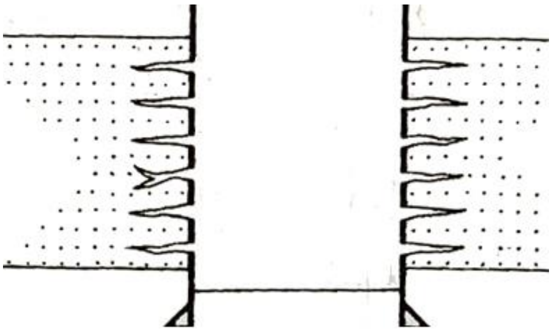
Well Testing

➢ Introduction

After production well is drilled and completed for produce oil, gas and sometime water produce… here comes the role of well testing to determine the ability of a formation to produce reservoir fluids.



➢ What is Well Test??

- Well testing is the technique and method for the evaluation of well conditions and reservoir characteristics.
- A well test is the measurement ***under controlled conditions*** of all factors relating to the production of oil, gas, and water from a well.

➢ Why We Test a Well?

To determine the following parameters measure;

- ✓ Initial reservoir pressure.
- ✓ Average reservoir pressure.
- ✓ Permeability (K).
- ✓ Formation flow capacity, kh.
- ✓ Formation damage due to drilling and completion (skin effect).
- ✓ Drainage area.
- ✓ Identifying fluid behavior.
- ✓ Identifying and confirming heterogeneities and boundaries.

➢ Types of Well Tests

1. Drawdown Test
2. Buildup Test
3. Injection Test
4. Falloff Test
5. Interference Test
6. Drill Stem Test (DST)

Gas Well Testing:

1. Flow after Flow Test.
2. Isochronal Test.
3. Modified Isochronal Test.

➢ How Do We Test Wells?

- Create a step ***change in flow rate*** by;
  - ✓ closing a flowing well (buildup test) or an injection well (falloff test).
  - ✓ opening a well previously shut in (drawdown test).
  - ✓ injecting in a well previously closed (injection test).
- This rate change creates a ***change in pressure*** in the same well (exploration or production testing) or in a different well (interference testing).

➢ How Do We Change of Flow Rates??

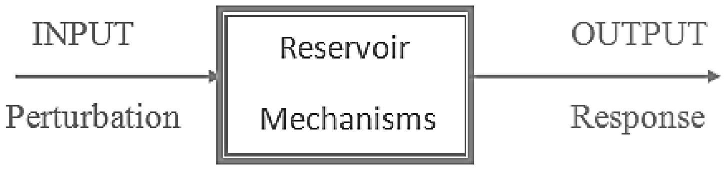
- A ***change in flow rate*** can be created
  - ***At the surface*** by shutting or opening the master valve….. or……
  - ***At the bottom of the well*** with a special down hole shut-in device.
- ***Wellhead shut-in*** is commonly used in wells already in production, whereas ***bottom hole shut-in*** is standard practice after drilling [a drill stem test (DST)].
- The way the rate signal is created is ***not important*** as far as ***well test analysis*** is concerned.

➢ General Methodology

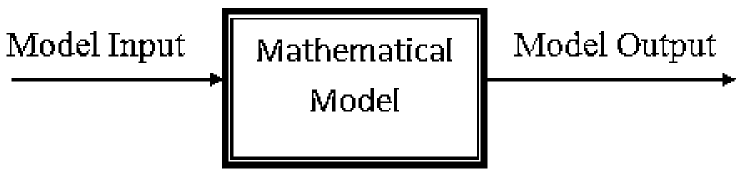
- ❖ Create a change in production rate (starting, stopping, or changing the rate of production)
- ❖ Measure the reservoir response (reservoir response may be pressure, rate, or both)
- ❖ Analyze the reservoir response by using the theoretical responses equations.

➢ How Do We Interpret Well Tests?

- During a well test, the response of a reservoir to changing flow rates conditions is observed. In most cases of well testing, the reservoir response that is measured is the **pressure response**, therefore, in many cases well test analysis means ***pressure transient analysis***.
- The **pressure transient** is due to changes in production or injection of fluids, so, we treat the ***flow rate transient*** as ***input*** and the ***pressure transient*** *as* ***output***.



- In well test interpretation, we use a ***mathematical model*** to relate pressure response (output) to flow rate history (input).



➢ Key Points of Well-Test Interpretation

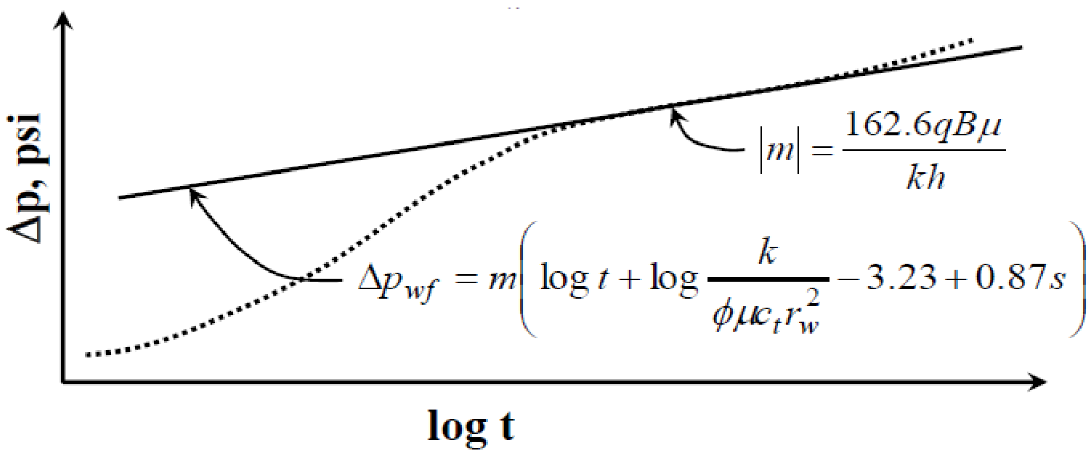
- ❖ For the success of the analysis, accurate data are essential.
- ❖ Measurements do not directly give the wanted information; the data need to be ***analyzed*** and ***interpreted***.
- ❖ Interpretation is based on the comparison of the measured responses with a theoretical model.
- ❖ The theoretical model is a solution of governing flow equation (diffusion equation).

➢ Methodology of Well Test Analysis

1. Straight Line Analysis
2. Pressure Derivative Analysis
3. Type-Curve Matching Analysis.

1) Straight Line Analysis

Theoretical models indicate **certain straight-line relations** when ***pressure is plotted as a function of time*** on specific coordinates. ***The slopes*** of the straight lines are functions of the properties required.



This is known as semi-log analysis

Basic of well testing is an understanding of the theory of ***fluid flow in porous media***.

➢ Fundamentals of Fluid Flow in Porous Media

❖ There are many equations that are designed to describe the flow of fluids through porous medium.

❖ The mathematical forms of these equations depending upon the characteristics of the reservoir.

❖ The primary reservoir characteristics that must be considered include:

✓ Types of fluids in the reservoir

✓ Flow regimes

✓ Reservoir geometry

✓ Number of flowing fluids in the reservoir

➢ Types of Fluids

In general, reservoir fluids are classified into three groups:

• ***Incompressible fluids*** • ***Slightly compressible fluids*** • ***Compressible fluids***

As we know the isothermal compressibility coefficient c is described mathematically by the following two equivalent expressions:

$$c = \frac{-1}{V}\frac{\partial V}{\partial p} \quad in\ term\ of\ fluid\ volume$$

$$c = \frac{-1}{\rho}\frac{\partial \rho}{\partial p} \quad in\ term\ of\ fluid\ density$$

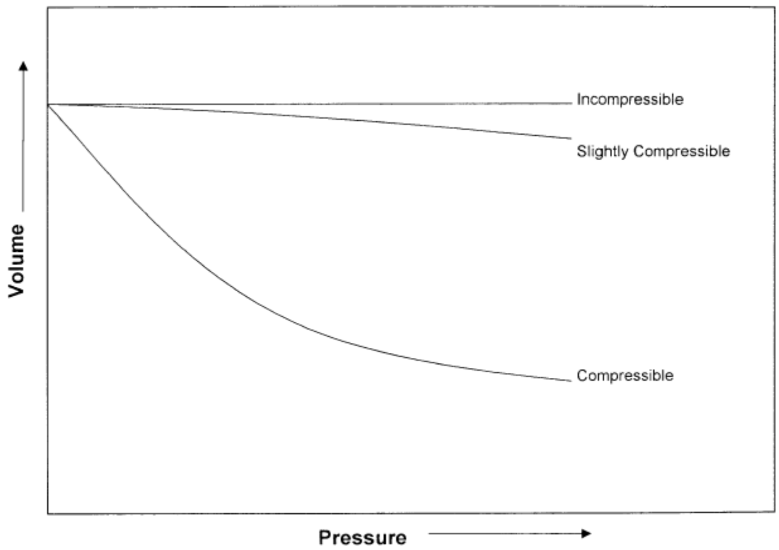
- **Incompressible Fluids:** volume (or density) does not change with pressure, i.e.:

$$\frac{\partial V}{\partial P} = 0 \quad , \quad \frac{\partial \rho}{\partial P} = 0$$

Incompressible fluids do not exist; this behavior, however, may be assumed in some cases to simplify the derivation and the final form of many flow equations.

- **Slightly Compressible Fluids:** These fluids exhibit small changes in volume, or density, with changes in pressure. crude oil and water systems fit into this group.

- **Compressible Fluids:** These fluids have large changes in volume as a function of pressure. All gases are considered compressible fluids.



➢ Flow Regimes

There are basically three types of flow regimes that must be recognized in order to describe the fluid flow behavior and reservoir pressure distribution as a function of time. There are three flow regimes:

• ***Steady-state flow*** • ***Unsteady-state flow*** • ***Pseudosteady-state flow***

- **Steady-State Flow:** The pressure at every location in the reservoir remains constant, i.e., does not change with time. $\left(\frac{\partial P}{\partial t}\right)_i = 0$

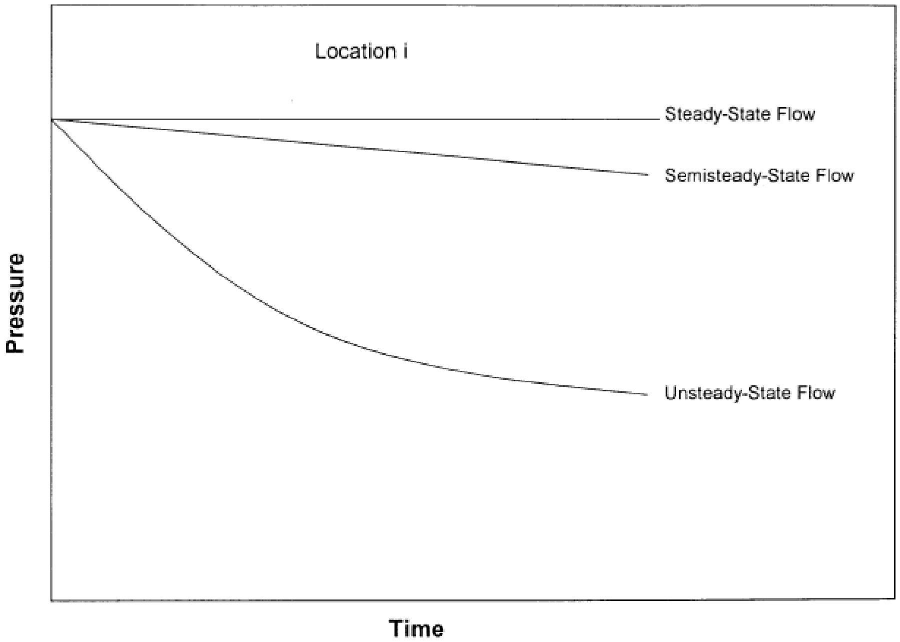
  The rate of change of pressure (p) with respect to time (t) at any location (i) is zero. In reservoirs, the steady-state flow condition can only occur when the reservoir is completely recharged and supported by strong aquifer or pressure maintenance operations.

- **Unsteady-State Flow (*transient flow*):** The rate of change of pressure with respect to time at any position in the reservoir is not zero or constant.
  $\left(\frac{\partial P}{\partial t}\right)_i = f(i,t)$
  The pressure derivative with respect to time is essentially a function of both position **i** and time **t**.

- **Pseudosteady-State Flow (semi-steady state or quasi-steady state flow):** The pressure at different locations in the reservoir is declining linearly as a function of time, i.e., at a constant declining rate.

$$\left(\frac{\partial P}{\partial t}\right)_i = constant$$

The rate of change of pressure with respect to time at every position is constant.



