

# FLUID MECHANICS

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# LECTURE 3

- 1- Compressibility of Fluids  
Bulk Modulus
- 2- Vapor Pressure.
- 3- Surface Tension



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# 1- Compressibility of Fluids (انضغاط السوائل)

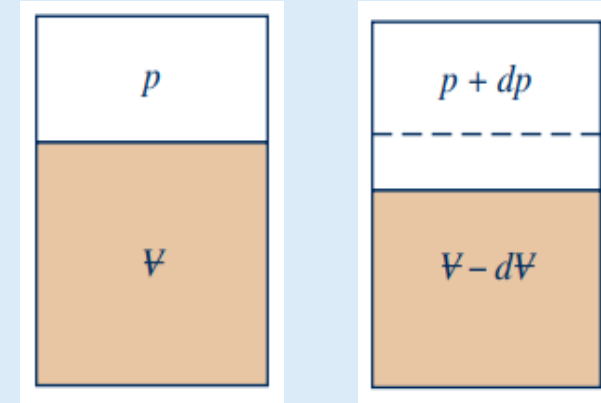
## - Bulk Modulus ( $E_v$ ) (معامل الحجم)

An important question to answer when considering the behavior of a particular fluid is how easily can the volume (and thus the density) of a given mass of the fluid be changed when there is a change in pressure? That is, how compressible is the fluid?

معامل الحجم: مقاومة المادة للضغط المنتظم.

A property that is commonly used to characterize compressibility is the **bulk modulus**,  $E_v$ , defined as

$$E_v = -\frac{dp}{dV/V}$$



Where,

$dp$  is the differential change in pressure needed to create a differential change in volume,  $dV$ , of a volume  $V$ .

The negative sign is included since an increase in pressure will cause a decrease in volume. Since a decrease in volume of a given mass, will result in an increase in density, is also expressed as

$$E_v = \frac{dp}{d\rho/\rho}$$

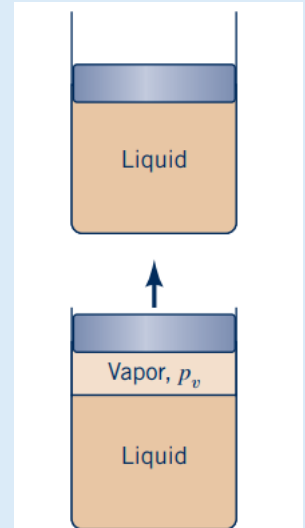
the bulk modulus has dimensions of pressure,  $FL^{-2}$ .

In BG units, the bulk modulus is  $lb/in^2$  (psi). And in SI units is  $N/m^2$  (Pa).

## 2- Vapor Pressure (ضغط البخار)

liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere.

Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere. If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules. When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed them, **vapor pressure,  $P_v$** .



## ***Vapor pressure does not depend upon:***

- ***Amount of liquid.***
- ***Volume of container.***
- ***Surface area of the liquid.***

## ***Vapor pressure affected by:***

- ***Vapor pressure increase with Temperature.***
- ***Stronger the intermolecular forces the lower the Vapor pressure.***

### 3- Surface Tension (الشد السطحي)

At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface that cause the surface to behave as if it were a “skin” or “membrane” stretched over the fluid mass.

Although such a skin is not actually present, this conceptual analogy allows us to explain several commonly observed phenomena.

For example, a steel needle or a razor blade will float on water if placed gently on the surface because the tension developed in the hypothetical skin supports it.



**Surface tension:** The intensity of the molecular attraction per unit length along any line in the surface. It is designated by the Greek symbol ( $\sigma$ ) (sigma).

The surface tension is affected by a temperature as well as the other fluid it is in contact with at the interface.

