Basics of Applied Petroleum Reservoir Engineering

Dr. Isam Najar

Omar Mohammed Al-Sultan

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By

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Omar Al-Sultan

Contents

Preface

Because of lacking a simplified textbook about the applied petroleum reservoir engineering. The principal author wanted to collect his lectures as a textbook to be a reference for teaching this subject to the students of third class in petroleum reservoir engineering departments.

Mr. Al-Sultan volunteered and supported Dr. Najar and helped him with great efforts in feeding the lectures using Word, Excel and Photoshop. Without Al-Sultan efforts this book would not have been come out with the current arrangement.

Senior author Isam Najar is primarily responsible for chapters 1-7, while Omer Al-Sultan is responsible of section 6.3.

This textbook contains seven chapters; The first chapter represents the review of some basic concepts in petroleum reservoir engineering necessary to understand the following six chapters. Chapter two deals with petroleum reservoir recovery mechanisms and reserve estimation methods. In chapter three we explain the Material Balance Equation. In chapter four we talk about natural water influx. In chapter five we displayed basic concepts about fluid flow in petroleum reservoirs. Chapter six represents fluid potential and datum plane. Finally, in chapter seven we demonstrated the important basics of gas reservoirs.

The authors hope that this book to be a very helpful source for students during their study.

NAJAR and AL-SULTAN

Some units Conversion

	To convert	**Into**	**Multiply by**
Length (L)	Feet	inches	12
	Feet	meters	0.3048
	meters	feet	3.28
	inch	cm	2.54
	ft	cm	30.48
	m	inch	39.7
Volume (V)	bbl	m^3	0.159
	bbl	ft^3	5.615
	m^3	bbl	6.29
	lt	ft^3	0.03531
	bbl	lt	159
	m^3	ft^3	35.3
Mass (m)	lb	kg	0.4536
	kg	lb	2.205
Density (ρ)	g/cm^3	lb/ft^3	62.43
	kg/m^3	lb/ft^3	$6.243x10^{-2}$
	lb/ft^3	g/cm^3	$1.602x10^{-2}$
	lb/ft^3	kg/m^3	16.02
Pressure (p)	atm	psi	14.7
	psi	atm	$6.805x10^{-2}$
	bar	atm	0.9869
	bar	Torr	750.061
	atm	pascal	$1x10^5$
	psi	pascal	47.88

List of Figures:

Chapter 1.

Chapter 2.

Chapter 3.

Chapter 4.

Chapter 5.

Chapter 6.

Chapter 7.

Chapter

1

Review of Some Basic Concepts in Petroleum Reservoir Engineering

1.1 Reservoir Fluid Properties

To understand and predict the volumetric behavior of oil and gas reservoirs as a function of pressure, knowledge of the physical properties of reservoir fluids must be gained. These fluid properties are usually determined by laboratory experiments performed on samples of actual reservoir fluids. In the absence of experimentally measured properties, it is necessary for the petroleum engineer to determine the properties from empirically derived correlations. The objective of this chapter is to present several of the well-established physical property correlations for the following reservoir fluids:

- Natural gases
- Crude oil systems
- Reservoir water systems

1.1.1 Natural gas properties

Gas is defined as a homogeneous fluid of low viscosity and density, which has no definite volume but expands to completely fill the vessel in which it is placed. Generally, the natural gas is a mixture of hydrocarbon and nonhydrocarbon gases. The hydrocarbon gases normally found in a natural gas are methane, ethane (light components as gases), propane, butanes, pentanes, and small amounts of hexanes (medium components) and heavier components (heptane, octane, ...etc).The medium and heavy components are normally liquids. The nonhydrocarbon gases, that is, impurities, include carbon dioxide, hydrogen sulfide, and nitrogen.

Knowledge of pressure-volume-temperature (PVT) relationships and other physical and chemical properties of gases are essential for solving problems in natural gas reservoir engineering. The properties of interest include:

- Isothermal gas compressibility coefficient, c_g.
- Gas formation volume factor, B_g.
- Gas expansion factor, E_g.
- Gas viscosity, μ_g
- Apparent molecular weight, M_a.
- Gas density, ρ_g.
- Specific gravity, γ_g.
- Specific volume ,v.
- Compressibility factor, Z.

The mathematical expressions define these properties are:

1 Apparent Molecular Weight (Ma) and Mole Fraction (y_i)

$$M_a = \sum_{i=1} y_i M_i \qquad eq. (1.1)$$

where:

M_a = apparent molecular weight of a gas mixture.
M_i = molecular weight of the i^{th} component in the mixture.
y_i = mole fraction of component i in the mixture.

$$y_i = \frac{n_i}{n} = \frac{n_i}{\sum_i n_i} \qquad eq. (1.2)$$

where:

n = number of moles of gas, *lb-mole.*

y_i = mole fraction

$$w_i = \frac{m_i}{m} = \frac{m_i}{\sum_i m_i} \qquad eq. (1.3)$$

where:

w_i = weight fraction.

m = total weight.

$$v_i = \frac{V_i}{V} = \frac{V_i}{\sum_i V_i} \qquad eq. (1.4)$$

where:

v_i = the volume fraction of a particular component.

V = total volume.

$$y_i = \frac{n_i}{n} = \frac{n_i}{1} = n_i \qquad (At\ n = 1)$$

$$m_i = n_i M_i = y_i M_i$$

$$w_i = \frac{m_i}{m} = \frac{m_i}{\sum_i m_i} = \frac{y_i M_i}{\sum_i y_i M_i} = \frac{y_i M_i}{M_a}$$

$$y_i = \frac{w_i / M_i}{\sum_i w_i / M_i} \qquad eq. (1.5)$$

2 Gas Density (ρ_g)

$$\rho_g = \frac{pM_a}{RT} \qquad eq.(1.6)$$

3 Specific Gravity, γ_g

$$\gamma_g = \frac{gas\ density\ @\ 14.7\ and\ 60°}{air\ density\ @\ 14.7\ and\ 60°} = \frac{\rho_g}{\rho_{air}} \qquad eq.(1.7)$$

$$\gamma_g = \frac{\frac{p_{sc}M_a}{RT_{sc}}}{\frac{p_{sc}M_{air}}{RT_{sc}}}$$

Or

$$\gamma_g = \frac{M_a}{M_{air}} = \frac{M_a}{28.96} \qquad eq.(1.8)$$

where:

γ_g = gas specific gravity, 60°/60°.

density of the air.=ρ_{air}

28.96 =M_{air} = apparent molecular weight of the air

M_a = apparent molecular weight of the gas.

standard pressure, psia.=P_{sc}

standard temperature, °R.=T_{sc}

4 Specific Volume, ν:

$$\nu = \frac{V}{m} = \frac{RT}{PM_a} = \frac{1}{\rho} \qquad eq.(1.9)$$

1. Determination of Z-factor Value

$$Z = \frac{p\left(V_{gas}\right)_{p,T}}{T}\frac{T_{sc}}{p_{sc}\left(V_{gas}\right)_{sc}} = \frac{V_R p_R T_{sc}}{V_{sc} p_{sc} T_R} \qquad eq.(1.10)$$

5 Isothermal Gas Compressibility, c_g:

$$c_g = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad eq.(1.11)$$

Where:

c_g = isothermal gas compressibility, 1/psi

1- Isothermal compressibility factor for **ideal gas** given by deriving the ideal gas law at constant temperature:

$$pV = nRT; \qquad T = constant$$

$$p\partial V + V\partial p = 0$$

$$p\partial V = -V\partial p \Longrightarrow \frac{\partial V}{\partial p} = \frac{-V}{p} \qquad eq.(1.12)$$

by substituting into Eq. (1.11):

$$c_g = \frac{-1}{V}\left(\frac{-V}{p}\right) \Longrightarrow c_g = \frac{1}{p}\ (psi^{-1}) \qquad eq.(1.13)$$

2- Isothermal compressibility factor for **real gas** given by deriving the real gas law at constant temperature:

$$pV = ZnRT; \qquad T = constant$$

$$p\partial V + V\partial p = nRT\partial Z$$

by dividing the equation above by (∂p):

$$p\frac{\partial V}{\partial p} + V\frac{\partial p}{\partial p} = nRT\frac{\partial Z}{\partial p}$$

$$p\frac{\partial V}{\partial p} = nRT\frac{\partial Z}{\partial p} - V$$

$$\frac{\partial V}{\partial p} = \frac{nRT}{p}\frac{\partial Z}{\partial p} - \frac{V}{p} \qquad eq.(1.14)$$

by substituting (*V=ZnRT/p*) from real gas law and Eq. (1.14) into Eq. (1.11):

$$c_g = \frac{ZnRT}{pZnRT} - \frac{1}{Z}\frac{\partial Z}{\partial p}$$

$$c_g = \frac{1}{p} - \frac{1}{Z}\left(\frac{\partial Z}{\partial p}\right)_T \qquad eq.(1.15)$$

Equation (1.15) is the isothermal compressibility factor for real gas.

From the law of corresponding state, we know that:

$$p_{pr} = \frac{p}{p_{pc}} \Rightarrow p = p_{pr}p_{pc}$$

By substituting $p = p_{pr}p_{pc}$ from the law of corresponding state into Eq. (1.15):

$$c_g = \frac{1}{p_{pr}p_{pc}} - \frac{1}{z}\left[\frac{\partial Z}{\partial\left(p_{pr}p_{pc}\right)}\right]_{T_{pr}}$$

Multiplying this equation by p_{pc} yields:

$$c_g p_{pc} = c_{pr} = \frac{1}{p_{pr}} - \frac{1}{Z}\left[\frac{\partial Z}{\partial p_{pr}}\right]_{T_{pr}} \qquad eq.(1.16)$$

The term c_{pr} is called the isothermal pseudoreduced compressibility, defined by the relationship:

$$c_{pr} = c_g p_{pc} \qquad eq.(1.17)$$

where:

c_{pr} = isothermal pseudoreduced compressibility.
c_g = isothermal gas compressibility, psi^{-1}.
p_{pc} = pseudoreduced pressure, psi.

6 Gas Formation Volume Factor, B_g

$$B_g = \frac{(V)_{p,T}}{V_{sc}} \qquad eq.(1.18)$$

where:

B_g = gas formation volume factor, ft^3/scf.
$V_{p,T}$ = volume of gas at pressure p and temperature T, ft^3.
V_{sc} = volume of gas at standard conditions.

Applying the real gas equation of state, (pV=ZnRT) and substituting for the volume V, gives:

$$B_g = \frac{\frac{ZnRT}{p}}{\frac{Z_{sc}nRT_{sc}}{p_{sc}}} = \frac{p_{sc}ZT}{T_{sc}p}$$

where:

Z_{sc} = Z-factor at standard conditions is equal to (1.0).
p_{sc}, T_{sc} = standard pressure and temperature.

