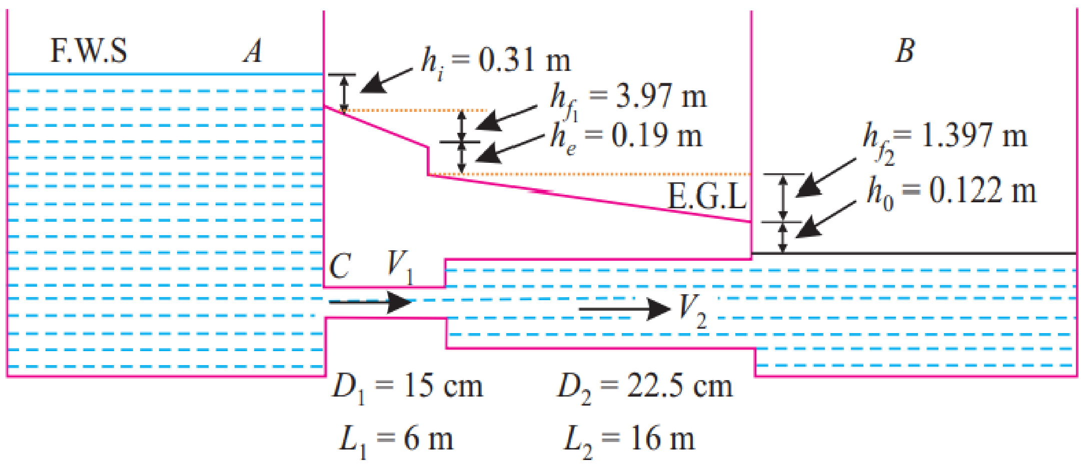
Example

Two reservoirs are connected by a pipeline consisting of two pipes, one of $15\ cm$ diameter and length $6\ m$ and other of diameter $22.5\ cm$ and $16\ m$ length. If the difference of water levels in the two reservoir is $6\ m$. Calculate the discharge and draw the total energy line.

Take $f = 0.04$



$$Q_1 = Q_2 \qquad \rightarrow \qquad V_1 \, . \, A_1 = V_2 \, . \, A_2$$

$$\frac{\pi}{4} \times (0.15)^2 \times V_1 = \frac{\pi}{4} \times (0.225)^2 \times V_2$$

$$V_1 = 2.25\, V_2$$

Loss of head at entrance to a pipe, $h_i = \frac{0.5\, V_1^2}{2g}$

Loss of head due to friction on $L_1 = h_{f1} = \frac{4fL_1V_1^2}{2g\times D_1}$

$$h_{f1} = \frac{4\times 0.04\times 6\times V_1^2}{0.15\times 2g} = 6.4\frac{V_1^2}{2g}$$

Loss of head due sudden enlargement $= h_e = \frac{(V_1 - V_2)^2}{2g}$

$$h_e = \frac{\left[V_1 - \frac{V_1}{2.25}\right]^2}{2g} = 0.308\ \frac{V_1^2}{2g}$$

Loss of head due friction on $L_2 = h_{f2} = \frac{4\times 0.04\times 16\times\left(\frac{V_1}{2.25}\right)^2}{0.225\times 2g} = 2.25\ \frac{V_1^2}{2g}$

Loss of head of the exit pipe $L_2 = h_o = \frac{V_2^2}{2g}$

$$h_o = \left(\frac{V_1}{2.25}\right)^2\times\frac{1}{2g} = 0.197\frac{V_1^2}{2g}$$

B.E. between A and B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + losses$$

