

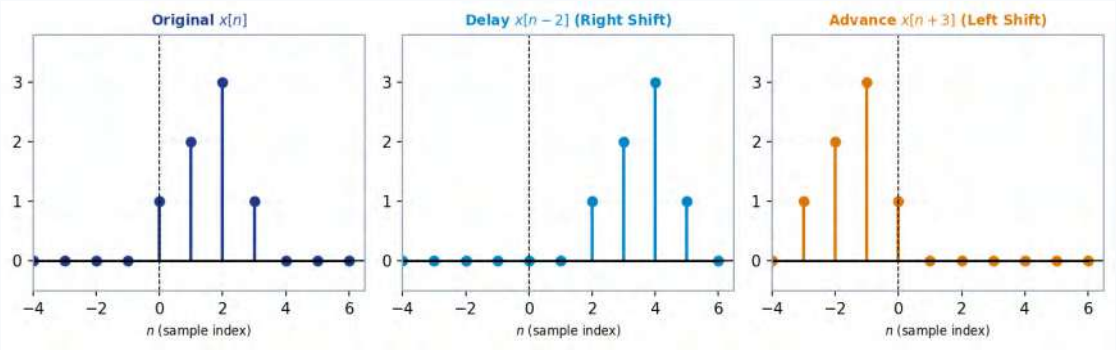
$x(t + T_0)$ [Left Shift]

Discrete-Time Time Shifting: Sample Index Displacement

- **Sample Right shift :** x[n - M] shifts all stem values M positions to the RIGHT.
- **Sample Left shift :** x[n + M] shifts all stem values M positions to the LEFT.

DISCRETE TIME-SHIFT EQUATIONS

x[n - M] [Right Shift]
x[n + M] [Left Shift]

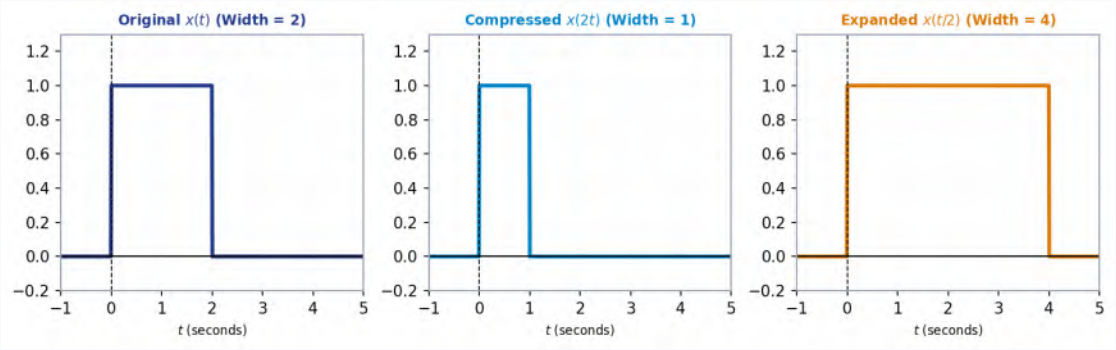


Continuous-Time Time Scaling: Compression vs. Expansion

- **Signal Compression (a > 1):** x(at) speeds up time, compressing the pulse duration horizontally by factor a.
- **Signal Expansion (0 < a < 1):** x(t/a) slows down time, stretching the pulse duration horizontally by factor 1/a.

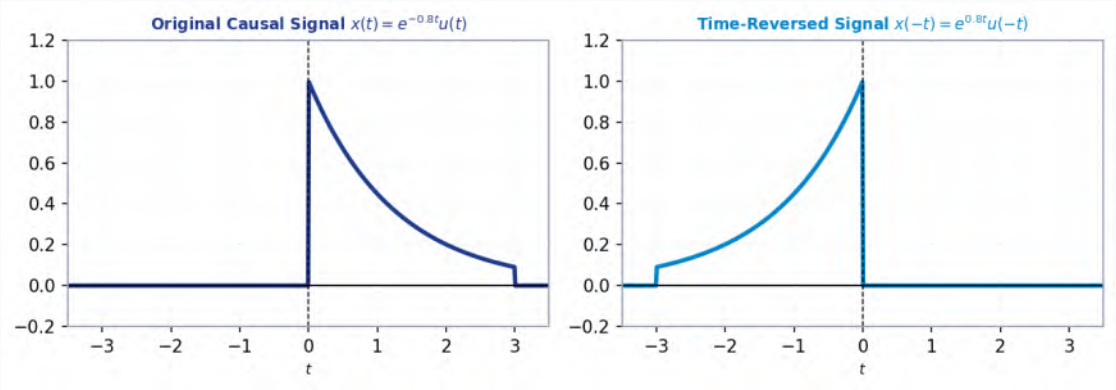
CONTINUOUS SCALING

$$x_{scaled}(t) = x(at)$$



Continuous-Time Time Reversal: Reflection Across Axis

- **Graphical Effect:** Flips the signal horizontally about the vertical t = 0 axis.



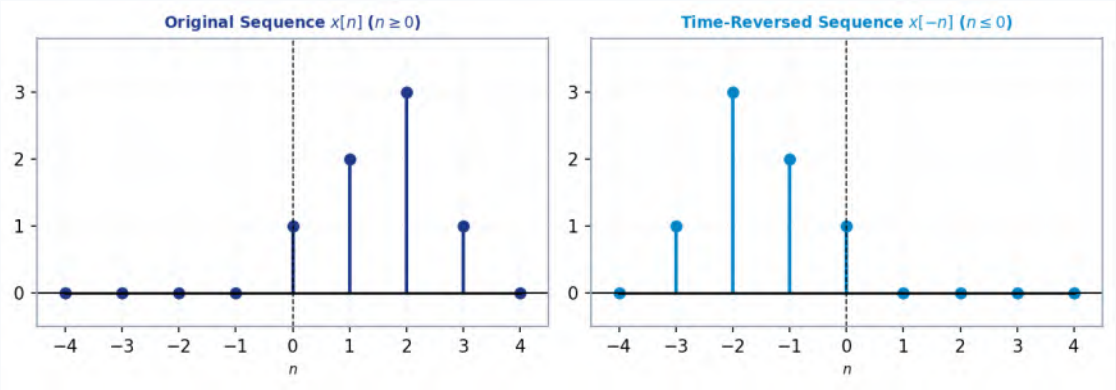
CONTINUOUS TIME-REVERSAL EQUATIONS

$$x_{reversed}(t) = x(-t)$$

Anchor Point: t = 0 remains stationary

Discrete-Time Time Reversal: Stem Sequence Reflection

- **Discrete Reflection:** Replacing sample index n with -n. the origin sample x[0] remains stationary.



