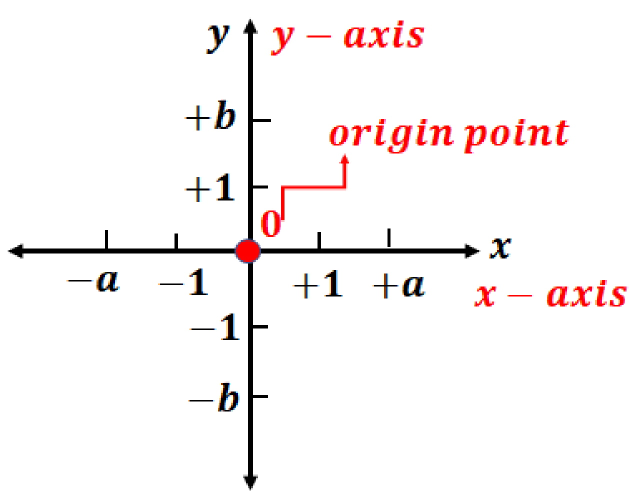
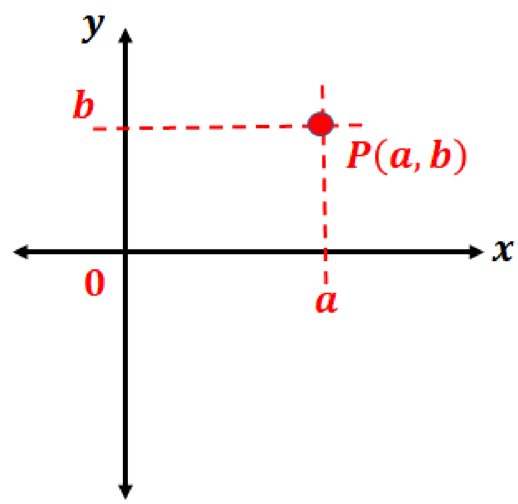
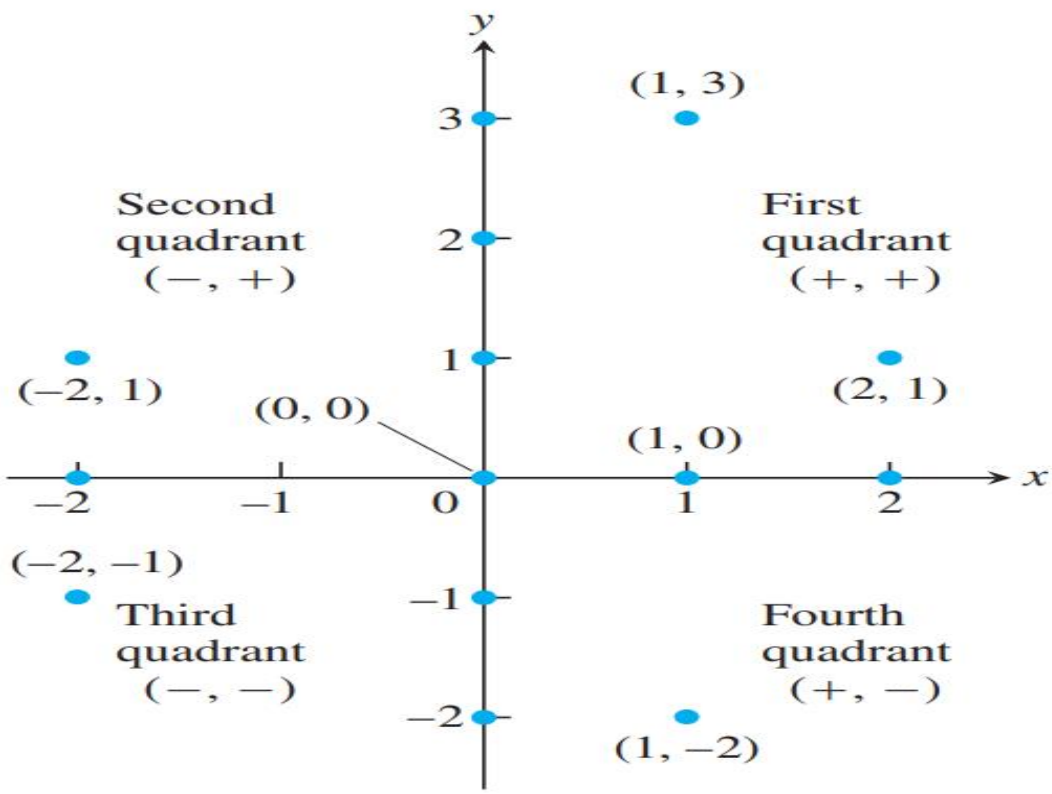
CHAPTER 1

