SINGLE TUNNELS

Although tunnel shape, size, and orientation may be fixed by overall design and construction considerations and thus there is no opportunity to reduce stress concentration, estimation of *stress concentration* factors in compression and tension (Kc, Kt) is still important as are determinations of *rock mass strengths* (Co, To) and pre-excavation principal stresses along the tunnel route. Rock mass strength is influenced by joints, of course, and thus depends on the strength of intact rock between joints, joint persistence, and orientation as well as joint strength. Both stress and strength are essential to calculating tunnel wall safety factors (Fc, Ft). Relatively high safety factors, say 4 in compression and 8 in tension, indicate a high degree of stability and little need for an engineered support system. Low safety factors, near 1, indicate the opposite and that some permanent support will be required.

In hard rock, drilling and blasting are necessary for excavation. An advantage, in this case, is the capability of fitting the blast design to result in an approved tunnel shape. Perhaps the most common shape is a rectangular section with a semi-circular top, as shown in Figure 1a. The bottom in Figure 1a is a semi-square. The *top* is also known as the (*back*) in *hard rock* tunneling and mining terms, while in *soft rock* mining, the (roof); *sidewalls* are (*ribs*), while the *bottom* is the (*bottom*) in hard rocks, and in *soft rock* mining, the (*floor*). Drilling and blasting occur at the (face). The semi-circular top in Figure 1a is an arched back and serves the purpose of reducing stress concentration and thus improving safety. This back has a radius equal to one-half the width of the tunnel. However, an arched back need not be a semicircle. A longer radius is possible as are other arch shapes than the circle.

The shape in Figure 1a is symmetric about a vertical axis, but there is no symmetry about any horizontal axis as there is for circular, elliptical, and rectangular sections. For this reason, no simple analytical formulas are available for estimating stress concentration factors for an arched back tunnel. Numerical methods serve the purpose instead.

Figures 1 b, c, and d show distributions of stress about the periphery of an arched back tunnel section with a width (W) to height (H) ratio of one. The radius of the semi-circular back is W /2; the height of the rectangular portion is also W /2. Figure 1b shows the stress distribution that results when the pre-excavation stress field is uniaxial and vertical (Sy = 1, Sx = 0); Figure 1c shows the result when the pre-excavation stress field is uniaxial and horizontal (Sy = 0, Sx = 1), and Figure 1d shows the hydrostatic case (Sy = Sx = 1). In all cases, *the tunnel axis is assumed to* *be parallel to a principal stress direction.* These results are based on the assumption of *elastic behavior* and do not depend on the construction sequence.

These results show that under vertical load tension nearly equal in magnitude to the applied compression appears at the center of the floor or bottom and the crown, while high compressive stresses appear at the bottom corners and along the shoulders at the top of the ribs in the vertical wall sections. In theory, a mathematically sharp, 90° corner, would result in an infinite compressive stress concentration at the corner points. In numerical analysis, the result is simply a high but not infinite stress concentration. A rounded corner done deliberately or left to nature would *reduce* an extraordinarily high-stress concentration. When left to nature, reduction in the stress concentration occurs through the yielding of the rock mass in the vicinity of the corner point. Yielding at a point elevates stress concentration at neighboring points. Thus, peak stress is lowered, while the average stress is raised in the vicinity of a point stressed beyond the elastic limit. As long as yielding is localized, the threat to tunnel stability is negligible. The distributions shown in Figure 1 suggest that this may be the case; the region of high-corner compressive stress does not extend far up the ribs. If the stress concentration at the shoulder or top of the rib is tolerable, then the corner stress is also likely to be tolerable. likely to be tolerable.

Under a uniaxial but horizontal compression (Sy = 0, Sx = 1); the floor and crown are under high compressive stress, while the rib is under tension equal in magnitude to the applied compression. When the applied vertical and horizontal stresses are equal (hydrostatic pre-excavation stress state), the results show no tension and reduced compressive stress peaks relative to the uniaxial cases.