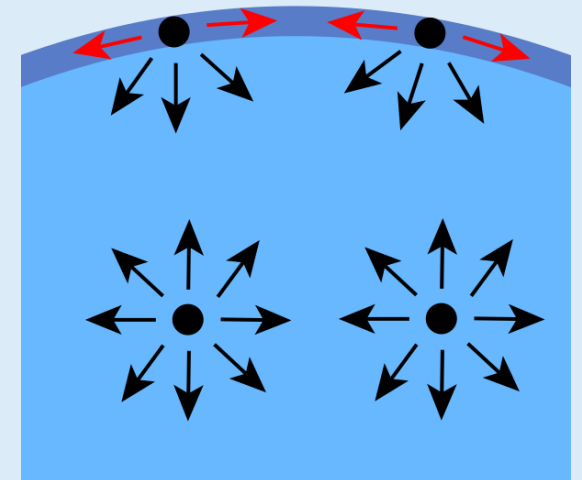
قوة مؤثرة عموديا على وحدة الطول من الطبقة السطحية للسائل

$$\sigma = F/L$$

The dimensions of surface tension are  $FL^{-1}$ .

The unit of surface tension is **lb/ft** in BG unit system, and **N/m** in SI unit system.

Table 7 & 8 shows the values of surface tension for different liquids.





## Table 9

Physical Properties of Water (BG Units)<sup>a</sup>

Temperature (°F)	Density, $\rho$ (slugs/ft <sup>3</sup> )	Specific Weight <sup>b</sup> , $\gamma$ (lb/ft <sup>3</sup> )	Dynamic Viscosity, $\mu$ (lb·s/ft <sup>2</sup> )	Kinematic Viscosity, $\nu$ (ft <sup>2</sup> /s)	Surface Tension <sup>c</sup> , $\sigma$ (lb/ft)	Vapor Pressure, $p_v$ [lb/in. <sup>2</sup> (abs)]	Speed of Sound <sup>d</sup> , $c$ (ft/s)
32	1.940	62.42	3.732 E - 5	1.924 E - 5	5.18 E - 3	8.854 E - 2	4603
40	1.940	62.43	3.228 E - 5	1.664 E - 5	5.13 E - 3	1.217 E - 1	4672
50	1.940	62.41	2.730 E - 5	1.407 E - 5	5.09 E - 3	1.781 E - 1	4748
60	1.938	62.37	2.344 E - 5	1.210 E - 5	5.03 E - 3	2.563 E - 1	4814
70	1.936	62.30	2.037 E - 5	1.052 E - 5	4.97 E - 3	3.631 E - 1	4871
80	1.934	62.22	1.791 E - 5	9.262 E - 6	4.91 E - 3	5.069 E - 1	4819
90	1.931	62.11	1.500 E - 5	8.233 E - 6	4.86 E - 3	6.979 E - 1	4960
100	1.927	62.00	1.423 E - 5	7.383 E - 6	4.79 E - 3	9.493 E - 1	4995
120	1.918	61.71	1.164 E - 5	6.067 E - 6	4.67 E - 3	1.692 E + 0	5049
140	1.908	61.38	9.743 E - 6	5.106 E - 6	4.53 E - 3	2.888 E + 0	5091
160	1.896	61.00	8.315 E - 6	4.385 E - 6	4.40 E - 3	4.736 E + 0	5101
180	1.883	60.58	7.207 E - 6	3.827 E - 6	4.26 E - 3	7.507 E + 0	5195
200	1.869	60.12	6.342 E - 6	3.393 E - 6	4.12 E - 3	1.152 E + 1	5089
212	1.860	59.83	5.886 E - 6	3.165 E - 6	4.04 E - 3	1.469 E + 1	5062

<sup>a</sup>Based on data from *Handbook of Chemistry and Physics*, 69th Ed., CRC Press, 1988. Where necessary, values obtained by interpolation.

<sup>b</sup>Density and specific weight are related through the equation  $\gamma = \rho g$ . For this table,  $g = 32.174 \text{ ft/s}^2$ .

<sup>c</sup>In contact with air.

<sup>d</sup>Based on data from R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., Inc., New York, 1984.