PRERQUISITES FOR CALCULUS

Coordinates and Graphs in the Plane



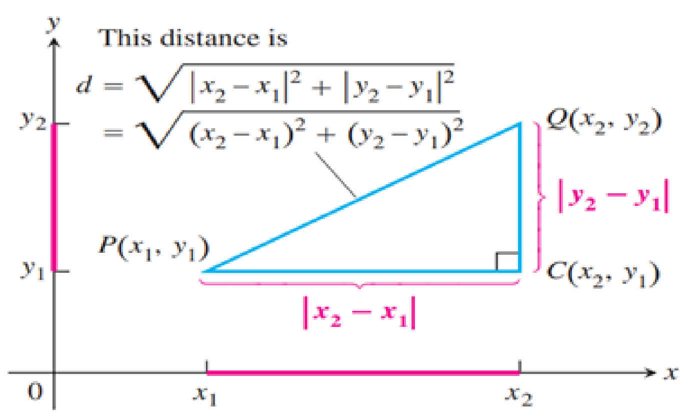




Distance Between Points

The distance between $P\ (x_1, y_1)$ **and** $Q\ (x_2, y_2)$ **is**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example

Find the distance between $P\ (-1\ ,2)$ and $Q\ (3\ ,4)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-1))^2 + (4 - 2)^2}$$

$$= \sqrt{(4)^2 + (2)^2} = \sqrt{20} = \sqrt{4 \times 5}$$

$$= 2\sqrt{5}$$

Example

Find the distance between $\left(2\sqrt{3}, 4\right)$ and $\left(-\sqrt{3}\ , 1\right)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(-\sqrt{3} - 2\sqrt{3}\right)^2 + (1 - 4)^2}$$

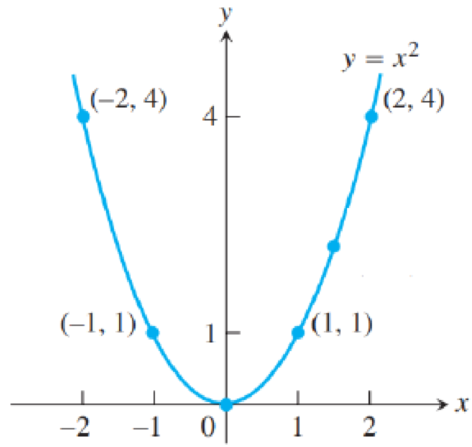
$$= \sqrt{\left(-3\sqrt{3}\right)^2 + (-3)^2} = \sqrt{27 + 9} = \sqrt{36} = 6$$

