Lecture one

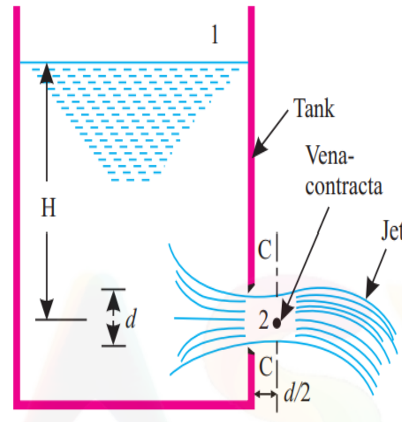
Type of orifices   أنواع الفتحة الحادة

1. According to size: a) small orifice , b) large orifice
2. According to shape: a) circular orifice , b) rectangular orifice ,

   c) square orifice, d) triangular orifice

3. According to shape of the edge: a) sharp-edged orifice, b) bell orifice

4. According to discharge: a) Fully submerged , b) partially submerged



Flow Through an Orifice
الجريان خلال الفتحة الحادة

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\gamma}$$

$$P_1 = P_2 = 0$$

$$z_1 = z_2 + H \quad \rightarrow \quad H = z_1 - z_2 \quad , \quad V_1 = 0$$

$$z_1 - z_2 + 0 + 0 = + \frac{V_2^2}{2g} + 0 \quad \rightarrow \quad \frac{V_2^2}{2g} = H$$

$$V_2 = \sqrt{2gH}$$

Hydraulic Coefficients

Coefficient of Contraction (C_c) معامل التخصر

$$C_c = \frac{Area\ of\ jet}{Area\ of\ orifice}$$

Coefficient of Velocity (C_v) معامل السرعة

$$C_v = \frac{Actual\ velocity}{Theortical\ velocity}$$

Coefficient of Discharge (C_d) معامل التصريف

$$C_d = \frac{Actual\ discharge}{Theortical\ discharge}$$

$$= \frac{Actual\ velocity \times Actual\ area}{Theortical\ velocity \times Theoretical\ area}$$

$$= C_v \times C_c$$

Example 8.1. *An orifice 50mm in diameter is discharging water under a head of 10 metres. If $C_d = 0.6$ and $C_v = 0.97$, find :*

(i) Actual discharge, and

(ii) Actual velocity of the jet at vena contracta.

Solution. Diameter of the orifice, $d = 50$ mm $= 0.05$ m

$\therefore$ Area of the orifice, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2 = 0.001963\ m^2$

Head, $H = 10$m; $C_d = 0.6$; $C_v = 0.97$

(i) Actual discharge

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = 0.6 \qquad \text{...(Given)}$$

But theoretical discharge = Area of orifice × theoretical velocity

$= a \times \sqrt{2gH}$

$= = 0.001963 \times \sqrt{2 \times 9.81 \times 10}$

$= 0.02749\ m^3/s$

$\therefore$ Actual discharge $= 0.6 \times 0.02749 =$ **$0.01649\ m^3/s$ (Ans.)**

(ii) Actual velocity

We know that , $C_v = \frac{\text{actual velocity}}{\text{theoretical velocity}}$

$\therefore$ Actual velocity $= C_v \times$ theoretical velocity

$= 0.97 \times \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 10} =$ **13.58m/s (Ans.)**

Example 8.2. *The head of water over the centre of an orifice of diameter 30 mm is 1.5m. The actual discharge through the orifice is 2.55 litres/sec. Find the co-efficient of discharge.*

Solution. Diameter of the orifice, $d = 30$ mm $= 0.03$m

$$\therefore \quad \text{Area, } a = \frac{\pi}{4} \times 0.03^2 = 0.0007068 \text{ m}^2$$

$$\text{Head, } H = 1.5 \text{ m}$$

Co-efficient of discharge, C_d

Actual discharge through the orifice, $Q = 2.55$ litres/sec.$= 0.00255$ m^3/s

$$\text{Theoretical discharge, } Q_{th} = \text{Area of orifice} \times \text{theoretical velocity.}$$

$$\text{But theoretical velocity, } V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1.5} = 5.425 \text{ m/s}$$

$$\therefore \quad Q_{th} = a \times V_{th}$$

$$= 0.0007068 \times 5.425 = 0.004166 \text{m}^3/\text{s}$$

$$\text{Hence,} \quad C_d = \frac{Q}{Q_{th}} = \frac{0.00255}{0.004166} = \mathbf{0.612} \text{ (Ans.)}$$