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$P_A = P_B = 0$

$V_A = V_B = 0$

$z_A - z_B = 6 \ \ m$

$losses = 6 \ \ \ m$

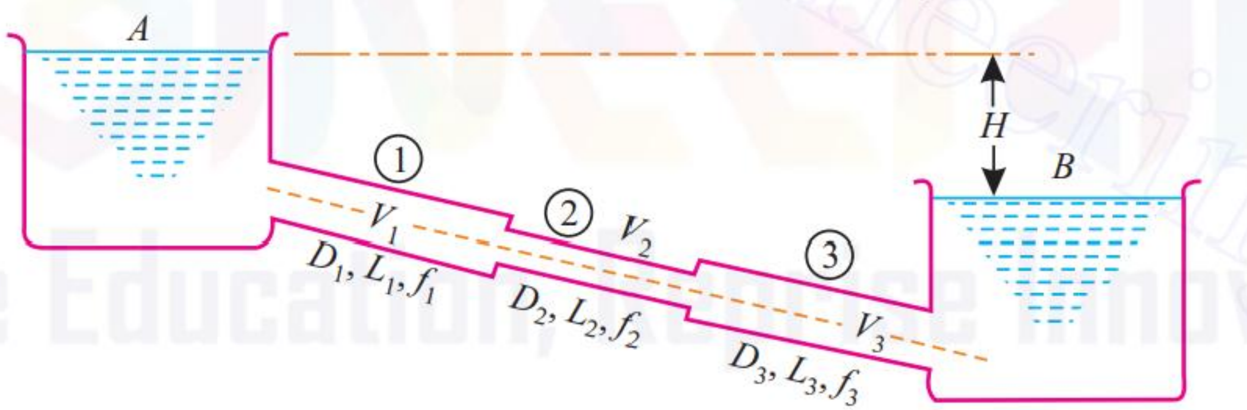
$h_i + h_{f1} + h_e + h_{f2} + h_o = 6$

$$\frac{0.5V_1^2}{2g} + 6.4 \ \frac{V_1^2}{2g} + 0.308 \ \frac{V_1^2}{2g} + 2.25 \ \frac{V_1^2}{2g} + 0.197 \ \frac{V_1^2}{2g} = 6$$

$V_1 = 3.49 \quad m/s$

$Q = A_1 \times V_1 = \frac{\pi}{4} \times (0.15)^2 \times 3.49 = 0.0617 \quad m^3/s$

Pipes in Series or Compound pipes الانابيب المتتالية او الانابيب المركبة



Let, D_1, D_2, D_3 = Diameters of pipes 1, 2 and 3 respectively,

L_1, L_2, L_3 = Lengths of pipes 1, 2 and 3 respectively,

V_1, V_2, V_3 = Velocities of flow through pipes 1, 2 and 3 respectively

f_1, f_2, f_3 = Co-efficients of friction for pipes 1, 2 and 3 respectively, and

H = Difference of water level in the two tanks.

As the rate of flow (Q) of water through each pipe is same, therefore,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

Also, The difference in liquid surface levels = Sum of the various head losses in the pipes

i.e., $$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^3}{2g}$$ ..

where, h_i = Head loss at entrance $= \frac{0.5V_1^2}{2g}$

h_{f_1} = Head loss due to friction in pipe 1 $= \frac{4f_1L_1V_1^2}{D_1 \times 2g}$

h_c = Head loss at contraction $= \frac{0.5V_2^2}{2g}$

h_{f_2} = Head loss due to friction in pipe 2 $= \frac{4f_2L_2V_2^2}{D_2 \times 2g}$

h_e = Head loss due to enlargement $= \frac{(V_2 - V_3)^2}{2g}$

h_{f_3} = Head loss due to friction in pipe 3 $= \frac{4f_3L_3V_3^2}{D_3 \times 2g}$

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$= \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g}$$

If *minor losses are neglected*, then above equation becomes:

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad ..$$

If, $f_1 = f_2 = f_3 = f,$ then:

$$H = \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_2V_2^2}{D_2 \times 2g} + \frac{4fL_3V_3^2}{D_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{D_1} + \frac{L_2V_2^2}{D_2} + \frac{L_3V_3^2}{D_3}\right]$$

Example 12.22. *Three pipes of diameters 300 mm, 200 mm and 400 mm and lengths 450 m, 255 m and 315 m respectively are connected in series. The difference in water surface levels in two tanks is 18 m. Determine the rate of flow of water if co-efficients of friction are 0·0075, 0·0078 and 0·0072 respectively considering :*

(*i*) *Minor losses also, and*

(*ii*) *Neglecting minor losses.*

Solution. Pipe 1 : $L_1 = 450$ m, $= D_1 = 300$ mm $= 0{\cdot}3$ m, $f_1 = 0{\cdot}0075$

Pipe 2 : $L_2 = 255$ m, $D_2 = 200$ mm $= 0{\cdot}2$ m, $f_2 = 0{\cdot}0078$

Pipe 3 : $L_3 = 315$ m, $D_3 = 400$ mm $= 0{\cdot}4$ m, $f_3 = 0{\cdot}0072$

Difference of water level, $H = 18$ m.

(*i*) Considering minor losses :

Let V_1, V_2 and V_3 be the velocities in Ist, 2nd, and 3rd pipe respectively.

