Strain الانفعال

$$\epsilon = \frac{\sigma}{E} = \frac{\text{الاجهاد}}{\text{معامل المرونة}}$$

الانفعال

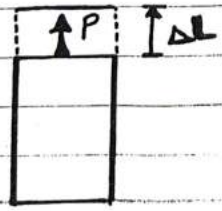
$$\epsilon = \frac{\Delta L}{L_o} = \frac{\text{مقدار التغير بالطول}}{\text{الطول الاصلي}}$$

$$\Delta L = \frac{P \cdot L}{E \cdot A}$$

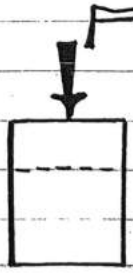
where P = axial force (N)
L = Length (mm)
E = modolus of elasteity (N/mm^2)
A = cross-section area (mm^2)

ΔL = deformation , change of Length (mm)
التشوه التغير بالطول

elongation الاستطالة



Shortining التقصر



$Mpa = \frac{N}{mm^2}$

$\frac{MN}{m^2} = \frac{N}{mm^2}$

$Gpa = 10^3 \frac{N}{mm^2}$

$kpa = 10^{-3} \frac{N}{mm^2}$

$pa = 10^{-6} \frac{N}{mm^2}$

$m = 1000mm$

$cm = 10mm$

$KN = 1000\ N$

$MN = 10^6\ N$

$$\Delta L = 0.1421\,\gamma \left[26180 \ln y - 26180 \frac{y^{-1}}{-1} \right]_{0.25}^{1.25}$$

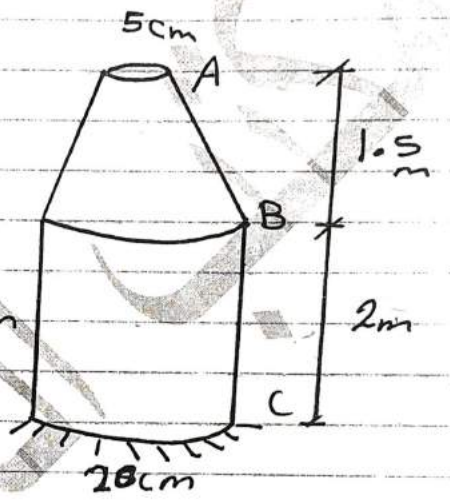
H.w

Ex:.

$\gamma = 10 \frac{kN}{m^3}$

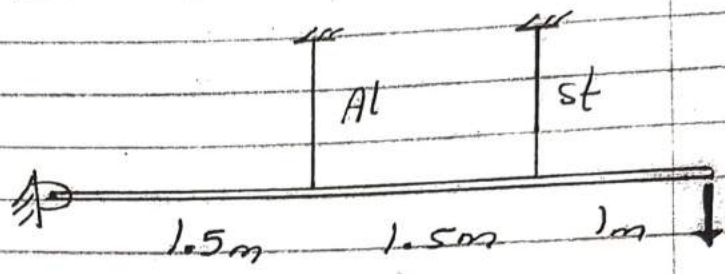
$E = 200 \frac{kN}{mm^2}$

Find the deformation at A ?



H.w

Ex. 13 Find the stresses in steel and Aluminume



Given	steel	Aluminum
	$E = 200\ Gpa$	$E = 70\ Gpa$
	$L = 1m$	$L = 1.2m$
	$Area = 200mm^2$	$Area = 300mm^2$

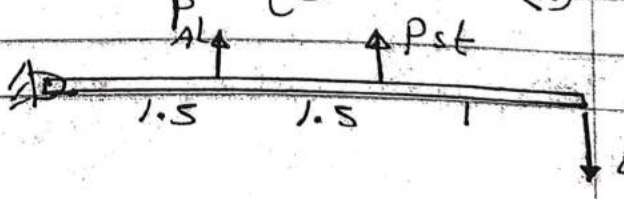
Sol:-

لاحظ أن عدد المجاهيل (4)
وعدد معادلات التوازن (3) $\Sigma F_x=0, \Sigma F_y=0, \Sigma M_z=0$

1. عدد المجاهيل = عدد معادلات التوازن في هذه الحالة تستخرج ردود الأفعال بدون الحاجة الى معادلة التشوه.
2. عدد المجاهيل أكبر من عدد معادلات التوازن. في هذه الحالة لا نستطيع أن نستخرج ردود الأفعال بأستخدام معادلات التوازن الطبيعية لذلك سنحتاج معادلات أضافية نستخرج من التشوه.

خطوات الحل:-

① نستخرج معادلة التوازن.

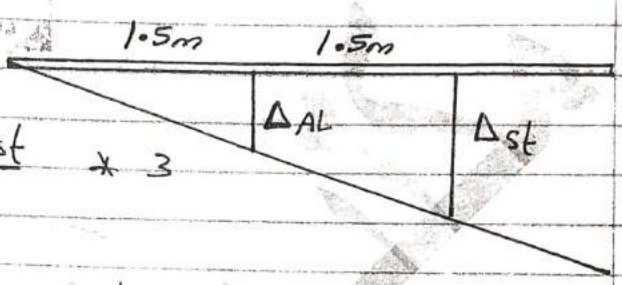


$+\circlearrowleft \Sigma M_A = 0$

$-P_{Al}(1.5) - P_{st}(3) + 40 * 4 = 0$

$1.5P_{AL} + 3P_{st} = 160$ ---- ①

⑤ نرسم المثلث



$$\frac{\Delta AL}{1.5} = \frac{\Delta st}{3} \times 3$$

$$2\,\Delta AL = \Delta st$$

$$2 * \frac{P_{AL} * 10^3 * 1200}{70 * 10^3 * 300} = \frac{P_{st} * 10^3 * 1000}{200 * 10^3 * 200}$$