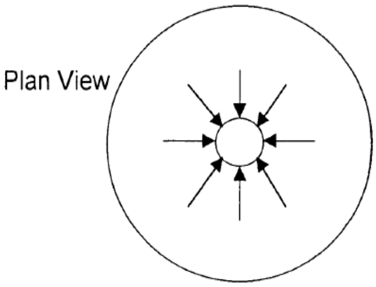
➢ Reservoir Geometry

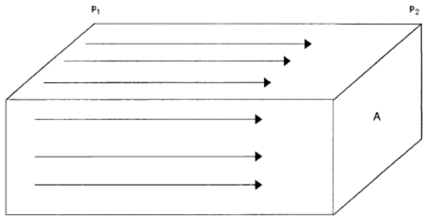
The flow geometry is represented by one of the following flow geometries:

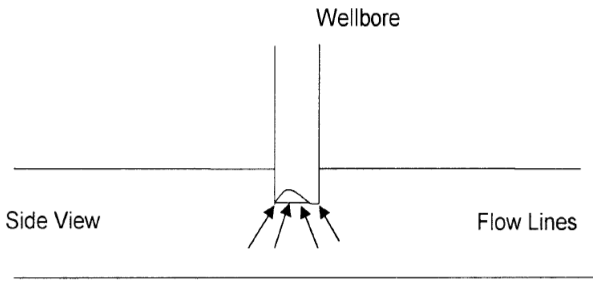
• ***Radial flow*** • ***Linear flow*** • ***Spherical and hemispherical flow***

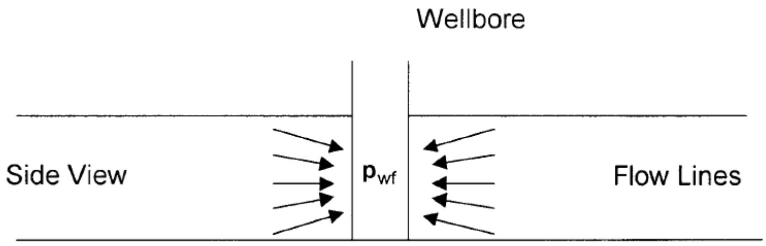
- **Radial Flow:** The fluids move toward the well from all directions and coverage at the wellbore.
- **Linear Flow:** Linear flow occurs when flow paths are parallel and the fluid flows in a single direction. In addition, the cross-sectional area to flow must be constant.
- **Spherical and Hemispherical Flow:** Depending upon the type of wellbore completion configuration.

A well with a *limited perforated interval* could result in **spherical flow**.
A well that only *partially penetrates the pay zone*, could result in **hemispherical flow**.









➢ Number of Flowing Fluids in The Reservoir

There are generally three cases of flowing systems:

- Single-phase flow (oil, water, or gas).
- Two-phase flow (oil-water, oil-gas, or gas-water).
- Three-phase flow (oil, water, and gas).

➢ Pressure Transient Analysis

- Pressure Transient Analysis Techniques is the ***measuring changes in bottom-hole pressure (BHP) as a function of time***, due to change producing rates, open the well to produce after its close or closed the well after its open to flow).
- The Pressure Transient Analysis Techniques are based on the **diffusivity equation** that describe the flow of fluids through porous medium.
- In the next section we will explain the following fluid flow equations that describe fluid flow in the porous media;

Flow type	Basic Equation	Fluid type	Flow geometry	Solution of Fluid flow equation
Unsteady state or transient	Diffusivity Equation	Slightly compressible	Radial	Ei function
Pseudo steady-State				Dimensionless solution

➢ Assumptions and Limitations Diffusivity Equation

1. Homogeneous and isotropic porous medium.
2. Uniform thickness.
3. Single phase flow.
4. Rock and fluid properties (such as fluids viscosity & compressibility) independent of pressure, i.e. remaining constant at all pressure.
5. The well completely penetrates the formation, & gravity forces are negligible.

➢ Derivation of the Diffusivity Equation

❖ Under the steady-state flowing condition, the same quantity of fluid enters the flow system as leaves it.

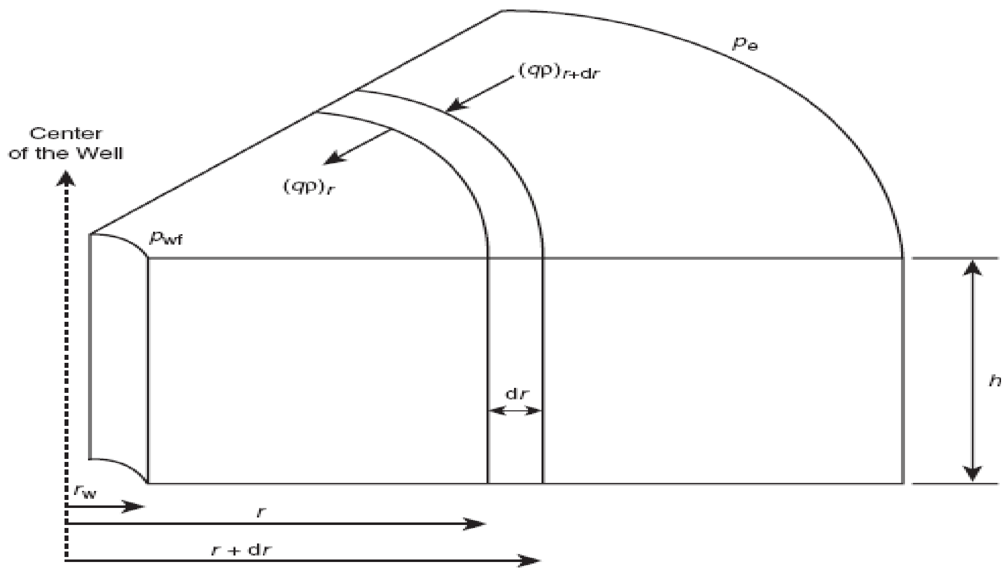
❖ In the unsteady-state flow condition, the flow rate into an element of volume of a porous media may not be the same as the flow rate out of that element. Accordingly, the fluid content of the porous medium changes with time.

❖ The mathematical formulation of the transient flow equation is based on combining three independent equations:

A. **Continuity Equation**
B. **Transport Equation**
C. **Compressibility Equation**

Consider the flow element shown in figure below. The element has a width of **dr** and is located at a distance of **r** from the center of the well. The porous element has a different volume of **dV**. According to the concept of the material balance equation, **the rate of mass flow into an element minus the rate of mass flow out of the element during a different time Δt must be equal to the mass rate of accumulation during that time interval**, or:

$$\begin{bmatrix}\text{mass entering}\\ \text{volume element}\\ \text{during interval } \Delta t\end{bmatrix} - \begin{bmatrix}\text{mass leaving}\\ \text{volume element}\\ \text{during interval } \Delta t\end{bmatrix} = \begin{bmatrix}\text{rate of mass}\\ \text{accumulation}\\ \text{during interval } \Delta t\end{bmatrix} \quad \text{---------------------} \quad \textbf{(3-1)}$$



Mass Entering the Volume Element During Time Interval Δt

$(\text{Mass})_{in} = \Delta t\,[A\nu\rho]_{r+dr}$ -- **(3-2)**

Where:

v= velocity of flowing fluid, ft/day

ρ= fluid density at (r+dr), lb/ft^3

A= Area at (r+dr)

Δt= time interval, days

The area of an element at the entering side is:

$A_{r+dr} = 2\pi\,(r + dr)\,h$ -- **(3-3)**

Combining eqs. (3-3) with (2-3) gives:

$(\text{Mass})_{in} = 2\pi\Delta t(r + dr)h\,[\nu\rho]_{r+dr}$ -- **(3-4)**

Mass Leaving the Volume Element

Adopting the same approach as that of the leaving mass gives:

$(\text{Mass})_{out} = 2\pi\,\Delta t\,r\,h\,[\nu\rho]_{r}$ -- **(3-5)**

Total Accumulation of Mass

The volume of some element with a radius of **r** is given by:

$V = \pi\,r^2 h$ -- **(3-6)**

Differentiating the above equation with respect to r gives:

$$\frac{dV}{dr} = 2\pi\,r\,h$$

Or:

$dV = 2\pi\,r\,h\,dr$ -- **(3-7)**

Total mass accumulation during $\Delta t = dV\,[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]$

Substituting for **dV** yields:

Total mass accumulation during $= (2\pi\,r\,h)\,dr\,[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]$

Replacing the terms of equation (3-1) with those of the calculated relationships gives:

$$2\pi\,h\,(r + dr)\Delta t\,[\nu\rho]_{r+dr} - 2\pi\,r\,h\,\Delta t\,[\nu\rho]_r = (2\pi\,r\,h)\,dr\,[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]$$

Dividing the above equation by **(2π r h) dr** and simplifying gives:

$$\frac{1}{r\,dr}[(r + dr)(\nu\rho)_{r+dr} - r\,(\nu\rho)_r] = \frac{1}{\Delta t}\,[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]$$

Or:

$\frac{1}{r}\,\frac{\partial}{\partial r}\,[r\,(\nu\,\rho)] = \frac{\partial}{\partial t}\,(\emptyset\rho)$ -- **(3-8)**

Eq. (3-8) is called the **continuity equation** and it provides the principle of conservation of mass in radial coordinates.

The transport equation must be introduced into the continuity equation to relate the fluid velocity to the pressure gradient within the control volume **dV**. Darcy`s law is essentially the basic motion equation, which states that the velocity is proportional to the pressure gradient $\partial P/\partial r$:

$$\nu = (5.615)(0.001127)\frac{k}{\mu} \quad \frac{\partial P}{\partial r}$$

$$= 0.006328 \quad \frac{k}{\mu} \quad \frac{\partial P}{\partial r} \quad \text{------------------} \quad \textbf{(3-9)}$$

Combining equations (3-9) with (3-8) results in:

$$\frac{0.006328}{r} \quad \frac{\partial}{\partial r} \quad \left[\frac{k}{\mu} \ (\rho\ r)\frac{\partial P}{\partial r}\right] = \frac{\partial}{\partial t} \quad (\emptyset\rho) \quad \text{------------------} \quad \textbf{(3-10)}$$

Expanding the right-hand side by taking the indicated derivatives eliminates the porosity from the partial derivative term on the right-hand side:

$$\frac{\partial}{\partial t} \quad (\emptyset\rho) = \emptyset \quad \frac{\partial \rho}{\partial t} \ + \ \rho \quad \frac{\partial \emptyset}{\partial t} \quad \text{------------------} \quad \textbf{(3-11)}$$

The porosity is related to the formation compressibility by the following:

$$C_f = \quad \frac{1}{\emptyset} \quad \frac{\partial \emptyset}{\partial P} \quad \text{------------------} \quad \textbf{(3-12)}$$

Applying the chain rule of differentiation to $\partial\emptyset/\partial t$:

$$\frac{\partial \emptyset}{\partial t} = \frac{\partial \emptyset}{\partial t} + \frac{\partial p}{\partial t}$$

Substituting eq. (3-12) into this equation:

$$\frac{\partial \emptyset}{\partial t} = \emptyset \ c_f \frac{\partial p}{\partial t}$$

Finally, substituting the above relation into eq. (3-11) and the result into eq. (3-10) gives:

$$\frac{0.006328}{r} \quad \frac{\partial}{\partial r} \quad \left[\frac{k}{\mu} \ (\rho\ r)\frac{\partial P}{\partial r}\right] = \rho \ \emptyset \ C_f \ \frac{\partial P}{\partial t} + \emptyset \quad \frac{\partial \rho}{\partial t} \quad \text{------------------} \quad \textbf{(3-13)}$$

- Equation (3-13) is the general partial differential equation used to describe the flow of any fluid flowing in a radial direction in porous media.
- In addition to the initial assumptions, Darcy`s equation has been added, which implies that the flow is **laminar**. Otherwise, the equation is not restricted to any type of fluid and is equally valid for gases or liquids. However, compressible and slightly compressible fluids must be treated separately in order to develop practical equations that can be used to describe the flow behavior of these two fluids. The treatments of the following systems are discussed below:
  - ✓ Radial flow of slightly compressible fluids.
  - ✓ Radial flow of compressible fluids.