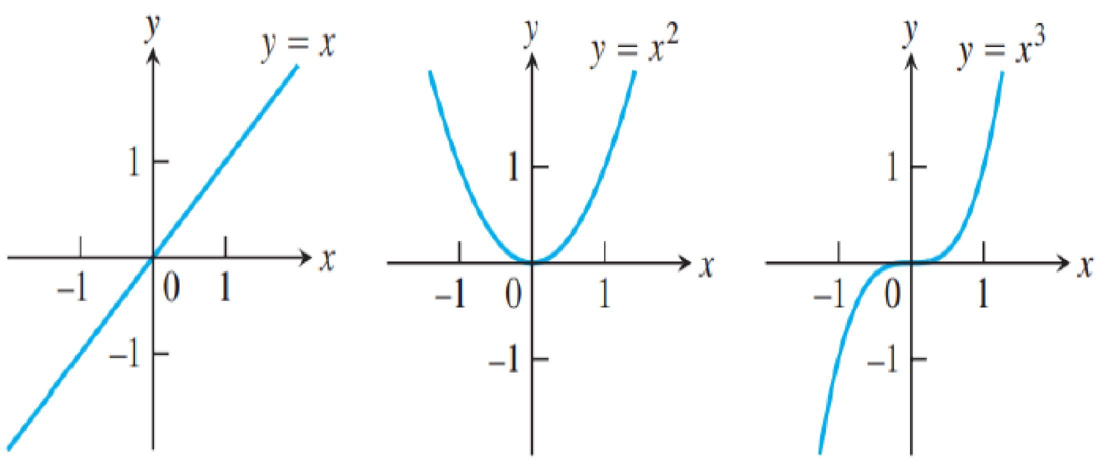
Graphs of Equations

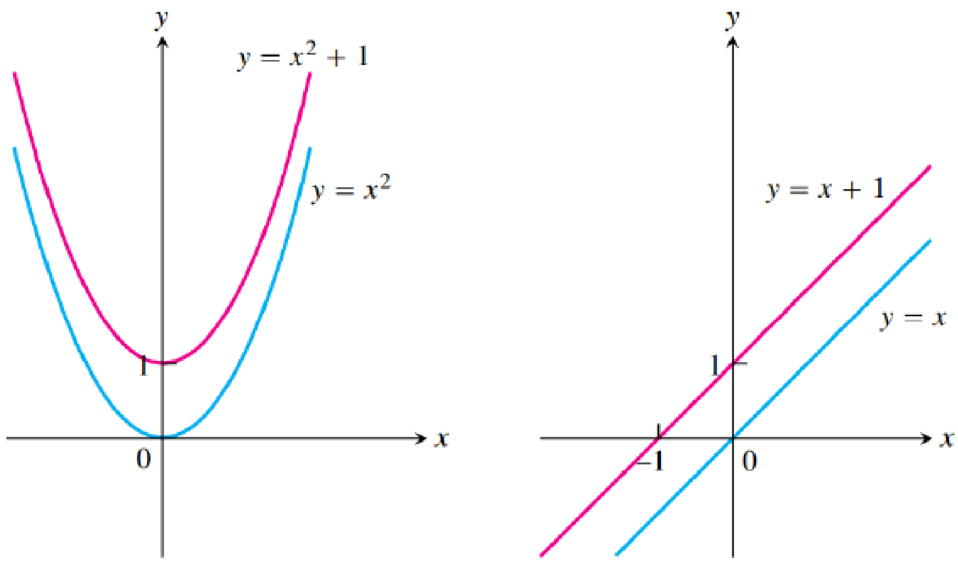
Example

Graph the equation $y = x^2$, for values of $x = -2$ to $x = 2$

$y = x^2$	x
4	-2
1	-1
0	0
1	1
4	2







Intercepts Points نقاط التقاطع

Example

Find the intercepts of the graph of equation $y = x^2 - 1$

The $x - intercepts$

$y = x^2 - 1$, $Let \quad y = 0$

$0 = x^2 - 1 \quad \rightarrow \quad x^2 = 1 \quad \rightarrow \quad x = \pm 1$

The $y - intercepts$

$y = x^2 - 1$, $Let \quad x = 0$

$y = 0 - 1 \quad \rightarrow \quad y = 1$

Example

Find the intercepts of the graph of equation $4x^2 + 4y^2 = 1$

The $x - intercepts$, $Let \quad y = 0$

$4x^2 + 4(0) = 1 \quad \rightarrow \quad 4x^2 = 1 \quad \rightarrow \quad x^2 = \frac{1}{4}$