$$P_{AL} = 0.21875\,P_{st} \quad ---- ②$$

نعوض معادلة ② في معادلة ①

$$1.5 * 0.21875\,P_{st} + 3P_{st} = 160$$

$$3.328\,P_{st} = 160$$

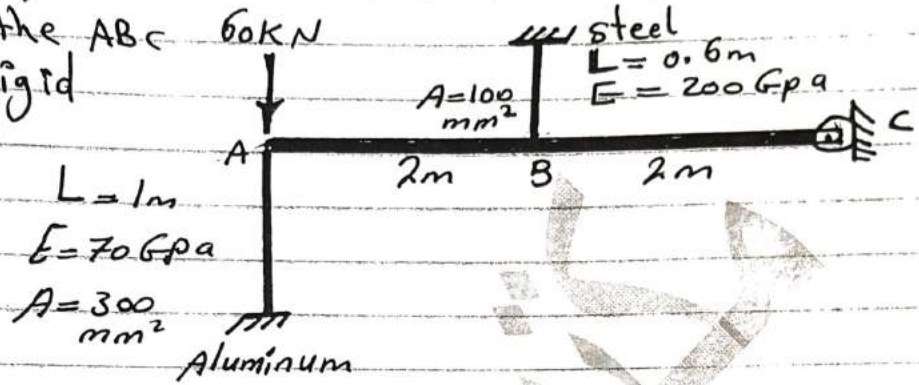
$$P_{st} = 48.07\ kN$$

$$\therefore P_{AL} = 0.21875\,P_{st} = 0.21875 * 48.07 = 10.5\ kN$$

$$\sigma_{AL} = \frac{P_{AL}}{area(AL)} = \frac{10.5 * 10^3}{300} = 35.05\ N/mm^2$$

$$\sigma_{st} = \frac{P_{st}}{area(st)} = \frac{48.07 * 10^3}{200} = 160.2\ N/mm^2$$

Ex 14 Find the stress in steel and Aluminum if the ABC is rigid bars?



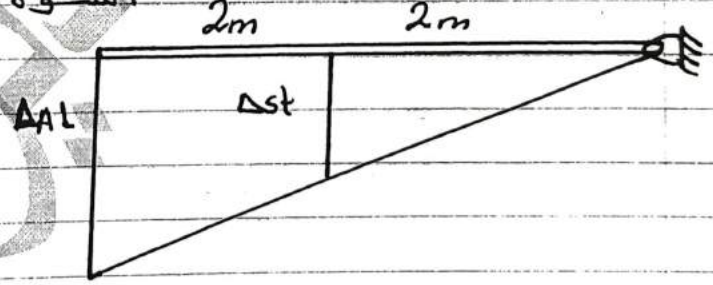
Sol. Unknown 4 > Equation of Equilibrium 3

we need the equation of deformation

تحتاج معادلة واحدة من التشوه.

* دائماً نبدأ بالتشوه.

** في هذا السؤال معادلة التشوه تكون تشابه مثلثات



$$\frac{\Delta AL}{4} = \frac{\Delta st}{2}$$

$$* 4$$

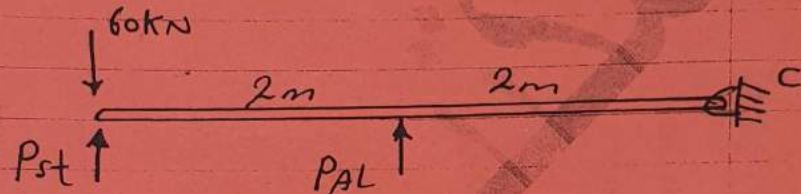
$$\Delta AL = 2\,\Delta st$$

$$\frac{PAL \cdot L}{E_{AL} \cdot A} = 2\,\frac{Pst \cdot L}{Est \cdot A}$$

$$\frac{PAL * 10^3 * 1000}{70 * 10^3 * 300} = \frac{Pst * 10^3 * 600}{200 * 10^3 * 100}$$

$P_{AL} = 0.63\, P_{st}$ ____ ①

- بعد الأنتهاء من معادلات الشوه نأخذ معادلات التوازن.
- في هذا السؤال ستكون معادلة التوازن هو أخذ عزم بالمسند وذلك للتخلص من ذلك المسند.
- رد الفعل يكون عكس الشوه.



$\curvearrowleft + \sum M_C = 0$

$P_{st}(4) + P_{AL}(2) - 60 * 4 = 0$

$4\, P_{st} + 0.63\, P_{st}(2) = 240$

$P_{st} = 45.6\ KN$

$P_{AL} = 0.63 P_{st} = 28.74\ KN$

$$\sigma_{AL} = \frac{P_{AL}}{A} = \frac{28.74 * 10^3}{300} = 95.8\ N/mm^2$$

$$\sigma_{st} = \frac{P_{st}}{A} = \frac{45.6 * 10^3}{100} = 456\ N/mm^2$$

الأزاحة

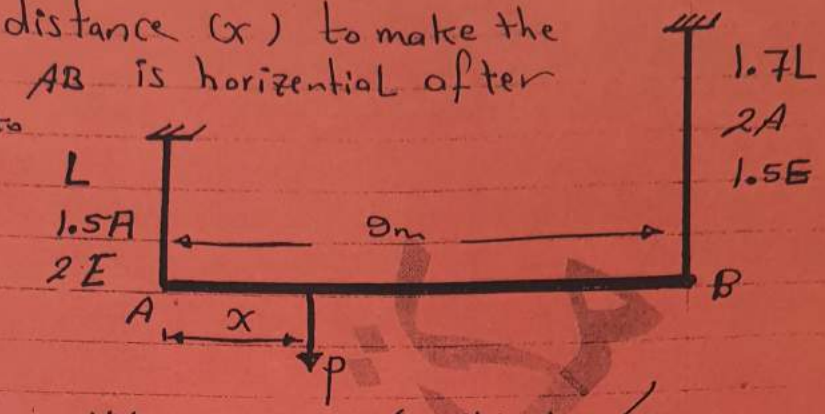
Find the displacement at B ?

الأزاحة = الشوه بالاتجاه العمودي.

ملاحظة: عندما يطلب أزاحة لنقطة وكانت هذه النقطة مرتبطة بسلك عمودي فأن الأزاحة هنا تساوي الشوه.

$$\Delta_B = \Delta_{st} = \frac{P_{st} \cdot L}{EA} = \frac{45.6 * 10^3 * 600}{200 * 10^3 * 100} = 1.368\ mm$$

Ex. 15 Find the distance (x) to make the rigid bar AB is horizontal after subject to the force P?