Assuming that the standard conditions are represented by p_{sc}=14.7 psia and T_{sc}=520, the preceding expression can be reduced to the following relationship:

$$B_g = 0.02827\frac{ZT}{p}\ (ft^3/scf) \qquad eq.(1.19)$$

where:

B_g = gas formation volume factor, ft^3/scf.
Z = gas compressibility factor.
T = temperature, °R.

In other field units, the gas formation volume factor can be expressed in *bbl/scf*, by dividing it on 5.614 ft^3/ bbl to give:

$$B_g = 0.005035\frac{ZT}{p}\ (bbl/scf) \qquad eq.(1.20)$$

And also:

$$c_g = \frac{-1}{B_g}\left(\frac{\partial B_g}{\partial p}\right)_T \qquad eq.(1.21)$$

7 Gas Expansion Factor, E_g

$$E_g = \frac{1}{B_g}$$

In terms of *scf/ft*^{3}, the gas expansion factor is:

$$E_g = 35.37\frac{p}{ZT}, scf/ft^3 \qquad eq.(1.22)$$

In other units:

$$E_g = 198.6\frac{p}{ZT}, scf/bbl \qquad eq.(1.23)$$

8 Gas Viscosity, μ_g:

1 centipoise = g mass / 100 sec cm

$$\begin{aligned} 1\,poise &= 100\ centipoises \\ &= 1 \times 10^6\ micropoises \\ &= 6.72 \times 10^{-2}\ \ (lb.mass/ft.sec) \\ &= 20.9 \times 10^{-3}\ \ (lb.sec/ft^2) \end{aligned}$$

❖ Carr-Kobayashi-Burrows's Method

$$\mu_1 = (\mu_1)_{uncorrected} + (\Delta\mu)_{N_2} + (\Delta\mu)_{CO_2} + (\Delta\mu)_{H_2S} \qquad eq.(1.24)$$

where:

μ_1 = “corrected” gas viscosity at 1 atm and reservoir temperature, cp.
$(\Delta\mu)_{N_2}$ = viscosity corrections due to the presence of N_2.
$(\Delta\mu)_{CO_2}$= viscosity corrections due to the presence of CO_2.
$(\Delta\mu)_{H_2S}$ = viscosity corrections due to the presence of H_2S.
$(\mu_1)_{\text{uncorrected}}$ = uncorrected gas viscosity, cp.

❖ Lee-Gonzalez-Eakin's Method

$$\mu_g = 10^{-4}K exp\left[X\left(\frac{\rho_g}{62.4}\right)^Y\right] \qquad eq.(1.25)$$

$$K = \frac{(9.4 + 0.02M_a)T^{1.5}}{209 + 19M_a + T} \qquad eq.(1.26)$$

$$X = 3.5 + \frac{986}{T} + 0.01M_a \qquad eq.(1.27)$$

$$Y = 2.4 - 0.2X \qquad eq.(1.28)$$

Where:

ρ = gas density at reservoir pressure and temperature, *lb/ft*^{3}.
T = reservoir temperature, °R.
M_a = apparent molecular weight of the gas mixture.

1.1.2 Crude Oil Properties

Petroleum (an equivalent term is crude oil) is a complex mixture consisting predominantly of hydrocarbons and containing sulfur, nitrogen, oxygen, and helium as minor constituents. The physical and chemical properties of crude oils vary considerably and depend on the concentration of the various types of hydrocarbons and minor constituents present.

An accurate description of physical properties of crude oils is of a considerable importance in the fields of both applied and theoretical science and especially in the solution of petroleum reservoir engineering problems. Physical properties of primary interest in petroleum engineering studies include:

- Crude oil gravity "γ_o".
- Specific gravity of solution gas "γ_g".
- Crude oil density "ρ_o".
- Gas solubility “R_s”.
- Bubble point pressure “p_b”.
- Oil formation volume factor “B_o”.
- Isothermal compressibility coefficient of crude oil “c_o”.
- Total formation volume factor “B_t”.
- Crude oil viscosity.

Data on most of these fluid properties is usually determined by laboratory experiments performed on samples of actual reservoir fluids. In the absence of experimentally measured properties of crude oils, it is necessary for the petroleum engineer to determine the properties from empirically derived correlations. The following equations represent these concepts:

1 Crude Oil API Gravity

$$\gamma_o = \frac{\rho_o}{\rho_w} \qquad eq.(1.29)$$

where:

γ_o = specific gravity of the oil.
ρ_o = density of the crude oil, *lb/ft*^{3}.
ρ_w = density of the water, *lb/ft*^{3}.

$$\gamma_o = \frac{\rho_o}{62.4}, 60°/60°$$

$$API = \frac{141.5}{\gamma_o} - 131.5 \qquad eq.(1.30)$$

2 Specific Gravity of the Solution Gas, "γ_g"

$$\gamma_g = \frac{\sum_{i=1}^{n}(R_{sep})_i(\gamma_{sep})_i + R_{st}\gamma_{st}}{\sum_{i=1}^{n}(R_{sep})_i + R_{st}} \qquad eq.(1.31)$$

where:

n = number of separators.
R_{sep} = separator GOR, scf/STB.
γ_{sep} = separator gas gravity.
R_{st} = GOR from the stock-tank, scf/STB.
γ_{st} = gas gravity from the stock-tank.

3 Crude Oil Density, "ρ_o":

I- Density Correlations Based on the Oil Composition.

II- Density Correlations Based on Limited PVT Data.

III- Standing's Method for Determining "ρ_o":

$$\rho_o = \frac{62.4\gamma_o + 0.0136R_s\gamma_g}{Bo}$$

$$\rho_o = \frac{62.4\gamma_o + 0.0136R_s\gamma_g}{0.9759 + 0.000120\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25(T-460)\right]^{1.2}} \qquad eq.(1.32)$$

where:

T = system temperature, °R.
γ_o = specific gravity of the stock-tank oil, 60°/60°.
γ_g = specific gravity of the gas.
R_s = gas solubility, scf/STB.
ρ_o = oil density, lb/ft^3.
Bo=oil formation volume factor, bbl/STB

4 Gas Solubility "R_s":

I- Standing's Correlation for Determining "R_s"

As shown in Figure (1.1).

$$R_s = \gamma_g \left[\left(\frac{p}{18.2} + 1.4\right) 10^x\right]^{1.2048} \qquad eq.(1.33)$$

with:

$$x = 0.0125API - 0.00091(T - 460) \qquad eq.(1.34)$$

where:

R_s = gas solubility, scf/STB.
T = temperature, °R.
p = system pressure, psia.
γ_g = solution gas specific gravity.
API = oil gravity, °API.

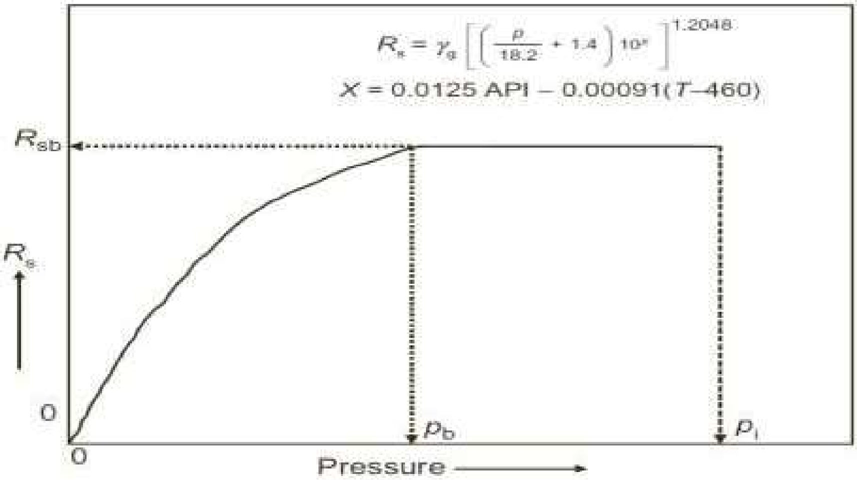

Figure (1.1): Typical gas solubility-pressure relationship.

5 Bubble Point Pressure "p_b":

$$p_b = f(R_s, \gamma_g, API, T)$$

I- Standing's Correlation for Determining "p_b"

$$p_b = 18.2\left[\left(R_s/\gamma_g\right)^{0.83}(10)^a - 1.4\right] \qquad eq.(1.35)$$

with:

$$a = 0.00091(T - 460) - 0.0125(API) \qquad eq.(1.36)$$

where:

R_s = gas solubility, scf/STB.
p_b = bubble point pressure, psia.
T = system temperature, °R.

6 Oil Formation Volume Factor "B_o"

$$B_o = \frac{(V_o)_{p,T}}{(V_o)_{sc}} \qquad eq.(1.37)$$

where:

B_o = oil FVF, bbl/STB.

$(V_o)_{p,T}$ = volume of oil, in *bbl*, under reservoir pressure, *p*, and temperature, *T*.

$(V_o)_{sc}$ = volume of oil is measured under SC, STB.

$$B_o = f\left(R_s, \gamma_g, \gamma_o, T\right)$$

I. Standing's Correlation for Determining "B_o"

$$B_o = 0.9759 + 0.000120\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25(T - 460)\right]^{1.2} \qquad eq.(1.38)$$

where:

T = temperature, °R.
γ_o = specific gravity of the stock-tank oil, 60°/60°.
γ_g = specific gravity of the solution gas.

As shown in Figure (1.2).

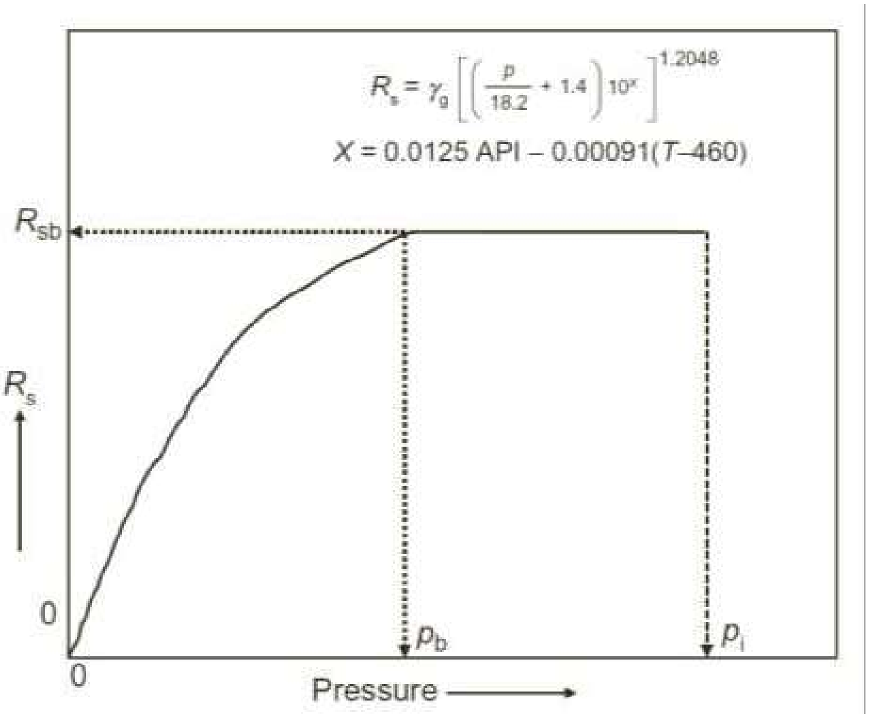

Figure (1.2): Typical oil formation volume factor-pressure relationship.