The shape of an excavation usually changes during construction and consequently so does the distribution of stress. Figures 1 e, f, g shows the same three cases for a higher tunnel that has a height-to-width ratio of 1.5; Figures 1 h, i, j show results for an even higher tunnel with a height-to-width ratio of 2. Under vertical load only, the peak tension in the floor and crown changes very little from the first case where the height-to-width ratio was 1 and the peak tension was nearly equal in magnitude to the applied compression. The high compressive corner stress declines rapidly with increasing tunnel height at a fixed width, while the high compressive rib stress declines rather slowly.

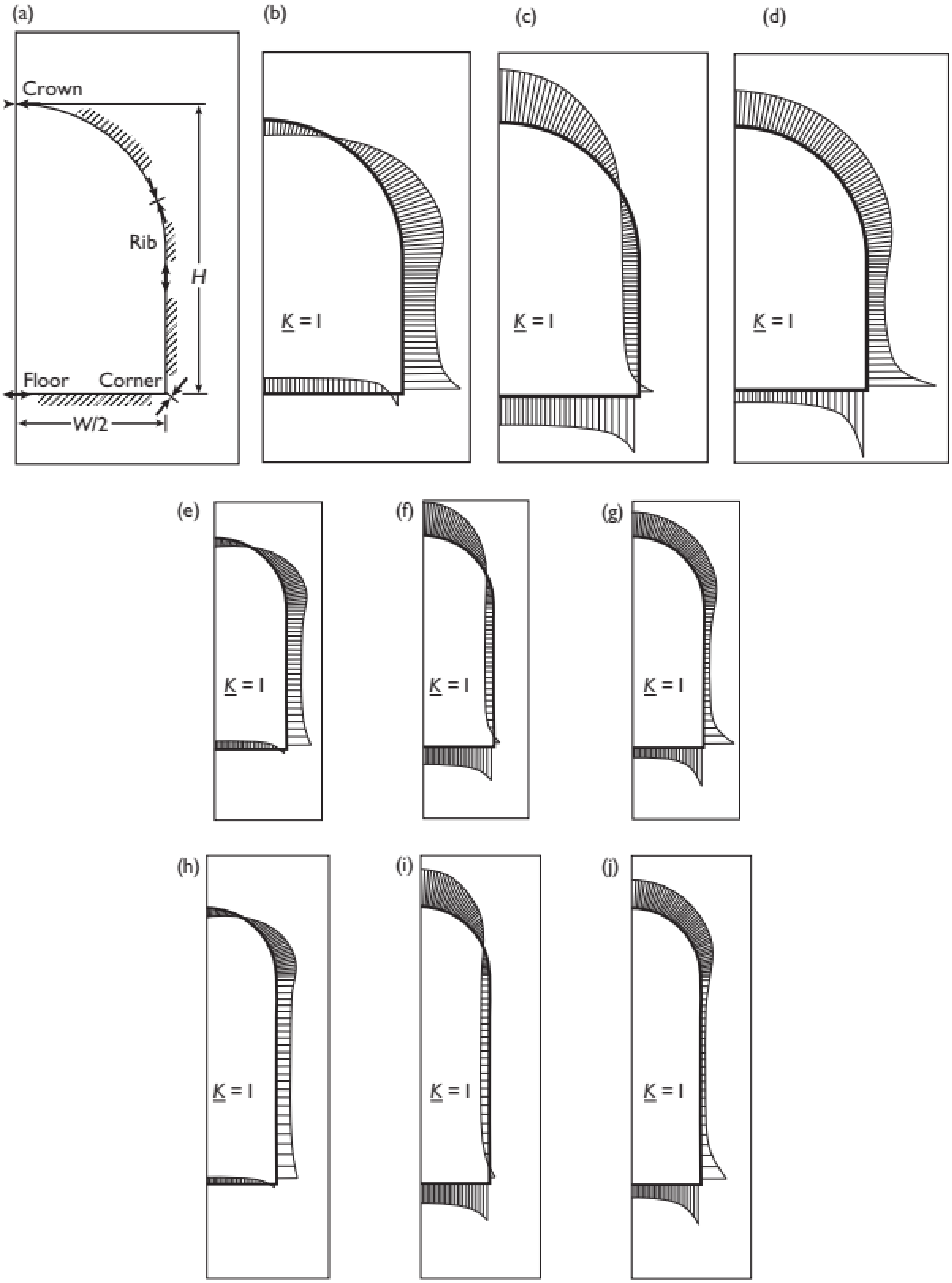


Figure 1 Stress distribution about arched back tunnel sections:

(a) section notation, (b) H/W = 1, Sy = l, Sx = 0,
(c) H/W = 1, Sy = 0, Sx = 1, (d) H/W = 1, Sy = 1, Sx = 1,
(e) H/W = 1.5, Sy = 1, Sx = 0, (f) H/W = 1.5, Sy = 0, Sx = 1,
(g) H/W = 1.5, Sy = 1, Sx = 1, (h) H/W = 2, Sy = 1, Sx = 0,
(i) H/W = 2, Sy = 0, Sx = 1, (j) H/W = 2, Sy = 1, Sx = 1

These trends are summarized in Figure 2 which shows stress concentrations as a function of height to width ratio (H/W). Under *vertical* load, *peak tensions* in the floor and crown and *rib compression* are largely unaffected by height to width ratio. Corner compression decreases nonlinearly with height to width ratio (but increases linearly with the reciprocal width-to-height ratio). Under *horizontal* load, *peak tension* in the rib remains largely unaffected, while compressive stresses at the floor, corner, and crown increase almost linearly with the height-to-width ratio, as seen in Figure 2b. Compressive stress concentration trends with a height-to-width ratio in the *hydrostatic case*, are also linear in height-to-width ratio, as seen in Figure 2c. *Tension is absent* in all hydrostatic cases.

Trend lines fitted to data in Figure 2 show close fits to linearity and for this reason, may be extrapolated to somewhat higher height-to-width ratios than the 2.5 ratio case shown. However, backward extrapolation to ratios less than 1.0 would be ill-advised, and for this reason, the trend lines are not plotted to smaller height-to-width ratios. In retrospect, numerical values in Figure 2 could have been estimated to a degree from consideration of stress concentration about rectangular and ovaloidal openings because of the hybrid shape of the arched back tunnel cases examined.

Superposition of uniaxial cases may be used to obtain stress concentrations associated with other load cases. Care must be exercised to insure that the same location and direction of stress are being considered.

Example 1 An arched back tunnel is planned with a height-to-width ratio of two. Estimate stress concentration factors in the floor and crown when the pre-excavation principal stress ratio M = 1 / 4 and (a) the vertical stress is S1, (b) the horizontal stress is S1.

Solution: The stress concentrations are combinations of vertical and horizontal loads and may be estimated from trend lines in Figures 2 a, and b using superposition.