➢ Radial Flow of Slightly Compressible Fluids

To simplify equation (3-13) assume that the ***permeability*** and ***viscosity*** are ***constant*** over pressure, time and distance ranges. This leads to:

$$\frac{0.006328\,k}{\mu\,r}\ \frac{\partial}{\partial r}\left[(\rho\,r)\frac{\partial P}{\partial r}\right] = \rho\,\varnothing\,C_f\,\frac{\partial P}{\partial t} + \varnothing\ \frac{\partial \rho}{\partial t} \text{ -- } \mathbf{(3\text{-}14)}$$

Expanding the above equation gives:

$$0.006328\left(\frac{k}{\mu}\right)\ \frac{\partial}{\partial r}\ \left[\left(\frac{\rho}{r}\right)\frac{\partial P}{\partial r} + \rho\frac{\partial^2 P}{\partial r^2} + \frac{\partial P}{\partial r}\frac{\partial \rho}{\partial r}\right] = \rho\,\varnothing\,C_f\,\frac{\partial P}{\partial t} + \varnothing\ \frac{\partial \rho}{\partial t}$$

Using the chain rule in the above relationship yields:

$$0.006328\left(\frac{k}{\mu}\right)\left[\left(\frac{\rho}{r}\right)\frac{\partial P}{\partial r} + \rho\frac{\partial^2 P}{\partial r^2} + \left(\frac{\partial P}{\partial r}\right)^2\frac{\partial \rho}{\partial P}\right] = \rho\,\varnothing\,C_f\,\frac{\partial P}{\partial t} + \varnothing\,\frac{\partial P}{\partial t}\left(\frac{\partial \rho}{\partial P}\right)$$

Dividing the above expression by the fluid density **ρ** gives:

$$0.006328\left(\frac{k}{\mu}\right)\left[\frac{1}{r}\frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial r^2} + \left(\frac{\partial P}{\partial r}\right)^2\left(\frac{1}{\rho}\frac{\partial \rho}{\partial P}\right)\right] = \varnothing\,C_f\,\frac{\partial P}{\partial t} + \varnothing\,\frac{\partial P}{\partial t}\left(\frac{1}{\rho}\,\frac{\partial \rho}{\partial P}\right)$$

Recalling that the compressibility of any fluid is related to its density by:

$$C = \frac{1}{\rho}\,\frac{\partial \rho}{\partial P}$$

Combining the above two equations gives:

$$0.006328\left(\frac{k}{\mu}\right)\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + C\left(\frac{\partial P}{\partial r}\right)^2\right] = \varnothing\,C_f\,\frac{\partial P}{\partial t} + \varnothing\,C\frac{\partial P}{\partial t}$$

The term $C\left(\frac{\partial P}{\partial r}\right)^2$ is considered very small and may be ignored:

$$0.006328\left(\frac{k}{\mu}\right)\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right] = \varnothing\ \frac{\partial P}{\partial t}(C_f + C) \text{ --- } \mathbf{(3\text{-}15)}$$

Defining total compressibility, $\mathbf{c_t}$, as:

$$C_t = C_f + C \text{ -- } \mathbf{(3\text{-}16)}$$

Combining eqs. (3-15) with (3-16) and rearranging gives:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\varnothing\,\mu\,C_t}{0.006328\,K}\frac{\partial P}{\partial t} \text{ -- } \mathbf{(3\text{-}17)}$$

Where the time **t** is expressed in days.

Or;

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\varnothing\,\mu\,C_t}{0.000264\,K}\frac{\partial P}{\partial t} \text{ -- } \mathbf{(3\text{-}18)}$$

Equation (3-17) is called the **diffusivity equation** and is considered one of the most important and widely used mathematical expressions in petroleum engineering. The equation is particularly used in the analysis of well testing data.

The term $[\mathbf{0.000264}\ \boldsymbol{k/\varphi\mu ct}]$ eq. (3-2) is called the **diffusivity constant** and is denoted by the symbol **η**, or:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t} \quad \textbf{(3-19)}$$

The diffusivity equation as represented by eq. (3-19) is essentially designed to determine the ***pressure*** *as a function of* ***time t*** *and* ***position r***.

Notice that for a **steady-state flow** condition, ***the pressure at any point in the reservoir is constant*** and does not change with time, i.e., ∂p/∂t = 0, and therefore eq. (3-18) reduces to:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = 0 \quad \textbf{(3-20)}$$

This equation called **Laplace's equation** for ***steady-state flow***.

➢ Solution of Diffusivity Equation

There are four solutions to Eq. (3-18) that are particularly useful in well testing:

1) The solution for an infinite reservoir with a well-considered to be a line source with zero wellbore radiuses.
2) The pseudo-steady – state solution.
3) The solution for a bounded cylindrical reservoir.
4) The solution that includes wellbore storage for a well in an infinite reservoir.

1. infinite cylindrical reservoir with line- source well

Assume that:

1. A well produces at a constant rate (qB).
2. The well has zero radius.
3. The reservoir is at uniform pressure, $\mathbf{P_i}$, before production begins.
4. The well drains an infinite area $(\mathbf{P \rightarrow P_i,\ as\ r \rightarrow \infty})$.
5. The well, with a wellbore radius of $\mathbf{r_w}$, is centered in a cylindrical reservoir of radius $\mathbf{r_e}$.
6. No flow across the outer boundary, i.e., at $\mathbf{r_e}$

Under those conditions the solution to Eq. (3-18) is:

$$P_{(r,t)} = pi + \left(\frac{70.6 Q_o \mu_o B_o}{Kh}\right) E_i \left(\frac{-948 \varnothing\, \mu_o\, C_t\, r^2}{K\,t}\right)$$ -- **(3-21)**

where $P_{(r,t)}$ = pressure at radius r from the well after t hours

t = time, hrs

k = permeability, md

Q_o = flow rate, STB/day

- The Ei solution, as expressed by eq. (3-21), is commonly referred to as the **line-source solution.**
- The mathematical function, Ei, is called the **exponential integral.**

$$E_i(-X) = -\int_x^{\infty} \frac{e^{-u}}{u} du$$

- The **exponential integral Ei** can be estimated by the following conditions;

✓ for **x ≤ 0.02**:

$$Ei(-x) = \ln(1.781\, x)$$ -- **(3-22)**

$$X = \frac{948\, \phi \mu c_t r^2}{kt}$$

✓ for **0.02 < x ≤ 10.9,** Use **table (1-1)** or **figure (3-1)** below.

✓ for **x > 10.9**

$$Ei(-x) = 0$$

The Ei function is not an accurate solution to flow equations until the flow time (t) be;

$$\frac{3.79 * 10^5\, \varnothing\, \mu\, C_t\, {r_w}^2}{k} < t < \frac{948\, \varnothing\, \mu\, C_t\, {r_e}^2}{k}$$

Inner condition effect

If the flow times less than this term, the assumption of zero well size (i.e., line source assuming) limits or reduced the accuracy of the eq. (3-21).

Outer(boundary) condition

If the flow times greater than this term, the reservoir's boundaries begin to affect the pressure distribution in the reservoir, so that the reservoir is no longer infinite acting (acting as finite or bounded reservoir).

TABLE 1.1* – VALUES OF THE EXPONENTIAL INTEGRAL, $-Ei(-x)$

$-Ei(-x)$, 0.000<0.209, interval – 0.001

x	0	1	2	3	4	5	6	7	8	9
0.00	+∞	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
0.01	4.038	3.944	3.858	3.779	3.705	3.637	3.574	3.514	3.458	3.405
0.02	3.355	3.307	3.261	3.218	3.176	3.137	3.098	3.062	3.026	2.992
0.03	2.959	2.927	2.897	2.867	2.838	2.810	2.783	2.756	2.731	2.706
0.04	2.681	2.658	2.634	2.612	2.590	2.568	2.547	2.527	2.507	2.487
0.05	2.468	2.449	2.431	2.413	2.395	2.377	2.360	2.344	2.327	2.311
0.06	2.295	2.279	2.264	2.249	2.235	2.220	2.206	2.192	2.178	2.164
0.07	2.151	2.138	2.125	2.112	2.099	2.087	2.074	2.062	2.050	2.039
0.08	2.027	2.015	2.004	1.993	1.982	1.971	1.960	1.950	1.939	1.929
0.09	1.919	1.909	1.899	1.889	1.879	1.869	1.860	1.850	1.841	1.832
0.10	1.823	1.814	1.805	1.796	1.788	1.779	1.770	1.762	1.754	1.745
0.11	1.737	1.729	1.721	1.713	1.705	1.697	1.689	1.682	1.674	1.667
0.12	1.660	1.652	1.645	1.638	1.631	1.623	1.616	1.609	1.603	1.596
0.13	1.589	1.582	1.576	1.569	1.562	1.556	1.549	1.543	1.537	1.530
0.14	1.524	1.518	1.512	1.506	1.500	1.494	1.488	1.482	1.476	1.470
0.15	1.464	1.459	1.453	1.447	1.442	1.436	1.431	1.425	1.420	1.415
0.16	1.409	1.404	1.399	1.393	1.388	1.383	1.378	1.373	1.368	1.363
0.17	1.358	1.353	1.348	1.343	1.338	1.333	1.329	1.324	1.319	1.314
0.18	1.310	1.305	1.301	1.296	1.291	1.287	1.282	1.278	1.274	1.269
0.19	1.265	1.261	1.256	1.252	1.248	1.243	1.239	1.235	1.231	1.227
0.20	1.223	1.219	1.215	1.210	1.206	1.202	1.198	1.195	1.191	1.187

$-Ei(-x)$, 0.00<x<2.09, interval = 0.01

0.0	+∞	4.038	3.335	2.959	2.681	2.468	2.295	2.151	2.027	1.919
0.1	1.823	1.737	1.660	1.589	1.524	1.464	1.409	1.358	1.309	1.265
0.2	1.223	1.183	1.145	1.110	1.076	1.044	1.014	0.985	0.957	0.931
0.3	0.906	0.882	0.858	0.836	0.815	0.794	0.774	0.755	0.737	0.719
0.4	0.702	0.686	0.670	0.655	0.640	0.625	0.611	0.598	0.585	0.572
0.5	0.560	0.548	0.536	0.525	0.514	0.503	0.493	0.483	0.473	0.464
0.6	0.454	0.445	0.437	0.428	0.420	0.412	0.404	0.396	0.388	0.381
0.7	0.374	0.367	0.360	0.353	0.347	0.340	0.334	0.328	0.322	0.316
0.8	0.311	0.305	0.300	0.295	0.289	0.284	0.279	0.274	0.269	0.265
0.9	0.260	0.256	0.251	0.247	0.243	0.239	0.235	0.231	0.227	0.223
1.0	0.219	0.216	0.212	0.209	0.205	0.202	0.198	0.195	0.192	0.189
1.1	0.186	0.183	0.180	0.177	0.174	0.172	0.169	0.166	0.164	0.161
1.2	0.158	0.156	0.153	0.151	0.149	0.146	0.144	0.142	0.140	0.138
1.3	0.135	0.133	0.131	0.129	0.127	0.125	0.124	0.122	0.120	0.118
1.4	0.116	0.114	0.113	0.111	0.109	0.108	0.106	0.105	0.103	0.102
1.5	0.1000	0.0985	0.0971	0.0957	0.0943	0.0929	0.0915	0.0902	0.0889	0.0876
1.6	0.0863	0.0851	0.0838	0.0826	0.0814	0.0802	0.0791	0.0780	0.0768	0.0757
1.7	0.0747	0.0736	0.0725	0.0715	0.0705	0.0695	0.0685	0.0675	0.0666	0.0656
1.8	0.0647	0.0638	0.0629	0.0620	0.0612	0.0603	0.0595	0.0586	0.0578	0.0570
1.9	0.0562	0.0554	0.0546	0.0539	0.0531	0.0524	0.0517	0.0510	0.0503	0.0496
2.0	0.0489	0.0482	0.0476	0.0469	0.0463	0.0456	0.0450	0.0444	0.0438	0.0432

2.0<x<10.9, interval = 0.1

x	0	1	2	3	4	5	6	7	8	9
2	4.89×10^{-2}	4.26×10^{-2}	3.72×10^{-2}	3.25×10^{-2}	2.84×10^{-2}	2.49×10^{-2}	2.19×10^{-2}	1.92×10^{-2}	1.69×10^{-2}	1.48×10^{-2}
3	1.30×10^{-2}	1.15×10^{-2}	1.01×10^{-2}	8.94×10^{-3}	7.89×10^{-3}	6.87×10^{-3}	6.16×10^{-3}	5.45×10^{-3}	4.82×10^{-3}	4.27×10^{-2}
4	3.78×10^{-3}	3.35×10^{-3}	2.97×10^{-3}	2.64×10^{-3}	2.34×10^{-3}	2.07×10^{-3}	1.84×10^{-3}	1.64×10^{-3}	1.45×10^{-3}	1.29×10^{-3}
5	1.15×10^{-3}	1.02×10^{-3}	9.08×10^{-4}	8.09×10^{-4}	7.19×10^{-4}	6.41×10^{-4}	5.71×10^{-4}	5.09×10^{-4}	4.53×10^{-4}	4.04×10^{-4}
6	3.60×10^{-4}	3.21×10^{-4}	2.86×10^{-4}	2.55×10^{-4}	2.28×10^{-4}	2.03×10^{-4}	1.82×10^{-4}	1.62×10^{-4}	1.45×10^{-4}	1.29×10^{-4}
7	1.15×10^{-4}	1.03×10^{-4}	9.22×10^{-5}	8.24×10^{-5}	7.36×10^{-5}	6.58×10^{-5}	5.89×10^{-5}	5.26×10^{-5}	4.71×10^{-5}	4.21×10^{-5}
8	3.77×10^{-5}	3.37×10^{-5}	3.02×10^{-5}	2.70×10^{-5}	2.42×10^{-5}	2.16×10^{-5}	1.94×10^{-5}	1.73×10^{-5}	1.55×10^{-5}	1.39×10^{-5}
9	1.24×10^{-5}	1.11×10^{-5}	9.99×10^{-6}	8.95×10^{-6}	8.02×10^{-6}	7.18×10^{-6}	6.44×10^{-6}	5.77×10^{-6}	5.17×10^{-6}	4.64×10^{-6}
10	4.15×10^{-6}	3.73×10^{-6}	3.34×10^{-6}	3.00×10^{-6}	2.68×10^{-6}	2.41×10^{-6}	2.16×10^{-6}	1.94×10^{-6}	1.74×10^{-6}	1.56×10^{-6}

*Adapted from Nisle, R.G.: "How To Use The Exponential Integral," *Pet. Eng.* (Aug. 1956) B171-173.