DISCRETE TIME-REVERSAL EQUATIONS

$$y[n] = x[-n]$$

$$y[0] = x[0],\ y[1] = x[-1],\ y[-1] = x[1]$$

Anchor Point: n = 0 remains stationary

Combined Transformations: Continuous-Time x(at - b)

- **The Order Problem:** Evaluating x(at - b) requires transformation sequence.
- **Method (Shift First):**

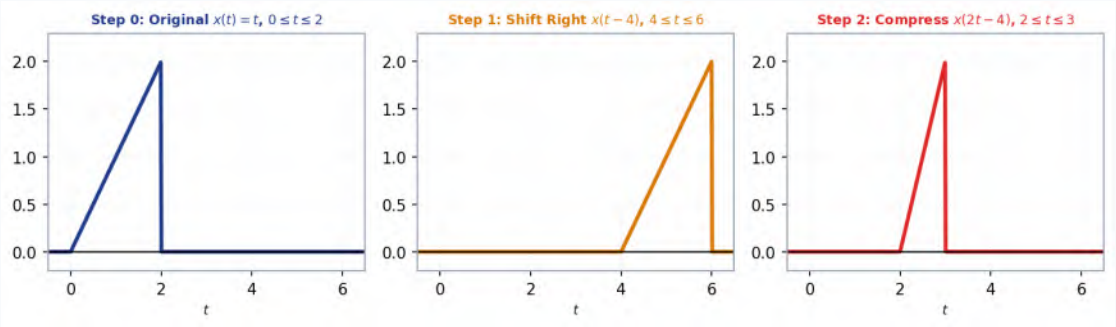
  1) Shift x(t) by b to get x(t - b).

  2) Replace t with at to get x(at - b).

Method A: x(t) ——Shift b——> x(t - b) ——Scale a——> x(at - b)

Step-by-Step Worked Example: Transforming y(t) = x(2t - 4)

- **Given Pulse:** x(t) = t for 0 ≤ t ≤ 2 (Ramp pulse of width 2, peak 2 at t=2).
- **Step 1 (Shift Right by 4):** x(t - 4) moves interval [0, 2] to [4, 6].
- **Step 2 (Compress by 2):** Replace t with 2t: x(2t - 4). Interval [4, 6] compresses to [2, 3].



BOUNDARY VERIFICATION CHECK

Set 2t - 4 = 0 ⇒ 2t = 4 ⇒ t = 2 (Start)

Set 2t - 4 = 2 ⇒ 2t = 6 ⇒ t = 3 (End)

Peak occurs at t = 3 with height = 2

Continuous-Time Signal Symmetries: Even vs. Odd

- **Even Signals (Symmetric):**

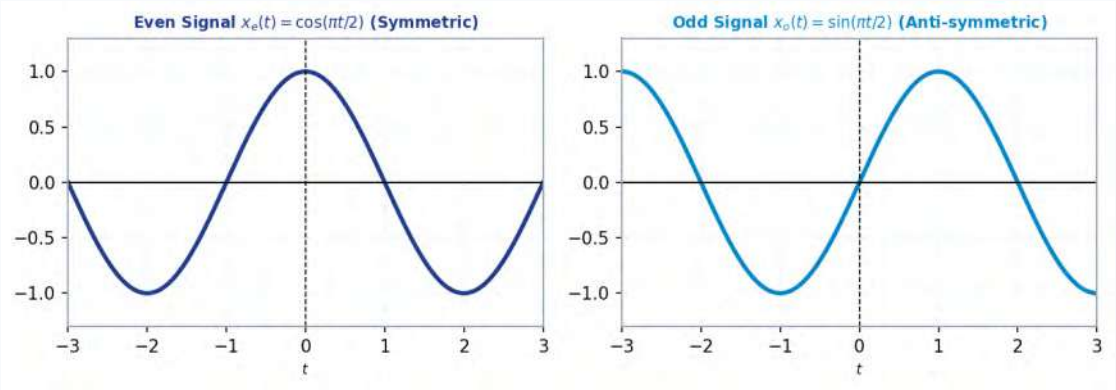
$x_e(t) = x_e(-t)$. Reflected across t = 0, the waveform is identical (e.g., $\cos(\omega_0 t)$).

- **Odd Signals (Anti-symmetric):**

$x_o(t) = -x_o(-t)$. Reflected across t = 0, amplitude flips sign (e.g., $\sin(\omega_0 t)$).

- **Origin Constraint:** Continuous odd signals MUST pass through the origin:

$x_o(0) = 0$.