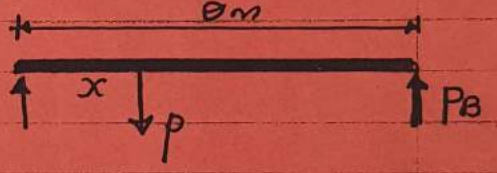


ذكر بالسؤال أن العارضة الأفقية AB سوف تبقى أفقية بعد تسليط القوة (P) هذا يعني أن

$$\Delta A = \Delta B$$

$$\frac{P_A \cdot L}{2E * 1.5A} = \frac{P_B \cdot (1.7L)}{1.5E * 2A}$$

$$P_A = 1.7\ P_B \quad ---\ (1)$$ علاقة القوة



$$\circlearrowleft^{+} \Sigma M_A = 0$$

$$P \cdot x - P_B(9) = 0$$

$$P_B = \frac{P \cdot x}{9} \quad (2)$$

$$\Sigma F_y = 0 \qquad P_A + P_B - P = 0$$

$$P_A + \frac{P \cdot x}{9} - P = 0 \qquad P_A = P\left(1 - \frac{x}{9}\right) \quad (3)$$

(1) معادلة و(2) معادلة و(3) معادلة نعوض

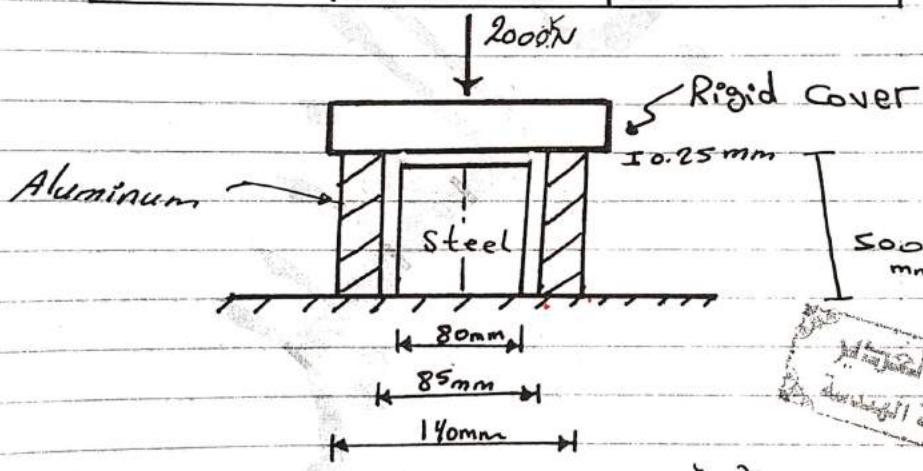
$$P_A = 1.7\ P_B$$

$$P\left(1 - \frac{x}{9}\right) = 1.7\ \frac{P \cdot x}{9}$$

$$1 = \frac{1.7x}{9} + \frac{x}{9} \rightarrow x = 3.83\ m$$

Q16:- Aright circular cylinder made of aluminum is surrounding another cylinder made of steel as shown in fig. The force 200kN is applied throught the rigid cover. If the aluminum cylinder was before application of the Load Longer than the steel cylinder by 0.25mm Calculate the vertical stress in each cylinder?

Property	steel	Aluminum
E	200×10^3 MN/m²	70×10^3 MN/m²



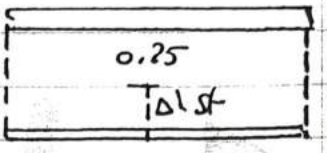
Sol:- لاحظ أن الـ cover في البداية يؤثر على الألمنيوم فقط لذلك سنحسب Δl لـ Al حتى لو كان - اقل من 0.25mm فان Al وحده يتأثر
واذا كان - أكبر من 0.25mm فان Al و st يتأثران.

$$\Delta l_{Al} = \frac{P.L}{E.A} = \frac{200\times10^3\times500}{70\times10^3\times\frac{\pi}{4}(140^2-85^2)} = 1.469\ mm > 0.25\ mm$$

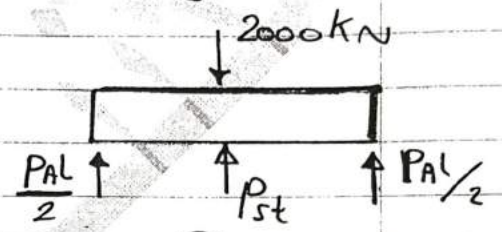
Al و st يتأثران

$$\Delta_{Al} = \Delta l_{st} + 0.25$$

$$\frac{P_{Al} * 10^3 * 500}{70 * 10^3 * (140^2 - 85^2)\frac{\pi}{4}} = \frac{P_{st} * 10^3 * 500}{200 * 10^3 * \frac{\pi}{4}(80)^2} + 0.25$$



$$7.35 * 10^{-4} P_{Al} = 4.9736 P_{st} + 0.25 \quad \text{---} (1)$$



$$\Sigma F_y = 0$$

$$P_{Al} + P_{st} = 2000$$

$$P_{Al} = 2000 - P_{st} \quad \text{---} (2)$$

(1) هي معادلة (2) تعويض معادلة

$$7.35 * 10^{-4} (2000 - P_{st}) = 4.9736 P_{st} + 0.25$$

$$10^{-3} * 1.232 P_{st} = 1.22 \Rightarrow P_{st} = 990.26 \text{ kN}$$

$$\therefore P_{Al} = 2000 - P_{st} = 2000 - 990.26$$

$$P_{AL} = 1009.75 \text{ kN}$$

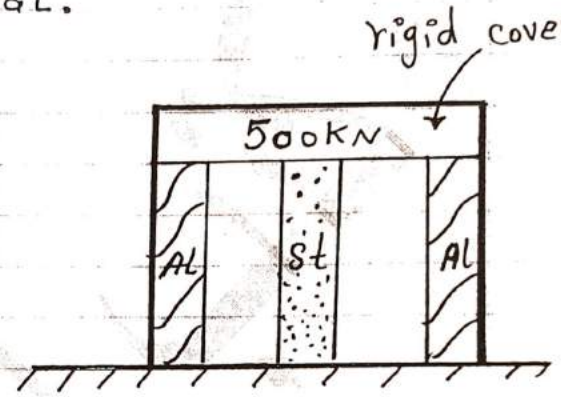
$$\sigma_{Al} = \frac{P_{Al}}{A} = \frac{1009.75 * 10^3}{\frac{\pi}{4}(140^2 - 85^2)}$$

$$\sigma_{st} = \frac{P_{st}}{A} = \frac{990.26 * 10^6}{\frac{\pi}{4}(80)^2}$$

Ex.17 Two aright circular cylinder made of aluminum and another cylinder made of steel as shown in fig. the force 500kN is applied throughth therigid cover. Calculate the stress in each material.