From continuity considerations, we have:

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{(\pi/4) \times D_1^2}{(\pi/4) \times D_2^2} \times V_1 = \frac{D_1^2}{D_2^2} \times V_1 = \left(\frac{0.3}{0.2}\right)^2 V_1 = 2.25\, V_1$$

and,

$$V_3 = \frac{A_1V_1}{A_3} = \frac{(\pi/4) \times D_1^2}{(\pi/4) \times D_3^2} \times V_1 = \frac{D_1^2}{D_3^2} \times V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\, V_1$$

We know that:

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g} \quad \text{...[Eqn. (12·9)]}$$

$$18 = \frac{0{\cdot}5V_1^2}{2g} + \frac{4 \times 0.0075 \times 450 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25\,V_1)^2}{2g} + \frac{4 \times 0.0078 \times 255 \times (2.25V_1)^2}{0.2 \times 2g}$$

$$+ \frac{(2.25\,V_1 - 0.5625\,V_1)^2}{2g} + \frac{4 \times 0.0072 \times 315 \times (0.5625\,V_1)^2}{0.4 \times 2g} + \frac{(0.5625\,V_1)^2}{2g}$$

$$18 = \frac{V_1^2}{2g}(0.5 + 45 + 2.53 + 201.4 + 2.847 + 7.176 + 0.316)$$

$$= 259.77\,\frac{V_1^2}{2g}$$

or, $$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{259.77}} = 1.166 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_1 \times V_1 = (\pi/4) \times 0{\cdot}3^2 \times 1{\cdot}166 = \mathbf{0{\cdot}0824\ m^3/s}$ **(Ans.)**

(*ii*) Neglecting minor losses :

We know that, $$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad \text{...[Eqn. (12·10)]}$$

$$18 = \frac{V_1^2}{2g}\left(\frac{4 \times 0.0075 \times 450}{0{\cdot}3} + \frac{4 \times 0.0078 \times 255 \times 2.25^2}{0{\cdot}2} + \frac{4 \times 0.0072 \times 315 \times (0.5625)^2}{0{\cdot}4}\right)$$

$$= \frac{V_1^2}{2g}(45 + 201.4 + 7.176) = 253.57 \times \frac{V_1^2}{2g}$$

or, $$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{253.57}} = 1.18 \text{ m}$$

$\therefore$ Discharge, $Q = A_1V_1 = (\pi/4) \times 0{\cdot}3^2 \times 1{\cdot}18 = \mathbf{0{\cdot}0834\ m^3/s}$ **(Ans.)**

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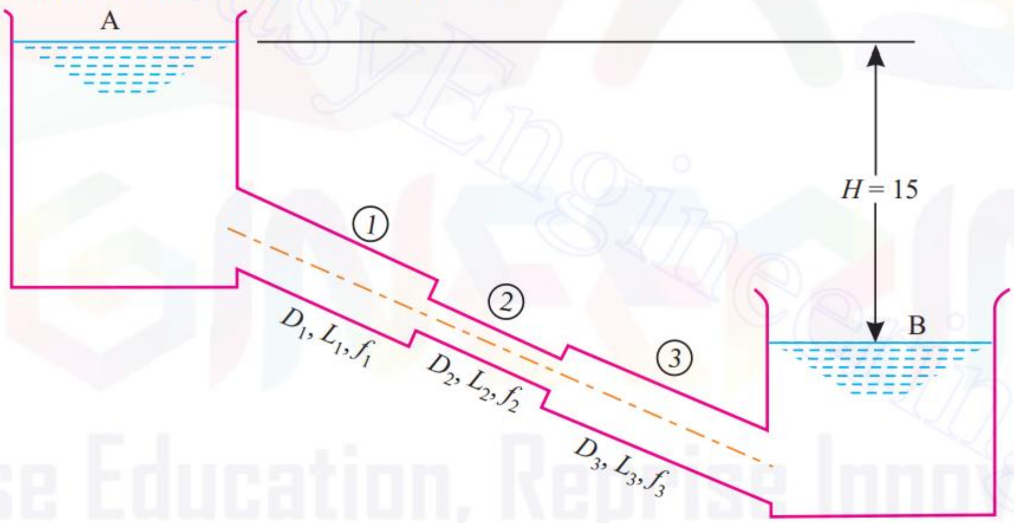
Example 12.23. *Two reservoirs with a difference in elevation of 15 m are connected by the three pipes in series. The pipes are 300 m long of diameter 30 cm, 150 m long of 20 cm diameter, and 200 m long of 25 cm diameter respectively. The friction factors (f) in the relation*

$$h_f = \frac{fLV^2}{D \times 2g}$$

for the three pipes are, respectively, 0·018, 0·020 and 0·019, and which account for friction and all losses. Further the contractions and expansions are sudden. Determine the flow rate in l/s. The loss co-efficient for sudden contraction from dia. 30 cm to 20 cm = 0·24. **(PTU)**