Discrete-Time Signal Symmetries: Stem Plot Symmetry

- **Discrete Even Sequences:**

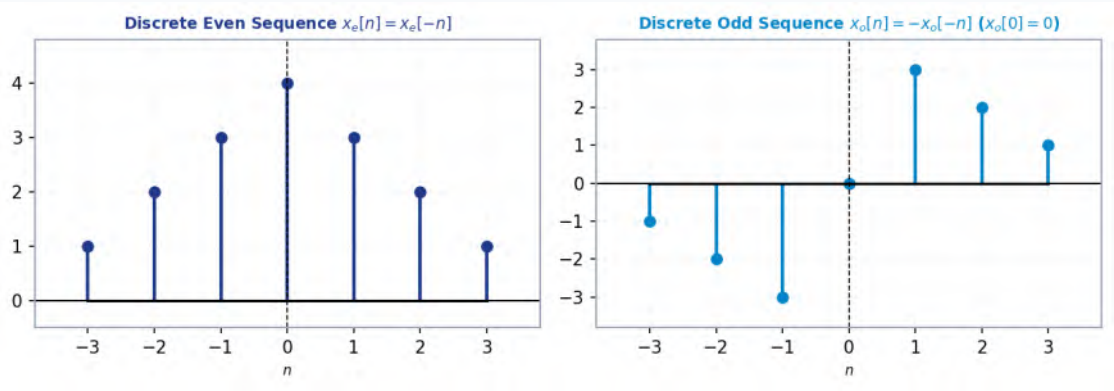
  $x_e[n] = x_e[-n]$. Sample values at +k and -k are equal.

- **Discrete Odd Sequences:**

  $x_o[n] = -x_o[-n]$. Sample values at +k and -k have equal magnitude but opposite sign.

- **Discrete Origin Constraint:**

  For any discrete odd sequence, $x_o[0]$ MUST equal 0.



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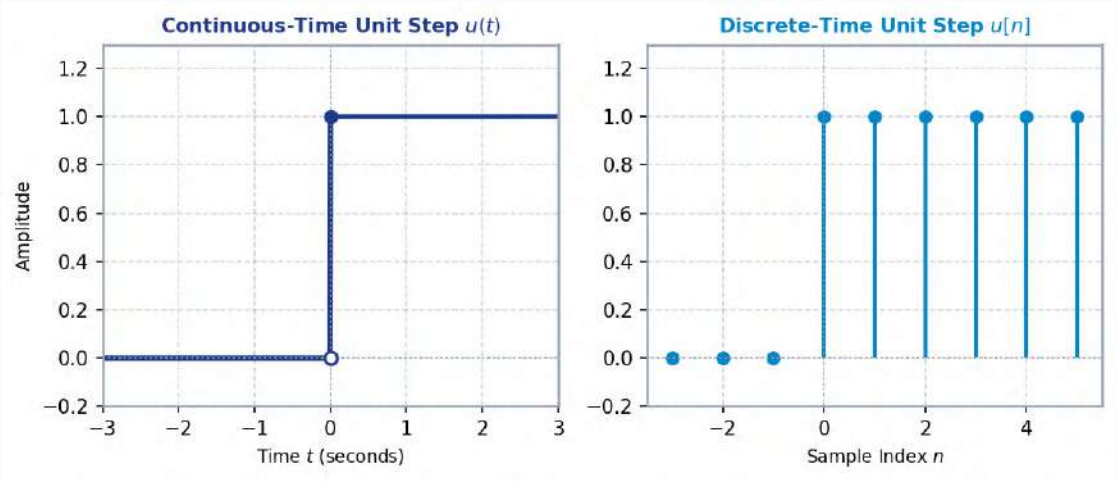
Essential Special Functions: Unit Step, Unit Impulse, & Signum

Continuous & Discrete Mathematical Models, Shifting Properties, & Worked Step-by-Step Examples

Dr. Omar Mustafa Ali

Unit Step Function: Fundamental Continuous & Discrete Definitions

- **Physical Intuition:** Models an ideal electrical switch turning ON at time t = 0, changing state instantly from 0 to 1.
- **Continuous-Time u(t):** Zero for negative time t < 0 and unity (1.0) for positive time t > 0. The jump discontinuity occurs at t = 0.
- **Discrete-Time u[n]:** Sequence equals 0 for n < 0 and 1 for n >= 0. Unlike u(t), u[n] is defined cleanly at n = 0 without ambiguity.
- **Causal Masking Role:** Multiplying any signal x(t) or x[n] by u(t) or u[n] enforces causality by zeroing out all negative-time values.



MATHEMATICAL MODELS

$$u(t) = \{ 1\ (t > 0),\ 0\ (t < 0) \}$$

$$u[n] = \{ 1\ (n >= 0), 0\ (n < 0) \}$$

Unit Step Function: Properties & Rectangular Pulse Generation

- **Time-Shifted Step u(t - t0):** Turns ON at time t = t_0.

For $t_0 > 0$, it represents a delayed turn-on; for $t_0 < 0$, an advanced turn-on.

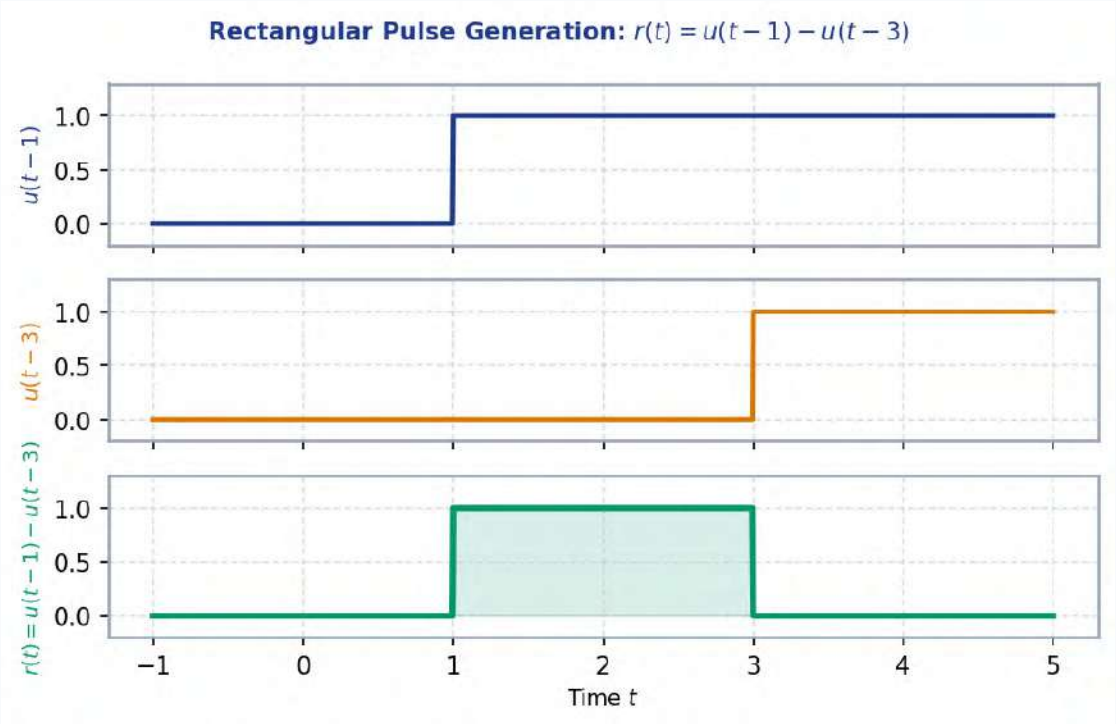
- **Rectangular Pulse Synthesis:** Subtracting two shifted steps u(t - t1) - u(t - t2) creates a finite rectangular window active over [t1, t2].