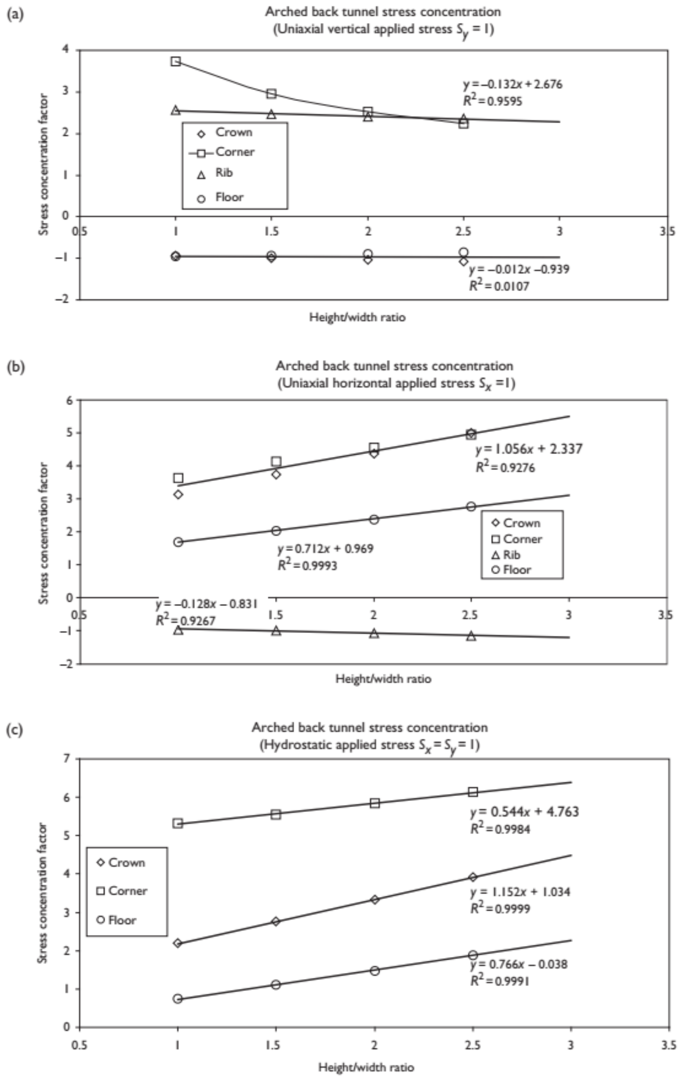


Figure 2 Stress concentration trends at key points of single arched back tunnel sections (a) vertical load, (b) horizontal load, and (c) hydrostatic load.

In case (a),

K (floor) = (1)[stress concentration from Figure 2a] + (1/4)[stress concentration from Figure 2b]

$$K_{\mathrm{f}} = (1)[(-0.012)(H/W) - 0.939] + (1/4)[(0.712)(H/W) + 0.969]$$

$$= (1)[-0.963] + (1/4)[2.393]$$

$$K_{\mathrm{f}} = -0.37$$

K (crown) = (1)[stress concentration from Figure 2a] + (1/4)[stress concentration from Figure 2b]

$$K_{cr} = (1)[(-0.012)(2) - 0.939] + (1/4)[(1.056)(2) + 2.337]$$
$$= (1)[-0.963] + (1/4)[4.449]$$
$$K_{cr} = 0.15$$

In case(b)

$$K_f = (1/4)[-0.963] + (1)[2.393] = 2.15$$
$$K_{cr} = (1/4)[-0.963] + (1)[4.449] = 4.21$$

The orientation effect is considerable. When the major compression is vertical a noticeable tension is induced in the floor, but only a small compression occurs in the crown. When the major compression is horizontal, the floor tension and crown experience substantial compressive stress concentrations.

Example 2 An arched back tunnel is planned with a height-to-width ratio of two. Estimate stress concentration factors in the ribs when the pre-excavation principal stress ratio M = 1 / 4 and (a) the vertical stress is S1, (b) the horizontal stress is S1. These are the same conditions given in Example 1.

Solution: In case (a) for the rib,

$$K_r = (1)[(-0.132)(2) + 2.676] + (1/4)[(-0.128)(2) - 0.831]$$
$$= (1)[2.41] + (1/4)[-1.087]$$
$$K_r = 2.14$$

In case (b) for the rib,

$$K_r = (1/4)[2.41] + (1)[-1.087]$$
$$K_r = -0.48$$

Again, there is a strong orientation effect as the rib stress concentration changes from compression to tension with a change in principal stress direction from vertical and parallel to the long axis of the tunnel (height dimension) to horizontal and perpendicular to the long dimension of the tunnel.

Estimation of the corner stress, which is often the peak compressive stress at the tunnel wall, is less easily done because of the change in orientation with applied stresses and height-to-width ratio. One might suppose that the principal stress at the corner is closely related to the floor and rib stresses very near the corner. These stresses appear as sharp spikes in the stress distributions plotted in the figures. The floor and rib spikes, Fx and Rx under uniaxial horizontal loading (Sy = 0, Sx = 1) are linear in the height-to-width ratio (H/W), while under uniaxial vertical loading (Sy = 1, Sx = 0), the floor and rib spikes, Fy and Ry are linear in the reciprocal ratio (W/H) as shown in Figures 3 a, b. The trend lines indicate a very good linear fit, so the trend line equations may be used to estimate floor and rib spikes under combined loading. These spikes are generally compressive.

