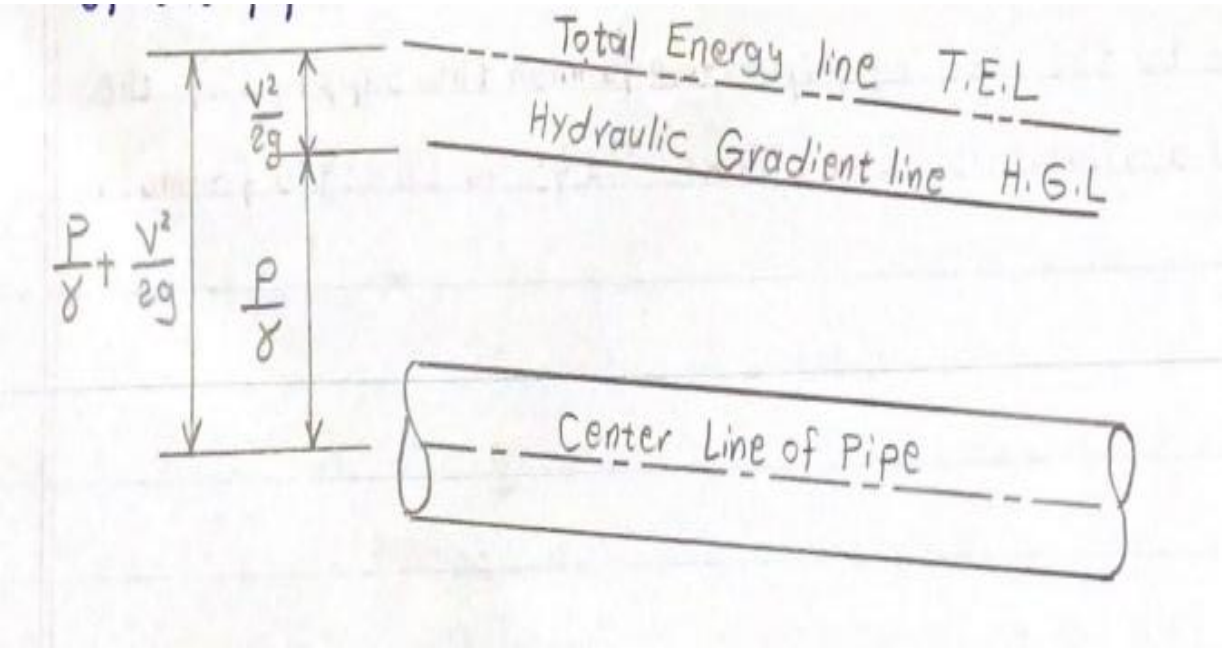
Lecture three

Hydraulic Gradient Line خط الانحدار الهيدروليكي

If pressure heads $\frac{P}{\gamma}$ of a liquid flowing in a pipe be plotted as vertical ordinates on the center line of the pipe.

Total Energy Line خط الطاقة الكلي

If the sum of pressure heads and velocity heads $\frac{P}{\gamma}+\frac{V^2}{2g}$ of a liquid flowing in a pipe.



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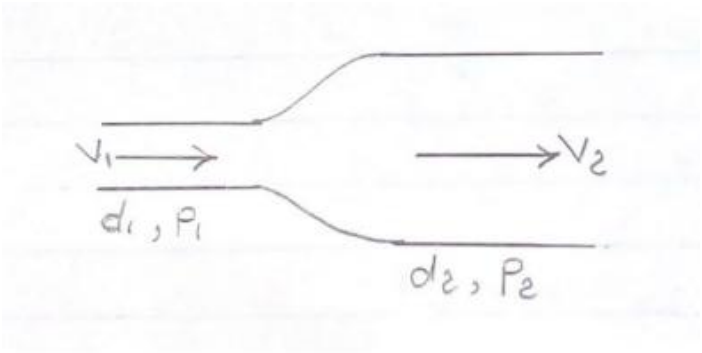
Minor Energy Losses الفواقد الصغرى

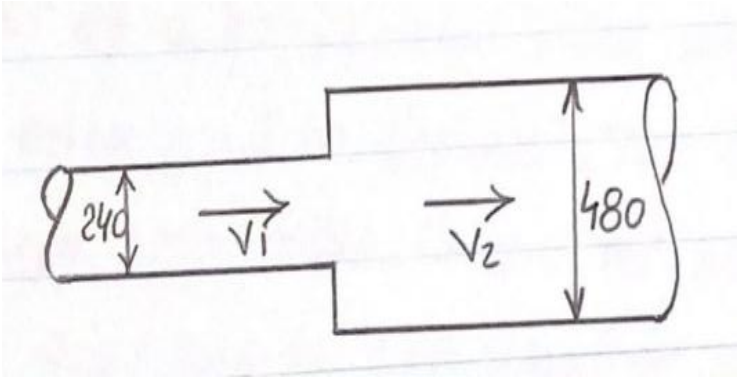
1. Loss of head due to sudden enlargement h_e الفواقد نتيجة التوسع الفجائي