Given :-

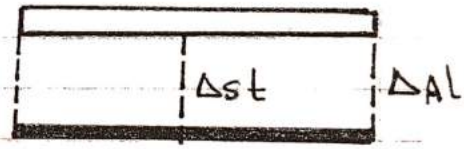
properties	Steel	Aluminum
$E(\frac{kN}{mm^2})$	200	70
Dimeter (mm)	30	20
Length (m)	1.2	1.2



Sol :- لأن المثكل متناظر يكون النزول متساوي

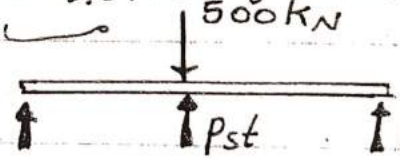
$$\Delta_{AL} = \Delta_{st}$$

$$\frac{P_{AL} * 10^3 * 1200}{70 * 10^3 * \frac{\pi}{4}(20)^2} = \frac{P_{st} * 10^3 * 1200}{200 * 10^3 * \frac{\pi}{4}(30)^2}$$



ملاحظة :- حتى هذا السؤال لاتستطيع أن تطبق معادلة العزم لعدم وجود مسافات.

$$P_{AL} = 0.155 P_{st} \quad \text{---------- (1)}$$



$$\Sigma F_y = 0$$

$$2P_{AL} + P_{st} - 500 = 0$$

$$2P_{AL} + P_{st} = 500$$

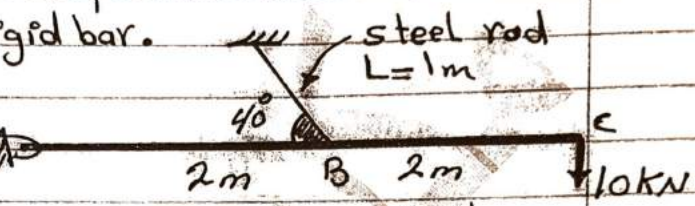
$$2 * 0.155 P_{st} + P_{st} = 500 \quad \therefore \boxed{P_{st} = 381.68 kN}$$

$$\therefore P_{AL} = 0.155 P_{st} = 0.155 * 381.68 \quad \boxed{P_{AL} = 59.16 \frac{kN}{}}$$

$$\sigma_{AL} = \frac{P_{AL}}{A_{AL}} = \frac{59.16 * 10^3}{\frac{\pi}{4}(20)^2} \quad , \quad \sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{381.68 * 10^3}{\frac{\pi}{4}(30)^2}$$

Ex.18 Find the displacement movement at C?

ABC is rigid bar.



$E_{st} = 210 * 10^3 \frac{N}{mm^2}$

$Area = 300 mm^2$.

* عندما يطلب أزاحة لا نستطيع أستخراجها مباشرة وذلك لأن الأزاحة ليس لها قانون لذلك سنستخرج الشوه الموجود بالاسلاك من قانون $\frac{P.L}{E.A}$ وبعد ذلك نربط علاقة بين الشوه والأزاحة.

$$\overset{+}{\curvearrowright} \sum M_A = 0$$

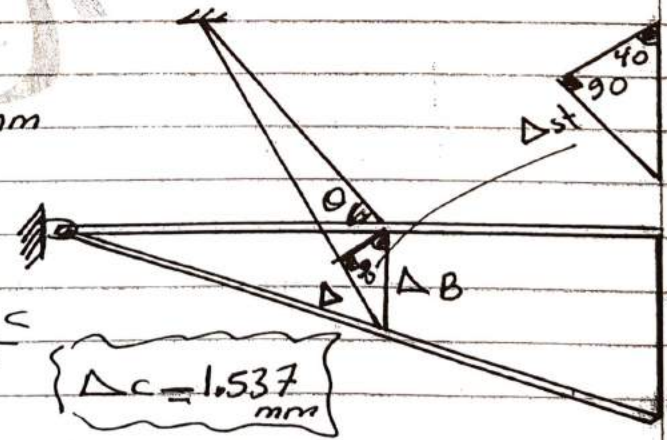
$$10 * 4 - P_{st} * \sin 40 * 2 = 0$$

$$P_{st} = 31.114 \, KN$$

$$\Delta l_{st} = \frac{P_{st} . L_{st}}{E . A} = \frac{31.114 * 10^3 * 1000}{210 * 10^3 * 300} = 0.494 mm$$

