NUMERICAL ANALYSIS

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CHAPETER ONE

DIFERENTIAL EQUATIONS

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Differential Equations

A differential equation is an equation with one or more derivatives of a function. The derivative of the function is given by dy/dx. In other words, it is defined as the equation that contains derivatives of one or more dependent variables with respect to one or more independent variables. For example,

$\frac{dy}{dx} = \frac{1+\ x^2}{1-\ y^2}$	$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0$
$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{dy}{dx}$	$x\frac{\partial\, u}{\partial\, y} + y\ \frac{\partial\, u}{\partial\, y} = nu$

المعادلات التي تحتوي على متغير واحد وليكن مثلا (y) ويعتبر اساسي ومتغير غير اساسي واحد وليكن على سبيل المثال (x).

Note:- $y' = \frac{dy}{dx}$; $y'' = \frac{d^2y}{dx^2}$; $y''' = \frac{d^3y}{dx^3}$

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Differential Equations

There are two types of differential equations:

1. Ordinary Differential Equation:

Is a differential equation involving derivative with respect to a single independent variable.

$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \dots \dots, \frac{d^ny}{dx^n} \right)$ (Standard Form) [*It is an equation containing function of "x", "y", and derivatives with respect to x*]

Example:

$\text{Sin}2x + e^{2y}\frac{dy}{dx} = 0,$ $\left[1 + \left[\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \left(\frac{d^3y}{dx^3}\right)^2\right.$

Key feature:

1- One independent variables i.e. (x).

2- It contains ordinary derivative with respect to (x) i.e. $\frac{(d^ny)^m}{(dx^n)}$.

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2. Partial Differential Equation:

Is a differential equation involving partial derivative with respect to more than one independent variables.

$F(x, y, z, \frac{\partial y}{\partial x}, \frac{\partial z}{\partial x}, \frac{\partial x}{\partial y}, \frac{\partial z}{\partial y}, \frac{\partial y}{\partial z}, \frac{\partial x}{\partial z}) = 0$ (Standard Form) [*The equation containing function of (x, y, z) and partial derivatives with respect to x, y, and z*].

Example:

$$y^2\frac{\partial z}{\partial x} + xy\frac{\partial x}{\partial y} = x^2 z \quad , \quad \frac{\partial y}{\partial z} + \frac{\partial x}{\partial y} + \frac{\partial z}{\partial y} = 0$$

Key features:

1- Two or more independent variables i.e. *x*, *y*, and *z*.

2- Two or more Partial Derivatives with respect to independent variables i.e. $(\frac{\partial z}{\partial x}, \frac{\partial x}{\partial y}, \frac{\partial y}{\partial z}, \frac{\partial x}{\partial y}, \frac{\partial z}{\partial y})$.

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Order and Degree of Differential Equations

The *order* of differential equation is the order of the highest differential co-efficient present in the equation.

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$1- \ L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=E\sin t,$

2- $\cos x\frac{d^2y}{dx^2}+\sin x\left(\frac{dy}{dx}\right)^2+8y=\tan x$

3- $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$

The ***order*** of the above equations are 2.

The *degree* of differential equation is the highest derivative after removing the radical sign and fraction. (In simple way is the power of the highest derivative)

The ***degree*** of the equation (1) and (2) is 1. The ***degree*** of the equation (3) is 2.

هي درجة المعادلة التفاضلية وهي اس اعلى مشتقة

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Formation of Differential Equations

The differential equations can be formed by differentiating the ordinary equation and eliminating the arbitrary constant.

Example 1: Form the differential equation by eliminating arbitrary constant, in the following cases and also write down the order of the differential equations obtained.