$x = \frac{1}{2}, \frac{-1}{2}$

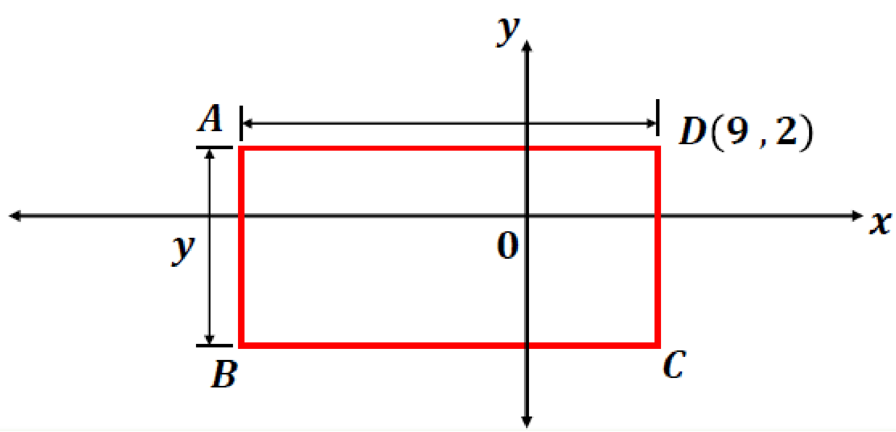
The $y - intercepts$, $Let \quad x = 0$

$4(0) + 4y^2 = 1 \quad \rightarrow \quad 4y^2 = 1 \quad \rightarrow \quad y^2 = \frac{1}{4}$

$y = \frac{1}{2}, \frac{-1}{2}$

Example

The rectangle in figure below has sides parallel to the axes. It is three times as long as it is wide. Its perimeter is 56 units. Find the coordinates of the vertices A , B and C.



$P = x + x + y + y = 2x + 2y$

$x = 3y$

$P = 2\,(3y) + 2y = 8y$

$56 = 8y \quad \rightarrow \quad y = 7\ units$

$x = 3\ \times 7 = 21\ units$

$A\,(-12, 2)$, $B(-12, -5)$, $C(9, -5)$

Example

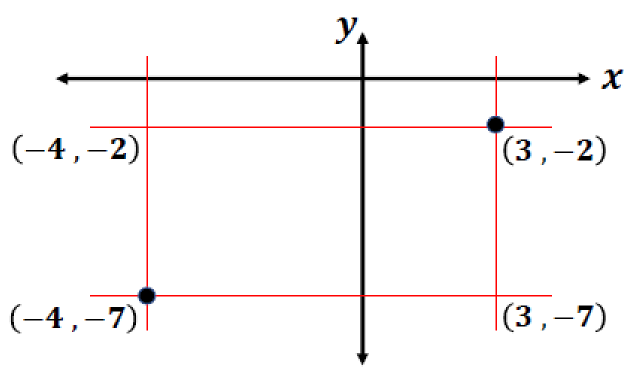
A rectangle with sides parallel to the axes has vertices at $(3, -2)$ and $(-4, -7)$

1) Find the coordinates of the other two vertices.

2) Find the area of the rectangle.

1) $(3\,, -7)\ , (-4\,, -2)$

2) $Area = 5\ \times 7 = 35$



Slope and Equations for Lines

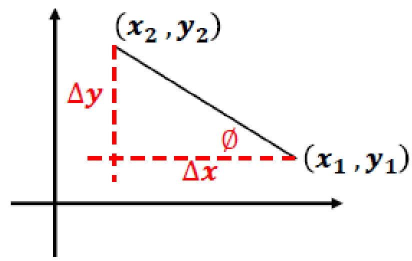
When a particle moves from (x_1 , y_1) to (x_2 , y_2) the increments are

$\Delta x = x_2 - x_1 \quad and \quad \Delta y = y_2 - y_1$

The slope of a no vertical line is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = tan\, \emptyset$$



For parallel lines $m_1 = m_2$

For perpendicular lines $m_2 = \frac{-1}{m_1}$

The equation $y - y_1 = m\,(x - x_1)$ **, is the point – slope equation of the line that passes through the point (x_1 , y_1) with slope m .**

Example

Write an equation for the line that passes through the point (2 , 3) with slope $\frac{-3}{2}$

$$y - y_1 = m\,(x - x_1)$$

$$y - 3 = \frac{-3}{2}\,(x - 2)$$

$$y = \frac{-3}{2}x + 3 + 3$$

$$y = \frac{-3}{2}x + 6$$

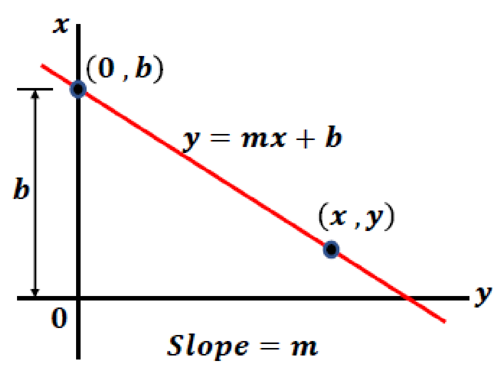
Slope – intercept Equations

$$(x_1 , y_1) = (0 , b)$$

$$y - b = m\,(x - 0)$$

$$y = mx + b$$

The equation $y = mx + b$ is the slope – intercept equation of the line with slope m and y intercept b