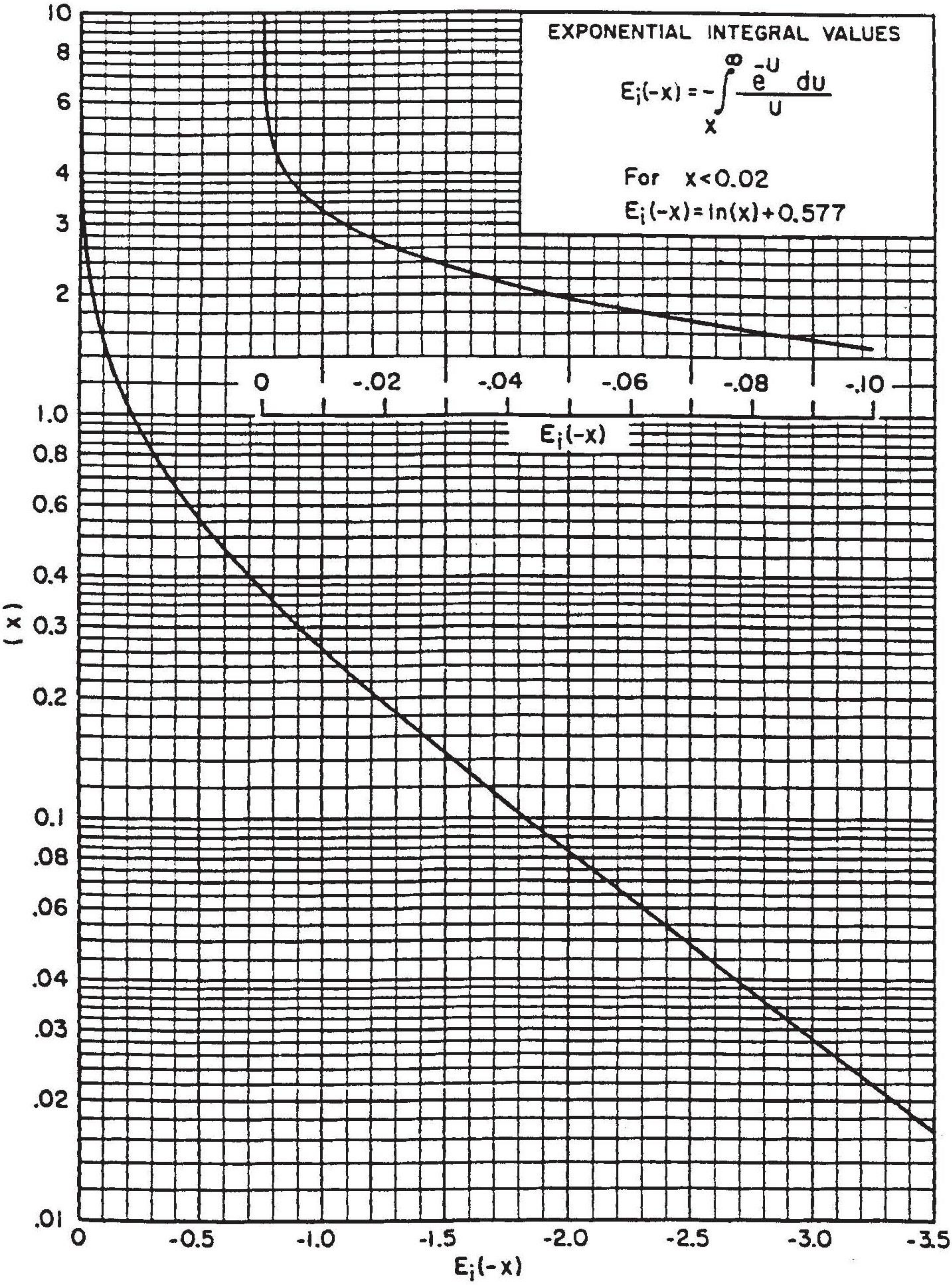

EXPONENTIAL INTEGRAL VALUES
$E_i(-x) = -\int_x^\infty \frac{e^{-u}}{u} du$
For x<0.02
$E_i(-x) = \ln(x) + 0.577$
0
-.02
-.04
-.06
-.08
-.10
$E_i(-x)$
10
8
6
4
3
2
1.0
0.8
0.6
0.4
0.3
0.2
0.1
.08
.06
.04
.03
.02
.01
(x)
0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
$E_i(-x)$

Figure (3-1)

Step 2: Perform the required calculations after one hour in the following tabulated form:

Elapsed Time t = 1 hr

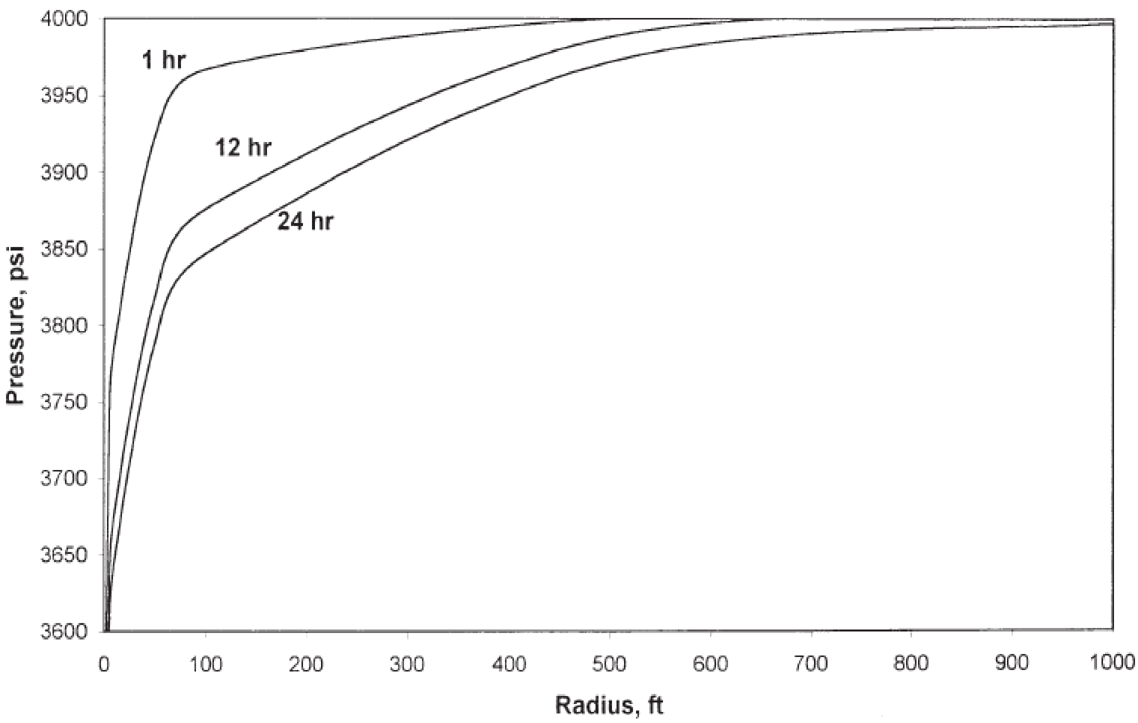
r, ft	$x = -42.6(10^{-6})\ \dfrac{r^2}{1}$	$E_i(-x)$	$p(r,1) = 4000 + 44.125\ E_i(-x)$
0.25	$-2.6625(10^{-6})$	−12.26*	3459
5	−0.001065	−6.27*	3723
10	−0.00426	−4.88*	3785
50	−0.1065	−1.76†	3922
100	−0.4260	−0.75†	3967
500	−10.65	0	4000
1000	−42.60	0	4000
1500	−95.85	0	4000
2000	−175.40	0	4000
2500	−266.25	0	4000

**As calculated from Equation*

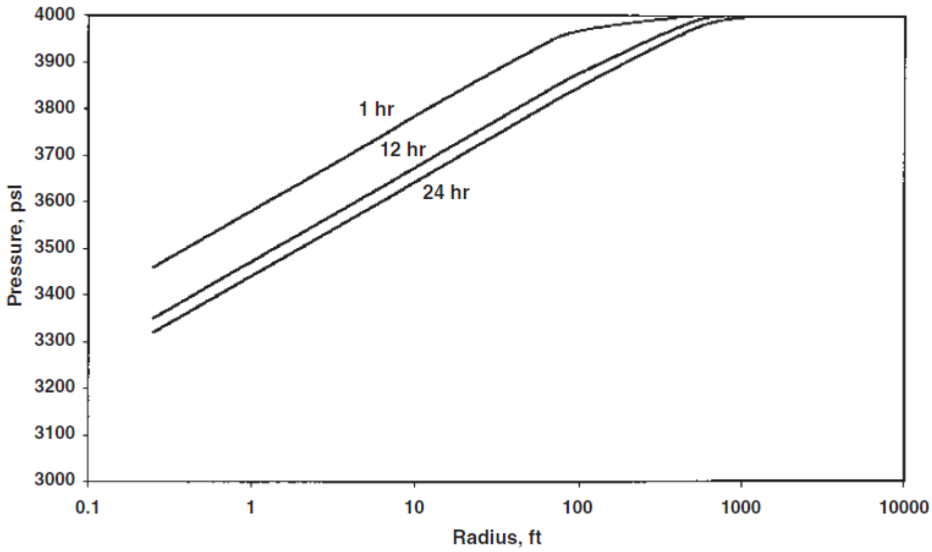
†From Figure

We show that most of the *pressure loss* occurs close to the wellbore; so, near-wellbore conditions will have the greatest influence on flow behavior.

Step 3: Show results of the calculation graphically.



Last Figure shows that the pressure profile and the drainage radius are continuously changing with time.



Step 4: Repeat the calculation for t = 12 and 24 hrs.

Elapsed Time t = 12 hrs

r, ft	$x = 42.6(10^{-6})\ \frac{r^2}{12}$	$E_i(-x)$	$p(r,12) = 4000 + 44.125\ E_i(-x)$
0.25	$0.222\ (10^{-6})$	−14.74*	3350
5	$88.75\ (10^{-6})$	−8.75*	3614
10	$355.0\ (10^{-6})$	−7.37*	3675
50	0.0089	−4.14*	3817
100	0.0355	−2.81†	3876
500	0.888	−0.269	3988
1000	3.55	−0.0069	4000
1500	7.99	$-3.77(10^{-5})$	4000
2000	14.62	0	4000
2500	208.3	0	4000

**As calculated from Equation*
†From Figure

Elapsed Time t = 24 hrs

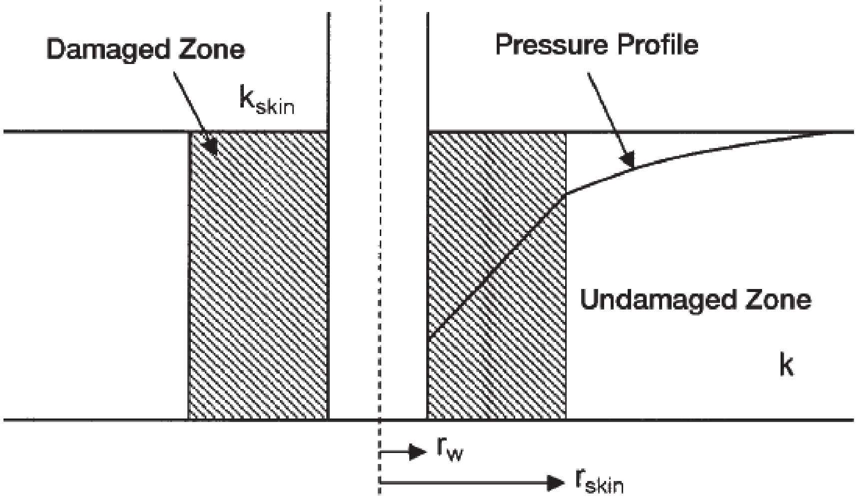
r, ft	$x = 42.6(10^{-6})\ \dfrac{r^2}{24}$	$E_i(-x)$	$p(r,24) = 4000 + 44.125\ E_i(-x)$
0.25	$-0.111\ (10^{-6})$	-15.44*	3319
5	$-44.38\ (10^{-6})$	-9.45*	3583
10	$-177.5\ (10^{-6})$	-8.06*	3644
50	-0.0045	-4.83*	3787
100	-0.0178	-3.458†	3847
500	-0.444	-0.640	3972
1000	-1.775	-0.067	3997
1500	-3.995	-0.0427	3998
2000	-7.310	$8.24\ (10^{-6})$	4000
2500	-104.15	0	4000

**As calculated from Equation*

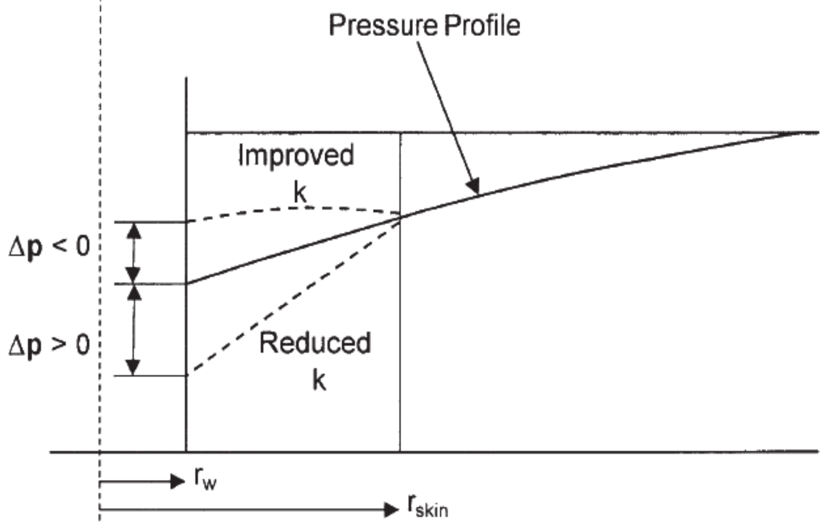
†From Figure

➢ Skin Effect

- ❖ Most wells have reduced permeability (damage) near the wellbore resulting from drilling or completion operations.
- ❖ Some materials such as mud filtrate, cement slurry, or clay particles, enters the formation during drilling, completion, or workover operations and reduce the permeability around the wellbore.
- ❖ This effect is commonly referred to as a ***wellbore damage*** and the region of altered permeability is called the ***skin zone****.*
- ❖ Many other wells are stimulated by acidizing or fracturing, which increase the permeability near the wellbore.
- ❖ Thus, the permeability near the wellbore is always different from the permeability away from the well where the formation has not been affected by damage or stimulation.