$$\sin 40 = \frac{\Delta st}{\Delta B}$$

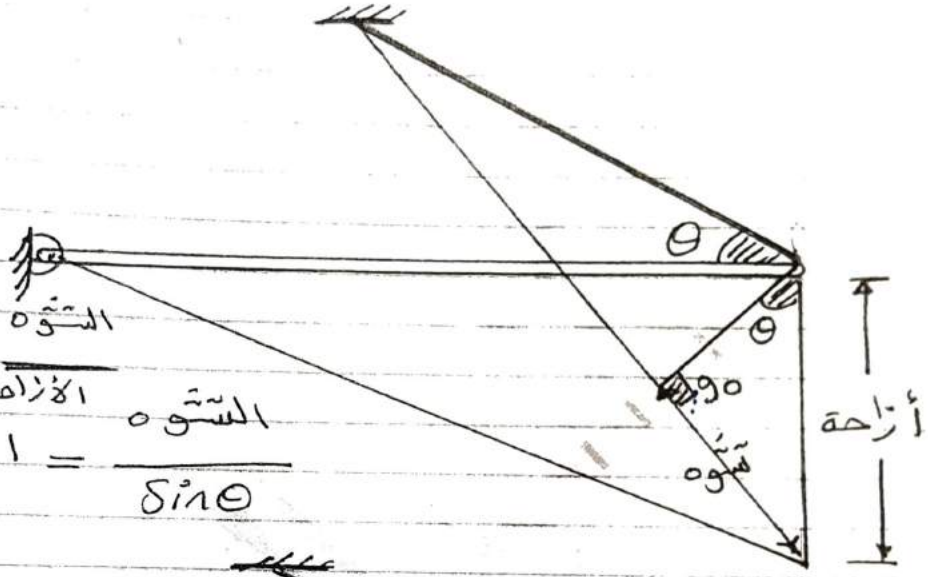
$$\Delta B = 0.768 \, mm$$



$$\frac{\Delta B}{2} = \frac{\Delta C}{4}$$

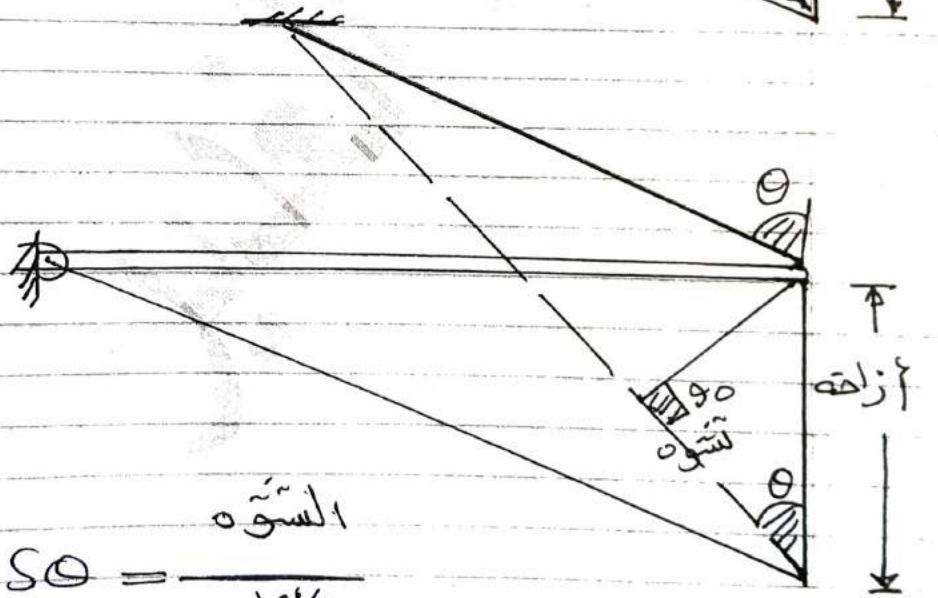
$$\frac{0.768}{2} = \frac{\Delta C}{4}$$

$$\Delta C = 1.537 \, mm$$



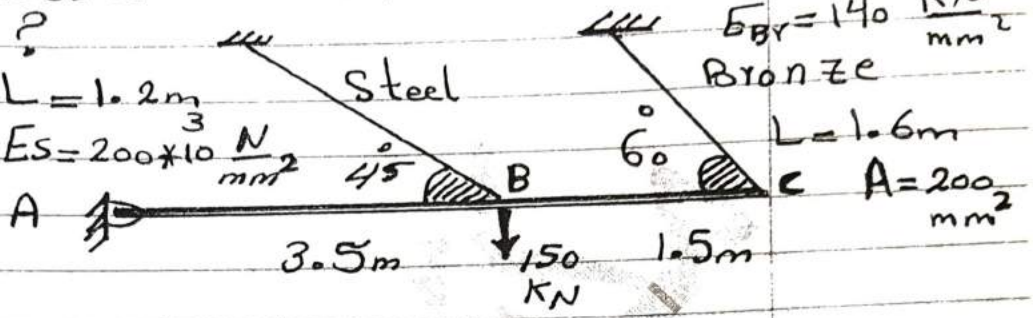
$$\sin\theta = \frac{\text{الشوه}}{\text{الأزاحة}}$$

$$\text{الأزاحة} = \frac{\text{الشوه}}{\sin\theta}$$

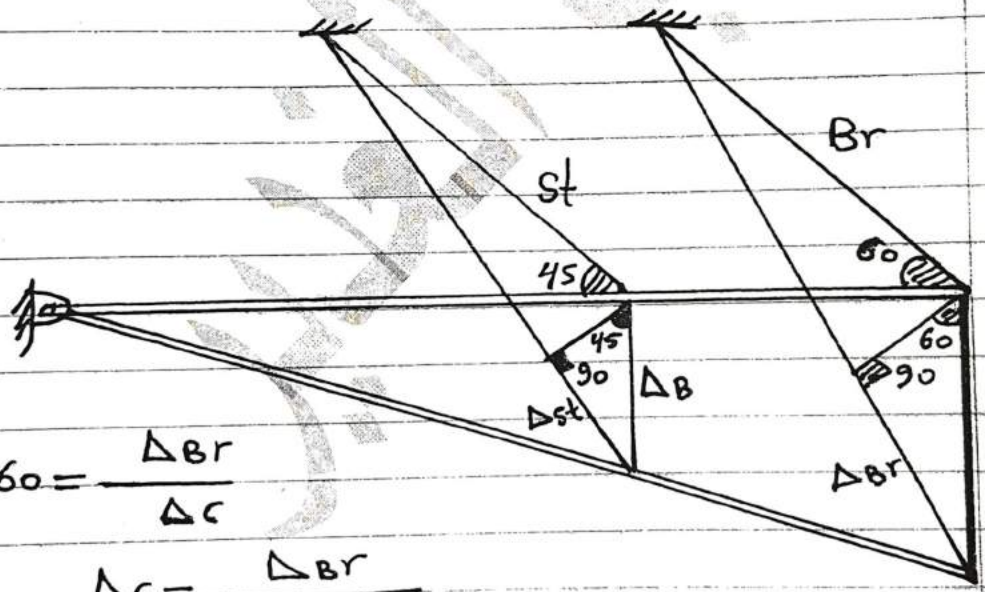


$$\cos\theta = \frac{\text{الشوه}}{\text{الأزاحة}}$$

Ex. 19 The bar ABC is rigid. Find the stresses in each rods steel and Bronze ?