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$



Example

At a sudden enlargement of a water from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Calculate the rate of flow.



$$\left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) = 10\ mm = 0.01\ m$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_e$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$Q_1 = Q_2 \quad \rightarrow \quad A_1 \,.\, V_1 = A_2 \,.\, V_2$$

$$\frac{\pi}{4} \times (0.24)^2 \times V_1 = \frac{\pi}{4} \times (0.48)^2 \times V_2$$

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$$V_1 = 4V_2$$

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$\frac{P_1}{\gamma} + \frac{(4V_2)^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{P_2}{\gamma} + z_2\right) - \left(\frac{P_1}{\gamma} + z_1\right)$$

$$\frac{6V_2^2}{2g} = 0.01 \qquad \rightarrow \qquad V_2 = 0.181 \ \ m/s$$

$$Q = A_2 \, . \, V_2 = \frac{\pi}{4} \times (0.48)^2 \times 0.181 = 0.03275 \ \ \ m^3/s$$

Example

The diameter of a horizontal pipe which is $300\ mm$ is suddenly enlarged to $600\ mm$. The rate of flow of water through this pipe is $0.4\ m^3/s$. If the intensity of pressure in smaller pipe is $125\ Kn/m^2$. Determine:

1. **Loss of head due to sudden enlargement.**
2. **Intensity of pressure in the large pipe.**

1. $$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.4}{\frac{\pi}{4} \times (0.3)^2} = 5.66 \ \ m/s$$

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$$V_2 = \frac{Q}{A_2} = \frac{0.4}{\frac{\pi}{4} \times (0.6)^2} = 1.414\ m/s$$

$$h_e = \frac{(5.66 - 1.414)^2}{2g} = 0.918\ m$$

2. $$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_e$$

$$z_1 = z_2$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_e$$

$$\frac{P_2}{\gamma} = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{125}{9.81} + \frac{(5.66)^2}{2 \times 9.81} - \frac{(1.414)^2}{2 \times 9.81} - 0.918 = 13.35\ m$$

$$P_2 = 9.81 \times 13.35 = 130.9\ \ Kn/m^2$$

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2. Loss of head due to sudden contraction h_c الفواقد نتيجة التخصر

$$h_c = \frac{{V_2}^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

C_c = Contraction cofficient معامل التخصر

Example

A horizontal pipe carries water at the rate of $0.04\ m^3/s$. Its diameter which is $300\ mm$ reduces to $150\ mm$. Calculate the head due to sudden contraction, take the contraction coefficient is 0.62

$$h_c = \frac{{V_2}^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{\frac{\pi}{4} \times (0.15)^2} = 2.26\ \ m/s$$

$$h_c = \frac{(2.26)^2}{2 \times 9.81}\left(\frac{1}{0.62} - 1\right)^2 = 0.098\ \ m$$

Example

When a sudden contraction in a horizontal pipeline from 500 mm to 250 mm diameter. The pressure change from 105 Kn/m^2 to 69 Kn/m^2. If the coefficient of contraction is assumed to be 0.62

Calculate the water flow rate.

$$h_c = \frac{{V_2}^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$$= \frac{{V_2}^2}{2g}\left(\frac{1}{0.65} - 1\right)^2 = 0.2899 \times \frac{{V_2}^2}{2g}$$

$$A_1.V_1 = A_2.V_2 \qquad \rightarrow \qquad V_1 = \frac{A_2}{A_1} \times V_2$$

$$V_1 = \frac{\frac{\pi}{4} \times (0.25)^2}{\frac{\pi}{4} \times (0.5)^2} \times V_2 = \frac{V_2}{4}$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_c$$

$$z_1 = z_2$$

$$\frac{105}{9.81} + \frac{\left(\frac{V_2}{4}\right)^2}{2 \times 9.81} = \frac{69}{9.81} + \frac{V_2^2}{2 \times 9.81} + 0.2899 \times \frac{V_2^2}{2 \times 9.81}$$

$$V_2 = 7.66 \ \ m/s$$

$$Q = A_2 \, . \, V_2 = \frac{\pi}{4} \times (0.25)^2 \times 7.66 = 0.376 \ \ m^3/s$$

----------Dr. Tahseen Gelmiran