7 Isothermal Compressibility Coefficient of Crude Oil "c_o":

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

$$c_o = -\frac{1}{V}\left(\frac{V_1 - V_2}{p_1 - p_2}\right)_T$$

I. Standing's Correlation for Determining "c_o"

$$c_o = 10^{-6} exp\left[\frac{\rho_{ob} + 0.004347(p - p_b) - 79.1}{0.0007141(p - p_b) - 12.938}\right] \qquad eq.(1.39)$$

where:

ρ_{ob} = oil density at the bubble point pressure, *lb/ft*^{3}.
p_b = bubble point pressure, psia.
c_o = oil compressibility, psia^{-1}.

II. The Vasquez-Beggs Correlation

$$c_o = \frac{-1433 + 5R_{sb}17.2(T - 460) - 1180\gamma_{gs} + 12.61°API}{10^5 p} \qquad eq.(1.40)$$

where:

T = temperature, °R
p = pressure above the bubble-point pressure, psia.
R_{sb} = gas solubility at the bubble-point pressure.
γ_{gs} = corrected gas gravity.

See (Ahmed T., 2019) for more details.

8 Total Formation Volume Factor "B_t"

$$B_t = \frac{(V_o)_{p,T} + \left(V_g\right)_{p,T}}{(V_o)_{sc}} \qquad eq.(1.41)$$

where:

B_t = total FVF, bbl/STB.
$(V_o)_{p,T}$ = volume of the oil at p and T, bbl.
$(V_g)_{p,T}$ = volume of the liberated gas at p and T, bbl.
$(V_o)_{sc}$ = volume of the oil at SC, STB.

$$B_t = B_o + (R_{sb} - R_s)B_g \qquad eq.(1.42)$$

where:

R_{sb} = gas solubility at the bubble point pressure, scf/STB.
R_s = gas solubility at any pressure, scf/STB.
B_o = oil FVF at any pressure, bbl/STB.
B_g = gas FVF, bbl/scf.

As shown in Figure (1.3).

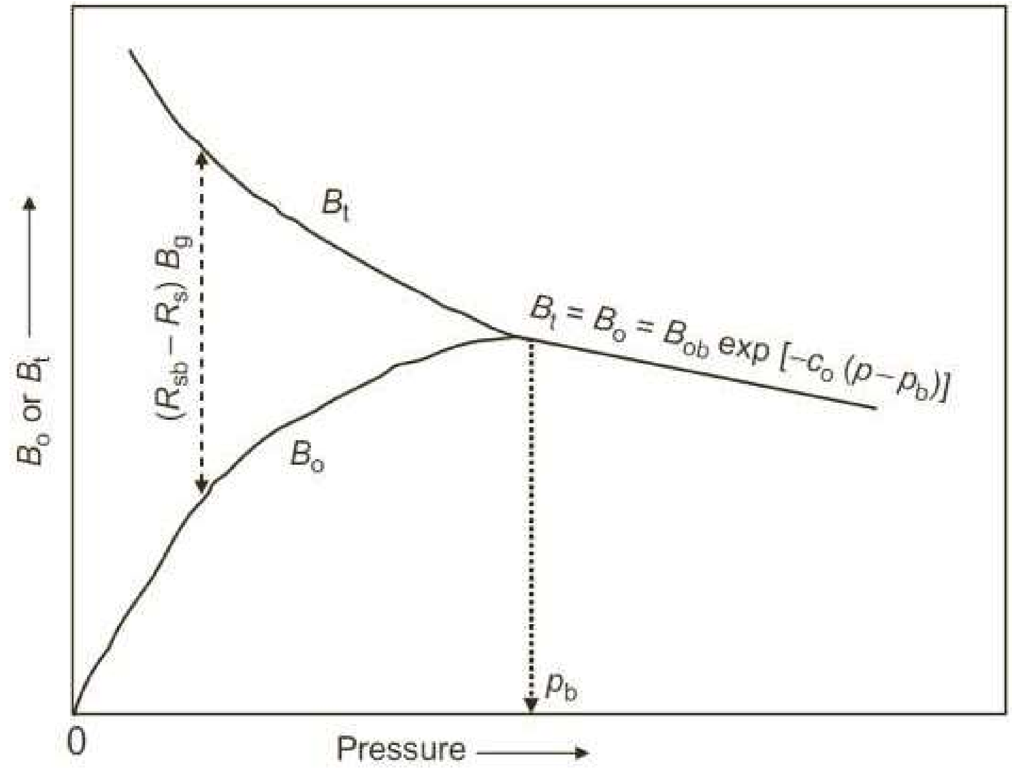

Figure (1.3): B_o and B_t vs. pressure.

9 Crude Oil Viscosity "μ_o":

1 Glaso's Correlation for Determining "μ_{od}"

$$\mu_{od} = [3.141(10^{10})](T-460)^{-3.444}[log(API)]^{A} \qquad eq.(1.43)$$

$$A = 10.313[log(T-460)] - 36.447 \qquad eq.(1.44)$$

2 Abu-Khamsin and Al-Marhoun Correlation for Determining "μ_{ob}"

$$\ln(\mu_{ob}) = 8.484462\rho_{ob}^{4} - 2.652294 \qquad eq.(1.45)$$

Where:

the saturated oil density, ρ_{ob}, is expressed in g/cm^3; that is, $\rho_{ob}/62.4$

3 Khan's Correlation for Determining undersaturated Oil Viscosity "μ_o"

$$\mu_o = \mu_{ob} exp[9.6(10^{-6})(p-p_b)] \qquad eq.(1.46)$$

1.2 Reservoir Rock Properties

Rock properties are determined by performing laboratory analysis on cores from the reservoir. Basically, two main categories of core analysis tests are performed on core samples regarding physical properties of reservoir rocks. These are:

Routine Core Analysis (RCAL):

- Porosity.
- Saturation.
- Permeability.

Special Core Analysis (SCAL):

- Overburden pressure (Chapter 6)
- Surface and interfacial tension.
- Wettability.
- Capillary pressure.
- Relative permeability.

  These rock properties data are essential for reservoir engineering calculations as they directly affect both the quantity and the distribution of hydrocarbons and, when combined with fluid properties, control the flow of the existing phases (i.e., gas, oil, and water) within the reservoir. The mathematical expressions define these properties are as follows:

1 Porosity:

$$\phi = \frac{pore\ volume}{bulk\ volume} = \frac{V_p}{V_b} = \frac{V_b - V_g}{V_b} \qquad eq.(1.47)$$

where:

$\phi =$ fractional porosity.
V_p = pore volume.
V_b= bulk volume.
V_g = grain volume.

2 Fluid Saturation

$$fluid\ saturation = \frac{total\ volume\ of\ fluid}{pore\ volume}$$

$$S_o = \frac{volume\ of\ oil}{pore\ volume} \qquad eq.(1.48)$$

$$S_g = \frac{volume\ of\ gas}{pore\ volume} \qquad eq.(1.49)$$

$$S_w = \frac{volume\ of\ water}{pore\ volume} \qquad eq.(1.50)$$

where:

S_o = oil saturation.
S_g = gas saturation.
S_w = water saturation.

$$S_o + S_g + S_w = 1.0 \qquad eq.(1.51)$$

3 Permeability:

i. Steady-state linear flow

$$q = \frac{0.001127kA}{\mu B_o}\frac{(p_1 - p_2)}{L} \qquad eq.(1.52)$$

where:

q = volumetric flowrate, STB/Day.
k = absolute permeability of the rock, millidarcy (md).
A = cross sectional area in the flow direction, ft^2.
p_1 = inlet pressure, psig.
p_2 = outlet pressure, psig.
μ = fluid viscosity, cp.
L = length of the rock, ft.
B_o = oil formation volume factor, bbl/STB.

ii. Radial flow of fluids

$$q = \frac{0.00708kh(p_e - p_w)}{\mu B_o \ln\left(\frac{r_e}{r_w}\right)} \qquad eq.(1.53)$$

where:

q = volumetric flowrate, STB/Day.
k = absolute permeability of the rock, millidarcy (md).

h = pay thickness, *ft*.
p_e= pressure at external radius, psig.
p_w = pressure at wellbore, psig.
μ = fluid viscosity, cp.
L = length of the rock, *ft*.
B_o = oil formation volume factor, bbl/STB.
r_e = external drainage area, *ft*.
r_w = wellbore radius, *ft*.
ln = natural logarithm.

4 Relative Permeability

$$k_{ro} = \frac{k_o}{k}$$

$$k_{rg} = \frac{k_g}{k}$$

$$k_{rw} = \frac{k_w}{k}$$

where:

k_{ro} = relative permeability to oil.
k_{rg} = relative permeability to gas.
k_{rw} = relative permeability to water.
k = absolute permeability.
k_o = effective permeability to oil for a given oil saturation.
k_g = effective permeability to gas for a given gas saturation.
k_w = effective permeability to water at some given water saturation.

5 Surface and Interfacial Tensions

$$\sigma_{gw} = \frac{rh\rho_w g}{2\cos\theta} \qquad eq.(1.54)$$

where:

σ_{gw} = surface tension between air (gas) and water (oil), dynes/cm.
r = radius, cm.
h = height to which the liquid is held, cm.
ρ_w = density of water, gm/cm^3.
θ = contact angle.

g = acceleration due to gravity, cm/sec^2.

$$\sigma_{ow} = \frac{rhg(\rho_w - \rho_o)}{2\cos\theta} \qquad eq. (1.55)$$

where:

ρ_o = density of oil, gm/cm^3.
σ_{ow} = interfacial tension between the oil and the water, dynes/cm.

6 Wettability

$$A_T = \sigma_{so} - \sigma_{sw} \qquad eq. (1.56)$$

where:

A_T = *adhesion tension.*
σ_{so} = the interfacial tension between the solid and the lighter fluid phase (oil in this case).
σ_{sw} = the interfacial tension between the solid and the denser fluid phase (water in this case).