Example

A 60 mm diameter orifice is discharge water under a head of 9 m. Calculate the actual discharge in l/sec and actual velocity of the jet in m/sec, if $C_d = 0.6$ and $C_v = 0.9$

$$Q_{th} = a \,.\, \sqrt{2\,g\,h}$$

$$= \frac{\pi}{4} \times (6)^2 \times \sqrt{2 \times 981 \times 900} = 37570 \ cm^3/sec$$

$$= 37.57 \ \ l/sec$$

$$Q_{ac} = C_d \times Q_{th} = 0.6 \times 37.57 = 22.54 \ \ l/sec$$

$$V_{th} = \sqrt{2\,g\,h}$$

$$= \sqrt{2 \times 981 \times 900} = 1329 \ cm/sec = 13.29 \ \ m/sec$$

$$V_{ac} = C_v \times V_{th} = 0.9 \times 13.29 = 11.96 \ \ m/sec$$

Determination of Coefficient of Velocity (C_v)

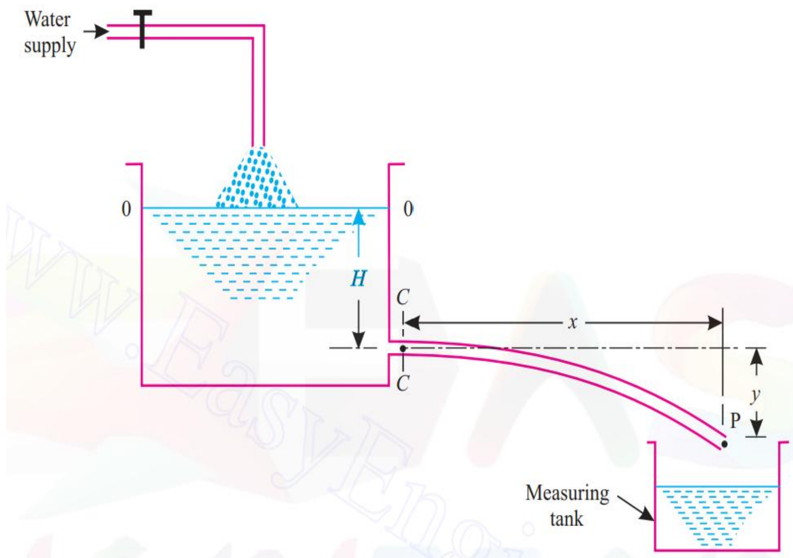
x = Horizontal distance

y = Vertical distance of the jet between C-C and P

V = Actual velocity of the jet

H = Constant water head

$$C_v = \sqrt{\frac{x^2}{4\,y\,H}}$$



Determination of Coefficient of Discharge (C_d)

$$Q = \frac{area\ measuring\ tank\ \times rise\ of\ water\ in\ the\ tank}{time\ (t)}$$

$$C_d = \frac{Q}{a\ \times\ \sqrt{2g\,H}}$$

$a = area\ of\ orifice$

Determination of Coefficient of Contraction (C_c)

$$C_d = C_c\ \times\ C_v$$

$$C_c = \frac{C_d}{C_v}$$

Example 8.3. *A vertical sharp-edged orifice 120 mm in diameter is discharging water at the rate of 98.2 litres/sec. under a constant head of 10 metres. A point on the jet, measured from the vena contracta of the jet has co-ordinates 4.5metres horizontal and 0.54 metre vertical. Find the following for the orifice.*

(*i*) *Co-efficient of velocity,* (*ii*) *Co-efficient of discharge, and*

(*iii*) *Co-efficient of contraction.*

Solution. Diameter of orifice, d = 120 mm = 0.12 m

$\therefore$ Area of orifice, $a = \frac{\pi}{4} \times 0.12^2 = 0.01131 \text{ m}^2$

