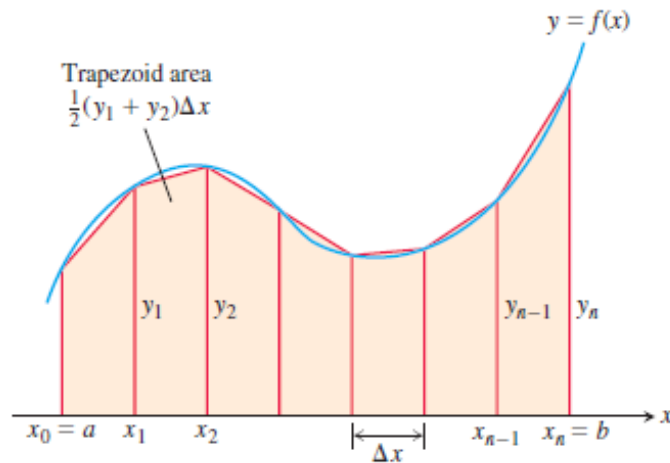


Lecture (2)

التكامل العددي Numerical Integration

1. Trapezoidal Approximations

$$\Delta x = \frac{b - a}{n}.$$



The Trapezoidal Rule

To approximate $\int_a^b f(x) dx$, use

$$T = \frac{\Delta x}{2} \left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n \right).$$

The y 's are the values of f at the partition points

$$x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \dots, x_{n-1} = a + (n - 1)\Delta x, x_n = b,$$

where $\Delta x = (b - a)/n$.

EXAMPLE 1 Use the Trapezoidal Rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. Compare the estimate with the exact value.

Solution Partition $[1, 2]$ into four subintervals of equal length (Figure 8.8). Then evaluate $y = x^2$ at each partition point (Table 8.2).

Using these y -values, $n = 4$, and $\Delta x = (2 - 1)/4 = 1/4$ in the Trapezoidal Rule, we have

$$\begin{aligned} T &= \frac{\Delta x}{2} \left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4 \right) \\ &= \frac{1}{8} \left(1 + 2\left(\frac{25}{16}\right) + 2\left(\frac{36}{16}\right) + 2\left(\frac{49}{16}\right) + 4 \right) \\ &= \frac{75}{32} = 2.34375. \end{aligned}$$

2. Simpson's Rule: Approximations Using Parabolas

Simpson's Rule

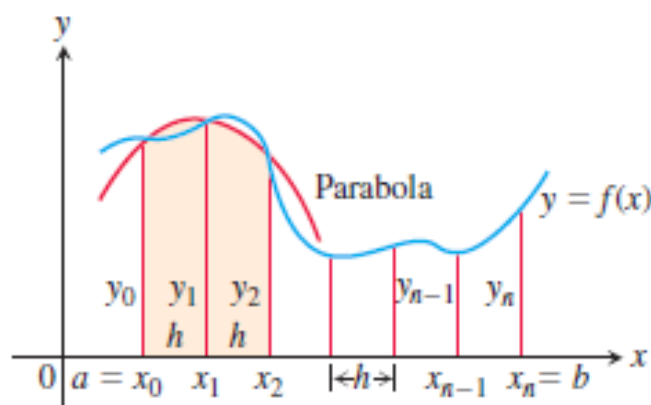
To approximate $\int_a^b f(x) dx$, use

$$S = \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n).$$

The y 's are the values of f at the partition points

$$x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \dots, x_{n-1} = a + (n-1)\Delta x, x_n = b.$$

The number n is even, and $\Delta x = (b - a)/n$.



EXAMPLE 2 Use Simpson's Rule with $n = 4$ to approximate $\int_0^2 5x^4 dx$.

Solution Partition $[0, 2]$ into four subintervals and evaluate $y = 5x^4$ at the partition points (Table 8.3). Then apply Simpson's Rule with $n = 4$ and $\Delta x = 1/2$:

$$\begin{aligned} S &= \frac{\Delta x}{3} \left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4 \right) \\ &= \frac{1}{6} \left(0 + 4\left(\frac{5}{16}\right) + 2(5) + 4\left(\frac{405}{16}\right) + 80 \right) \\ &= 32 \frac{1}{12}. \end{aligned}$$

H.W.

Estimate the integral with $n = 4$ steps using Simpson's Rule

3. $\int_{-1}^1 (x^2 + 1) dx$

4. $\int_{-2}^0 (x^2 - 1) dx$

5. $\int_0^2 (t^3 + t) dt$

6. $\int_{-1}^1 (t^3 + 1) dt$

23. **Volume of water in a swimming pool** A rectangular swimming pool is 30 ft wide and 50 ft long. The accompanying table shows the depth $h(x)$ of the water at 5-ft intervals from one end of the pool to the other. Estimate the volume of water in the pool using the Trapezoidal Rule with $n = 10$ applied to the integral

$$V = \int_0^{50} 30 \cdot h(x) dx.$$

Position (ft) x	Depth (ft) $h(x)$	Position (ft) x	Depth (ft) $h(x)$
0	6.0	30	11.5
5	8.2	35	11.9
10	9.1	40	12.3
15	9.9	45	12.7
20	10.5	50	13.0
25	11.0		