- Formation damage can produce additional pressure drop during flow. This additional pressure drop is commonly referred to as **ΔP_{skin}.** while, well stimulation methods will enhance the properties of the formation and increase the permeability around the wellbore, so that a decrease in pressure drop is observed.
- The resulting effect of altering the permeability around the well bore is called the **skin effect**.
- Figure below compares the differences in the skin zone pressure drop for three possible outcomes:



- **ΔP_{skin}** > 0

indicates an additional pressure drop due to wellbore damage.

i.e., **$k_{skin} < k$.**

- **ΔP_{skin}** < 0

indicates less pressure drop due to wellbore improvement.

i.e., **$k_{skin} > k$.**

- **Δp_{skin}** = 0

indicates no changes in the wellbore condition,

i.e., $k_{skin} = k$.

Hawkins proposed the following approach:

$\Delta p_{skin} = [\Delta p \text{ in skin zone due to } k_{skin}] - [\Delta p \text{ in skin zone due to k}]$ ---------------- **(3-23)**

Applying Darcy's equation in radial form

$$Q_o = \frac{0.00708\, kh(p_e - p_w)}{\mu_o B_o \ln\left({r_e}/{r_w}\right)}$$

rearrange Darcy's equation Eq.

$$(P_e - P_w) = \Delta P = 141.2 \frac{q\,\mu\,B}{k\,h} \ln\left(\frac{r_e}{r_w}\right)$$

Substitute above Eq. in Eq. (3-23)

$$\Delta P_{skin} = 141.2 \left(\frac{q\,\mu\,B}{K_s h}\right) \ln\left(\frac{r_s}{r_w}\right) - 141.2 \left(\frac{q\,\mu\,B}{Kh}\right) \ln\left(\frac{r_s}{r_w}\right)$$

Simplify…….

$$\Delta P_{skin} = 141.2 \left(\frac{q\,\mu\,B}{k\,h}\right) \left(\frac{K}{K_s} - 1\right) \ln\left(\frac{r_s}{r_w}\right) \quad \text{------} \quad \textbf{(3-24)}$$

Where:

k=permeability of the formation, md.

K_{skin}=permeability of the formation, md.

❖ The skin factor (s) is defined from eq. (3-24) as;

$$S = \left(\frac{K}{K_s} - 1\right) \ln\left(\frac{r_s}{r_w}\right) \quad \text{------} \quad \textbf{(3-25)}$$

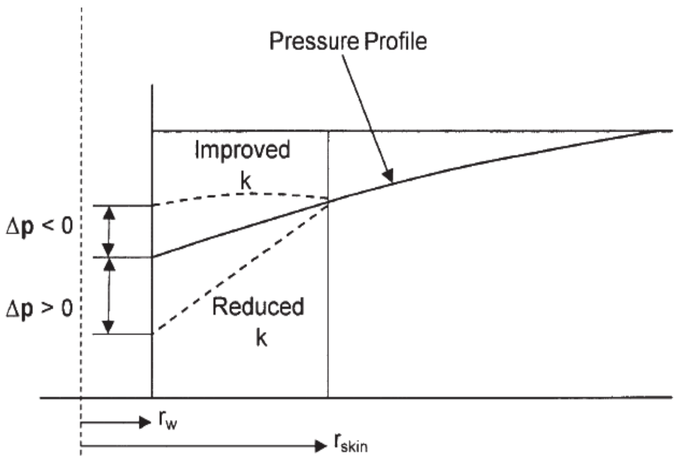
So;

$$\Delta P_{skin} = 141.2 \left(\frac{q\,\mu\,B}{k\,h}\right) S \quad \text{------} \quad \textbf{(3-26)}$$

❖ Eq. (3-26) can be applied to all flow regimes to account for the skin zone around the wellbore.

- From Eq. (3-26), there are only three possible outcomes in evaluating the skin factor s;

- **Positive Skin Factor, s > 0**
  - ✓ Damage exist,
  - ✓ $k_{skin} < k$
- **Negative Skin Factor, s < 0**
  - ✓ improved conditions exist (stimulation)
  - ✓ $k_{skin} > k$
- **Zero Skin Factor, s = 0**
  - ✓ $k_{skin} = k$.



❖ In Unsteady-State Radial Flow for Slightly Compressible Fluids:

$$p_i - p = -\left(\frac{70.6\,q\,\mu\,B}{Kh}\right) E_i\left(\frac{-948\emptyset\,\mu\,C_t r^2}{K\,t}\right) + \Delta P_s \quad \text{------} \quad \textbf{(3-27)}$$

$$= -\left(\frac{70.6\,q\,\mu\,B}{Kh}\right) E_i\left(\frac{-948\emptyset\,\mu\,C_t r^2}{K\,t}\right) + \left(\frac{141.2\,q\,\mu\,B}{Kh}\right) s$$

$$= -\left(\frac{70.6\, q\, \mu\, B}{Kh}\right)\left[E_i\left(\frac{-948\, \emptyset\, \mu\, C_t r^2}{K\, t}\right) - 2s\right]$$

❖ For $r = r_w$, $P = P_{wf}$, the argument of the Ei functions sufficiently small after a short time that we can use the logarithmic approximation

$$p_i - p_{wf} = -\left(\frac{70.6\, q\, \mu\, B}{Kh}\right)\left[\ln\left(\frac{1688\, \emptyset\, \mu\, C_t r_w{}^2}{K\, t}\right) - 2S\right] \quad \text{-------------------------------------- } \mathbf{(3\text{-}28)}$$

Eq. (3-28) is used to calculate the sand-face pressure (P_{wf}) at of a well with an altered zone.

While the equation;

$$P_{(r,t)} = pi + \left(\frac{70.6 Q_o \mu_o B_o}{Kh}\right) E_i\left(\frac{-948\emptyset\, \mu_o\, C_t\, r^2}{K\, t}\right)$$

is used to calculate pressure beyond the altered zone in the formation surrounding the well.

Problem (3-2)

An oil well is producing at a constant flow rate of 300 STB/day under unsteady-state flow conditions. The reservoir has the following rock and fluid properties:

$B_o = 1.25$ bbl/STB	$\mu_o = 1.5$ cp	$c_t = 12 \times 10^{-6}$ psi^{-1}
$k_o = 60$ md	$h = 15$ ft	$p_i = 4000$ psi
$\phi = 15\%$	$r_w = 0.25$ ft	

1. Calculate pressure at radii of 0.25, 5, 10, 50, 100, 500, 1,000, 1,500, 2,000, and 2,500 feet, for 1 hour.
   Plot the results as:

   a. Pressure versus logarithm of radius
   b. Pressure versus radius

2. Repeat part 1 for t = 12 hours and 24 hours. Plot the results as pressure versus logarithm of radius.

Solution:

Step 1. From Equation (3-21)

$$p(r,t) = 4000 + \left[\frac{70.6\,(300)\,(1.5)\,(1.25)}{(60)\,(15)}\right]$$

$$\times E_i\left[\frac{-948\,(.15)(1.5)\,(12 \times 10^{-6})\,r^2}{(60)\,(t)}\right]$$

$$p(r,t) = 4000 + 44.125\,E_i\left[-42.6(10^{-6})\frac{r^2}{t}\right]$$

TABLE 1.1* – VALUES OF THE EXPONENTIAL INTEGRAL, $-Ei(-x)$

$-Ei(-x)$, 0.000<0.209, interval – 0.001

x	0	1	2	3	4	5	6	7	8	9
0.00	$+\infty$	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
0.01	4.038	3.944	3.858	3.779	3.705	3.637	3.574	3.514	3.458	3.405
0.02	3.355	3.307	3.261	3.218	3.176	3.137	3.098	3.062	3.026	2.992
0.03	2.959	2.927	2.897	2.867	2.838	2.810	2.783	2.756	2.731	2.706
0.04	2.681	2.658	2.634	2.612	2.590	2.568	2.547	2.527	2.507	2.487
0.05	2.468	2.449	2.431	2.413	2.395	2.377	2.360	2.344	2.327	2.311
0.06	2.295	2.279	2.264	2.249	2.235	2.220	2.206	2.192	2.178	2.164
0.07	2.151	2.138	2.125	2.112	2.099	2.087	2.074	2.062	2.050	2.039
0.08	2.027	2.015	2.004	1.993	1.982	1.971	1.960	1.950	1.939	1.929
0.09	1.919	1.909	1.899	1.889	1.879	1.869	1.860	1.850	1.841	1.832
0.10	1.823	1.814	1.805	1.796	1.788	1.779	1.770	1.762	1.754	1.745
0.11	1.737	1.729	1.721	1.713	1.705	1.697	1.689	1.682	1.674	1.667
0.12	1.660	1.652	1.645	1.638	1.631	1.623	1.616	1.609	1.603	1.596
0.13	1.589	1.582	1.576	1.569	1.562	1.556	1.549	1.543	1.537	1.530
0.14	1.524	1.518	1.512	1.506	1.500	1.494	1.488	1.482	1.476	1.470
0.15	1.464	1.459	1.453	1.447	1.442	1.436	1.431	1.425	1.420	1.415
0.16	1.409	1.404	1.399	1.393	1.388	1.383	1.378	1.373	1.368	1.363
0.17	1.358	1.353	1.348	1.343	1.338	1.333	1.329	1.324	1.319	1.314
0.18	1.310	1.305	1.301	1.296	1.291	1.287	1.282	1.278	1.274	1.269
0.19	1.265	1.261	1.256	1.252	1.248	1.243	1.239	1.235	1.231	1.227
0.20	1.223	1.219	1.215	1.210	1.206	1.202	1.198	1.195	1.191	1.187

$-Ei(-x)$, 0.00<x<2.09, interval = 0.01

x	0	1	2	3	4	5	6	7	8	9
0.0	$+\infty$	4.038	3.335	2.959	2.681	2.468	2.295	2.151	2.027	1.919
0.1	1.823	1.737	1.660	1.589	1.524	1.464	1.409	1.358	1.309	1.265
0.2	1.223	1.183	1.145	1.110	1.076	1.044	1.014	0.985	0.957	0.931
0.3	0.906	0.882	0.858	0.836	0.815	0.794	0.774	0.755	0.737	0.719
0.4	0.702	0.686	0.670	0.655	0.640	0.625	0.611	0.598	0.585	0.572
0.5	0.560	0.548	0.536	0.525	0.514	0.503	0.493	0.483	0.473	0.464
0.6	0.454	0.445	0.437	0.428	0.420	0.412	0.404	0.396	0.388	0.381
0.7	0.374	0.367	0.360	0.353	0.347	0.340	0.334	0.328	0.322	0.316
0.8	0.311	0.305	0.300	0.295	0.289	0.284	0.279	0.274	0.269	0.265
0.9	0.260	0.256	0.251	0.247	0.243	0.239	0.235	0.231	0.227	0.223
1.0	0.219	0.216	0.212	0.209	0.205	0.202	0.198	0.195	0.192	0.189
1.1	0.186	0.183	0.180	0.177	0.174	0.172	0.169	0.166	0.164	0.161
1.2	0.158	0.156	0.153	0.151	0.149	0.146	0.144	0.142	0.140	0.138
1.3	0.135	0.133	0.131	0.129	0.127	0.125	0.124	0.122	0.120	0.118
1.4	0.116	0.114	0.113	0.111	0.109	0.108	0.106	0.105	0.103	0.102
1.5	0.1000	0.0985	0.0971	0.0957	0.0943	0.0929	0.0915	0.0902	0.0889	0.0876
1.6	0.0863	0.0851	0.0838	0.0826	0.0814	0.0802	0.0791	0.0780	0.0768	0.0757
1.7	0.0747	0.0736	0.0725	0.0715	0.0705	0.0695	0.0685	0.0675	0.0666	0.0656
1.8	0.0647	0.0638	0.0629	0.0620	0.0612	0.0603	0.0595	0.0586	0.0578	0.0570
1.9	0.0562	0.0554	0.0546	0.0539	0.0531	0.0524	0.0517	0.0510	0.0503	0.0496
2.0	0.0489	0.0482	0.0476	0.0469	0.0463	0.0456	0.0450	0.0444	0.0438	0.0432