7 Capillary pressure

$$p_c = p_{nw} - p_w \qquad eq. (1.57)$$

where:

p_c = capillary pressure, *psi (lb/in^2) = lb/144 ft^2; 1 ft=12 in.*
p_{nw} = pressure of the nonwetting phase.
p_w = pressure of the wetting phase.

$$p_c = \left(\frac{h}{144}\right)\Delta\rho \qquad eq. (1.58)$$

where:

h = capillary rise, *ft*.
$\Delta\rho$ = density difference, *lb/ft^3*.

Chapter 2

Petroleum Reservoir Recovery Mechanisms and Reserve Estimation Methods

2.1 Oil Recovery Methods

Oil recovery methods are illustrated in Table (2.1) below. **Primary recovery**, production depends on the natural energy of the reservoir itself. **Secondary recovery**, when natural drive energy is depleted or too small for economic oil recovery, energy must be added to the reservoir to permit additional oil recovery. That additional energy is usually in the form of injected water or gas. **Tertiary recovery**, when secondary recovery is no longer economic, supplemental energy of a different kind permits additional oil recovery. Enhanced fluid flow conditions within the reservoir are usually induced by addition of heat, chemical interaction between the injected fluid and the reservoir oil, mass transfer, and/or changing of oil properties in such a way that the process facilitates oil movement through the reservoir. Tertiary recovery processes generally include thermal, chemical, gas miscible and microbial. They are also often referred to as **enhanced oil recovery (EOR)** processes.

Improved oil recovery (IOR) refers to any practice used to increase oil recovery. This can include EOR and secondary recovery processes such as waterflooding and gas pressure maintenance, as well as practices to increase sweep such as infill drilling, horizontal wells and polymers for mobility control or improved conformance.

Table 2.1: oil recovery methods.

Primary Recovery (oil recovery less than 30%)			Natural flow	- Rock and liquid expansion drive. - Depletion drive. - Gas-cap drive. - Water drive. - Gravity drainage drive. - Combination drive.
			Artificial lift	- Pump - Gas lift - Other
Improved Oil Recovery - IOR	**Secondary Oil Recovery** (oil recovery: 30-50%)		Water flood	
			Pressure Maintenance	Water/Gas Reinjection
	Enhanced Oil Recovery - EOR	**Tertiary Oil Recovery** (oil recovery more than 50% and up to 80%)	Thermal	- Combustion - Steam soak/cyclic - Huff-and-puff - Steam drive/flood - Hot water drive - Electromagnetic
			Gas miscible/ immiscible	- CO_2 - Nitrogen - Flue gas - Hydrocarbon
			Chemical & other	- Alkaline - Micellar-Polymer - Microbial/foam

2.2 Reservoir Primary Recovery Mechanisms

The recovery of oil by any of the natural drive mechanisms is called primary recovery. The term refers to the production of hydrocarbons from a reservoir without the use of any process (such as fluid injection) to supplement the natural energy of the reservoir.

For a proper understanding of reservoir behavior and predicting future performance, it is necessary to have knowledge of the driving mechanisms that control the behavior of fluids within reservoirs. The overall performance of oil reservoirs is largely determined by the nature of the energy, i.e., driving mechanism, available for moving the oil to the wellbore. There are basically six driving mechanisms that provide the natural energy necessary for oil recovery:

- Rock and liquid expansion drive.
- Depletion drive.
- Gas-cap drive.
- Water drive.
- Gravity drainage drive.
- Combination drive.

2.2.1 Rock and Liquid Expansion

When an oil reservoir initially exists at a pressure higher than its bubble-point pressure, the reservoir is called an undersaturated-oil reservoir. At pressures above the bubble-point pressure, crude oil, connate- water, and rock are the only materials present. As the reservoir pressure declines, the rock and fluids expand due to their individual compressibilities. The reservoir rock compressibility is the result of two factors:

- Expansion of the individual rock grains.
- Formation compaction.

As the expansion of the fluids and reduction in the pore volume occur with decreasing reservoir pressure, the crude oil and water will be forced out of the pore space to the wellbore. Because liquids and rocks are only slightly compressible, the reservoir will experience a rapid pressure decline. The oil reservoir under this driving mechanism is characterized by a constant gas-oil ratio that is equal to the gas solubility at the bubble point pressure.

This driving mechanism is considered the least efficient driving force and usually results in the recovery of only a small percentage of the total oil-in-place.

2.2.2The Depletion-Drive Mechanism

This driving form may also be referred to by the following various terms:

- Solution gas drive.
- Dissolved gas drive.
- Internal gas drive.

In this type of reservoir, the principal source of energy is a result of gas liberation from the crude oil and the subsequent expansion of the solution gas as the reservoir pressure is reduced. As pressure falls below the bubble-point pressure, gas bubbles are liberated within the microscopic pore spaces. These bubbles expand and force the crude oil out of the pore space, see figure (2.2).

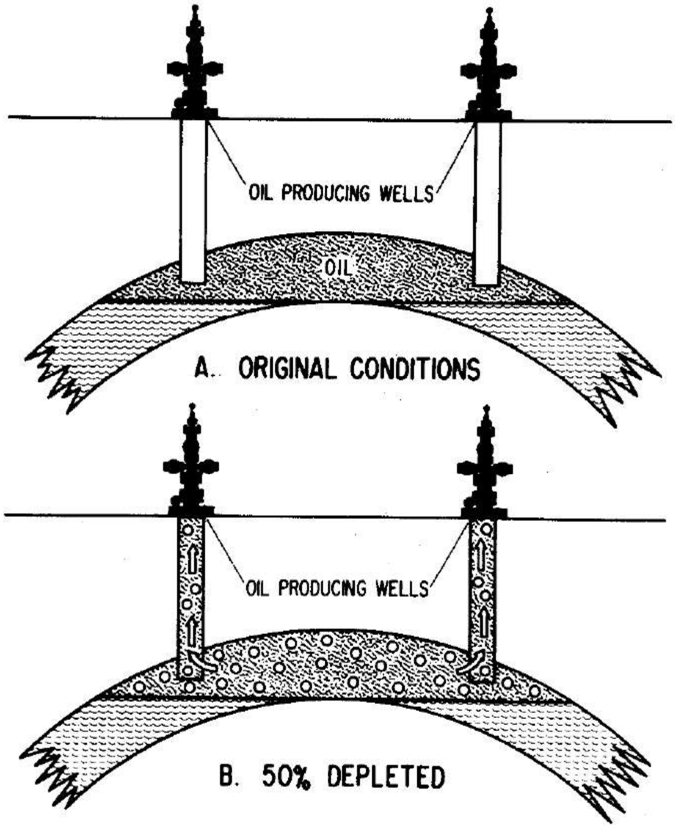

Figure (2.2): Dissolved gas drive reservoir, (Clark, 1969).

2.2.3 Gas-Cap Drive

Gas-cap-drive reservoirs can be identified by the presence of a gas cap with little or no water drive. Due to the ability of the gas cap to expand, these reservoirs are characterized by a slow decline in the reservoir pressure. The natural energy available to produce the crude oil comes from the following two sources:

- Expansion of the gas-cap gas.
- Expansion of the solution gas as it is liberated, see figure (2.3).

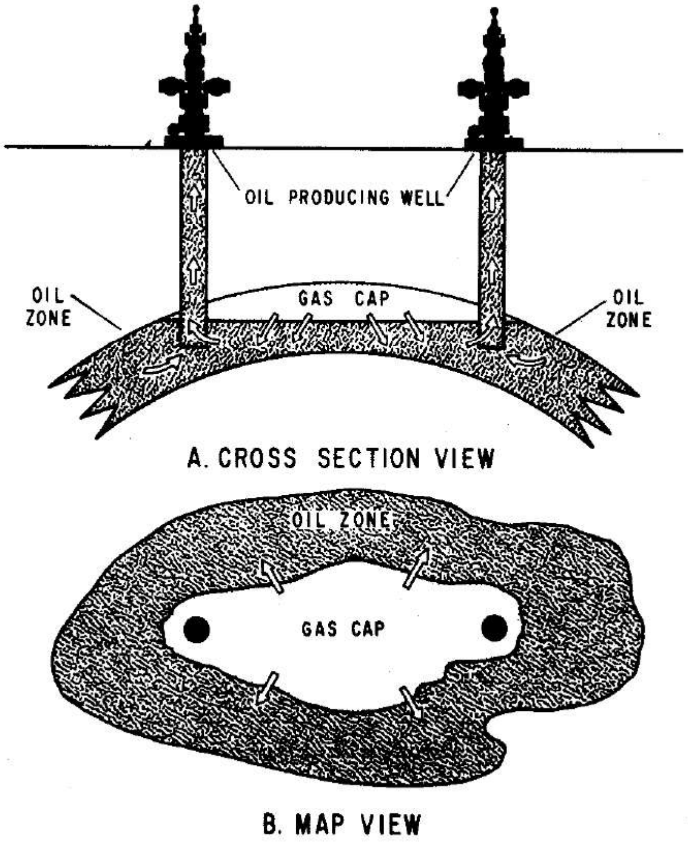

Figure (2.3): Gas cap drive reservoir, (Clark, 1969).

2.2.4 The Water-Drive Mechanism

Many reservoirs are bounded on a portion or all of their peripheries by water bearing rocks called aquifers. Although water is considered incompressible, the total compressed volume is quite large when such great quantities of total water volume are involved. Even the great volume of rock in which the water exist is influenced by water pressure. As oil is produced, pressure declines at the point where oil is withdrawn from the reservoir. Water then moves in to replace the oil as it is produced because of expansion of the minutely compressed water; a reservoir producing in this manner is termed a ***water drive reservoir*** (Figure 2.4). Many of the most important reservoir in the world are producing by energy supplied by water drives.

Notable examples are the east Texas field; Alberta in Canada; Burgan field in Kuwait.