- **Signal Truncation:** Multiplying a signal x(t) by

r(t) = u(t - t1) - u(t - t2) isolates x(t) strictly inside the interval t1 <= t <= t2.

- **Discrete Windowing:** Similarly,

w[n] = u[n - N1] - u[n - N2] extracts a finite sub-sequence of length N2 - N1.



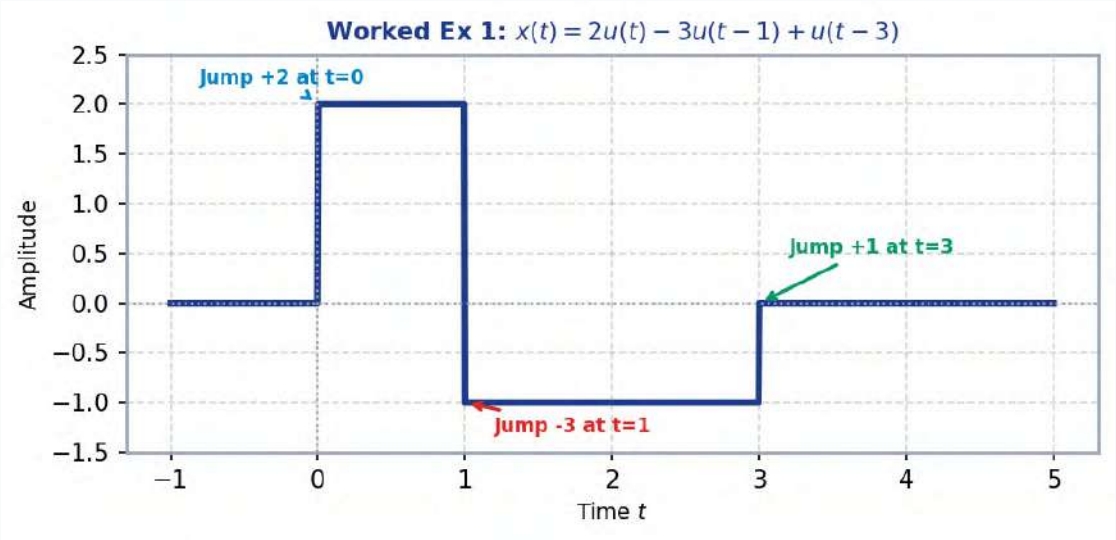
RECTANGULAR PULSE SYNTHESIS FORMULA

r(t) = A [u(t - t1) - u(t - t2)]

Active Interval: t1 <= t < t2, Height = A

Worked Example 1: Piecewise Signal Decomposition via Step Functions

- **Problem Statement:** Express a piecewise constant signal x(t) with jumps: +2 at t=0, -3 at t=1, and +1 at t=3 as a sum of step functions.
- **Jump Analysis:**
  - At t=0: Jump from 0 to +2 ------> +2 u(t)
  - At t=1: Jump from +2 to -1 ------> -3 u(t-1)
  - At t=3: Jump from -1 to 0 ------> +1 u(t-3)
- **Verification:** For t > 3: x(t) = 2(1) - 3(1) + 1(1) = 0. Matches the waveform perfectly!



PIECEWISE STEP REPRESENTATION

$$x(t) = 2\,u(t) - 3\,u(t-1) + u(t-3)$$

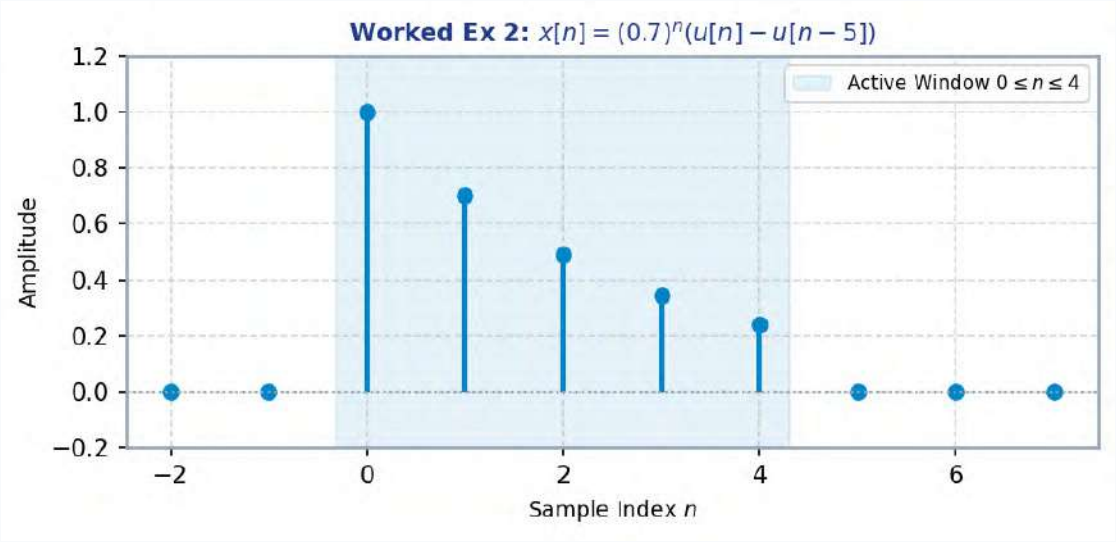
Worked Example 2: Discrete Windowed Exponential Sequences