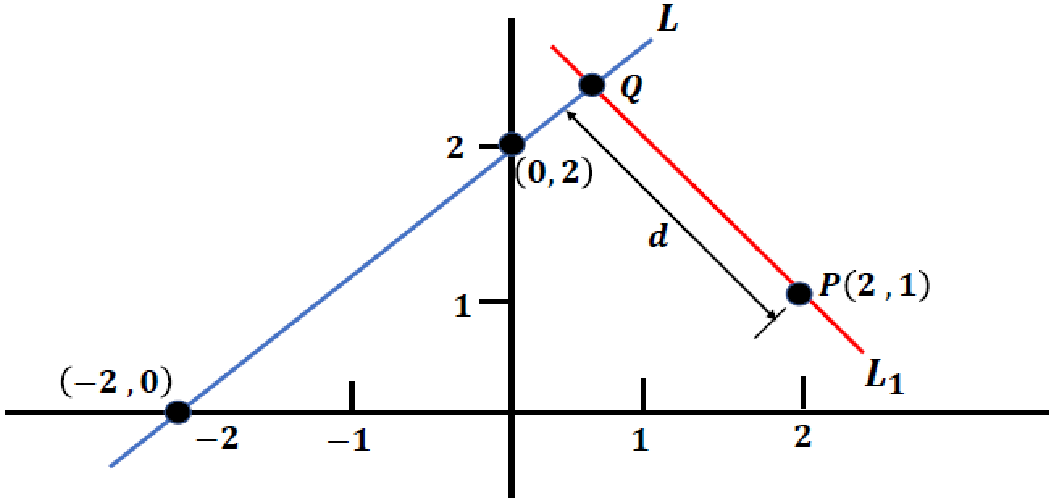
Example

Find the slope and y – intercept of the line $8x + 5y = 20$

$$8x + 5y = 20$$
$$5y = -8x + 20$$
$$y = \frac{-8}{5}x + 4$$
$$y = mx + b$$
$$m = \frac{-8}{5} \quad , \qquad b = 4$$

Example

Find the distance from the point $p\ (2\ ,1)$ to the line $y = x + 2$



$y = x + 2 \quad , \quad y = mx + b$

$m = 1$ **slope of line L**

Slope of line $L_1 \quad , \quad m_1 = \frac{-1}{m} = \frac{-1}{1} = -1$

$y - y_1 = m\,(x - x_1) \quad , \quad P\,(2, 1)$

$y - 1 = -1\,(x - 2)$

$y = -x + 2 + 1 \quad \rightarrow \quad y = -x + 3$

$x + 2 = -x + 3 \quad \rightarrow \quad x + x = 3 - 2$

$2x = 1 \quad \rightarrow \quad x = \frac{1}{2}$

$y = x + 2$

$y = \frac{1}{2} + 2 = \frac{5}{2} \quad , \quad Q\left(\frac{1}{2}, \frac{5}{2}\right)$

Distance between $p\,(2\,,1)$ **and** $Q\,\left(\frac{1}{2}\,,\frac{5}{2}\right)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{1}{2} - 2\right)^2 + \left(\frac{5}{2} - 1\right)^2} = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$$

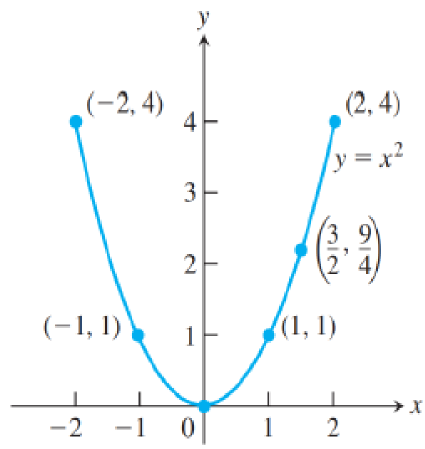
$$= \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2} = \frac{3}{\sqrt{2}}$$