Sol:- لاحظ أن عدد المجاهيل أكبر من عدد معادلات التوازن لذلك نتاج معادلة أختلافية من الشكل.



$$\sin 60 = \frac{\Delta_{Br}}{\Delta_c}$$

$$\Delta_c = \frac{\Delta_{Br}}{\sin 60}$$

$$\sin 45 = \frac{\Delta_{st}}{\Delta_B}$$

$$\Delta_B = \frac{\Delta_{st}}{\sin 45}$$

$$\frac{\Delta_B}{3.5} = \frac{\Delta_c}{5}$$

$$\frac{\Delta B}{3.5} = \frac{\Delta c}{5}$$

$$\frac{\Delta st}{2.5 \sin 45} = \frac{\Delta BR}{5 * \sin 60}$$

$$\frac{P_{st} * 10^3 * 1200}{3.5 * \sin 45 * 200 * 10^3 * 100} = \frac{P_{BR} * 10^3 * 1600}{5 * \sin 60 * 140 * 10^3 * 200}$$

$$P_{st} = 0.5443 \; P_{BR} \quad ---(1)$$

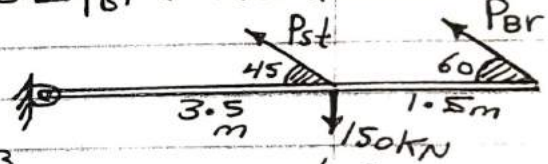
$$+\circlearrowleft \Sigma M_A = 0$$

$$150 * 3.5 - P_{st} \cdot \sin 45 * 3.5 - P_{BR} \cdot \sin 60 * 5 = 0$$

$$525 - 0.5443 P_{BR} \cdot \sin 45 * 3.5 - P_{BR} * \sin 60 * 5 = 0$$

$$P_{BR} = 92.475 \text{ KN}$$

$$P_{st} = 50.334 \text{ KN}$$

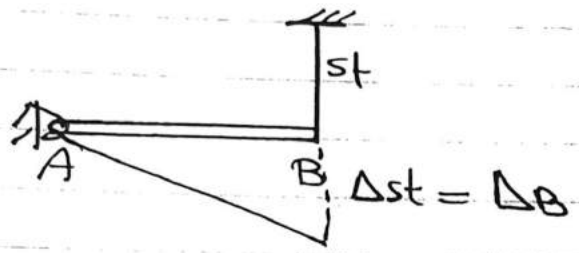


$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{50.334 * 10^3}{100} = 503.34 \text{ N/mm}^2$$

$$\sigma_{BR} = \frac{P_{BR}}{A_{BR}} = \frac{92.475 * 10^3}{200} = 1849.5 \text{ N/mm}^2$$

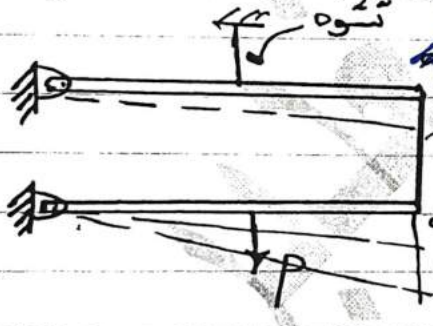
Strain.33 Strength of material

١. أذا كان الـ rod مربوط بـ Fixed من جهة و الـ Rigid من الجهة الأخرى.

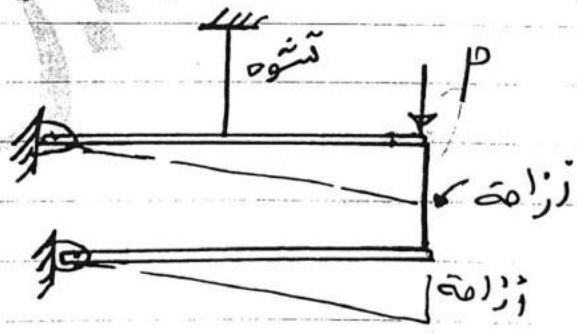


تشوه فقط
AB rigid

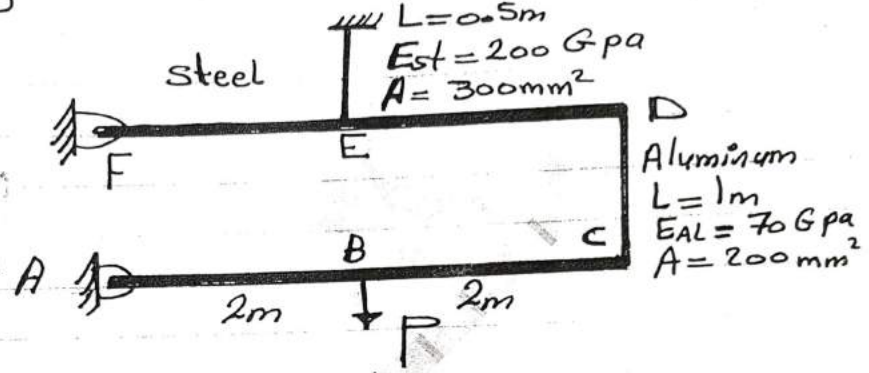
٢. أذا كان rod مربوط بـ Rigid من الاعلى و Rigid من الأسفل أو مربوط من جهتين واحده هنا احتمالين



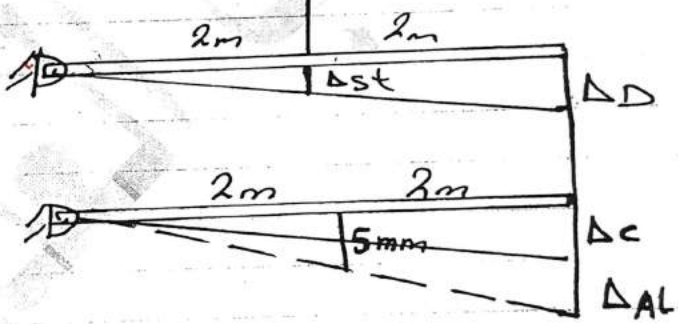
أ. تشوه + أزاحة.
ب. أزاحة.



Ex:- The bar ABC and bar DEF is rigid bar . Find the force P if the movement at point B is 5mm ?



Sol:-



$\Delta c = \Delta D$

$$\frac{5}{2} = \frac{\Delta c + \Delta Al}{4}$$

$$10 = \Delta c + \Delta Al$$

* العلاقة يجب ان تحوي تشوه فقط فمفروض أزاحة يجب ان تحول لدلالة تشوه سلك .

* من الملاحظ أن الـ (A) موجود بالعلاقة وعدم وجود الحديد لذلك توجد علاقة بين الأزاحة وتشوه الحديد .