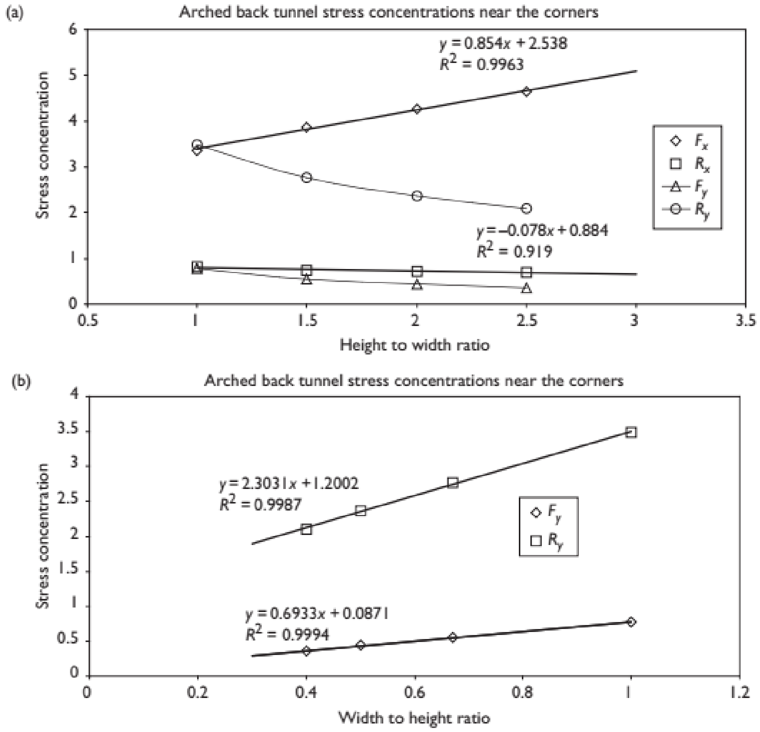


Figure 3 Arched back tunnel stress concentration trends near the bottom or floor corners, R = rib, F = floor, x = horizontal, y =vertical (a) horizontal and vertical loading results vs H/W, (b) vertical loading results vs the reciprocal, W/H.

Further examination of floor and rib stress spikes near the corner suggests a simple, empirical procedure for estimating the peak stress at the corner (K). The procedure is to add 10% of the floor and rib spikes to the greater of the two. A smaller addition at lower height-to-width ratios and a somewhat greater percentage addition at higher height-to-width ratios than the 2 used would improve these estimates. However, the estimates produced by the 10% rule are certainly of sufficient accuracy to allow for a preliminary analysis of safety concerning strength failure at points of relatively high-stress concentration at the corners of an arched back tunnel floor.

Example 3 Consider the applied stress: $Sy = 1$ and $Sx = 1/4$ and a height-to-width ratio of 2, as in Examples 1 and 2. Estimation of corner stress concentration may be done using data from Figure 3.

The floor and rib corner stresses induced by the vertical stress are:

$Ry = 2.303(1/2) + 1.200$ and

$Fy = 0.693(1/2) + 0.087$, that is,

$Ry = 2.35$ and

$Fy = 0.43$.

Under $Sx = 1$ or full horizontal stress,

$Rx = - 0.078(2) + 0.884$ and

$Fx = 0.854(2) + 2.54$, that is,

$Rx = 0.73$ and

$Fx = 15$.

Superposition of the y and x loading give

$Ry + Rx = (1)\,(2.35) + (1/4)\,(0.73) = 2.53$, and

$Fy + Fx = (1)\,(0.43) + (1/4)\,(15) = 1.49$.