- **Problem Statement:** Represent a 5-sample truncated geometric sequence $x[n] = (0.7)^n$ for $0 \le n \le 4$ and 0 elsewhere using u[n].
- **Discrete Window Formulation:** A finite window active for $0 \le n \le 4$ is constructed using discrete steps:

w[n] = u[n] - u[n - 5].

- **Notice the Index N=5:** To include sample n=4, the subtracting step must turn on at n=5 so that

u[4] - u[-1] = 1 - 0 = 1.



TRUNCATED SEQUENCE FORMULA

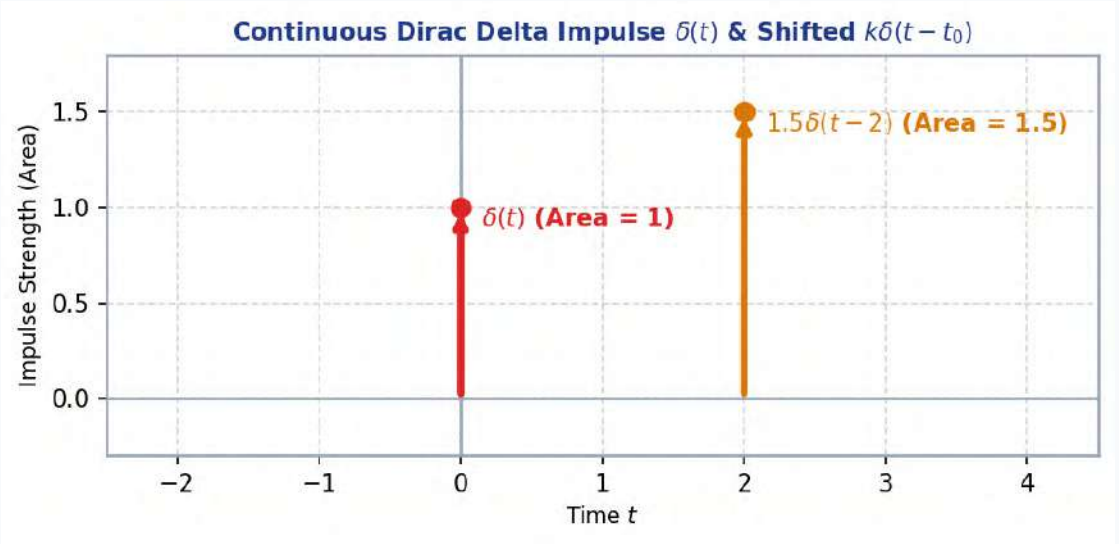
$$x[n] = (0.7)^n [\, u[n] - u[n-5] \,]$$

Active Samples: n in {0, 1, 2, 3, 4}

Unit Impulse Function: Continuous Dirac Delta -----> delta(t)

- **Mathematical Idealization:** Defined as the limit of a rectangular pulse of width epsilon and height 1/epsilon as epsilon ----> 0. Has infinite peak height and zero duration.
- **Unit Area Characteristic:** The key defining feature of delta(t) is its total integrated area equal to 1. In diagrams, drawn as an arrow labeled with its area (strength).
- **Derivative of Unit Step:** The Dirac delta is the generalized derivative of the unit step: d/dt [u(t)] = δ(t).
- **Integral of Impulse:** Integrating delta(t) yields the unit step:

$$\int_{-\infty}^{\infty} \delta(t)\,dt = u(t)$$

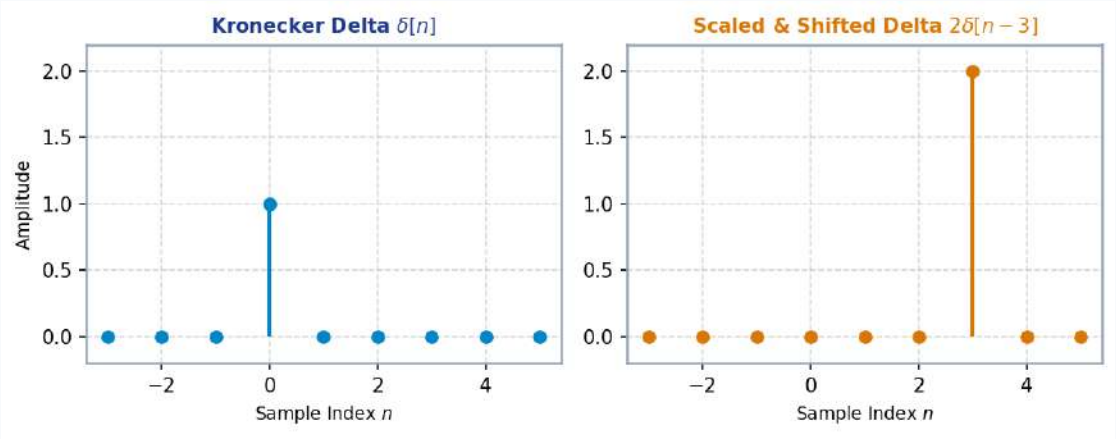


Unit Impulse Function: Discrete Kronecker Delta delta[n]