2.0<x<10.9, interval = 0.1

x	0	1	2	3	4	5	6	7	8	9
2	4.89×10^{-2}	4.26×10^{-2}	3.72×10^{-2}	3.25×10^{-2}	2.84×10^{-2}	2.49×10^{-2}	2.19×10^{-2}	1.92×10^{-2}	1.69×10^{-2}	1.48×10^{-2}
3	1.30×10^{-2}	1.15×10^{-2}	1.01×10^{-2}	8.94×10^{-3}	7.89×10^{-3}	6.87×10^{-3}	6.16×10^{-3}	5.45×10^{-3}	4.82×10^{-3}	4.27×10^{-2}
4	3.78×10^{-3}	3.35×10^{-3}	2.97×10^{-3}	2.64×10^{-3}	2.34×10^{-3}	2.07×10^{-3}	1.84×10^{-3}	1.64×10^{-3}	1.45×10^{-3}	1.29×10^{-3}
5	1.15×10^{-3}	1.02×10^{-3}	9.08×10^{-4}	8.09×10^{-4}	7.19×10^{-4}	6.41×10^{-4}	5.71×10^{-4}	5.09×10^{-4}	4.53×10^{-4}	4.04×10^{-4}
6	3.60×10^{-4}	3.21×10^{-4}	2.86×10^{-4}	2.55×10^{-4}	2.28×10^{-4}	2.03×10^{-4}	1.82×10^{-4}	1.62×10^{-4}	1.45×10^{-4}	1.29×10^{-4}
7	1.15×10^{-4}	1.03×10^{-4}	9.22×10^{-5}	8.24×10^{-5}	7.36×10^{-5}	6.58×10^{-5}	5.89×10^{-5}	5.26×10^{-5}	4.71×10^{-5}	4.21×10^{-5}
8	3.77×10^{-5}	3.37×10^{-5}	3.02×10^{-5}	2.70×10^{-5}	2.42×10^{-5}	2.16×10^{-5}	1.94×10^{-5}	1.73×10^{-5}	1.55×10^{-5}	1.39×10^{-5}
9	1.24×10^{-5}	1.11×10^{-5}	9.99×10^{-6}	8.95×10^{-6}	8.02×10^{-6}	7.18×10^{-6}	6.44×10^{-6}	5.77×10^{-6}	5.17×10^{-6}	4.64×10^{-6}
10	4.15×10^{-6}	3.73×10^{-6}	3.34×10^{-6}	3.00×10^{-6}	2.68×10^{-6}	2.41×10^{-6}	2.16×10^{-6}	1.94×10^{-6}	1.74×10^{-6}	1.56×10^{-6}

*Adapted from Nisle, R.G.: "How To Use The Exponential Integral," *Pet. Eng.* (Aug. 1956) B171-173.

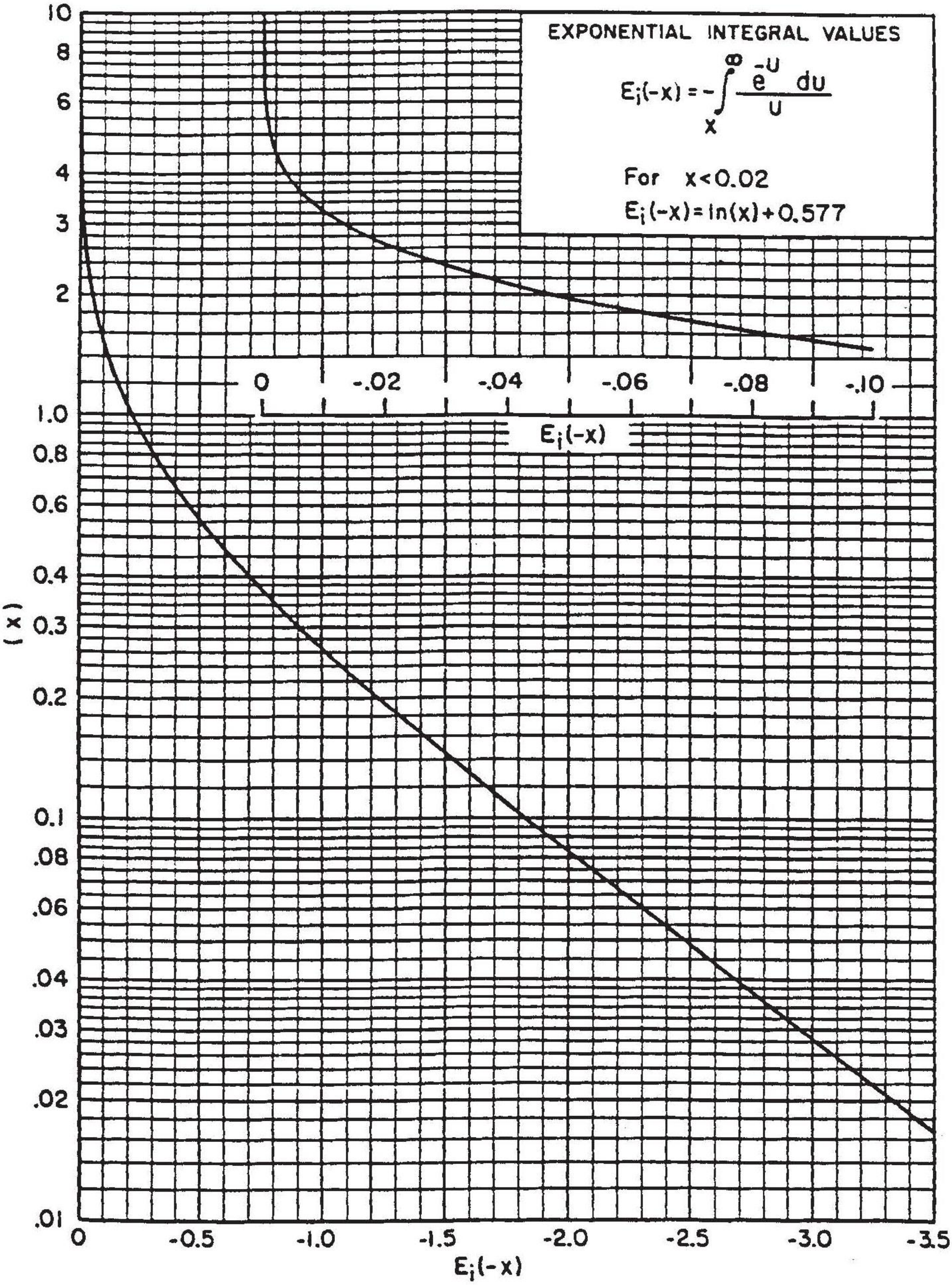

EXPONENTIAL INTEGRAL VALUES
$E_i(-x) = -\int_x^\infty \frac{e^{-u}}{u} du$
For x<0.02
$E_i(-x) = \ln(x) + 0.577$
0
-.02
-.04
-.06
-.08
-.10
$E_i(-x)$
10
8
6
4
3
2
1.0
0.8
0.6
0.4
0.3
0.2
0.1
.08
.06
.04
.03
.02
.01
(x)
0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
$E_i(-x)$

Figure (3-1)

Step 2: Perform the required calculations after one hour in the following tabulated form:

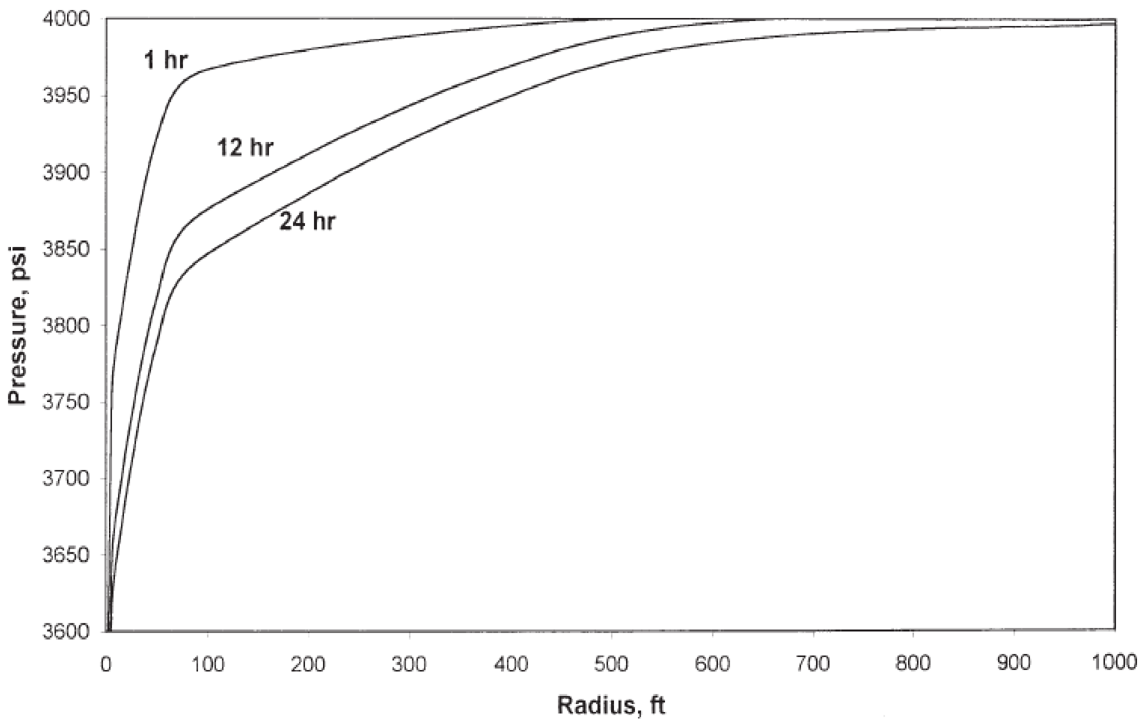
Elapsed Time t = 1 hr

r, ft	$x = -42.6(10^{-6})\ \frac{r^2}{1}$	$E_i(-x)$	$p(r,1) = 4000 + 44.125\ E_i(-x)$
0.25	$-2.6625(10^{-6})$	−12.26*	3459
5	−0.001065	−6.27*	3723
10	−0.00426	−4.88*	3785
50	−0.1065	−1.76†	3922
100	−0.4260	−0.75†	3967
500	−10.65	0	4000
1000	−42.60	0	4000
1500	−95.85	0	4000
2000	−175.40	0	4000
2500	−266.25	0	4000

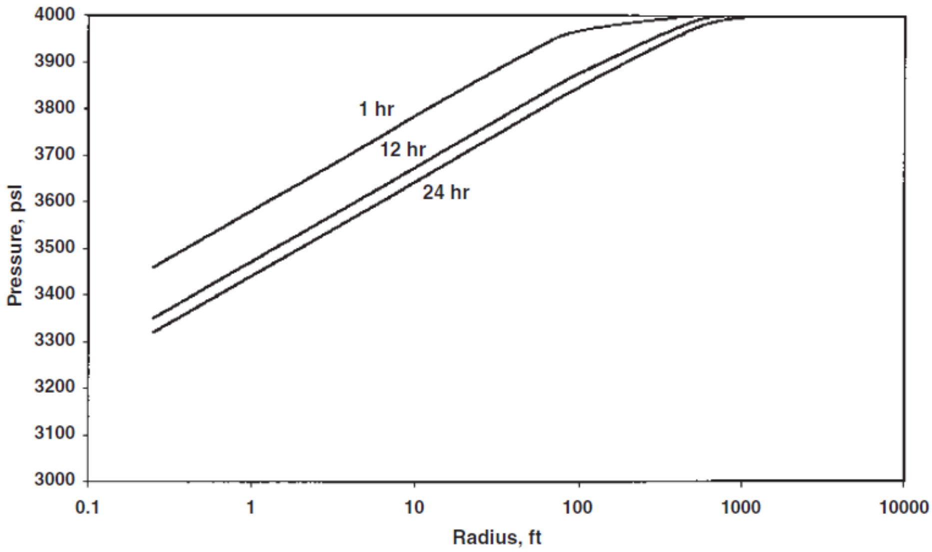
**As calculated from Equation*
†*From Figure*

We show that most of the *pressure loss* occurs close to the wellbore; so, near-wellbore conditions will have the greatest influence on flow behavior.

Step 3: Show results of the calculation graphically.



Last Figure shows that the pressure profile and the drainage radius are continuously changing with time.