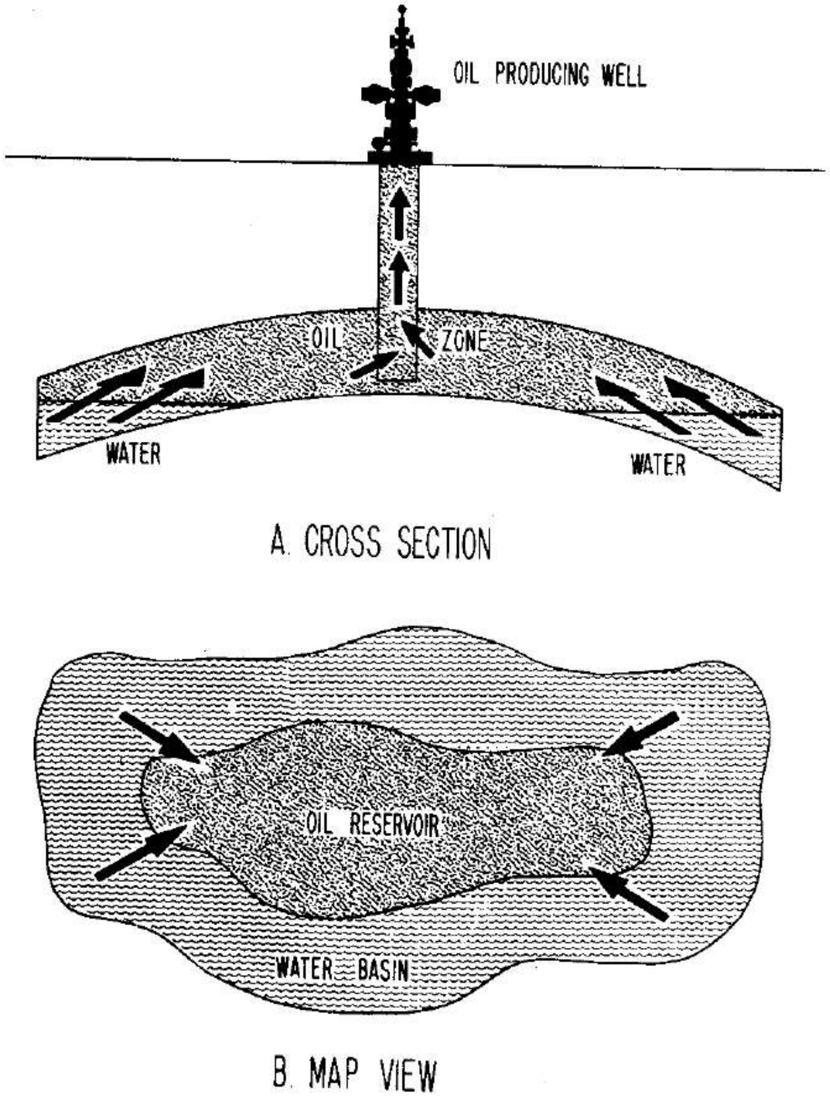

Figure (2.4): Water drive reservoir, (Clark, 1969).

2.2.5 The Gravity-Drainage-Drive Mechanism

The mechanism of gravity drainage occurs in petroleum reservoirs as a result of differences in densities of the reservoir fluids. The effects of gravitational forces can be simply illustrated by placing a quantity of crude oil and a quantity of water in a jar and agitating the contents. After agitation, the jar is placed at rest, and the denser fluid (normally water) will settle to the bottom of the jar, while the less dense fluid (normally oil) will rest on top of the denser fluid. The fluids have separated as a result of the gravitational forces acting on them.

The fluids in petroleum reservoirs have all been subjected to the forces of gravity, as evidenced by the relative positions of the fluids, i.e., gas on top, oil underlying the gas, and water underlying oil. Due to the long periods of time involved in the petroleum accumulation-and-migration process, it is generally assumed that the reservoir fluids are in equilibrium. If the reservoir fluids are in equilibrium, then the gas-oil and oil-water contacts should be essentially horizontal. Although it is difficult to determine precisely the reservoir fluid contacts, best available data indicate that, in most reservoirs, the fluid contacts actually are essentially horizontal.

Gravity segregation of fluids is probably present to some degree in all petroleum reservoirs, but it may contribute substantially to oil production in some reservoirs, see Figure (2.5).

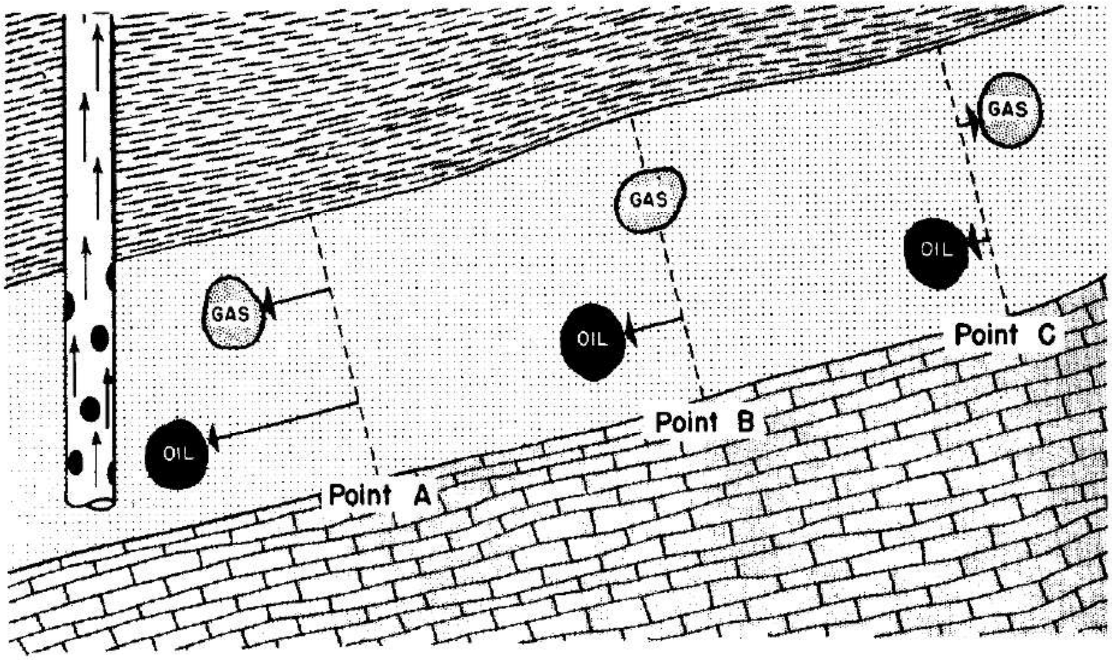

Figure (2.5): Gravitational segregation showing relative movement of gas and oil along the structure at various distance from a producing well, (Clark, 1969).

2.2.6 The Combination-Drive Mechanism

Reservoirs are seldom found to fit exactly one type of drive classification. The most common type of drive encountered, is a combination drive see Figure (2.6). Production problems are exceedingly complicated because of the infinite number of combinations characterizing the various reservoirs occurring naturally.

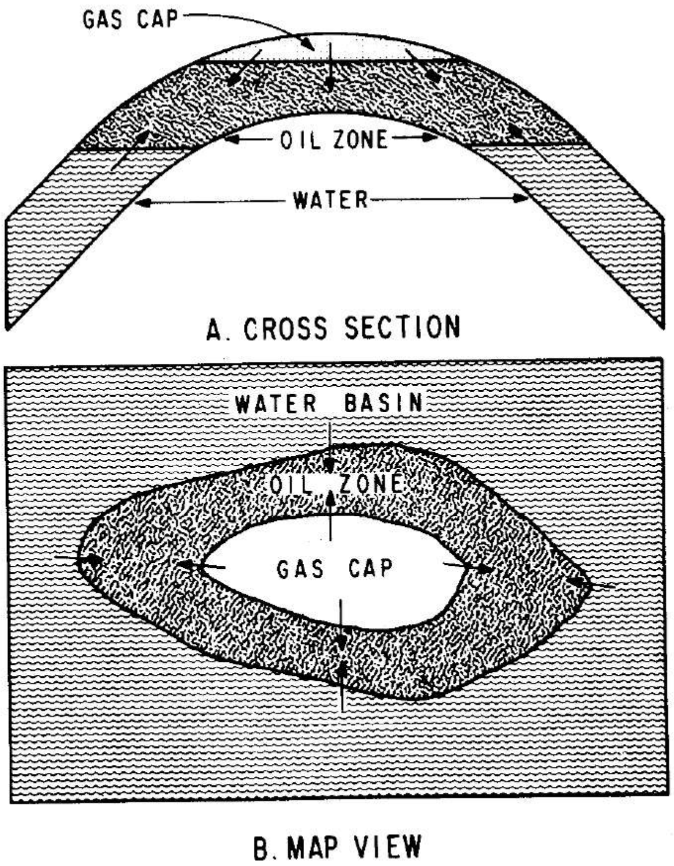

Figure (2.6): Combination drive reservoir, (Clark, 1969).

2.3 Reserve Estimation Methods

The term "*reserves*" means different things to different subjects. To the oil and gas industry, reserves are the amount of crude oil, natural gas, and associated substances that can be produced profitably in the future from subsurface reservoirs.

Estimates of oil and/or gas reserves are inherently uncertain. The degree of uncertainty in estimates of reserves depends mainly on:

- The degree of geologic complexity.
- Maturity of the property.
- The quality and quantity of geologic and engineering data.
- The operating environment.
- The skill, experience and integrity of the estimators.

Over the years, there have been numerous classifications of oil and/or gas reserves published by various individuals and organizations. The principle purposes of a classification system are: (1) to promote uniformity and (2) to qualify the degree of risk for each class of reserve.

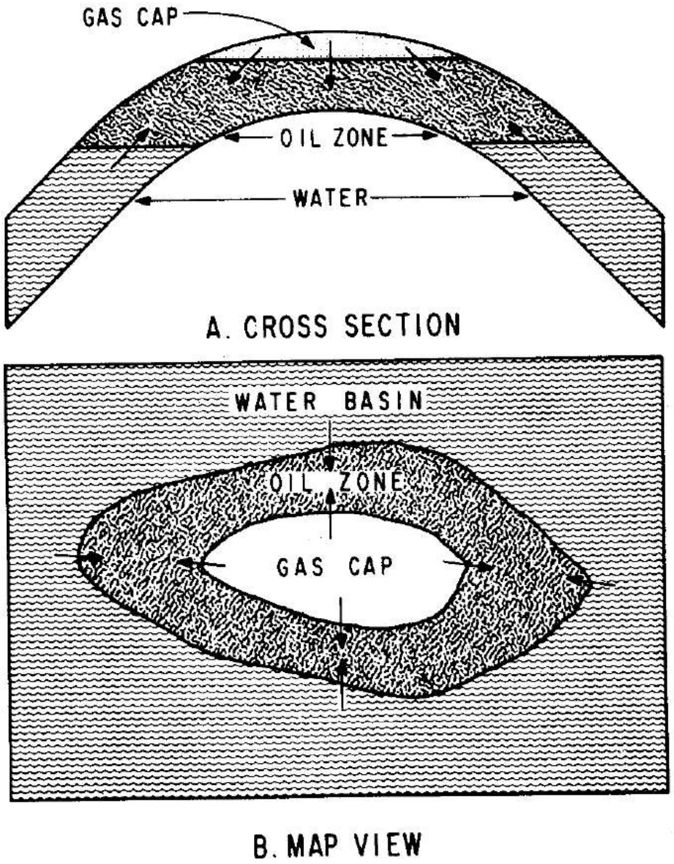

Figure (2.6): Combination drive reservoir, (Clark, 1969).