$A)\quad y = Ax + A^2$

Solution:

$\frac{dy}{dx} = A \quad , \quad \left(\frac{dy}{dx}\right)^2 = A^2$ By putting the value of A in (1), we gate

$$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

$order = 1 \quad , \quad degree = 2$

يتم تكوين المعادلة التفاضلية من خلال اشتقاق المعادلة المعطاة عدد من المرات مساوي لعدد الثوابت التي تتضمنها المعادلة التفاضلية الى حين التخلص من كل الثوابت الموجودة في المعادلة (يكون الناتج النهائي معادلة تفاضلية خالية من الثوابت)، يلاحظ ان عدد الثوابت في المعادلة يشير الى رتبة المعادلة التفاضلية الناتجة.

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$B)\quad y = A\cos x + B\sin x$

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x = -(A\cos x + B\sin x) = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

This is differential equation of ***order*** 2 and ***degree*** 1 eliminating two constants A and B.

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$C) \quad y^2 = a\,x^2 + b\,x + c$

On differentiation,

$$2y\,\frac{dy}{dx} = 2a\,x + b$$

Again differentiating,

$$2y\,.\,\frac{d^2y}{dx^2} + 2\,\frac{dy}{dx}\,.\,\frac{dy}{dx} = 2\,a$$

On Again differentiating,

$$2y\,\frac{d^3y}{dx^3} + 2\,\frac{d^2y}{dx^2}\,.\,\frac{dy}{dx} + 2\,\frac{dy}{dx}\,.\,\frac{d^2y}{dx^2} + 2\,\frac{dy}{dx}\,.\,\frac{d^2y}{dx^2} = 0$$

divided all by 2

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$$y\frac{d^3y}{dx^3} + \frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} = 0$$

OR,

$$y\frac{d^3y}{dx^3} + 3\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} = 0$$

$$order = 3$$
$$degree = 1$$

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$D)\ a\,x^2 + b\,y^2 = 1$

$2ax + 2by\,\frac{dy}{dx} = 0 \quad \longrightarrow \quad ax + by\,\frac{dy}{dx} = 0$ ....(1)

$a + by\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\,b\,\frac{dy}{dx} = 0 \quad \longrightarrow \quad a + by\,\frac{d^2y}{dx^2} + b\left(\frac{dy}{dx}\right)^2 = 0$................(2)

Form (2) ..

$a = -b[y\,\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2] = 0$ substitute into the equation (1)

$-x\not{b}[y\,\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2] + \not{b}y\,\frac{dy}{dx} = 0$

$x\,y\,\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\,\frac{dy}{dx} = 0$

2^{nd} *order*, 1^{st} *degree*

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Solution of Differential Equations
Differential Equations of First Order and First Degree

حل المعادلات التفاضلية من المرتبة الاولى و الدرجة الاولى

The standard methods of solving the differential equations of the following types:

1. **Equations solvable by separation of variables** فصل المتغيرات
2. **Homogeneous Equations** المعادلات المتجانسة
3. **Linear Equations of first order** المعادلات الخطية
4. **Exact Differential Equations** المعادلات التامة

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1- Separation of Variables   فصل المتغيرات

If the differential equation can be written in the form:

$$f\,(y)dy = \emptyset\,(x)dx$$

We say that variables are separable (قابل للفصل), ***y*** on the left hand side and ***x*** on the right hand side.

The equation will solve by integrating both sides of equation.

Working Rule:

1- Separate the variables as the equation above.

2- Integrate both sides as : $\int f\,(y)dy = \int \emptyset\,(x)dx$.

3- Add an arbitrary constant like C on right hand side.

يمكن حل المعادلات التفاضلية باستخدام طريقة فصل المتغيرات في حالة الحصول على الصيغة 2 اعلاه:

اي اذا تمكنا من فصل المتغير y مع dy والمتغير x مع dx يمكن الحل بهذه الطريقة.