- **Ordinary Function:** Unlike continuous $\delta(t)$, discrete $\delta[n]$ is an ordinary sequence with finite amplitude 1 at n = 0 and 0 everywhere else.
- **First Difference of Step:** Discrete delta is the first backward difference of the discrete unit step:

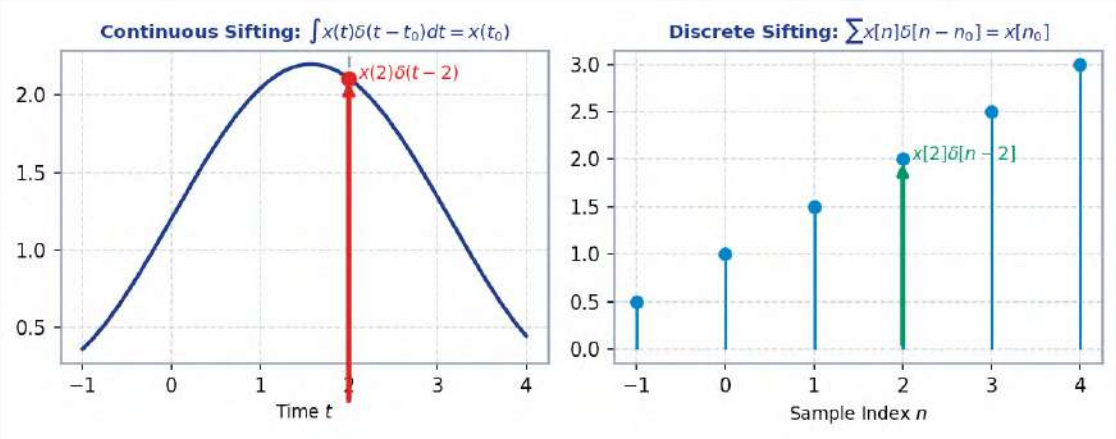
$\delta[n] = u[n] - u[n - 1]$.

- **Shifted Impulse delta[n - n0]:** Represents a single discrete sample spike located at sample index $n = n_0$.



Shifting & Sampling Properties of the Unit Impulse

- **Sifting Property (Continuous):** Integrating a signal x(t) multiplied by delta(t - t_0) extracts / shifts out the exact value of x(t) at instant t = t_0.
- **Sifting Property (Discrete):** Summing x[n] δ[n - n_0] over all n extracts the single sample value x[n_0].



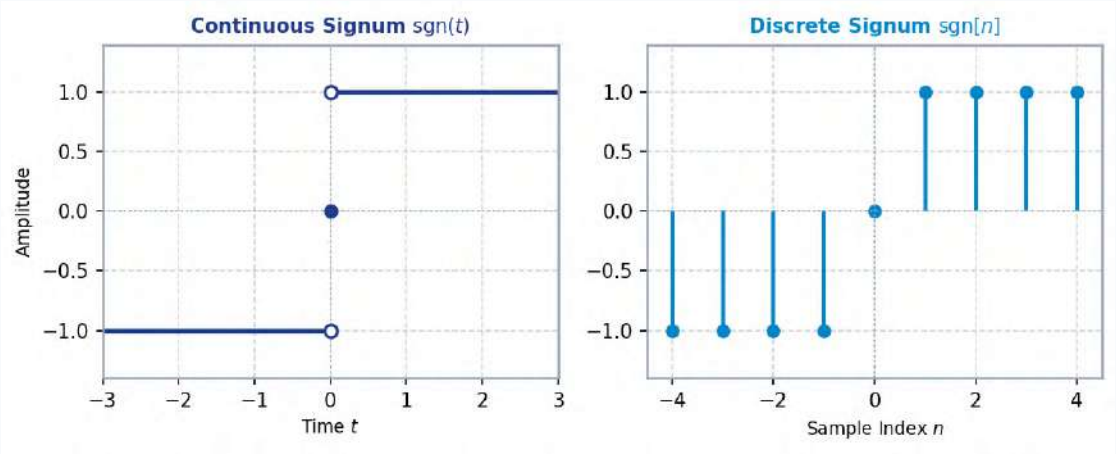
Signum Function: Continuous & Discrete Mathematical Models

- **Sign Extractor Role:** The signum function sgn(t) or sgn[n] extracts the algebraic sign (+1 or -1) of its independent variable.
- **Continuous Model sgn(t):** Equals +1 for t > 0, -1 for t < 0, and 0 at t = 0. Has a total step jump of size 2 at origin.
- **Discrete Model sgn[n]:** Equals +1 for n > 0, -1 for n < 0, and 0 for n = 0.
- **Pure Odd Symmetry:** Satisfies

sgn(-t) = -sgn(t)

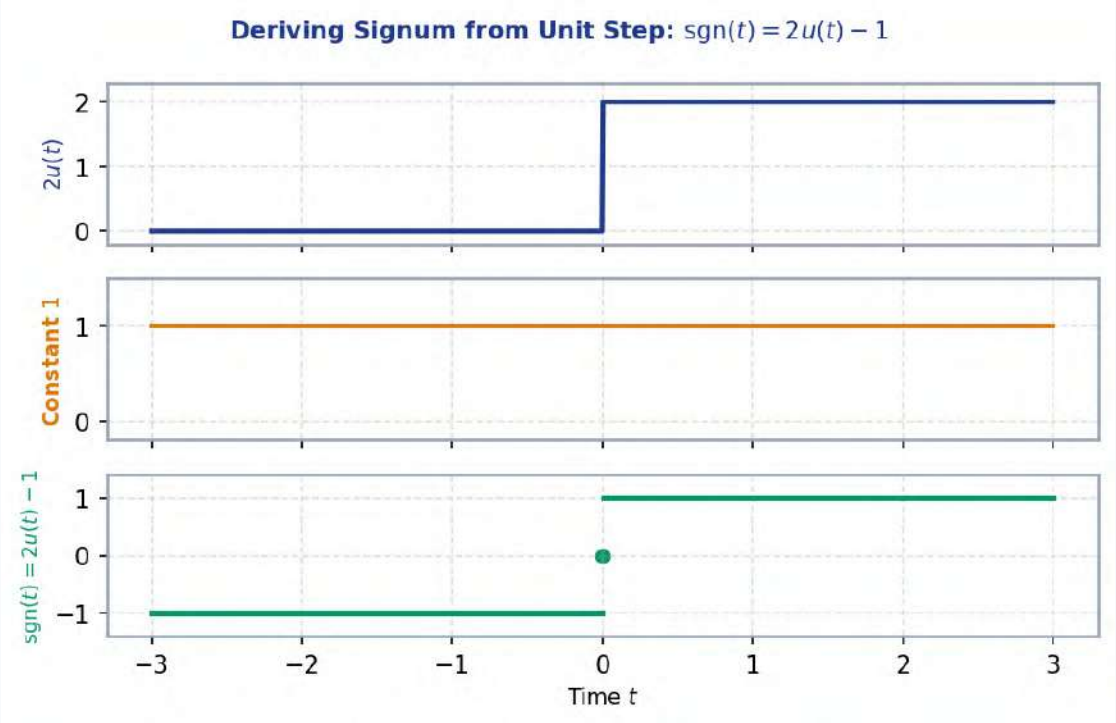
sgn[-n] = -sgn[n]