## Table 10

Physical Properties of Water (SI Units)<sup>a</sup>

Temperature (°C)	Density, $\rho$ (kg/m <sup>3</sup> )	Specific Weight <sup>b</sup> , $\gamma$ (kN/m <sup>3</sup> )	Dynamic Viscosity, $\mu$ (N·s/m <sup>2</sup> )	Kinematic Viscosity, $\nu$ (m <sup>2</sup> /s)	Surface Tension <sup>c</sup> , $\sigma$ (N/m)	Vapor Pressure, $p_v$ [N/m <sup>2</sup> (abs)]	Speed of Sound <sup>d</sup> , $c$ (m/s)
0	999.9	9.806	1.787 E - 3	1.787 E - 6	7.56 E - 2	6.105 E + 2	1403
5	1000.0	9.807	1.519 E - 3	1.519 E - 6	7.49 E - 2	8.722 E + 2	1427
10	999.7	9.804	1.307 E - 3	1.307 E - 6	7.42 E - 2	1.228 E + 3	1447
20	998.2	9.789	1.002 E - 3	1.004 E - 6	7.28 E - 2	2.338 E + 3	1481
30	995.7	9.765	7.975 E - 4	8.009 E - 7	7.12 E - 2	4.243 E + 3	1507
40	992.2	9.731	6.529 E - 4	6.580 E - 7	6.96 E - 2	7.376 E + 3	1526
50	988.1	9.690	5.468 E - 4	5.534 E - 7	6.79 E - 2	1.233 E + 4	1541
60	983.2	9.642	4.665 E - 4	4.745 E - 7	6.62 E - 2	1.992 E + 4	1552
70	977.8	9.589	4.042 E - 4	4.134 E - 7	6.44 E - 2	3.116 E + 4	1555
80	971.8	9.530	3.547 E - 4	3.650 E - 7	6.26 E - 2	4.734 E + 4	1555
90	965.3	9.467	3.147 E - 4	3.260 E - 7	6.08 E - 2	7.010 E + 4	1550
100	958.4	9.399	2.818 E - 4	2.940 E - 7	5.89 E - 2	1.013 E + 5	1543

<sup>a</sup>Based on data from *Handbook of Chemistry and Physics*, 69th Ed., CRC Press, 1988.

<sup>b</sup>Density and specific weight are related through the equation  $\gamma = \rho g$ . For this table,  $g = 9.807 \text{ m/s}^2$ .

<sup>c</sup>In contact with air.

<sup>d</sup>Based on data from R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., Inc., New York, 1984.

Table 9 & 10 show the surface tension of water at various temperature.

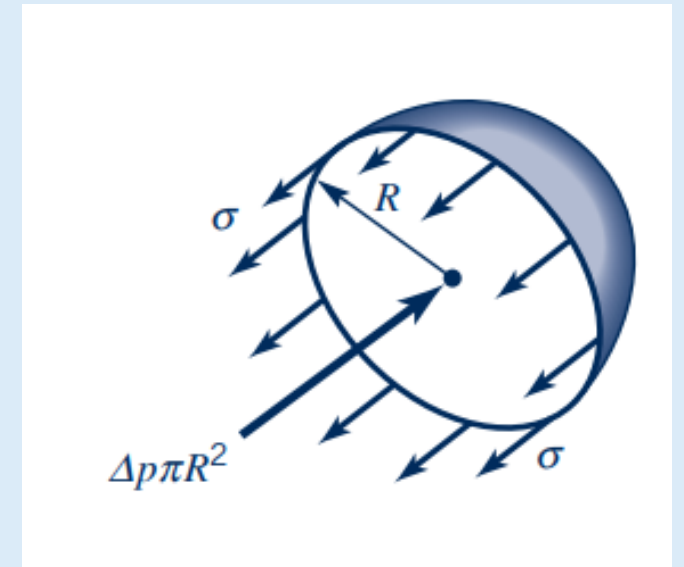
The pressure inside a drop of fluid can be calculated using the free-body diagram as shown in figure. If the spherical drop is cut in half, the force developed around the edge due to surface tension is  $2\pi R\sigma$