Graphs of Functions

Example

Graph the function $y = x^2$, over the interval $[-2, 2]$

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4



Domain $-2 \le x \le 2$ *or* $[-2, 2]$

Range $0 \le y$ *or* $[0, 4]$

Example

Graph the function $y = \sqrt{4 - x}$

The domain of $y = \sqrt{4 - x}$

$4 - x \ge 0 \quad \rightarrow \quad x \le 4$

The graph stars at $x = 4$ and less

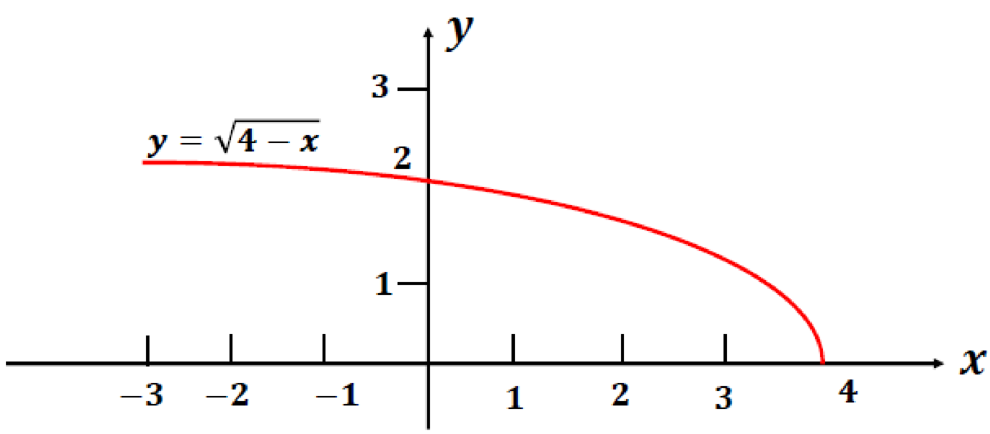
The x – intercept

$0 = \sqrt{4 - x} \quad \rightarrow \quad 0 = 4 - x \quad \rightarrow \quad x = 4$

The y – intercept

$y = \sqrt{4 - 0} \quad \rightarrow \quad y = \sqrt{4} \quad \rightarrow \quad y = 2$

$y = \sqrt{4-x}$	x
0	4
$y = \sqrt{4-3.75} = \sqrt{0.25} = 0.5$	3.75
$y = \sqrt{4-2} = \sqrt{2} = 1.4$	2
$y = \sqrt{4-0} = \sqrt{4} = 2$	0
$y = \sqrt{4-(-2)} = \sqrt{6} = 2.4$	-2