making it a purely odd signal with zero DC average value.



Mathematical Equivalence Between Signum & Unit Step

- **Deriving sgn(t) from u(t):** Subtracting 1 from 2 u(t) shifts the step level down from [0, 2] to [-1, +1], yielding sgn(t) = 2 u(t) - 1.
- **Expressing u(t) via sgn(t):** Inversely, the unit step can be expressed in terms of signum: u(t) = 1/2 [1 + sgn(t)].



FUNDAMENTAL EQUIVALENCE EQUATIONS

sgn(t) = 2 u(t) - 1 <===> u(t) = 1/2 [1 + sgn(t)]

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Signal Power, Energy

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1. Energy & Power Mathematical Foundations (CT vs. DT)

Continuous-Time (CT) Metrics

Continuous Signal Energy (E_x):

$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \ dt$ (Joules or V^2s)

Continuous Average Power (P_x):

$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \ dt$ (Watts or V^2)

Periodic Signal Power (Period T_0):

$$P_x = \frac{1}{T_0} \int_{0}^{T_0} |x(t)|^2 \ dt$$

Classification:

Energy Signal: $0 < E_x < \infty \implies P_x = 0$

Power Signal: $0 < P_x < \infty \implies E_x = \infty$

Discrete-Time (DT) Metrics

Discrete Sequence Energy (E_x):

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Discrete Average Power (P_x):

$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

Periodic Sequence Power (Period N_0):

$P_x = \frac{1}{N_0} \sum_{n=0}^{N_0-1} |x[n]|^2$

Key Physical Insight: Energy measures finite-duration transient pulses. Power measures everlasting or periodic waves.

2. CT Energy Example 1 — Causal Exponential Decay Pulse

Problem Statement & Derivation

Signal Model: $x(t) = Ae^{-at}u(t)$ for $a > 0$

Step 1: Set up the Energy Integral:

$$E_x = \int_0^{\infty} (Ae^{-at})^2 \, dt = A^2 \int_0^{\infty} e^{-2at} \, dt$$

Step 2: Integrate Square Amplitude:

$$E_x = A^2 \left[\frac{-e^{-2at}}{2a}\right]_0^{\infty}$$

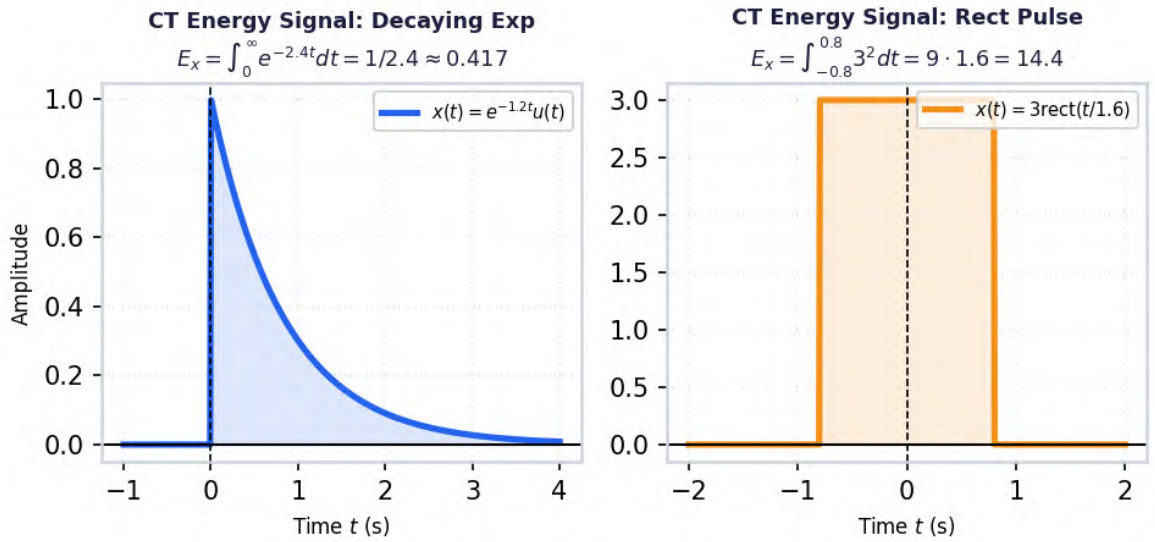
Step 3: Evaluate Limits:

$$E_x = A^2 \left(0 - \left(-\frac{1}{2a}\right)\right) = \frac{A^2}{2a}$$

Step 4: Average Power Calculation:

$$P_x = \lim_{T\to\infty} \frac{1}{T}\left(\frac{A^2}{2a}\right) = 0$$

Conclusion: Finite Energy $\Longrightarrow$ Energy Signal.