This force must be balanced by the pressure difference,  $\Delta P$ , between the internal pressure,  $P_i$ , and the external pressure  $P_e$ , acting over the circular area,  $\pi R^2$ . Thus,

$$2\pi R\sigma = \Delta P\pi R^2$$

Or

$$\Delta P = P_i - P_e = \frac{2\sigma}{R}$$

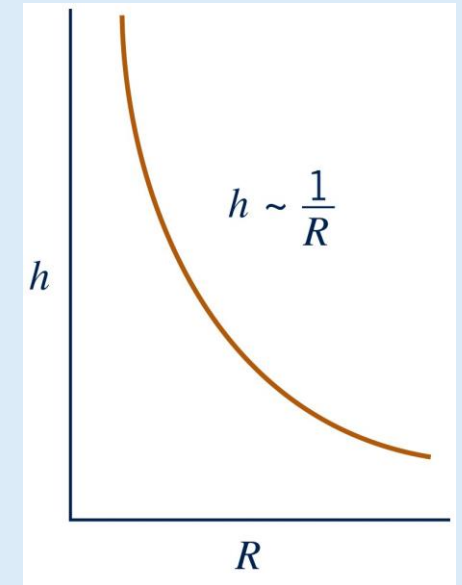


Forces acting on one-half of a liquid drop

Among common phenomena associated with surface tension is the rise (or fall) of a liquid in a capillary tube.

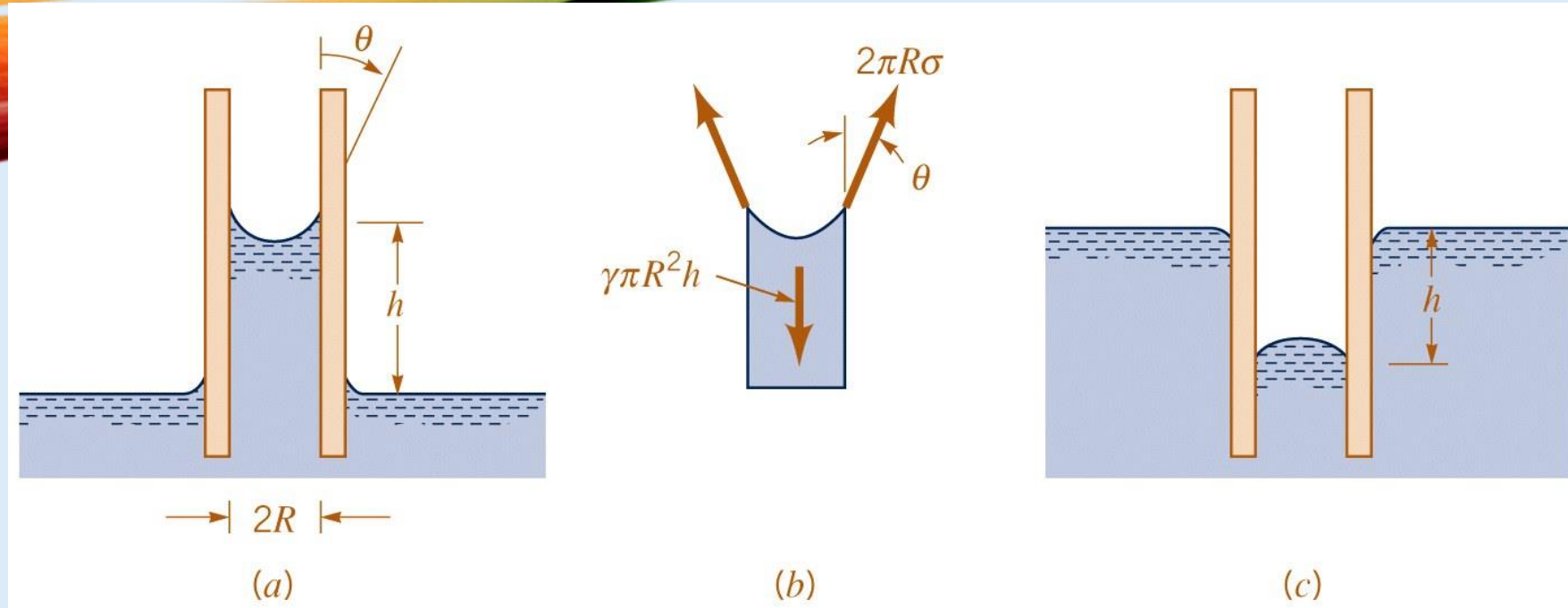
If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube.