Domain $x \leq 4$

Range $0 \leq y$

Even Functions and Odd Functions

Even function of x if $f(-x) = f(x)$

Odd function of x if $f(-x) = -f(x)$

Example
Even , odd and neither

$f(x) = x^2$

Even function

$(-x)^2 = x^2 \quad for\ all \quad x$

$f(x) = x^2 + 1$

Even function

$(-x)^2 + 1 = x^2 + 1 \quad for\ all \quad x$

$f(x) = x$

Odd function

$(-x) = -x \quad for\ all \quad x$

$f(x) = x + 1$

Not odd

$f(-x) = -x + 1$

$-f(x) = -x - 1$

$f(-x) \neq -f(x)$

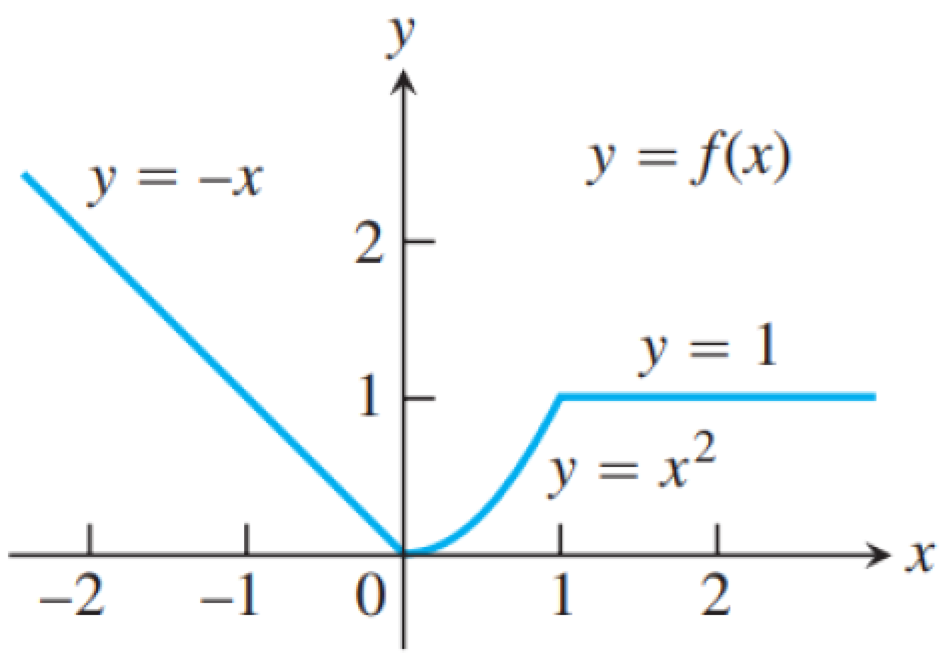
Not even

$(-x) + 1 \neq x + 1$

Functions Defined in Pieces

Example
The values of the function

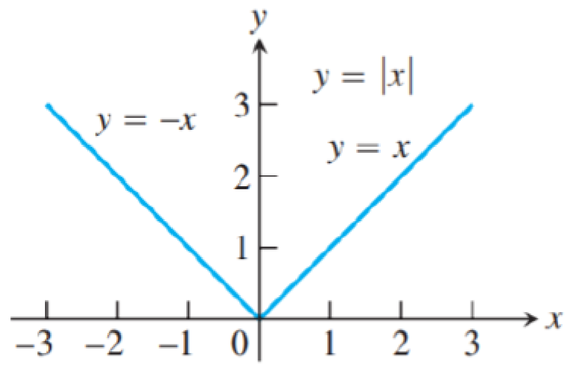
$$y = f(x) = \begin{cases} -x \\ x^2 \\ 1 \end{cases}$$



Example

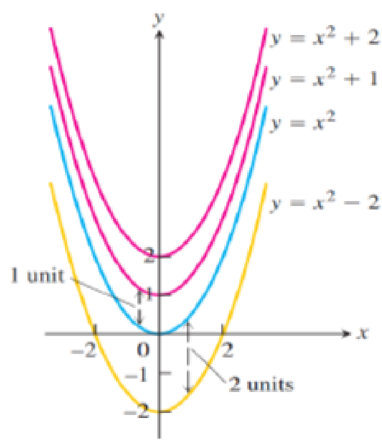
The values of the function

$$y = f(x) = \begin{cases} x \\ -x \end{cases}$$



Shifts, Circles and Parabolas

To shift the graph of a function $y = f(x)$ straight up, we add a positive constant to the right-hand side of $y = f(x)$



Equations for Circles

The standard equation for the circle of radius a centered at the point $(h\,,k)$

$$(x-h)^2 + (y-k)^2 = a^2$$

Example

The standard equation for the circle of radius 2 , centered at the point (3 , 4)

$$(x-h)^2 + (y-k)^2 = a^2$$

$$(x-3)^2 + (y-4)^2 = (2)^2$$

$$(x-3)^2 + (y-4)^2 = 4$$

Example

Find the center and radius of the circle

$$(x-1)^2 + (y+5)^2 = 3$$

$$(x-h)^2 + (y-k)^2 = a^2$$

$$(x-1)^2 + (y+5)^2 = 3$$

$$-h = -1 \quad \rightarrow \quad h = 1$$

$$-k = 5 \quad \rightarrow \quad k = -5$$

$$a^2 = 3 \quad \rightarrow \quad a = \sqrt{3}$$

The center is the point $(h, k) = (1, -5)$

The radius is $a = \sqrt{3}$

The Standard Equation for the Circle of Radius a Centered at the Origin

$$x^2 + y^2 = a^2$$

Example

If the circle $x^2 + y^2 = 25$, is shifted two units to the left and three units up

$$(x - h)^2 + (y - k)^2 = a^2$$

$$\left(x - (-2)\right)^2 + (y - 3)^2 = 25$$

$$(x + 2)^2 + (y - 3)^2 = 25$$