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Procedures to estimate and classify reserves (ECR) have been described as ***deterministic*** or ***probabilistic***. These procedures to estimate reserves, or a part of such reserves, may be classified as:

1. ***Proved reserves*** (with certainty of 90%).
2. Or ***unproved reserves:*** this type of reserves divided into two categories: ***probable reserves*** (with certainty of 50%) or ***possible reserves*** (with certainty at least of 10%) based on the engineer's judgment and relevant guidelines regarding the probability of actually producing such reserves.

While it has been recognized that there is always some degree of uncertainty in estimating reserves, most of the published literature on reserve estimating has focused on deterministic methods. Probabilistic calculations, however, have received continuing attention in the geologic literature in the evaluation of exploration "plays".

2.3.1 Reserve Status Categories

Reserve status categories define the development and producing status of wells and reservoirs.

Developed: Developed reserves are expected to be recovered from existing wells including reserves behind pipe. Improved recovery reserves are considered developed only after the necessary equipment has been installed, or when the costs to do so are relatively minor. Developed reserves may be subcategorized as producing or non-producing.

Producing: Reserves subcategorized as producing are expected to be recovered from completion intervals which are open and producing at the time of the estimate. Improved recovery reserves are considered producing only after the improved recovery project is in operation.

Non-producing: Reserves subcategorized as non-producing include shut-in and behind-pipe reserves. Shut-in reserves are expected to be recovered from (1) completion intervals which are open at the time of the estimate but which have not started producing, (2) wells which were shut-in for market conditions or pipeline connections, or (3) wells not capable of production for mechanical reasons. Behind-pipe reserves are expected to be recovered from zones in existing wells, which will require additional completion work or future recompletion prior to the start of production.

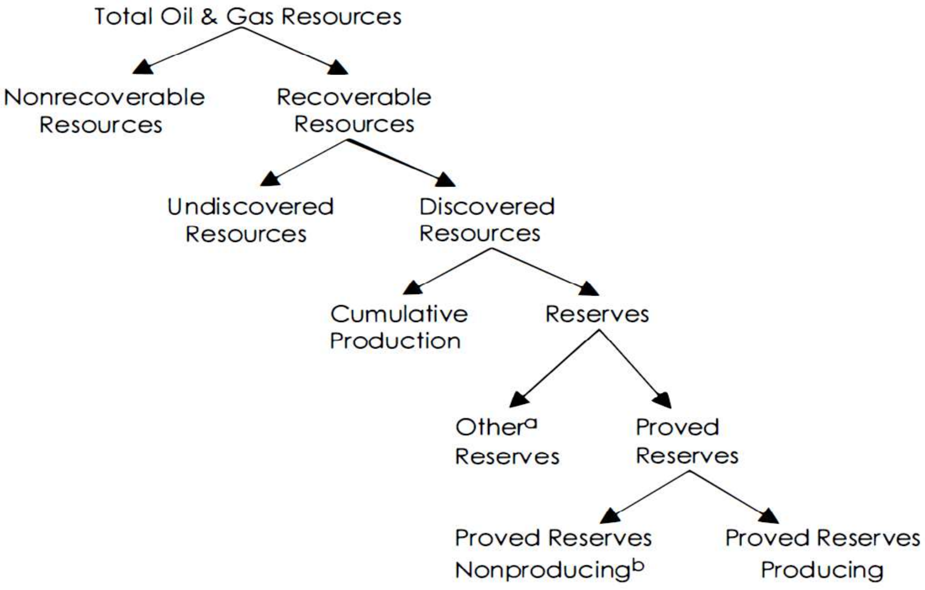
Undeveloped Reserves: Undeveloped reserves are expected to be recovered: (1) from new wells on undrilled acreage, (2) from deepening existing wells to a different reservoir, or (3) where a relatively large expenditure is required to (a) recomplete an existing well or (b) install production or transportation facilities for primary or improved recovery projects.

Terminology for oil and gas resources classification is illustrated by Figure (2.1).

2.3.2 Reserve Estimation Methods

Reserve estimation methods is discussed briefly in this chapter. Which may be classified as:

- Analogical Methods.
- Volumetric Methods.
- Performance Methods.
    - Material Balance.
    - Computer Simulation.
    - Performance/Decline Trend Analysis.



a. In reporting to EIA, "other" reserves include only "indicated additional" reserves that do not meet the criteria necessary to be classified as proved.

b. Nonproducing reserves include those that did not produce during the prior reporting year.

Figure (2.1): Terminology for oil and gas resources.

2.4 Volumetric Methods

The volumetric method involves calculating: (a) the amount of oil and gas initially in place by a combination of volumetric (geologic) mapping, petrophysical analysis, and reservoir engineering and (b) the fractions of oil, gas, and associated products initially in place that are expected to be recovered commercially-i.e., the recovery efficiencies-using analytical methods and/or analogy.

Procedures to estimate oil and gas initially in place and recovery efficiency are discussed in the following sections.

2.4.1 Oil Reservoirs

For an oil reservoir, or for the oil column of an oil reservoir with a gas cap, oil initially in place (OIIP) may be calculated as:

$$OIIP = N_i = \frac{7758\phi_o(1 - S_{wo})A_o h_{no}}{B_{oi}} \qquad eq.(2.1)$$

where:

N_i = oil initially in place, STB.
7758 = (unit conversion constant) barrels in an acre foot (1 acre=4046.85642 m^2 ≅4047 m^2).
ϕ_o = average porosity in the oil zone, fraction.
S_{wo} = average water saturation in the oil zone, fraction.
A_o = area of the oil zone, acres.
h_{no} = average net oil pay, feet.
B_{oi} = average initial formation volume factor, bbl/STB.

If the units for area and net pay are hectares and meters, respectively, the constant in Eq. (2.1) becomes 1.0, and the units for OIP are (10^4 m^3).

Porosity, water saturation, and oil formation volume factor should be *volume-weighted averages* in the *oil zone*.

Solution gas dissolved in the oil at initial reservoir conditions may be calculated as:

$$G_{Si} = N_i R_{si} \qquad eq.(2.2)$$

where:

G_{Si} = solution gas initially in place, scf.
R_{si} = average initial solution gas/oil ratio, scf/STB.

In some areas, GOR units are cubic meters of gas per cubic meter of oil (m^3/m^3). The conversion factor is (5.614 scf/STB).

Example 2.1:

An oil reservoir exists at its bubble-point pressure of (3,000 psia) and temperature of (160°F). The oil has an API gravity of (42°) and gas-oil ratio of (600 scf/STB).

The specific gravity of the solution gas is (0.65). The following additional data are also available:

- Reservoir area = 640 acres.
- Average thickness = 10 *ft*.
- Connate water saturation = 0.25
- Effective porosity = 15%.

Calculate the initial oil in place in STB.

Solution:

Step 1. Determine the specific gravity of the stock-tank oil from **Lee-Gonzalez-Eakin's Method (**Read Fundamentals of Petroleum Reservoir Engineering NAJAR & AL-NASER. Pag.60**).**

$$API = \frac{141.5}{\gamma_o} - 131.5$$

$$\gamma_o = \frac{141.5}{42 + 131.5} = 0.8156$$

Step 2. Calculate the initial oil formation volume factor by applying Standing's equation, i.e., Equation (1.38), to give:

$$B_o = 0.9759 + 0.000120\left[600\left(\frac{0.65}{0.8156}\right)^{0.5} + 1.25(160)\right]^{1.2} = 1.396\ bbl/STB$$

Step 3. Calculate the initial oil in place by using equation (2.1):

$$OIIP = N_i = \frac{7758(0.15)(1 - 0.25)(640)(10)}{1.396} = 4001260.74\,STB$$

2.4.2 Gas Reservoirs

For a non-associated gas reservoir, or for a gas cap, free gas initially in place may be calculated as:

$$G_{Fi} = \frac{43560\phi_g(1 - S_{wg})A_g h_{ng}}{B_{gi}} \qquad eq.(2.3)$$

where:

G_{Fi} = free gas initially in place, scf.
43560 = (unit conversion constant) cubic feet in an acre foot.
ϕ_g = average porosity in the free gas zone, fraction.
S_{wg} = average water saturation in the free gas zone, fraction.
A_g = area of gas cap or gas reservoir, acres.

h_{ng} = average net thickness of gas cap or gas reservoir, feet.
B_{gi} = average initial formation volume factor, *ft³/scf.*

Condensate (also called distillate) in the vapor phase at initial reservoir conditions (but measured as a liquid at surface conditions) may be calculated as:

$$C_i = C_{Fi} R_{ci} \qquad eq.(2.4)$$

where:

C_i = condensate (distillate) initially in place, STB.
R_{ci} = initial condensate/gas ratio (CGR), STB condensate/MMscf.
C_{Fi} = free gas initially in place, MMscf.

Example 2.2:

The following data are given for the Bell Gas Field:

- Area = 160 acres.
- Net productive thickness = 40 *ft.*
- Porosity = 22%.
- Connate water = 23%.
- Initial gas FVF = 0.00533 *ft³/SCF*.

Find the gas initially in place.

Solution:

Calculating initially gas in place by using Eq. (2.3):

$$G_{Fi} = \frac{43560(0.22)(1-0.23)(160)(40)}{0.00533} = 8860\ MMSCF$$

Note: Each "M" is equal 10^3, So "MM" equals 10^6.

2.5 Material Balance Equation

A material balance is an application of conservation of mass to the analysis of physical system. By accounting for material entering and leaving a system, mass flows can be identified which might have been unknown, or difficult to measure without this technique. We will talk about this subject in details in chapter three.

Chapter 3

The Material Balance Equation

3.1 The general form of Material Balance Equation

The material balance equation (MBE) has long been recognized as one of the basic tools of reservoir engineers for interpreting and predicting reservoir performance. The MBE, when properly applied, can be used to:

- Estimate initial hydrocarbon volumes in place.
- Predict future reservoir performance.
- Predict ultimate hydrocarbon recovery under various types of primary driving mechanisms.

The equation is structured to simply keep inventory of all materials entering, leaving, and accumulating in the reservoir. The concept of the material balance equation was presented by Schilthuis in 1941. In its simplest form, the equation can be written on a volumetric basis as:

Initial volume = volume remaining + volume removed

Since oil, gas, and water are present in petroleum reservoirs, the material balance equation can be expressed for the total fluids or for any one of the fluids presents.