In this situation we have a liquid–gas–solid interface. For the case illustrated there is an attraction (adhesion **الالتصاق**) between the wall of the tube and liquid molecules which is strong enough to overcome the mutual attraction (cohesion **التماسك**) of the molecules and pull them up the wall.



قوة التماسك : وهي قوة التجاذب الناشئة بين جزيئات السائل نفسه. Adhesion

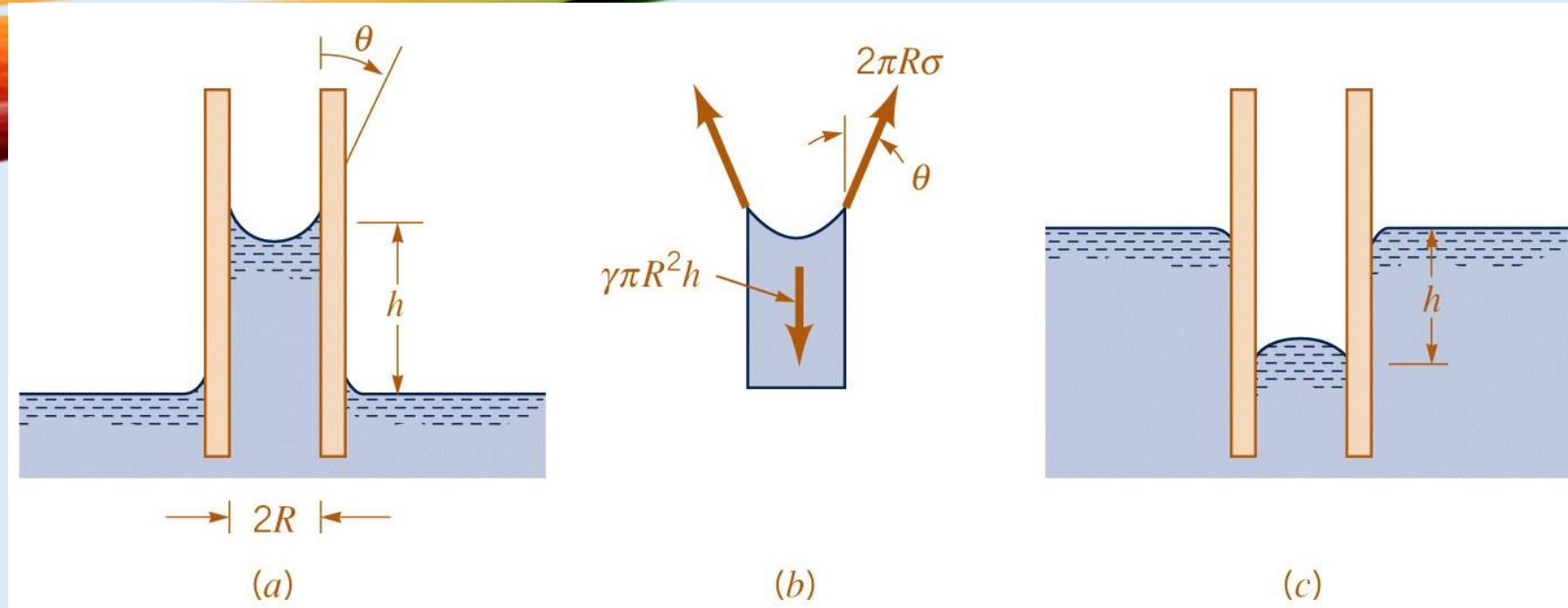
قوة الالتصاق : وهي قوة التجاذب الناشئة بين جزيئات السائل وجزيئات الإناء الحاوي على السائل. Cohesion



To measure the height  $h$ :-

The height,  $h$ , is governed by the value of the surface tension,  $\sigma$ , the tube radius,  $R$ , the specific weight of the liquid  $\gamma$ , and the angle of contact,  $\vartheta$ , between the fluid and tube.