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Example 2:

Solve the equation $(x+1)\frac{dy}{dx} = x\left(y^2+1\right)$

$$\int \frac{1}{1+x^2}dx = tan^{-1}x + C$$

$$\int \frac{1}{x}\,dx = \ln x + C$$

$$\int 1\,dx = x + C$$

Solution:

$$(x+1)\,dy = x\left(y^2+1\right)dx$$

$$\frac{dy}{y^2+1} = \frac{x\,dx}{x+1}$$

$$\int \frac{1}{y^2+1}\,dy = \int \frac{x+1-1}{x+1}\,dx \quad \rightarrow \int \frac{1}{y^2+1}\,dy = \int\left(1-\frac{1}{x+1}\right)dx$$

$$tan^{-1}\,y = x - ln\,(x+1) + C$$

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Example 3:

Solve the equation $(x\,y^2 + x)\,dx + (y\,x^2 + y)\,dy = 0$

Solution:

$$x\,(y^2 + 1)\,dx + y\,(x^2 + 1)\,dy = 0$$

$$\frac{x}{x^2 + 1}\,dx = -\frac{y}{y^2 + 1}\,dy$$

$$\frac{1}{2}\int \frac{2x}{x^2 + 1}\,dx = -\frac{1}{2}\int \frac{2y}{y^2 + 1}\,dy$$

$$\frac{1}{2}\,ln\,(x^2 + 1) = \frac{1}{2}\,ln\,(y^2 + 1) + \frac{1}{2}\,c$$

$$ln\,(x^2 + 1) + ln\,(y^2 + 1) = c$$

$$ln\,(x^2 + 1)(y^2 + 1) = c$$

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Example 4:

Solve the equation $(x^2 - ay)dx = (ax - y^2)\,dy$

Solution:

$$x^2\,dx - a\,y\,dx = a\,x\,dy - y^2\,dy$$

$$x^2\,dx + y^2 dy = a\,x\,dy + a\,y\,dx$$

$$\int x^2\,dx + \int y^2\,dy = a\int (x\,dy + y\,dx)$$

$$\frac{x^3}{3} + \frac{y^3}{3} = axy + c$$

$$x^3 + y^3 = 3axy + 3c$$

$$(xy)' = xdy + ydx$$

$$\int xdy + ydx = xy$$

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Example 5:

Solve the equation $y\,(1+\,x^2)^{\frac{1}{2}}\,dy + x\,\sqrt{1+\,y^2}\,dx = 0$

Solution:

$$\frac{y\,(1+\,x^2)^{\frac{1}{2}}}{dx} + \frac{x\,\sqrt{1+\,y^2}}{dy} = 0$$

$$\frac{(1+\,x^2)^{\frac{1}{2}}}{x\,dx} = -\frac{\sqrt{1+\,y^2}}{y\,dy} \quad \rightarrow \quad \frac{x\,dx}{\sqrt{1+\,x^2}} = -\frac{y\,dy}{\sqrt{1+\,y^2}}$$

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$$x\left(1+x^2\right)^{\frac{-1}{2}}dx=-y\left(1+y^2\right)^{\frac{-1}{2}}dy$$

$$2x\left(1+x^2\right)^{\frac{-1}{2}}dx=-2y\left(1+y^2\right)^{\frac{-1}{2}}dy$$

$$\int 2x\left(1+x^2\right)^{\frac{-1}{2}}dx=-\int 2y\left(1+y^2\right)^{\frac{-1}{2}}dy$$

$$2\left(1+x^2\right)^{\frac{1}{2}}=-2\left(1+y^2\right)^{\frac{1}{2}}+c$$

Example 6:

Solve the equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

Solution:

$$\frac{dy}{dx} = \frac{e^x}{e^y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = \frac{e^x}{e^y} + \frac{x^2}{e^y} \quad \rightarrow \quad \frac{dy}{dx} = \frac{e^x + x^2}{e^y} \quad \rightarrow \quad e^y dy = e^x\, dx + x^2\, dx$$

$$\int e^y\, dy = \int e^x\, dx + \int x^2\, dx \quad \rightarrow \quad e^y = e^x + \frac{x^3}{3} + c$$

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