Before deriving the material balance equation, it is convenient to denote certain terms by symbols for brevity. The symbols used conform where possible to the standard nomenclature adopted by the Society of Petroleum Engineers.

p_i	Initial reservoir pressure, psi
p	Volumetric average reservoir pressure
Δp	Change in reservoir pressure = $p_i -$ p, psi
P_p	Bubble point pressure, psi
N	Initial (original) oil-in-place, STB
N_p	Cumulative oil produced, STB
G_p	Cumulative gas produced, scf
W_p	Cumulative water produced, bbl
R_p	Cumulative gas-oil ratio, scf/STB
GOR	Instantaneous gas-oil ratio, scf/STB
R_{si}	Initial gas solubility, scf/STB
R_s	Gas solubility, scf/STB
B_{oi}	Initial oil formation volume factor, bbl/STB
B_o	Oil formation volume factor, bbl/STB
B_{gi}	Initial gas formation volume factor, bbl/scf

B_g	Gas formation volume factor, bbl/scf
W_{inj}	Cumulative water injected, STB
G_{inj}	Cumulative gas injected, scf
W_e	Cumulative water influx, bbl
m	Ratio of initial gas-cap gas reservoir volume to initial reservoir oil volume, bbl/bbl
G	Initial gas-cap gas, scf
P.V	Pore volume, bbl
c_w	Water compressibility, psi^{-1}
c_f	Formation (rock) compressibility, psi^{-1}

Several of the material balance calculations require the total pore volume (P.V) as expressed in terms of the initial oil volume *N* and the volume of the gas cap. The expression for the total pore volume can be derived by conveniently introducing the parameter *m* into the relationship as follows:

Defining the ratio *m* as:

$$m = \frac{Initial\ volume\ of\ gas\ cap}{Volume\ of\ oil\ initially\ in\ place} = \frac{G\ B_{gi}}{N\ B_{oi}} \qquad eq. (3.1)$$

Solving for the volume of the gas cap gives:

$$Initial\ volume\ of\ the\ gas\ cap = G\ B_{gi} = m\ N\ B_{oi}$$

The total volume of the hydrocarbon system is then given by:

$$\begin{aligned} &Initial\ oil\ volume + initial\ gas\ cap\ volume \\ &\qquad = (P.V)(1 - S_{wi})NB_{oi} + mNB_{oi} = (P.V)(1 - S_{wi}) \end{aligned}$$

or,

$$P.V = \frac{NB_{oi}(1 + m)}{1 - S_{wi}} \qquad eq. (3.2)$$

where:

S_{wi} = initial water saturation.
N = initial oil-in-place, STB.
P.V = total pore volume, bbl.
m = ratio of initial gas-cap gas reservoir volume to initial reservoir oil volume, bbl/bbl.

Treating the reservoir pore as an idealized container as illustrated in Figure (3.1a-b), volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir.

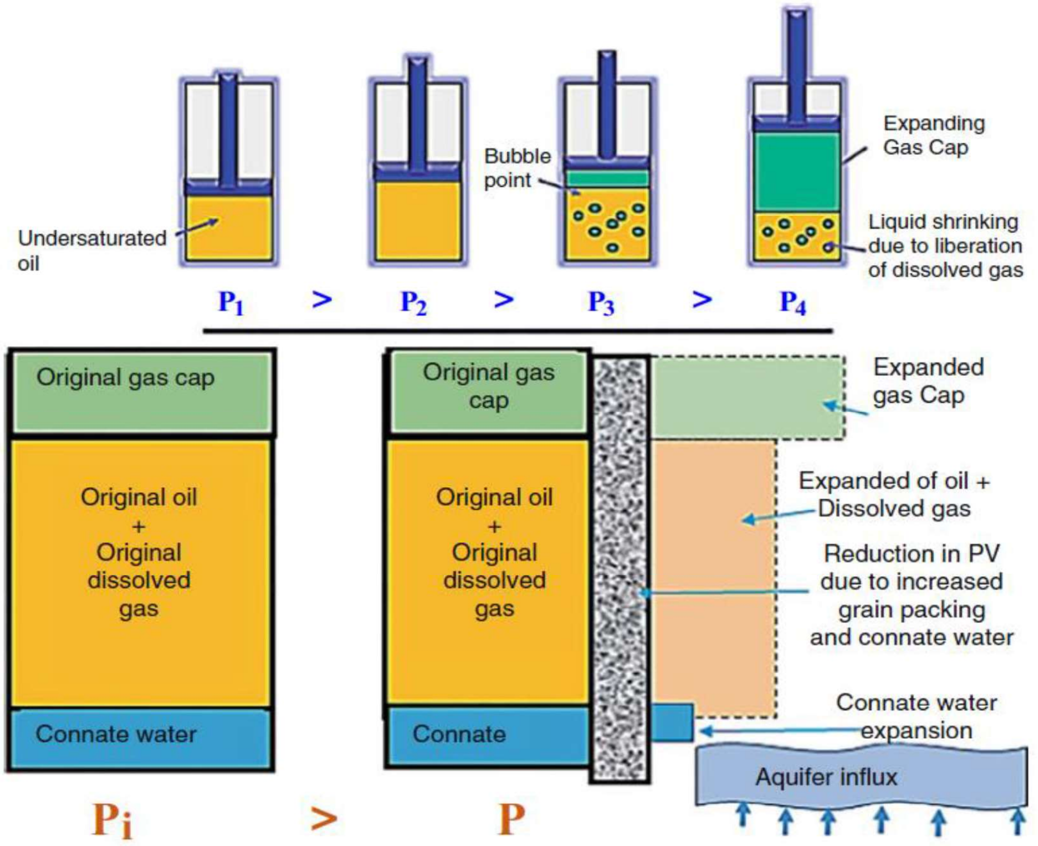


Figure (3.1a): Oil material balance system setup.

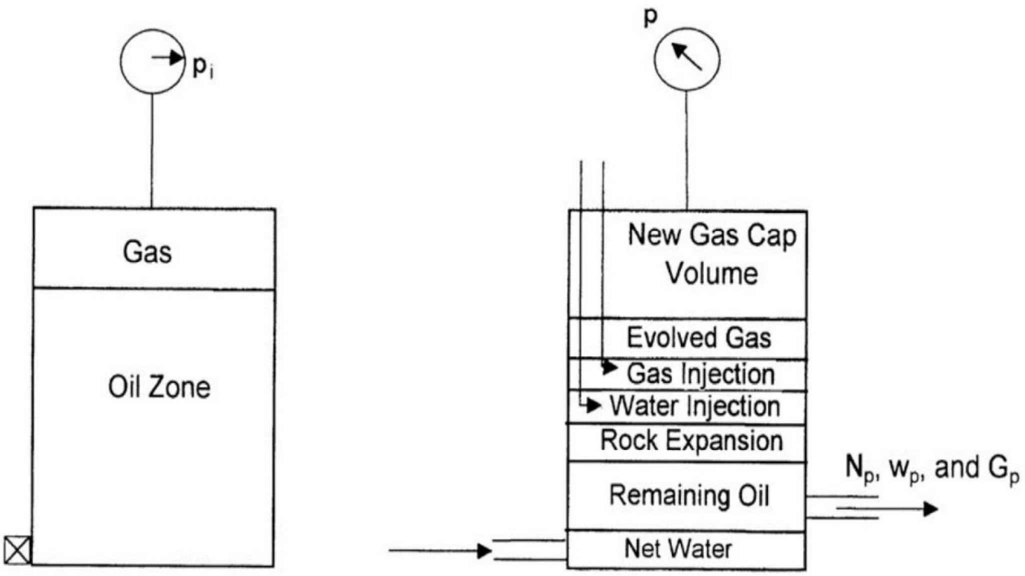


Figure (3.1b): Tank-model concept.

The MBE can be written in a generalized form as follows:

$$\overset{(1)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{oil} \\ \text{initially} \\ \text{in place} \\ \text{at } p_i \end{bmatrix}} + \overset{(2)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{gas} \\ \text{in the} \\ \text{gas cap} \\ \text{at } p_i \end{bmatrix}} = \overset{(3)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{remaining} \\ \text{oil at } p \end{bmatrix}} + \overset{(4)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{gas} \\ \text{in the} \\ \text{gas cap} \\ \text{at } p \end{bmatrix}} + \overset{(5)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{evolved} \\ \text{solution} \\ \text{gas} \\ \text{at } p \end{bmatrix}} + \overset{(6)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{net} \\ \text{water} \\ \text{influx} \\ \text{at } p \end{bmatrix}} + \overset{(7)}{\begin{bmatrix} \text{Change} \\ \text{in pore} \\ \text{volume} \\ \text{due to} \\ \text{connat} \\ \text{water} \\ \text{expansion} \\ \text{and rock} \\ \text{expansion} \end{bmatrix}} + \overset{(8)}{\begin{bmatrix} \text{Pore} \\ \text{volume} \\ \text{occupied} \\ \text{by the} \\ \text{injected} \\ \text{gas} \\ \text{at } p \end{bmatrix}} + \overset{(9)}{\begin{bmatrix} \text{Pore} \\ \text{voleum} \\ \text{occupied} \\ \text{by the} \\ \text{injected} \\ \text{water} \\ \text{at } p \end{bmatrix}}$$

$$[NB_{oi}] + [mNB_{oi}] = [(N - N_p)B_o] + \left[\left(\frac{mNB_{oi}}{B_{gi}}\right)B_g\right] + [NR_{si}B_g - N_pR_pB_g - (NR_sB_g - N_pR_sB_g)] + [W_e - W_pB_w] + \left[NB_{oi}(1+m)\left(\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right)\Delta p\right] + [G_{inj}B_{ginj}] + [W_{inj}B_w] \quad Eq.(3.3)$$

The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

(1) Pore Volume Occupied by the Oil Initially in Place:

$$Volume\ occupied\ by\ initial\ oil - in - place = NB_{oi} \qquad eq.(3.4)$$

where:

N = oil initially in place, STB.
B_{oi} = oil formation volume factor at initial reservoir pressure p_i, bbl/STB.

(2) Pore Volume Occupied by the Gas in the Gas Cap:

$$Volume\ of\ gas\ cap = mNB_{oi} = GB_{gi} \qquad eq.(3.5)$$

where m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

(3) Pore Volume Occupied by the Remaining Oil:

$$Volume\ of\ the\ remaining\ oil = \left(N - N_p\right)B_o \qquad eq.(3.6)$$

where:

N_p = cumulative oil production, STB.
B_o = oil formation volume factor at reservoir pressure p, bbl/STB.

(4) Pore Volume Occupied by the Gas Cap at Reservoir Pressure p:

As the reservoir pressure drops to a new level p, the gas in the gas cap expands and occupies a larger volume. Assuming no gas is produced from the gas cap during the pressure decline, the new volume of the gas cap can be determined as:

$$Volume\ of\ the\ gas\ cap\ at\ p = \left[\frac{mNB_{oi}}{B_{gi}}\right]B_g \qquad eq.(3.6)$$

where:

B_{gi} = gas formation volume factor at initial reservoir pressure, bbl/scf.
B_g = current gas formation volume factor, bbl/scf.