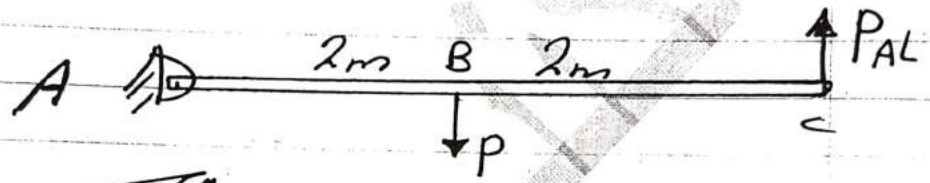
$$\frac{\Delta D}{24} = \frac{\Delta st}{2} \rightarrow \Delta D = 2\Delta st$$

$$10 = \Delta c + \Delta AL$$

$$10 = 2\Delta st + \Delta AL$$

$$10 = 2 \cdot \frac{P_{st} * 10^3 * 500}{200 * 10^3 * 300} + \frac{P_{AL} * 10^3 * 10^3}{70 * 10^3 * 200}$$

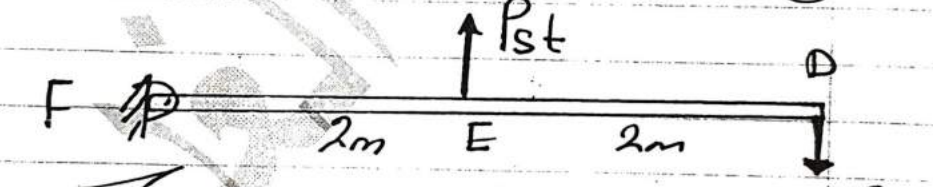
$$10 = \frac{P_{st}}{60} + \frac{P_{AL}}{14} \quad \text{---} (1)$$



$$(+\circlearrowleft) \Sigma M_A = 0$$

$$P(2) - P_{AL}(4) = 0$$

$$P_{AL} = \frac{P}{2} \quad \text{---} (2)$$



$$(+\circlearrowleft) \Sigma M_F = 0$$

$$-P_{st}(2) + \frac{P}{2} * 4 = 0$$

$$P_{AL} = \frac{P}{2}$$

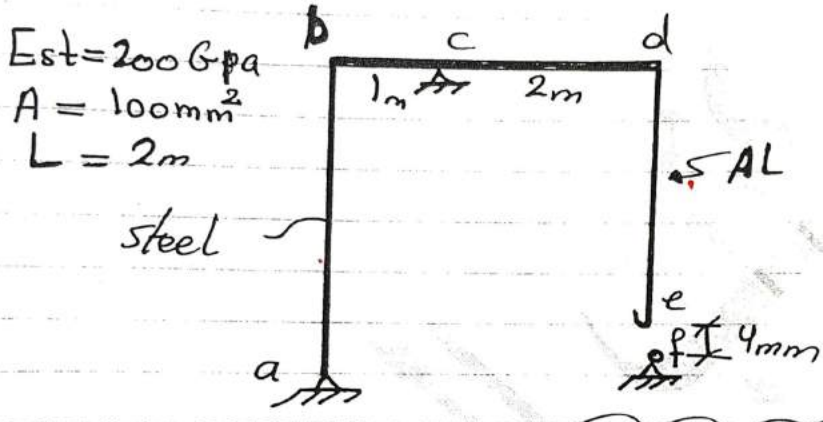
$$P_{st} = P \quad \text{---} (3)$$

$$10 = \frac{P_{st}}{60} + \frac{P_{AL}}{14}$$

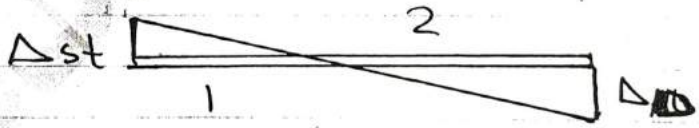
$$10 = \frac{P}{60} + \frac{P}{14 * 2}$$

$$P = 113.5 \text{ KN}$$

Ex:- Find the force to make point (e) contact point (f). Rigid (bcd).



Sol:- $\Delta AL + \Delta D = 4$



$\frac{\Delta D}{2} = \frac{\Delta st}{1}$

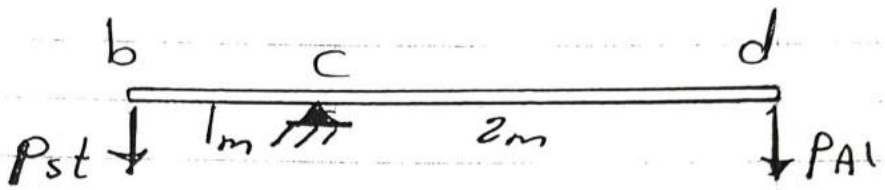
$\Delta D = 2\Delta st$

$\Delta AL + 2\Delta st = 4$ --- (1)

$2m = 2000$

$\frac{PAL * 10^3 * (2000-4)}{70*10^3 * 200} + 2\frac{Pst * 10^3 * 2000}{200*10^3 * 100} = 4$

$\frac{PAL}{7} + \frac{Pst}{5} = 4$ --- (1)



$$(+\circlearrowleft) \sum M_C = 0$$

$$-P_{st}(1) + P_{Al}(2) = 0$$

$$P_{st} = 2P_{Al} \quad \text{---} (2)$$

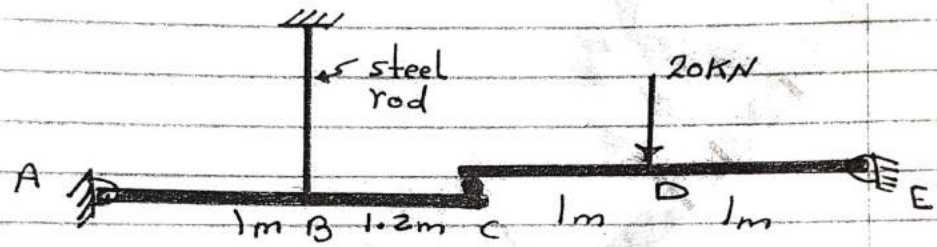
(1) في معادلة (2) نعوض معادلة

$$\frac{P_{Al}}{7} + \frac{P_{st}}{5} = 4$$

$$\frac{P_{Al}}{7} + \frac{2P_{Al}}{5} = 4$$

$$P_{Al} = 7.368 \text{ kN}$$

Ex:- the bar ABC and CDE is rigid. The modulus of elastiaty of the steel rod is $20 * 10^4$ N/mm² and has a length is 1.3m and diameter 20 mm Calculate the movement at point C?



