NUMERICAL ANALYSIS

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CHAPETER ONE DIFERENTIAL EQUATIONS







Differential Equations

A differential equation is an equation with one or more derivatives of a function. The derivative of the function is given by dy/dx. In other words, it is defined as the equation that contains derivatives of one or more dependent variables with respect to one or more independent variables. For example,

$$\frac{dy}{dx} = \frac{1+x^2}{1-y^2}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{dy}{dx}$$

$$x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial y} = nu$$

المعادلات التي تحتوي على متغير واحد وليكن مثلا (y) ويعتبر اساسي ومتغير غير اساسي واحد وليكن على سبيل المثال (x).

Note:-
$$y' = \frac{dy}{dx}$$
; $y'' = \frac{d^2y}{dx^2}$; $y''' = \frac{d^3y}{dx^3}$







Differential Equations

There are two types of differential equations:

1. Ordinary Differential Equation:

Is a differential equation involving derivative with respect to a single independent variable.

F(x, y,
$$\frac{dy}{dx}$$
, $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, ..., $\frac{d^ny}{dx^n}$) (Standard Form) [It is an equation containing function of "x", "y", and derivatives with respect to x]

Example:

$$\sin 2x + e^{2y} \frac{dy}{dx} = 0,$$
 $\left[1 + \left[\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \left(\frac{d^3y}{dx^3}\right)^2\right]$

Key feature:

- 1- One independent variables i.e. (x).
- 2- It contains ordinary derivative with respect to (x) i.e. $\frac{(d^n y)^m}{(dx^n)}$.







2. Partial Differential Equation:

Is a differential equation involving partial derivative with respect to more than one independent variables.

$$F(x, y, z, \frac{\partial y}{\partial x}, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial z}{\partial y}, \frac{\partial z}{\partial z}, \frac{\partial z}{\partial z}) = 0$$
 (Standard Form) [The equation containing

function of (x, y, z) and partial derivatives with respect to x, y, and z].

Example:

$$y^2 \frac{\partial z}{\partial x} + xy \frac{\partial x}{\partial y} = x^2 z$$
 , $\frac{\partial y}{\partial z} + \frac{\partial x}{\partial y} + \frac{\partial z}{\partial y} = 0$

Key features:

- 1- Two or more independent variables i.e. x, y, and z.
- 2- Two or more Partial Derivatives with respect to independent variables i.e.

$$(\frac{\partial z}{\partial x}, \frac{\partial x}{\partial y}, \frac{\partial y}{\partial z}, \frac{\partial x}{\partial y}, \frac{\partial z}{\partial y}).$$







Order and Degree of Differential Equations

The order of differential equation is the order of the highest differential co-efficient present in the equation.

هی رتبهٔ المعادلة التفاضلیة و هی اعلی مشتقة

$$1 - L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = E \sin t,$$

$$3 - \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

2-
$$\cos x \frac{d^2y}{dx^2} + \sin x \left(\frac{dy}{dx}\right)^2 + 8y = \tan x$$

The *order* of the above equations are 2.

The *degree* of differential equation is the highest derivative after removing the radical sign and fraction. (In simple way is the power of the highest derivative)

The *degree* of the equation (1) and (2) is 1. The *degree* of the equation (3) is 2.

هي درجة المعادلة التفاضلية وهي اس اعلى مشتقة







Formation of Differential Equations

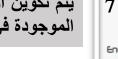
The differential equations can be formed by differentiating the ordinary equation and eliminating the arbitrary constant.

Example 1: Form the differential equation by eliminating arbitrary constant, in the following cases and also write down the order of the differential equations obtained.

$$A) \quad y = Ax + A^2$$

$$\frac{dy}{dx} = A$$
 , $\left(\frac{dy}{dx}\right)^2 = A^2$ By putting the value of A in (1), we gate

$$y = x \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$
 $order = 1$, $degree = 2$







$$B)$$
 $y = A \cos x + B \sin x$

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x = -(A\cos x + B\sin x) = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

This is differential equation of *order* 2 and *degree* 1 eliminating two constants A and B.







(c)
$$y^2 = a x^2 + b x + c$$

On differentiation,

$$2y \frac{dy}{dx} = 2a x + b$$

Again differentiating,

$$2y \cdot \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \cdot \frac{dy}{dx} = 2 a$$

On Again differentiating,

$$2y \frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} \cdot \frac{dy}{dx} + 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 0$$

divided all by 2







$$y \frac{d^3y}{dx^3} + \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 0$$

OR,

$$y \frac{d^3y}{dx^3} + 3 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 0$$

$$order = 3$$

 $degree = 1$







$$D) a x^2 + b y^2 = 1$$

$$2ax + 2by \frac{dy}{dx} = 0$$

$$ax + by \frac{dy}{dx} = 0$$

$$a + by \frac{d^2y}{dx^2} + \frac{dy}{dx}b\frac{dy}{dx} = 0$$

$$a + by \frac{d^2y}{dx^2} + b\left(\frac{dy}{dx}\right)^2 = 0$$
Form (2)...
(2)

$$a = -b\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = 0$$
 substitute into the equation (1)

$$-xb\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] + by\frac{dy}{dx} = 0$$