In consideration of the 10% empirical estimation suggested,

$$K(\text{corner}) = 2.53 + (0.1)(2.53 + 1.49) = 2.93.$$

Example 4 Consider the stress concentration factors obtained in Examples 1, 2, and 3 for a tunnel with a height-to-width ratio of two in a rock mass with unconfined compressive strength and tensile strength of 50 and 5 MPa, respectively. Estimate safety factors concerning compression and tension with $M = 1/4$ and $S1$ is vertical. Assume a tunnel depth of 1,000 m where $S1 = 20$ MPa. Sketch the results. **Solution:** From the previous examples in case (a) – vertical,

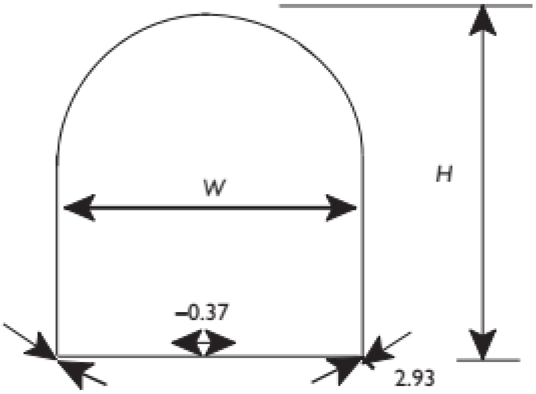
$$\begin{aligned} K(\text{floor}) &= -0.37 \\ K(\text{crown}) &= 0.15 \\ K(\text{rib}) &= 2.14 \\ K(\text{corner}) &= 2.93 \end{aligned}$$

Safety factors in compression and tension are

$$FS_c = \frac{50}{(2.93)(20)} = 0.85$$

$$FS_t = \frac{5}{(0.37)(20)} = 0.66$$

which indicates a potential for compressive failure at the floor corners and tensile failure at the floor center for the planned tunnel. The rib has a safety factor of 1.16 which is low and suggests the need to consider reinforcement.



An arched back tunnel is not a favorable shape in regions of high horizontal stress, (e.g. where Sx = 1 and Sy = 0). High horizontal stress is often encountered in relatively deep underground mines. The reasons are evident in Figures 1 c, f, and i which show high compressive stress concentrations over the back and floor and extensive regions of high tensile stress in the ribs. This inference is not surprising in consideration of the fact that the long dimension of the opening is perpendicular to the major pre-tunnel principal stress (compressive is positive here). Recall that the rule of thumb for minimizing stress concentration is to orient the long axis of the cross-section parallel to the major principal stress before excavation. This rule also applies to the hybrid shape of the arched back tunnel. Of course, in the case of hydrostatic pre-excavation stress (Sx = 1, Sy = 1, Figures 1d, g, j), there is no preferred orientation.

Arching the back of a rectangular section does produce a favorable tunnel shape in cases where the pre-excavation stress is largely vertical (e.g. where Sx = 0, Sy = 1). This is often the case in shallow-ground highways and railroad tunnels. Data in Figures 1 b, e, and h show arched back tunnel sections with H /W ≥ 1 in favorable orientation. Comparisons of stress distributions in backs and floors show peak tensions of similar magnitude at crown points and floor centerlines. Thus, there is no advantage to an arched back relative to peak tension. However, the high floor tension extends nearly the entire width of the section, almost from rib to rib, while the high back tension is contained in a small region adjacent to the crown point. Most of the back is in compression that reaches a high value over the shoulder of the section near the ribs. The ribs are stressed almost uniformly in compression from the shoulders to near the floor corners where the compression rises to a peak. One benefit of arching a tunnel back under high vertical stress is therefore reducing the extent of the tensile zone near the crown. A related benefit follows from the compression induced in the back which may assist in mobilizing resistance to frictional slip on joints and fractures that are usually present. The floor tension is a disadvantage and may be a threat to stability depending on the associated safety factor. If floor safety is not assured, then an alternative design is indicated, just as unstable ribs under high horizontal stress indicate an alternative design should be considered. An alternative design may retain the original proposed shape but then include additional support and reinforcement.