Discharge, Q = 98.2 litres/sec. $= \frac{98.2}{1000} = 0.0982 \text{ m}^3/\text{s}$

Head, H = 10 m

Horizontal distance of a point on the jet from vena contracta, x = 4.5 m

Vertical distance, y = 0.54 m

Now theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$

Theoretical discharge,

$$Q_{th} = \text{Area of orifice } (a) \times V_{th}$$

$$= 0.01131 \times 14 = 0.1583 \text{ m}^3/\text{s}$$

(*i*) Co-efficient of velocity, C_v:

$$C_v = \frac{x}{\sqrt{4yH}}$$

$$= \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = \mathbf{0.968}$$ **(Ans.)**

(*ii*) Co-efficient of dischrge, C_d:

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

$$= \frac{0.0982}{0.1583} = \mathbf{0.62}$$ **(Ans.)**

(*iii*) Co-efficient of contraction, C_c:

$$= \frac{C_d}{C_v} = \frac{0.62}{0.968} = \mathbf{0.64}$$ **(Ans.)**

Example 8.4. *A large tank has a sharp edged circular orifice of 930 mm² area at a depth of 3 m below constant water level. The jet issues horizontally and in a horizontal distance of 2.4 m, it falls by 0.53 m, the measured discharge is 4.3 lit/s. Derermine coefficients of velocity, contraction and discharge for the orfice.*

Solution. *Given :* Area of the orifice, $a = 930\ \text{mm}^2$; $H = 3$ m; $x = 2.4$ m; $y = 0.53$ m; $Q = 4.3$ litres/sec. $= 0.0043\ \text{m}^3/\text{s}$

C_v, C_c and C_d:

Theoretical volocity, $V = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 3}$
$= 7.67$ m/s

Theoretical discharge $= a \times V_{th}$
$= 930 \times 10^{-6} \times 7.67$
$= 0.00713\ \text{m}^3/\text{s}$

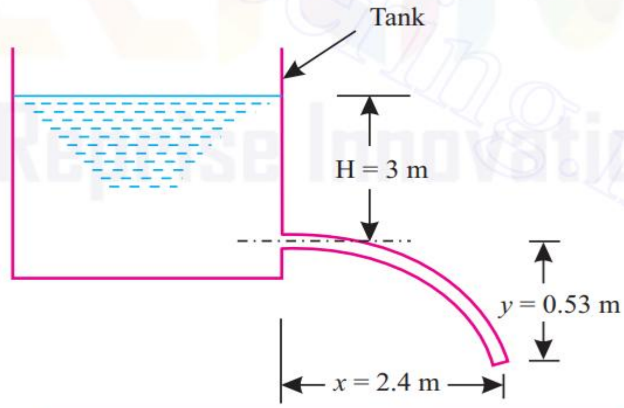


Fig. 8.3

Co-efficient of velocity,

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{2.4}{\sqrt{4 \times 0.53 \times 3}} = \mathbf{0.952}\ \textbf{(Ans.)}$$

Co-efficient of discharge,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.0043}{0.00713} = \mathbf{0.603}\ \textbf{(Ans.)}$$

Co-efficient of contraction,

$$C_c = \frac{C_d}{C_v} = \frac{0.603}{0.952} = \mathbf{0.633}\ \textbf{(Ans.)}$$

Example 8.5. *The head of water over an orifice of diameter 100 mm is 12 m. The water coming out from the orifice is collected in a rectangular tank 2 m × 0.9 m. The rise of water level in this tank is 1.2 m in 30 seconds. Find the co-efficient of discharge.*

Solution. Head of water, $H = 12$ m

Diameter of orifice, $d = 100\ \text{mm} = 0.1$ m

$\therefore$ Area, $a = \frac{\pi}{4} \times 0.1^2 = 0.00785\ \text{m}^2$

Area of the measuring tank, $A = 2 \times 0.9 = 1.8\ \text{m}^2$

Rise of water level (in $t = 30$ s), $h = 1.2$ m

Co-efficient of discharge, C_d

Theoretical velocity, $V_{th} = \sqrt{2gH}$
$= \sqrt{2 \times 9.81 \times 12} = 15.34\ \text{m/s}$