Step 4: Repeat the calculation for t = 12 and 24 hrs.

Elapsed Time t = 12 hrs

r, ft	$x = 42.6(10^{-6})\ \dfrac{r^2}{12}$	$E_i(-x)$	$p(r,12) = 4000 + 44.125\ E_i(-x)$
0.25	$0.222\ (10^{-6})$	−14.74*	3350
5	$88.75\ (10^{-6})$	−8.75*	3614
10	$355.0\ (10^{-6})$	−7.37*	3675
50	0.0089	−4.14*	3817
100	0.0355	−2.81†	3876
500	0.888	−0.269	3988
1000	3.55	−0.0069	4000
1500	7.99	$-3.77(10^{-5})$	4000
2000	14.62	0	4000
2500	208.3	0	4000

**As calculated from Equation*

†From Figure

Elapsed Time t = 24 hrs

r, ft	$x = 42.6(10^{-6})\ \frac{r^2}{24}$	$E_i(-x)$	$p(r,24) = 4000 + 44.125\ E_i(-x)$
0.25	$-0.111\ (10^{-6})$	−15.44*	3319
5	$-44.38\ (10^{-6})$	−9.45*	3583
10	$-177.5\ (10^{-6})$	−8.06*	3644
50	−0.0045	−4.83*	3787
100	−0.0178	−3.458†	3847
500	−0.444	−0.640	3972
1000	−1.775	−0.067	3997
1500	−3.995	−0.0427	3998
2000	−7.310	$8.24\ (10^{-6})$	4000
2500	−104.15	0	4000

**As calculated from Equation*
†From Figure

Problem (3-2)

A well and reservoir have the following characteristics: the well is producing only oil; it is producing at a constant rate of 20 STB/D. Data describing the well and formation are: $\mu = 0.72$ cp, $k = 0.1$ md, $c_t = 1.5 * 10^{-5}\ psi^{-1}$, $p_i = 3000$ psi, $r_e = 3000$ ft, $r_w = 0.5$ ft, $B_o = 1.475$ RB/STB, $h = 150$ ft, $Ø = 0.23$, and $s = 0$.

Calculate the reservoir pressure at a radius of 1 ft after 3 hours of production; then, calculate the pressure at radii of 10 and 100 ft after 3 hours of production.

Solution:

The Ei function is not an accurate solution to flow equations until…..

$$\frac{3.79 * 10^5\ Ø\ \mu\ C_t\ r_w^{\ 2}}{k} < t < \frac{948\ Ø\ \mu\ C_t\ r_e^{\ 2}}{k}$$

$$\frac{3.79 * 10^5\ Ø\ \mu\ C_t\ r_w^{\ 2}}{k} = \frac{(3.79 * 10^5)(0.23)\ (0.72)\ (1.5 * 10^{-5})\ (0.5)^2}{(0.1)} = 2.35 < t = 3 \text{ hours.}$$

$$\frac{948\ Ø\ \mu\ C_t\ r_e^{\ 2}}{k} = \frac{948\ (0.23)(0.72)(1.5 * 10^{-5})(3000)^2}{(0.1)} = 211900 \text{ hrs.} > t = 3 \text{ hrs.}$$

Thus, we can use Eq. (3-21).

- At a radius of 1 ft. and t=3,

$$p(1,3) = 3000 + \left(\frac{70.6 * 20 * 1.475 * 0.72}{0.1 * 150}\right) E_i \left(\frac{-948 * 0.23 * 0.72 * 1.5 * 10^{-5} * 1^2}{0.1 * 3}\right)$$

$$p = 3000 + 100 \, Ei(-0.007849)$$

$$p = 3000 + 100 \ln[(1.781)(0.007849)]$$

$$p = 3000 + 100(-4.27)$$

$$p = 2573 \text{ psi}$$

- At a radius of 10 ft and t=3,

$$p(10,3) = 3000 + (100)E_i \left(\frac{-948 * 0.23 * 0.72 * 1.5 * 10^{-5} * 10^2}{0.1 * 3}\right)$$

$$p = 3000 + 100 \, Ei(-0.7849)$$

$$p = 3000 + 100(-0.318)$$

$$p = 2968 \text{ psi}$$

- at a radius of 100 ft. and t=3,

$$p(100,3) = 3000 + (100)E_i \left(\frac{-948 * 0.23 * 0.72 * 1.5 * 10^{-5} * 100^2}{0.1 * 3}\right)$$

p = 3000 + 100 Ei(-78.49)

p = 3000 psi.

➢ Infinite-Acting Reservoir & Finite-Radial Reservoir

A. Infinite-Acting Reservoir

- ✓ The reservoir boundaries and the shape of the drainage area not influence the wellbore pressure response.
- ✓ Represent the transient flow case (***infinite acting state)***.
- ✓ Reservoir is unbounded (***infinite size***).

B. Finite-Radial Reservoir

- ✓ Represent the end of the ***transient flow period*** and the beginning of the semi ***(pseudo)-steady state***.
- ✓ The reservoir boundaries and the shape of the drainage area effect the wellbore pressure response.
- ✓ Reservoir is bounded. (finite size).

➢ Pseudo-steady-State Solution

❖ The ***pseudo-steady-state solution*** describes pressure behavior with time for a well centered in a cylindrical reservoir of radius $\mathbf{r_e}$. And when;

$$t > \frac{948\, Ø\, \mu\, C_t\, r_e^{\,2}}{k}$$

$$p_{wf} = p_i - 141.2\, \frac{q\,B\,\mu}{kh}\left[\frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4}\right] \text{--} \mathbf{(3\text{-}29)}$$

Where:

$$t_D = 0.000264\,\frac{K\,t}{\varphi\,\mu\,C_t r_w^2} \qquad \text{and} \qquad r_{eD} = \frac{r_e}{r_w}$$

So;

$$p_{wf} = p_i - 141.2\, \frac{q\,B\,\mu}{kh}\left[\frac{0.000527\,K\,t}{\varphi\,\mu\,C_t r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4}\right] \text{---------------------------------} \mathbf{(3\text{-}30)}$$

Where:

r_{eD} = dimensionless external radius

t_D = dimensionless time

t = time, hr

r_e = drainage radius, ft

r_w = wellbore radius, ft

❖ The importance of dimensionless variables is that they simplify the diffusivity equation and its solution by combining the reservoir parameters (such as permeability, porosity, etc.) and thereby reduce the total number of unknowns.

❖ By differentiating Eq. (3-30) with respect to time,

$$\frac{\partial P_{wf}}{\partial t} = \frac{-\,0.0744\,qB}{Ø\,c_t\;h\;r_e^2}$$

Since the liquid-filled pore volume of the reservoir, v_p (ft^3) is

$$v_p = \pi\,Ø\,h\,r_e^2 \qquad \text{sub. in eq. above}$$

$$\frac{\partial P_{wf}}{\partial t} = \frac{-\,0.234\,qB}{v_p\,c_t} \text{---} \mathbf{(3\text{-}31)}$$

here $\qquad \dfrac{\partial P_{wf}}{\partial t}\,\alpha\,\dfrac{1}{v_p}$

Eq. (3-31) leads to a form of well testing called "**reservoir limit testing**".

❖ Another form of Eq. (3-30) is useful in replacing (**P_i**, **original reservoir presser**) with (**P_r**) **average pressure**. The volumetric average pressure **P_r** can be found from material balance. (**P_r** depends on production volume during specific time).

❖ The pressure decrease ($P_i - P_r$) resulting from removal of **qB** RB/D of fluid for **t** hours (a total volume removed of 5.615 qB (t/24) ft^3) is;

$$P_i - P_r = \frac{\Delta v}{v\, c_t} = \frac{5.615\, q\, B \left(\frac{t}{24}\right)}{c_t\, (\pi\, \emptyset\, h\, r_e^2)} = \frac{0.0744\, qB\, t}{c_t\, \emptyset\, h\, r_e^2}$$

Substituting in Eq. (3-30)

$$p_{wf} = p_r + \frac{0.0744\, qB\, t}{c_t\, \emptyset\, h\, r_e^2} - \frac{0.0744\, qB\, t}{c_t\, \emptyset\, h\, r_e^2} - 141.2\, \frac{q\, B\, \mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]$$

Or

$$p_r - p_{wf} = 141.2\, \frac{q\, B\, \mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right] \text{ -- } \mathbf{(3\text{-}32)}$$

Adding the effect of the skin **factor** to Eq. (3-30) and (3-32),

$$p_{wf} = p_i - 141.2\, \frac{q\, B\, \mu}{kh} \left[\frac{0.000527\, K\, t}{\varphi\, \mu\, C_t r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4}\right] + \Delta P_s$$

where; $\Delta P_s = 141.2\, \dfrac{q\, B\, \mu}{kh}\, s$

$$p_{wf} = p_i - 141.2\, \frac{q\, B\, \mu}{kh} \left[0.000527 \frac{K\, t}{\varphi\, \mu\, C_t r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4}\right] + 141.2\, \frac{q\, B\, \mu}{kh}\, s$$

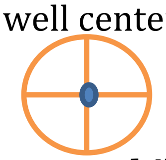
$$p_i - p_{wf} = 141.2\, \frac{q\, B\, \mu}{kh} \left[\frac{0.000527\, K\, t}{\varphi\, \mu\, C_t r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right] \text{ ------------------------ } \mathbf{(3\text{-}33)}$$

And ....

$$p_r - p_{wf} = 141.2\, \frac{q\, B\, \mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right] + \Delta P_s$$

$$p_r - p_{wf} = 141.2\, \frac{q\, B\, \mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right] \text{ -- } \mathbf{(3\text{-}34)}$$

Equation (3-33) and (3-34) used for well centered in a circular drainage area.



❖ Further, we can define an **average permeability**, k_{avg}, such that

$$p_r - p_{wf} = 141.2\, \frac{q\, B\, \mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right] \qquad \text{(A)}$$

$$= 141.2\, \frac{q\, B\, \mu}{k_{avg} h} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right] \qquad \text{(B)}$$

Equating Eq. (A) and (B), solving for K_{avg}

$$k_{avg} = K\, \frac{\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]}{\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \text{ -- } \mathbf{(3\text{-}35)}$$

$K > k_{avg} \rightarrow$ damage $\rightarrow s = +ve$

$K < k_{avg} \rightarrow$ stimulation $\rightarrow s = -ve$

$K = k_{avg} \rightarrow$ no damage, no stimulation $\rightarrow s = 0$

- Sometimes we can estimate the permeability of a well from productivity-index (PI) measurement, and since the productivity index J (STB/D/psi), of an oil well is defined as;

$$PI = J = \frac{q}{P_r - p_{wf}} = \frac{k_{avg}\,h}{141.2\ B\,\mu\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]} \quad \text{------------------} \quad \textbf{(3-36)}$$

$$FE = \frac{k_{avg}}{K} \quad \text{------------------} \quad \textbf{(3-37)}$$

$$s = \left(\frac{K}{K_{avg}} - 1\right)\ln\left(\frac{r_e}{r_w}\right) - 0.75 \quad \text{------------------} \quad \textbf{(3-38)}$$

Problem (3-3):

A well **produces 100 STB/D** oil at a measured flowing bottom-hole pressure **(BHP) of 1500 psi**. A recent pressure survey showed that **average reservoir pressure is 2000 psi**. Logs indicate a net sand **thickness of 10 ft**. the well drains an area with **drainage radius, r_e, of 1000 ft**; the **borehole radius is 0.25** ft. Fluid samples indicate that, at current reservoir pressure, **oil viscosity is 0.5 cp** and **formation volume factor is 1.5 RB/STB**.

1- Estimate the productivity index for the tested well.
2- Estimate formation permeability from these data.
3- Core data from the will indicate an effective permeability to oil of 50 md. Does this imply that the well is either damaged or stimulated? What is the apparent skin factor?

Solution:

1. $PI = J = \frac{q}{P_r - p_{wf}} = \frac{100}{2000-1500} = 0.25\,\frac{STB/day}{psi}$

2. we do not have sufficient information to estimate formation permeability; we can calculate average permeability, k_J, only, which is not necessarily a good approximation of formation permeability, k, from Eq. (3-36);

$$PI == \frac{k_{avg}\,h}{141.2\ B\,\mu\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]}$$

$$0.25 == \frac{k_{avg}\,(10)}{141.2\,(1.5)(0.5)\left[\ln\left(\frac{1000}{0.25}\right) - 0.75\right]}$$

$k_{avg} = 16$ md

3. Core data frequently provide a better estimate of formation permeability than do permeabilities derived from the productivity index, particularly for a well that is

badly damaged. Since cores indicate a permeability of **50 md**, we conclude that this well is damaged. Eq. (3-36) provides a method for estimating the skin factor **S**:

$$s = \left(\frac{K}{K_{avg}} - 1\right)\ln\left(\frac{r_e}{r_w}\right) - 0.75 = \left(\frac{50}{16} - 1\right)\ln\left(\frac{1000}{0.25}\right) - 0.75 = 16$$

➢ Flow equations for generalized reservoir geometry

Eq. (3-33) and (3-34) are limited to a well ***centered in a circular drainage area***, for more general reservoir shapes a correction factor called the **shape factor, C_A**, was introduced to Eq. (3-33) and (3-34) as:

$$p_r - p_{wf} = 141.2\ \frac{q\,B\,\mu}{kh}\left[\frac{1}{2}\ln\left(\frac{10.06\ A}{C_A\ r_w^2}\right) - 0.75 + s\right] \quad \textbf{(3-39)}$$

Where:

A = drainage area, ft^2

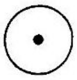
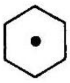
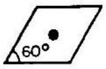
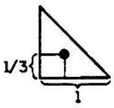
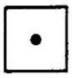
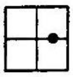
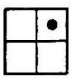
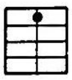
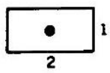
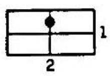
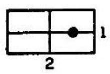
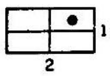
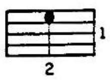
C_A = shape factor for specific drainage area shape and well location, dimensionless. Values of C_A are given in table (2).