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

2nd order, 1st degree







Solution of Differential Equations Differential Equations of First Order and First Degree

حل المعادلات التفاضلية من المرتبة الاولى و الدرجة الاولى

The standard methods of solving the differential equations of the following types:

1. Equations solvable by separation of variables

فصل المتغيرات

- 2. Homogeneous Equations
- 3. Linear Equations of first order
- 4. Exact Differential Equations

المعادلات المتجانسة المعادلات الخطية المعادلات التامة







1- Separation of Variables

فصل المتغيرات

If the differential equation can be written in the form:

$$f(y)dy = \emptyset(x)dx$$

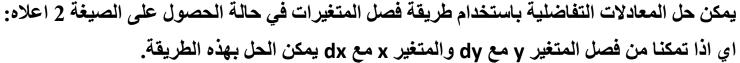
We say that variables are separable (قابل للفصل), y on the left hand side and x on the right hand side.

The equation will solve by integrating both sides of equation.

Working Rule:

- 1- Separate the variables as the equation above.
- 2- Integrate both sides as: $\int f(y)dy = \int \emptyset(x)dx$.
- 3- Add an arbitrary constant like C on right hand side.









 $\int \frac{1}{1+x^2} dx = tan^{-1}x + C$

 $\int \frac{1}{x} dx = \ln x + C$

 $\int 1 dx = x + C$

Example 2:

Solve the equation

$$(x+1) \frac{dy}{dx} = x \left(y^2 + 1 \right)$$

$$(x+1) dy = x (y^2+1) dx$$

$$\frac{dy}{v^2+1}=\frac{x\,dx}{x+1}$$

$$\int \frac{1}{v^2 + 1} dy = \int \frac{x + 1 - 1}{x + 1} dx \qquad \to \int \frac{1}{v^2 + 1} dy = \int \left(1 - \frac{1}{x + 1}\right) dx$$

$$tan^{-1} y = x - ln (x+1) + C$$







Example 3:

Solve the equation

$$(x y^2 + x) dx + (y x^2 + y) dy = 0$$

$$x(y^2+1) dx + y(x^2+1) dy = 0$$

$$\frac{x}{x^2+1}\ dx=-\frac{y}{v^2+1}\ dy$$

$$\frac{1}{2} \int \frac{2x}{x^2+1} \ dx = -\frac{1}{2} \int \frac{2y}{y^2+1} \ dy$$

$$\frac{1}{2} \ln (x^2 + 1) = \frac{1}{2} \ln (y^2 + 1) + \frac{1}{2} c$$

$$ln(x^2+1)+ln(y^2+1)=c$$

$$ln(x^2+1)(y^2+1)=c$$







Example 4:

Solve the equation

$$(x^2 - ay)dx = (ax - y^2) dy$$

$$x^{2} dx - a y dx = a x dy - y^{2} dy$$

$$x^{2} dx + y^{2} dy = a x dy + a y dx$$

$$\int x^{2} dx + \int y^{2} dy = a \int (x dy + y dx)$$

$$\frac{x^{3}}{3} + \frac{y^{3}}{3} = axy + c$$

$$x^3 + y^3 = 3axy + 3c$$

$$(xy)' = xdy + ydx$$

$$\int xdy + ydx = xy$$







Example 5:

Solve the equation

$$y(1+x^2)^{\frac{1}{2}}dy + x\sqrt{1+y^2}dx = 0$$

$$\frac{y\left(1+x^2\right)^{\frac{1}{2}}}{dx}+\frac{x\sqrt{1+y^2}}{dy}=0$$

$$\frac{(1+x^2)^{\frac{1}{2}}}{x\,dx} = -\frac{\sqrt{1+y^2}}{y\,dy} \qquad \Rightarrow \qquad \frac{x\,dx}{\sqrt{1+x^2}} = -\frac{y\,dy}{\sqrt{1+y^2}}$$







$$x(1+x^2)^{\frac{-1}{2}}dx = -y(1+y^2)^{\frac{-1}{2}}dy$$

$$2x(1+x^2)^{\frac{-1}{2}}dx = -2y(1+y^2)^{\frac{-1}{2}}dy$$

$$\int 2x \left(1+x^2\right)^{\frac{-1}{2}} dx = -\int 2y \left(1+y^2\right)^{\frac{-1}{2}} dy$$

$$2(1+x^2)^{\frac{1}{2}} = -2(1+y^2)^{\frac{1}{2}} + c$$







Example 6:

Solve the equation

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = \frac{e^x}{e^y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = \frac{e^x}{e^y} + \frac{x^2}{e^y} \qquad \rightarrow \qquad \frac{dy}{dx} = \frac{e^x + x^2}{e^y} \qquad \rightarrow \qquad e^y dy = e^x dx + x^2 dx$$

$$\int e^y dy = \int e^x dx + \int x^2 dx \rightarrow e^y = e^x + \frac{x^3}{3} + c$$