$\therefore$ Theoretical discharge, $Q_{th} = a \times V_{th}$
$= 0.00785 \times 15.34 = 0.1204\ \text{m}^3/\text{s}$

Actual discharge, $Q = \frac{A \times h}{t} = \frac{1.8 \times 1.2}{30} = 0.072\ \text{m}^3/\text{s}$

$\therefore$ Co-efficient of discharge, $C_d = \frac{Q}{Q_{th}} = \frac{0.072}{0.1204} \simeq \mathbf{0.6}$ **(Ans.)**

Example 8.6. *A tank has two identical orifices in one of its vertical sides, The upper orifice is 1.5 m below the water surface and the lower one is 3 m below the water surface as shown in Fig. 8.4. Find the point, at which the two jets will intersect, if the co-efficient of velocity is 0.92 for both the orifices.*

Solution. Height of water above the upper orifice, $H_1 = 1.5$ m

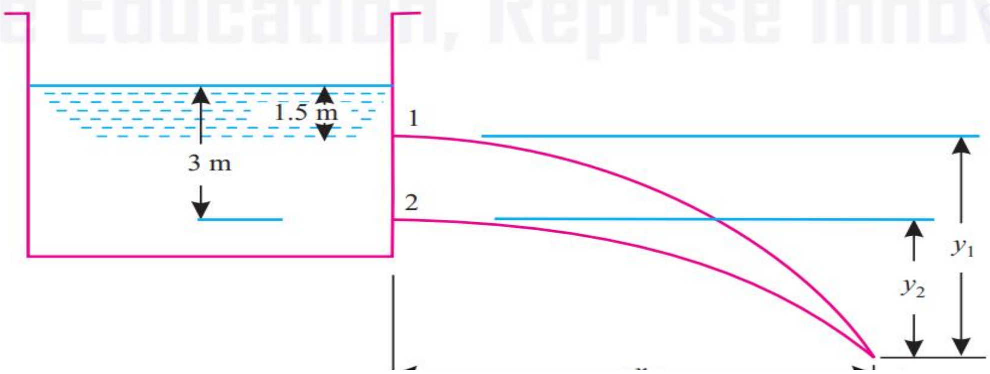
Height of water above the lower orifice, $H_2 = 3$ m

Co-efficient of velocity, $C_v = 0.92$

Let, x = Horizontal co-ordinate of the point of intersection *A*,

y_1 = Vertical co-ordinate of the point of intersection A from the orifice 1,and

y_2 = Vertical co-ordinate of the point of intersection A from the orifice 2.



Using the relation,

$$C_v = \sqrt{\frac{x^2}{4yH}}, \text{ with usual notations, we have:}$$

$$C_{v1} = \sqrt{\frac{x^2}{4y_1 \times 1.5}} \quad \text{...}(i)$$

and,

$$C_{v2} = \sqrt{\frac{x^2}{4y_2 \times 3.0}} \quad \text{...}(ii)$$

Since the two orifice are identical, therefore equating (*i*) and (*ii*), we get:

$$\sqrt{\frac{x^2}{4y_1 \times 1.5}} = \sqrt{\frac{x^2}{4y_2 \times 3.0}}$$

$$\therefore \quad y_1 = 2y_2 \quad \text{...}(iii)$$

From the geometry of the tank, we know that,

$$y_1 = y_2 + (3 - 1.5) = y_2 + 1.5 \quad \text{...}(iv)$$

Solving eqns. (*iii*) and (*iv*), we get:

$$y_2 = 1.5 \text{ m and } y_1 = 3 \text{ m}$$

Again, using the relation,

$$C_v = \sqrt{\frac{x^2}{4y_1 \times H_1}}, \text{ with usual notations, we get:}$$

$$0.92 = \sqrt{\frac{x^2}{4 \times 3 \times 1{\cdot}5}} = \sqrt{\frac{x^2}{18}} = \frac{x}{\sqrt{18}}$$

$$\therefore \quad x = 0.92 \times \sqrt{18} = \mathbf{3.9\ m}$$ **(Ans.)**