Solution. Refer to Fig. 12.16. *Given :* $D_1 = 30$ cm $= 0·3$ m; $L_1 = 300$ m; $D_2 = 20$ cm $= 0·2$ m; $L_2 = 150$ m; $D_3 = 25$ cm $= 0·25$ m; $L_3 = 200$ m; $f_1 = 0·018$; $f_2 = 0·020$, $f_3 = 0·019$.

Loss co-efficient for sudden contraction = 0·24



Various types of losses which occur in the pipelines 1, 2 and 3 are :

(*i*) Head loss at entrance, $h_i = 0.5 \times \frac{V_1^2}{2g} = \frac{0.5}{2 \times 9.81} \times \left[\frac{4Q}{\pi \times 0.30^2}\right] = 5.1\, Q^2$

(*ii*) Head loss due to friction in pipe 1, $h_{f_1} = \frac{f_1 L_1 V_1^2}{D_1 \times 2g} = \frac{f_1 \times L_1}{D_1 \times 2g}\left[\frac{4Q}{\pi\, D_1^2}\right]^2$

$$= \frac{0.018 \times 300}{0.3 \times 2 \times 9.81}\left[\frac{4Q}{\pi \times 0.3^2}\right]^2 = 183.6\, Q^2$$

(*iii*) Head loss at contraction, $h_c = 0.24\,\frac{V_2^2}{2g} = \frac{0.24}{2g}\left[\frac{4Q}{\pi D_2^2}\right]^2$

$$= \frac{0.24}{2 \times 9.81}\left[\frac{4Q}{\pi \times 0.2^2}\right]^2 = 12.394\, Q^2$$

(*iv*) Head loss due to friction in pipe 2, $h_{f_2} = \frac{f_2 L_2 V_2^2}{D_2 \times 2g} = \frac{f_2 \times L_2}{D_2 \times 2g}\left[\frac{4Q}{\pi D_2^2}\right]^2$

$$= \frac{0.02 \times 150}{0.2 \times 2 \times 9.81}\left[\frac{4Q}{\pi \times 0.2^2}\right]^2 = 774.267\, Q^2$$

(*v*) Head loss due to sudden enlargement, $h_e = \frac{(V_2 - V_3)^2}{2g} = \frac{1}{2g}\left[\frac{4Q}{\pi D_2^2} - \frac{4Q}{\pi D_3^2}\right]^2$

$$= \frac{16Q^2}{2 \times 9.81 \times \pi^2}\left[\frac{1}{0.2^2} - \frac{1}{0.25^2}\right]^2 = 6.69\, Q^2$$

(*vi*) Head loss due to friction in pipe 3, $h_{f_3} = \frac{f_3 L_3 V_3^2}{D_3 \times 2g} = \frac{f_3 L_3}{D_3 \times 2g}\left[\frac{4Q}{\pi D_3^2}\right]^2$

$$= \frac{0.019 \times 200}{0.25 \times 2 \times 9.81}\left[\frac{4Q}{\pi \times 0.25^2}\right]^2 = 321.518\, Q^2$$

(*vii*) Head loss at the exit, $h_0 = \frac{V_3^2}{2g} = \frac{1}{2g}\left[\frac{4Q}{\pi D_3^2}\right]^2 = \frac{1}{2 \times 9.81}\left[\frac{4Q}{\pi \times 0.25^2}\right]^2 = 21.152\, Q^2$

Applying the Bernoulli's equation between the water surfaces of the two reservoirs, we get:

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{w} + \frac{V_B^2}{2g} + z_B + \text{all losses.}$$

$$0 + 0 + 15 = 0 + 0 + 0 + (5{\cdot}1 + 183{\cdot}6 + 12{\cdot}394 + 774{\cdot}267 + 6{\cdot}69 + 321{\cdot}518 + 21{\cdot}152)\, Q^2$$

or, $Q = 0{\cdot}1064\ \text{m}^3/\text{s}$ or **106·4 *l/s*** **(Ans.)**