Sol:-

$\curvearrowleft +\Sigma M_A = 0$

$10 * 2.2 - Pst(1) = 0$

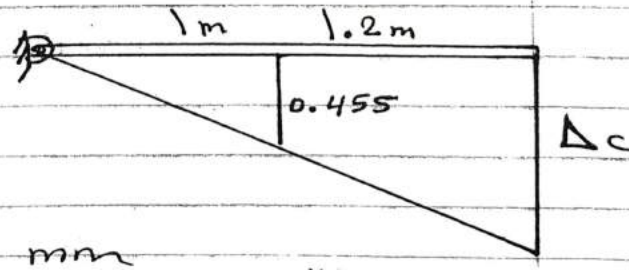
$Pst = 22\ KN$

$\curvearrowleft +\Sigma M_E = 0$

$Cy * 2 - 20 * 1 = 0$

$Cy = 10\ KN$

$$\Delta st = \frac{Pst \cdot L}{E \cdot A} = \frac{22 * 10^3 * 1300}{20 * 10^4 * \frac{\pi}{4}(20)^2} = 0.455\ mm$$



$$\frac{0.455}{1} = \frac{\Delta c}{2.2}$$

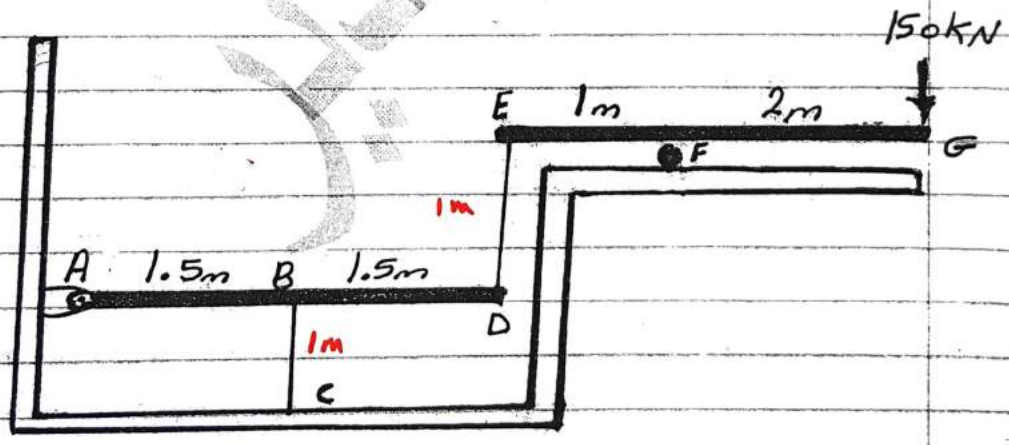
$\Delta c = 1.0014$ mm

Ex. Q. :- In the structure shown in fig member ABD is a solid rigid member pinned to the wall at A, supported by steel cable BC and connected member EFG by steel DE. (Cables BC and DE each have across sectional area of $3cm^2$). Member EFG is supported by a roller at F and is loaded with 150 kN at G. For this structure:

(A). Draw a free Body Diagram showing all support forces and Loads.

(B) Determine the axial stress in cable (BC).

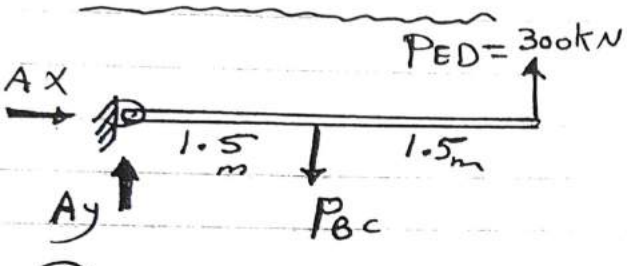
(C.) Determine the movement of point G due to the applied Load.



$E_{st} = 20 * 10^6 \frac{N}{cm^2}$

Sol:- ①

For the member ABD



$+\circlearrowleft \Sigma M_A = 0$

$P_{BC}(1.5) - 300*3 = 0$

$P_{BC} = 600\,kN$

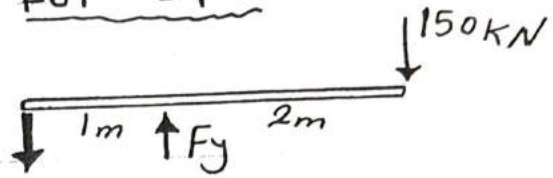
$+\rightarrow \Sigma F_x = 0 \quad A_x = 0$

$+\uparrow \Sigma F_y = 0$

$A_y + 300\,kN - P_{BC} = 0$

$A_y = 300\,kN \uparrow$

For EFE



$+\circlearrowleft \Sigma M_F = 0$

$-P_{ED}(1) + 150(2) = 0$

$P_{ED} = 300\,KN$

$+\uparrow \Sigma F_y = 0$

$F_y - 150 - 300 = 0$

$F_y = 450\,KN \uparrow$

② Determine the axial stress in cable BC.

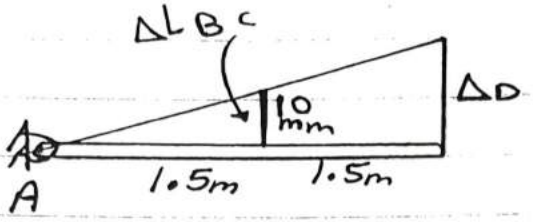
$$\sigma_{BC} = \frac{P_{BC}}{Area} = \frac{600*10^3}{300} = 2000\ N/mm^2$$

③ The movement at G.

$$\Delta L_{BC} = \frac{600*10^3*1000}{20*10^6*10^{-2}*300} = 10mm$$

$$\Delta L_{ED} = \frac{300*10^3*1000}{20*10^6*10^{-2}*300} = 5mm$$

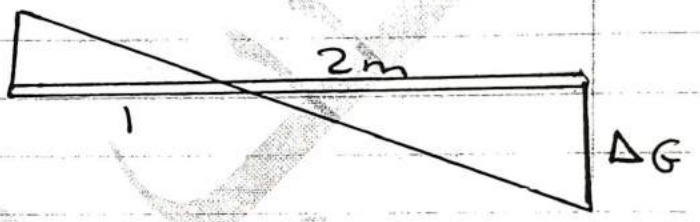
Strain.41 Strength of material



$\frac{\Delta D}{3} = \frac{10}{1.5}$ $\Delta D = 20mm$. displacement at D

$\Delta D + \Delta L_{ED}$

$20 + 5 = 25mm$



$\frac{25}{1} = \frac{\Delta G}{2}$

$\Delta G = 50mm$