From the free-body diagram as shown in (b), we see that **the vertical force due to the surface tension** is equal to  $2\pi R \sigma \cos\theta$ , and **the weight** is  $\gamma \pi R^2 h$  and these two forces must balance for equilibrium.



Thus, so that the height is given by the relationship

$$\gamma \pi R^2 h = 2 \pi R \sigma \cos \theta$$

So,

$$h = \frac{2 \sigma \cos \theta}{\gamma R}$$

The angle of contact is a function of both the liquid and the surface. For water in contact with clean glass  $\theta$  almost  $0^\circ$ .

If adhesion of molecules to the solid surface is weak compared to the cohesion between molecules, the liquid will not wet the surface and the level in a tube placed in a nonwetting liquid will actually be depressed, as shown in Fig.(c).

Mercury is a good example of a nonwetting liquid when it is in contact with a glass tube.

For nonwetting liquids, the angle of contact is greater than  $90^\circ$ , and for mercury in contact with clean glass  $\theta$  almost  $130^\circ$  .

## EXAMPLE 1.8 Capillary Rise in a Tube

**GIVEN** Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube.

**FIND** What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than  $h = 1.0$  mm?

### SOLUTION

From Eq. 1.22

$$h = \frac{2\sigma \cos \theta}{\gamma R}$$

so that

$$R = \frac{2\sigma \cos \theta}{\gamma h}$$

For water at 20 °C (from Table 10)  $\sigma = 0.0728$  N/m and  $\gamma = 9.789$  kN/m<sup>3</sup>. Since  $\theta \approx 0^\circ$  it follows that for  $h = 1.0$  mm,

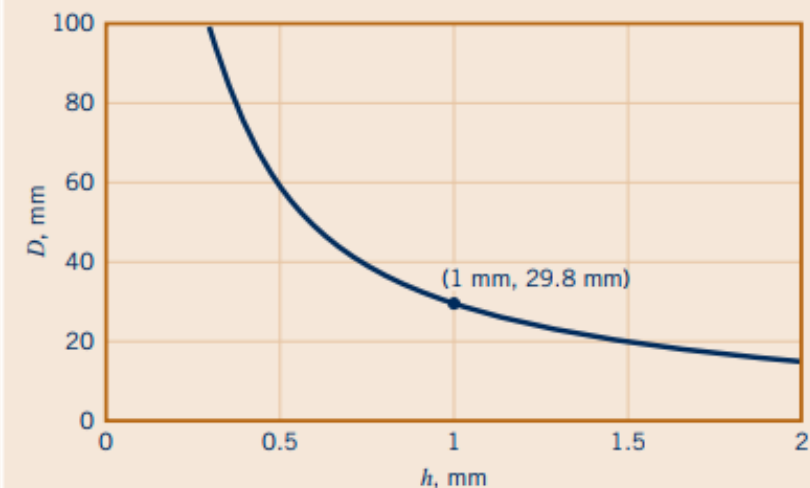
$$\begin{aligned} R &= \frac{2(0.0728 \text{ N/m})(1)}{(9.789 \times 10^3 \text{ N/m}^3)(1.0 \text{ mm})(10^{-3} \text{ m/mm})} \\ &= 0.0149 \text{ m} \end{aligned}$$

and the minimum required tube diameter,  $D$ , is

$$D = 2R = 0.0298 \text{ m} = 29.8 \text{ mm} \quad (\text{Ans})$$

**COMMENT** By repeating the calculations for various values of the capillary rise,  $h$ , the results shown in Fig. E1.8 are obtained.

Note that as the allowable capillary rise is decreased, the diameter of the tube must be significantly increased. There is always some capillarity effect, but it can be minimized by using a large enough diameter tube.



■ Figure E1.8