Electromagnetic Fields

University of Mosul

Course layout (15 Weeks)

- Vectors
- Force and Electric Field
- Gauss's law
- Potential Difference & Potential
- Capacitances
- Magnetostatics, Magnetic fields, Biot-Savart law
- Ampere's law
- Faraday's law and induced Electromotive Force
- Maxwell Equations

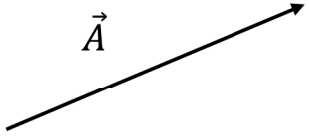
Books and References

- William H. Hayt “Engineering Electromagnetics” 2nd edition
- Schaum’s outline of theory and problems of Electromagnetics

1.1 Vector Analysis

A vector represents a physical quantity which has magnitude and direction

Can be written : $\vec{A} = A\, a_A$



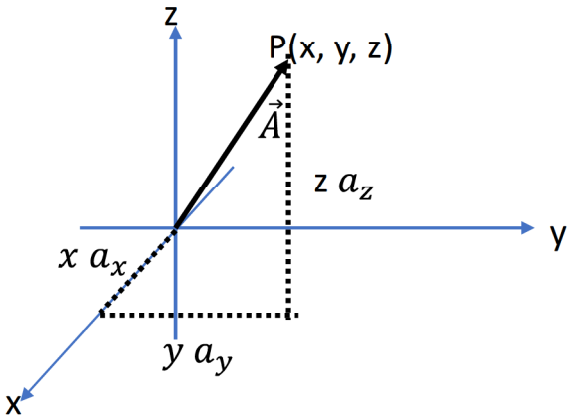
Where $A = |A|$= magnitude and $a_A = \frac{\vec{A}}{|A|}$ = unite vector represents the direction of $\vec{A}$

Position vector

A position vector is directed from the origin to the point P(x, y, z) as shown. It can be expressed as:

$$\vec{A} = x\, a_x + y\, a_y + z\, a_z$$

$$|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$$



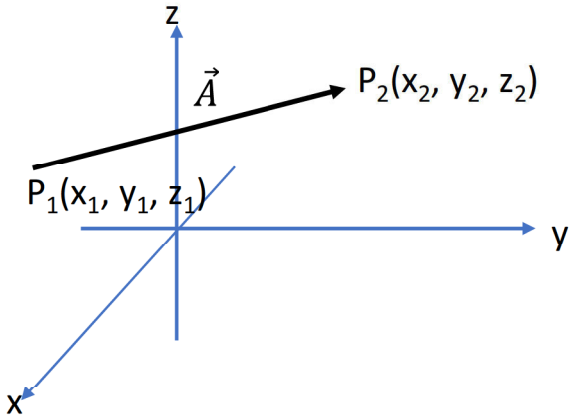
1.1 Vector Analysis

Distance Vector

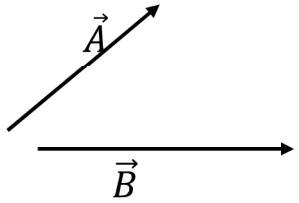
A distance vector is joining two points. It can be expressed as:

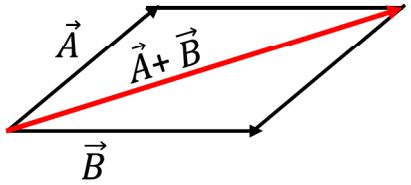
$$\vec{A}_{12} = (x_2 - x_1)a_x + (y_2 - y_1)\,a_y + (z_2 - z_1)a_z$$

$$|\vec{A}_{12}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

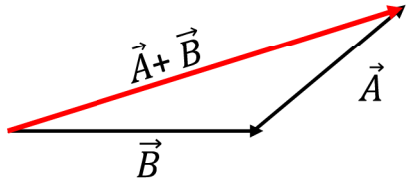


1.2 Vector Addition





Parallelogram



Head to Tail

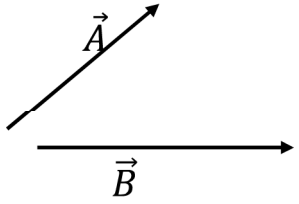
1.2 Vector Properties

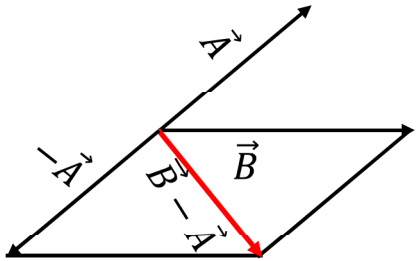
Commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Associative $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

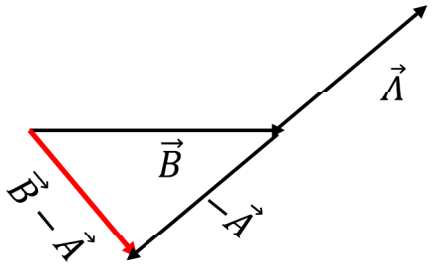
1.3 Vector Subtraction

Given two vectors $\vec{A}$ and $\vec{B}$ they can be subtracted as:





Parallelogram



Head to Tail

The vector addition is carried out component by component. Thus,

If $\vec{A} = A_x a_x + A_y a_y + A_z a_z$ and $\vec{B} = B_x a_x + B_y a_y + B_z a_z$ then

$$\vec{A} + \vec{B} = (A_x + B_x)\, a_x + (A_y + B_y)\, a_y + (A_z + B_z)\, a_z$$

Vector subtraction is carried out as

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$\therefore \vec{A} - \vec{B} = (A_x - B_x)\, a_x + (A_y - B_y)\, a_y + (A_z - B_z)\, a_z$$

Example/ Given points *M*(-1, 2, 1), *N*(3, -3, 0), and *P*(-2, -3, -4), find:
(*a*) $\vec{R}_{MN}$; (*b*) $\vec{R}_{MN} + \vec{R}_{MP}$; (*c*) $\vec{R}_M$; (*d*) a_{MP}.

Solution

$$\vec{R}_{MN} = (x_N - x_M)a_x + (y_N - y_M)\, a_y + (z_N - z_M)a_z =$$
$$(3 - (-1))a_x + (-3 - 2)\, a_y + (0 - 1)a_z = 4\, a_x - 5\, a_y - a_z$$

$$\vec{R}_{MP} = (x_P - x_M)a_x + (y_P - y_M)\, a_y + (P - z_M)a_z =$$
$$(-2 - (-1))a_x + (-3 - 2)\, a_y + (-4 - 1)a_z = -a_x - 5\, a_y - 5\, a_z$$

$$\vec{R}_{MN} + \vec{R}_{MP} = (4 - 1)\, a_x\big(-5 + (-5)\big)a_y\big(-1 + (-5)\big)a_z = 3\, a_x - 10\, a_y - 6a_z$$

$$\vec{R}_M = -a_x + 2\, a_y + a_z\ , \qquad \left|\vec{R}_M\right| = \sqrt{(-1)^2 + (2)^2 + (1)^2} = \sqrt{6}$$

$$a_{MP} = \frac{\vec{R}_{MP}}{\left|\vec{R}_{MP}\right|}\ , \left|\vec{R}_{MP}\right| = \sqrt{(-1)^2 + (-5)^2 + (-4)^2} = \sqrt{42}$$

$$a_{MP} = \frac{-a_x - 5\, a_y - 4\, a_z}{\sqrt{42}}$$

1.4 Products of Vectors

A scalar times a vector $\mathrm{k}\vec{A} = kA\ a_A$

1.5 Scalar or Dot Product

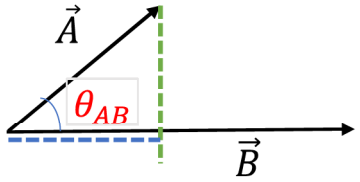
$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$$

$\vec{A}.\vec{A} = A^2$ $\quad |\vec{A}| = \sqrt{\vec{A}.\vec{A}}$

Properties of the Dot product

Commutative $\quad \vec{A}.\vec{B} = \vec{B}.\vec{A}$

Associative $\quad \vec{A}.(\vec{B}+\vec{C}) = (\vec{A}+\vec{B}).\vec{C}$



Projection of vector $\vec{A}$ on the vector $\vec{B}$

Let $\vec{A} = A_x a_x + A_y a_y + A_z a_z$ and $\vec{B} = B_x a_x + B_y a_y + B_z a_z$ then

Because the angle between two different unit vectors of the rectangular coordinate system is 90°, we then have

$$a_x.a_y = a_y.a_x = a_x.a_z = a_z.a_x = a_y.a_z = a_z.a_y = 0$$

The remaining three terms involve the dot product of a unit vector with itself, which is unity, giving finally:

$$\vec{A}.\vec{B} = A_x.B_x + A_y.B_y + A_z.B_z$$

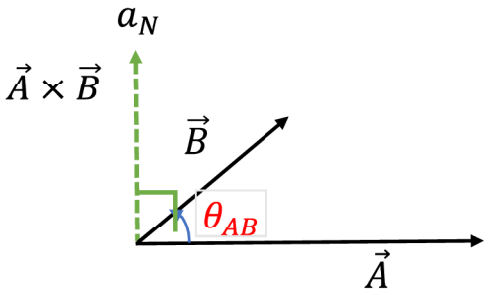
1.6 Vector Cross Product

Vector cross product $\vec{A} \times \vec{B}$ is a vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, i. e

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta_{AB}\, a_N$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

To find the direction of cross product we need to apply a right-handed screw as figure shown



$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$