(5) Pore Volume Occupied by the Evolved Solution Gas:

This volumetric term can be determined by applying the following material balance on the solution gas:

$$\begin{bmatrix} volume \\ of\ the \\ evolved \\ solution \\ gas \end{bmatrix} = \begin{bmatrix} volume \\ of\ gas \\ initially \\ in \\ solution \end{bmatrix} - \begin{bmatrix} volume \\ of\ gas \\ produced \end{bmatrix} - \begin{bmatrix} volume \\ of\ gas \\ remaining \\ in \\ solution \end{bmatrix}$$

or,

$$\begin{bmatrix} volume \\ of\ the \\ evolved \\ solution \\ gas \end{bmatrix} = \left[NR_{si} - N_pR_p - \left(N - N_p\right)R_s\right]B_g \qquad eq.(3.7)$$

where:

N_p = cumulative oil produced, STB.
R_p = net cumulative produced gas-oil ratio, scf/STB.
R_s = current gas solubility factor, scf/STB.
B_g = current gas formation volume factor, bbl/scf.
R_{si} = gas solubility at initial reservoir pressure, scf/STB.

(6) Pore Volume Occupied by the Net Water Influx:

$$net\ water\ influx = W_e - W_pB_w \qquad eq.(3.8)$$

where:

W_e = cumulative water influx, bbl.
W_p = cumulative water produced, STB.
B_w = water formation volume factor, bbl/STB.

(7) Change in Pore Volume Due to Initial Water and Rock Expansion:

The component describing the reduction in the hydrocarbon pore volume due to the expansion of initial (connate) water and the reservoir rock cannot be neglected for an undersaturated-oil reservoir. The water compressibility c_w and rock compressibility c_f are generally of the same order of magnitude as the compressibility of the oil. The effect of these two components, however, can be generally neglected for the gas-cap-drive reservoir or when the reservoir pressure drops below the bubble-point pressure.

The compressibility coefficient *c*, which describes the changes in the volume (expansion) of the fluid or material with changing pressure, is given by:

$$c = \frac{-1}{V}\frac{\partial V}{\partial p}$$

or,

$$\Delta V = V\ c\ \Delta p$$

where ΔV represents the net changes or expansion of the material as a result of changes in the pressure. Therefore, the reduction in the pore volume due to the expansion of the connate-water in the oil zone and the gas cap is given by:

$$Connate - water\ expansion = [(pore\ volume)S_{wi}]c_w\Delta p$$

Substituting for the pore volume (P.V) with Equation (3.2) gives:

$$Exapantion\ of\ connate\ water = \frac{NB_{oi}(1+m)}{1-S_{wi}}S_{wi}c_w\Delta p \qquad eq.(3.9)$$

where:

Δp = change in reservoir pressure, (pi − p).
c_w = water compressibility coefficient, psi^{-1}.
m = ratio of the volume of the gas-cap gas to the reservoir oil volume, bbl/bbl

Similarly, the reduction in the pore volume due to the expansion of the reservoir rock is given by:

$$Change\ in\ pore\ volume = \frac{NB_{oi}(1+m)}{1-S_{wi}}c_f\Delta p \qquad eq.(3.10)$$

Combining the expansions of the connate-water and formation as represented by Equations (3.9) and (3.10) gives:

$$\begin{bmatrix} Total\ changes\ in \\ the\ pore\ volume \end{bmatrix} = NB_{oi}(1+m)\left(\frac{S_{wi}c_w + c_f}{1-S_{wi}}\right)\Delta p \qquad eq.(3.11)$$

(8) & (9) Pore Volume Occupied by the Injection Gas and Water:

Assuming that G_{inj} volumes of gas and W_{inj} volumes of water have been injected for pressure maintenance, the total pore volume occupied by the two injected fluids is given by:

$$Total\ volume = G_{inj}B_{ginj} + W_{inj}B_w \qquad eq.(3.12)$$

where:

G_{inj} = cumulative gas injected, scf.
B_{ginj} = injected gas formation volume factor, bbl/scf.

W_{inj} = cumulative water injected, STB.
B_w = water formation volume factor, bbl/STB.

Combining Equations (3.4) through (3.12) with Equation (3.3) and rearranging gives:

$$N = \frac{N_p B_o + (G_p - N_p R_s) B_g - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi} c_w + c_f}{1 - S_{wi}}\right]\Delta p} \qquad eq.(3.13)$$

where:

N = initial oil-in-place, STB.
G_p = cumulative gas produced, scf.
N_p = cumulative oil produced, STB.
R_{si} = gas solubility at initial pressure, scf/STB.
m = ratio of gas-cap gas volume to oil volume, bbl/bbl.
B_{gi} = gas formation volume factor at p_i, bbl/scf.
B_{ginj} = gas formation volume factor of the injected gas, bbl/scf.

The cumulative gas produced G_p can be expressed in terms of the cumulative gas-oil ratio R_p and cumulative oil produced N_p by:

$$G_p = R_p N_p \qquad eq.(3.14)$$

Combining Equation (3.14) with Equation (3.13) gives:

$$N = \frac{N_p[B_o + (R_p - R_s) B_g] - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_{oi} - B_o) + (R_{si} - R_s) B_g + m B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi} c_w + c_f}{1 - S_{wi}}\right]\Delta p} \qquad eq.(3.15)$$

The above relationship is referred to as the **material balance equation** (MBE). A more convenient form of the MBE can be determined by introducing the concept of the total (two-phase) formation volume factor B_t into the equation. This oil PVT property is defined as:

$$B_t = B_o + (R_{si} - R_s) B_g \qquad eq.(3.16)$$

Introducing B_t into Equation (3.15) and assuming, for the sake of simplicity, no water or gas injection gives:

$$N = \frac{N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right] - \left(W_e - W_pB_w\right)}{\left(B_t - B_{ti}\right) + mB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p} \qquad eq.(3.17)$$

Example 3.1:

The Big Butte field is a combination-drive reservoir. The current reservoir pressure is estimated at (2,500 psi). The reservoir production data and PVT information are given below:

	Initial Reservoir Conditions	**Current Reservoir Conditions**
p, psi	3000	2500
B_o, bbl/STB	1.35	1.33
R_s, scf/STB	600	500
N_p, MMSTB	0	5
G_p, MMMscf	0	5.5
B_w, bbl/STB	1.00	1.00
W_e, MMbbl	0	3
W_p, MMbbl	0	0.2
B_g, bbl/scf	0.0011	0.0015
c_w, c_f	0	0

The following additional information is available:

Volume of bulk oil zone = 100,000 ac-ft.
Volume of bulk gas zone = 20,000 ac-ft.

Calculate the initial oil-in-place.

Solution:

Step 1. Assuming the same porosity and connate-water for the oil and gas zones, calculate *m*:

$$m = \frac{Initial\ volume\ of\ gas\ cap}{Volume\ of\ oil\ initially\ in\ place} = \frac{20{,}000}{100{,}000} = 0.2$$

Step 2. Calculate the cumulative gas-oil ratio R_p:

$$R_p = \frac{G_p}{N_p} = \frac{5.5 \times 10^9}{5 \times 10^6} = 1100\ scf/STB$$

Step 3. Solve for the initial oil-in-place by applying Equation (2.20):

$$N = \frac{N_p\left[B_o + \left(R_p - R_s\right)B_g\right] - \left(W_e - W_pB_w\right)}{\left(B_{oi} - B_o\right) + \left(R_{si} - R_s\right)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]}$$

$$N = \frac{5 \times 10^6[1.33 + (1100 - 500)0.0015] - (3 \times 10^6 - 0.2 \times 10^6)}{(1.35 - 1.33) + (600 - 500)0.0015 + (0.2)(1.35)\left[\frac{0.0015}{0.0011} - 1\right]}$$

$$N = 31.14MMSTB$$

Example 3.2:

A combination-drive reservoir contains 10 MMSTB of oil initially in place. The ratio of the original gas-cap volume to the original oil volume, i.e., m, is estimated as 0.25. The initial reservoir pressure is 3,000 psia at 150°F. The reservoir produced 1 MMSTB of oil, 1,100 MMscf of 0.8 specific gravity gas, and 50,000 STB of water by the time the reservoir pressure dropped to 2,800 psi. The following PVT is available:

	3000 psi	2800 psi
B_o, bbl/STB	1.58	1.48
R_s, scf/STB	1040	850
B_g, bbl/scf	0.00080	0.00092
B_t, bbl/STB	1.58	1.655
B_w, bbl/STB	1.000	1.000

The following data are also available:

$S_{wi} = 0.20$ $\quad c_w = 1.5 \times 10^{-6} psi^{-1}$ $\quad c_f = 1 \times 10^{-6} psi^{-1}$

Calculate:

a. Cumulative water influx.
b. Net water influx.

Solution:

Because the reservoir contains a gas cap, the rock and fluid expansion can be neglected, i.e., set c_f and c_w = 0. For illustration purposes, however, the rock and fluid expansion term will be included in the calculations.

Part A. Cumulative water influx.

Step 1. Calculate cumulative gas-oil ratio R_p:

$$R_p = \frac{G_p}{N_p} = \frac{1100 \times 10^6}{1 \times 10^6} = 1100\ scf/STB$$

Step 2. Arrange Equation (2.22) to solve for We:

$$\begin{aligned} W_e &= N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right] \\ &\quad -N\left[\left(B_g - B_{ti}\right) + mB_{ti}\left(\frac{B_g}{B_{gi}} - 1\right) + B_{ti}(1+m)\left(\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right)\Delta p\right] + W_pB_{wp} \end{aligned}$$

$$\begin{aligned} W_e &= 10^6[1.655 + (1100 - 1040)0.00092] \\ &\quad - 10^7\left[(1.655 - 1.58) + 0.25(1.58)\left(\frac{0.00092}{0.00080} - 1\right)\right. \\ &\quad \left. + 1.58(1 + 0.25)\left(\frac{0.2(1.5 \times 10^{-6})}{1 - 0.2}\right)(3000 - 2800)\right] + 50000 \end{aligned}$$

$$W_e = 411281\ bbl$$

Neglecting the rock and fluid expansion term, the cumulative water influx is 417,700 bbl.

Part B. Net water influx:

$$Net\ water\ influx = W_e - W_pB_w = 411281 - 50000 = 361281\ bbl$$

3.2 Uses and Limitations of the Material Balance Method

The material balance equation derived in the previous section has been in general use for many years, mainly for the following:

1. Determining the initial hydrocarbon in place.
2. Calculating water influx.
3. Predicting reservoir pressures.

Although in some cases it is possible to solve simultaneously to find the initial hydrocarbon and the water influx, generally one or the other must be known from data or methods that do not depend on the material balance calculations. One of the most important uses of the equations is predicting the effect of cumulative production and/or injection (gas or water) on reservoir pressure; therefore, it is very desirable to know in advance the initial oil and the ratio *m* from good core and log data. The presence of an aquifer is usually indicated by geologic evidence; however, the material balance may be used to detect the existence of a water drive by