3. CT Energy Example 2 — Rectangular & Triangular Pulses

A. Rectangular Pulse $x_1(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$

Definition: $x_1(t) = A$ for $|\,t\,| \leq T/2$, 0 elsewhere.

Energy Calculation:

$$E_{x_1} = \int_{-T/2}^{T/2} A^2\ dt = A^2[t]_{-T/2}^{T/2} = A^2T$$

Insight: Energy is proportional to Pulse Width (T) and Square Height (A^2).

B.Triangular Pulse $x_2(t) = A\left(1 - \frac{|t|}{\tau}\right)$

Definition: Base width 2τ(from $-\tau$ to $+\tau$), peak height A.

Symmetric Energy Calculation:

$$E_{x_2} = 2\int_0^{\tau} A^2\left(1 - \frac{t}{\tau}\right)^2\ dt =$$

$$2A^2\left[-\frac{\tau}{3}\left(1 - \frac{t}{\tau}\right)^3\right]_0^{\tau} = \frac{2}{3}A^2\tau$$

Comparison: Triangular pulse has $2/3$ the energy of a rectangular pulse of equal peak and total duration.

4. CT Power Example 1 — Real Sinusoids & Constant DC

Derivation for x(t) = A cos(ωo t + θ)

Signal: $x(t) = A\cos(\omega_0 t + \theta)$

Period: $T_0 = \frac{2\pi}{\omega_0}$

Step 1: Apply Single-Period Power Formula:

$$P_x = \frac{1}{T_0}\int_0^{T_0} A^2\cos^2(\omega_0 t + \theta)\,dt$$

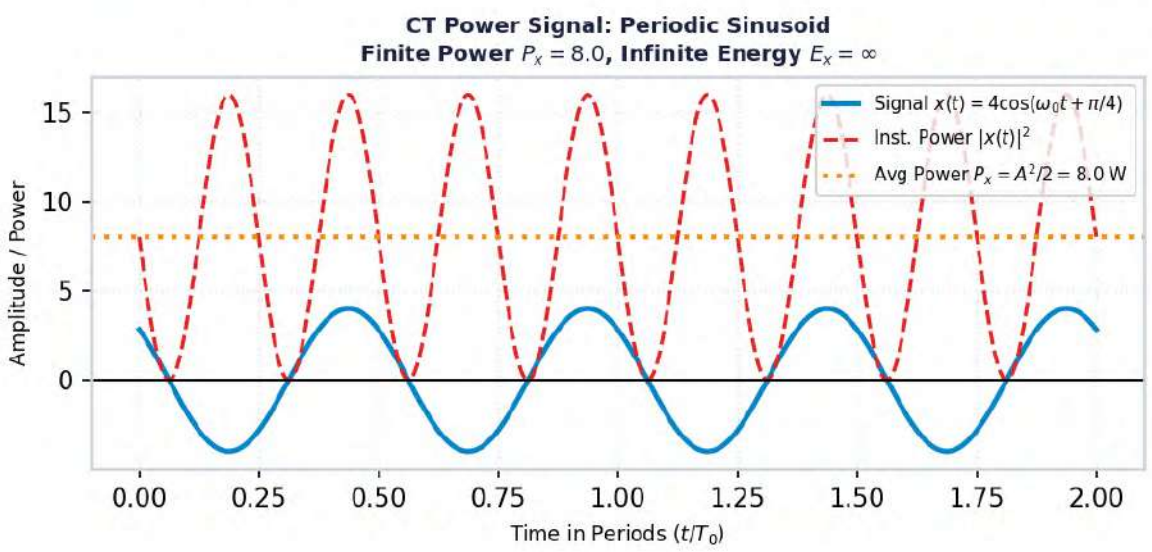
Step 2: Use Trig Identity $\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$**:**

$$P_x = \frac{A^2}{2T_0}\int_0^{T_0}[1 + \cos(2\omega_0 t + 2\theta)]\,dt$$

Step 3: Sinusoid over full period integrates to 0:

$$P_x = \frac{A^2}{2T_0}[T_0 + 0] = \frac{A^2}{2}$$

Critical Rule: Average power depends ONLY on amplitude A. Phase θ and frequency ω_0 do NOT affect power.



5. CT Power Example 2 — Multi-Tone Sinusoidal Sums

Case A: Distinct Frequencies ($\omega_1 \neq \omega_2$)

Signal Model:

$$x(t) = A_1\cos(\omega_1 t + \theta_1) + A_2\cos(\omega_2 t + \theta_2)$$

Orthogonality Property: Cross-terms integrate to 0 over long time intervals for distinct frequencies.

Total Average Power: $P_x = \frac{A_1^2}{2} + \frac{A_2^2}{2}$

Case B: Identical Frequencies ($\omega_1 = \omega_2$)

Phasor Addition Theorem: Phase difference $\Delta\theta = \theta_1 - \theta_2$ creates constructive or destructive interference.

Total Combined Power Formula:

$$P_x = \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1 A_2 \cos(\theta_1 - \theta_2)$$

Complex Exponential Signal:

$$x(t) = Ae^{j(\omega_0 t + \theta)} \implies |x(t)|^2 = A^2 \implies P_x = A^2$$

6. DT Energy Worked Examples — Geometric Series & Finite Windows

Problem Derivations & Solutions

Problem 1: Causal Geometric Sequence

$x_1[n] = a^n u[n]$

$$E_{x_1} = \sum_{n=0}^{\infty} |a^n|^2 = \sum_{n=0}^{\infty} (a^2)^n$$

Using Geometric Sum Formula (for $|a| < 1$):

$$E_{x_1} = \frac{1}{1-a^2}$$

Problem 2: Finite Window

$x_2[n] = A$ for $0 \le n \le N-1$

$$E_{x_2} = \sum_{n=0}^{N-1} A^2 = A^2 N$$