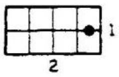
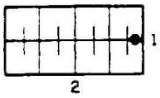
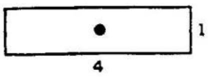
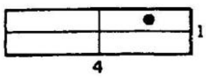
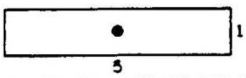
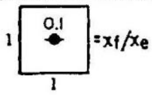
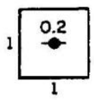
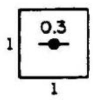
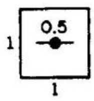
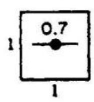
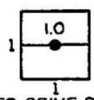
Shape factor, C_A is designed to account for the deviation of the drainage area from the ideal circular form.

- Productivity index, **J**, can be expressed for general drainage-area geometry as;

$$PI = J = \frac{q}{P_r - p_{wf}} = \frac{0.00708\ K\,h}{B\,\mu\left[\frac{1}{2}\ln\left(\frac{10.06\ A}{C_A\ r_w^2}\right) - 0.75 + S\right]} \quad \textbf{(3-40)}$$

Table 2
Shape Factors for Various Single-Well Drainage Areas (*After Earlougher, R.,* Advances in Well Test Analysis, *permission to publish by the SPE, copyright SPE, 1977*)

In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less Than 1% Error For t_{DA} >	Use Infinite System Solution with Less Than 1% Error for t_{DA} <
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
	27.1	3.2995	−1.2452	0.2	0.07	0.09
	21.9	3.0865	−1.1387	0.4	0.12	0.08
	0.098	−2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	−1.3106	0.1	0.05	0.09
	12.9851	2.5638	−0.8774	0.7	0.25	0.03
	4.5132	1.5070	−0.3490	0.6	0.30	0.025
	3.3351	1.2045	−0.1977	0.7	0.25	0.01
	21.8369	3.0836	−1.1373	0.3	0.15	0.025
	10.8374	2.3830	−0.7870	0.4	0.15	0.025
	4.5141	1.5072	−0.3491	1.5	0.50	0.06
	2.0769	0.7309	−0.0391	1.7	0.50	0.02
	3.1573	1.1497	−0.1703	0.4	0.15	0.005

In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less Than 1% Error For t_{DA} >	Use Infinite System Solution with Less Than 1% Error for t_{DA} <
	0.5813	−0.5425	+0.6758	2.0	0.60	0.02
	0.1109	−2.1991	+1.5041	3.0	0.60	0.005
	5.3790	1.6825	−0.4367	0.8	0.30	0.01
	2.6896	0.9894	−0.0902	0.8	0.30	0.01
	0.2318	−1.4619	+1.1355	4.0	2.00	0.03
	0.1155	−2.1585	+1.4838	4.0	2.00	0.01
	2.3606	0.8589	−0.0249	1.0	0.40	0.025
IN VERTICALLY FRACTURED RESERVOIRS	Use $(x_e/x_f)^2$ in place of A/r_w^2 for fractured systems					
	2.6541	0.9761	−0.0835	0.175	0.08	cannot use
	2.0348	0.7104	+0.0493	0.175	0.09	cannot use
	1.9986	0.6924	+0.0583	0.175	0.09	cannot use
	1.6620	0.5080	+0.1505	0.175	0.09	cannot use
	1.3127	0.2721	+0.2685	0.175	0.09	cannot use
	0.7887	−0.2374	+0.5232	0.175	0.09	cannot use
IN WATER-DRIVE RESERVOIRS						
	19.1	2.95	−1.07	—	—	—
IN RESERVOIRS OF UNKNOWN PRODUCTION CHARACTER						
	25.0	3.22	−1.20	—	—	—

Other numerical constants tabulated in table (2) allow us to calculate:

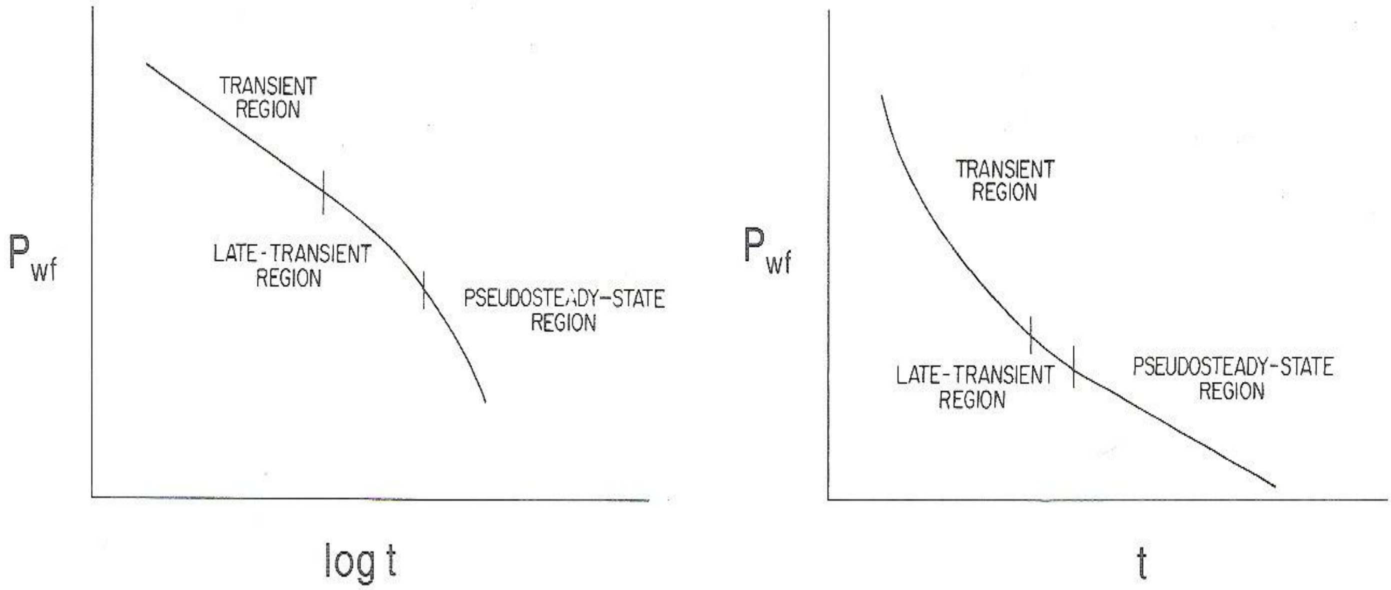
1. The maximum elapsed time during which a reservoir is infinite acting (so that the Ei-function solution can be used).
2. The time required for the pseudo-steady state solution to predict pressure draw down within 1% accuracy.
3. Time required for the pseudo-steady state solution to be exact.

- For a given reservoir geometry, **the maximum time a reservoir is infinite acting** can be determined using the entry in the last (**column 7**)
  "Use infinite system solution with less than 1% error for t_{DA} <."
  Since
  $$t_{DA} = 0.000264 \frac{K\,t}{\varphi\,\mu\,C_t A}$$
  this means that the *time in hours* is calculated from;
  $$t < \frac{\varphi\,\mu\,C_t A\,t_{DA}}{0.000264\,K}$$
- **Time required for the pseudo-steady state equation to be accurate** within 1% can be found from the entry in the (**column 6**)
  "less than 1% error for t_{DA} >"
  and the relationship is:
  $$t > \frac{\varphi\,\mu\,C_t A\,t_{DA}}{0.000264\,K}$$
- Finally, time required for the pseudo-steady state equation to be exact is found from the entry in the (**column 5**)
  "exact for t_{DA} >".

- At this point, it is helpful to describe graphically the flow regimes that occur in different time ranges.

✓ In the **transient region**, the reservoir ***is infinite acting***, and is modeled by Eq. (3-28), which implies that $\mathbf{p_{wf}}$, is a linear function of log **t**.

✓ In the **pseudo-steady state region**, the reservoir is modeled by Eq. (3-39).



- ✓ At times between the **end of the transient region** and the **beginning of the pseudo-steady state region,** this is a **transition region**, sometimes called the **late-transient region**. No simple equation is available to predict the relationship between BHP and time in this region.
  This region is small or nonexistent for a **well centered** in a circular, square, or hexagonal drainage area, as table (2) indicates. However, for a well off-center in its drainage area, the late-transient region can extent a significant time region, as table (2) also indicates.

Problem (3-4):

For each of the following reservoir geometries, calculate the ***time in hours*** for which.

A. The reservoir is infinite acting,
B. The pseudo-steady state is exact,
C. The pseudo-steady state equation is accurate to within 1%.
   1. Well centered in circular drainage area.
   2. Well centered in square drainage area.
   3. Well centered in one quadrant of square drainage area.

In each case,

$A = 17.42 * 10^6\ ft^2$ (40 acres), $Ø = 0.2$, $\mu = 1$ cp, $c_t = 1* 10^{-5}\ psi^{-1}$ $k = 100$ md.

Solution:

$$t > \frac{\varphi\ \mu\ C_t A}{0.000264\ K}\ t_{DA}$$

$$t > \frac{(0.2)(1)(10^{-5})(17.42*10^6)}{0.000264\ (100)}\ t_{DA}$$

$t > 1320\ t_{DA}$

Prepare the follwing table (vlaues from table (2))

	Infinite soluution		Pseudo-stead state (Approximate)		Pseudo-steady state (Exact)	
Geometry	t_{DA}	t (hrs)	t_{DA}	t (hrs)	t_{DA}	t (hrs)
Circular Centered	0.1	132	0.06	79.2	0.1	132
Square Centered	0.09	119	0.05	66	0.1	132
quadrant of square	0.025	33	0.3	396	0.6	792

Problem (3-5):

A. For each of the wells in **problem (3-4)**, estimate PI and stabilized production rate with $p_r - p_{wf} = 500$ psi, if $h = 10$ ft, $s = 3$, $r_w = 0.3$ ft $B = 1.2$ bbl/Stb.

B. For the well centered in one of the quadrants of a square, write equations relating constant flow rate and well-bore pressure drops at elapsed times of 30, 200, and 400 hours.

Solution:

A.

$$PI = J = \frac{q}{P_r - p_{wf}} = \frac{0.00708\ K\ h}{B\ \mu\left[\frac{1}{2}\ln\left(\frac{10.06\ A}{C_A\ r_w^2}\right) - 0.75 + S\right]}$$

$$PI = \frac{0.00708\ (100)(10)}{(1.2)\ (1)\left[\frac{1}{2}\ln\left(\frac{10.06*17.42*10^6}{C_A\ (0.3)^2}\right) - 0.75 + 3\right]} = \frac{5.9}{12.94 - 0.5\ln C_A}$$

and

$$J = \frac{q}{P_r - p_{wf}} = \frac{q}{500} \quad \rightarrow q = 500\ J$$

Thus, can prepare the following table:

Geometry	C_A	J(STB/D/psi)	q (stb/d)
Circular Centered	31.62	0.526	263
Square Centered	30.88	0.526	263
quadrant of square	4.513	0.484	242

B. For **t = 30 hours**, the reservoir is infinite acting, and

$$p_i - p_{wf} = -\left(\frac{70.6\, q\, \mu\, B}{Kh}\right)\left[\ln\left(\frac{1688\, Ø\, \mu\, C_t r_w^{\,2}}{K\, t}\right) - 2S\right]$$

For **t = 200 hours**, the reservoir is no longer infinite acting and the pseudo-steady state equation is not yet accurate. Accordingly, no simple equation can be written.

For **t = 400 hours**, the pseudo-steady state equation is accurate, and

$$p_r - p_{wf} = 141.2\ \frac{q\, B\, \mu}{kh}\left[\frac{1}{2}\ln\left(\frac{10.06\, A}{C_A\ r_w^{\,2}}\right) - 0.75 + s\right]$$

➢ Radius of investigation (r_i)

It's a distance that a pressure transient has moved into a formation following a rate change in a well, this distance is related to formation rock and fluid properties and time elapsed since the rate change.

$$r_i = \left(\frac{kt}{948\, Ø\mu\, ct}\right)^{1/2}$$

r_i = investigation radius, ft. k = permeability, md. t = flow time, hours. Ø = porosity. μ = viscosity, cp.
c_t = total compressibility, psi^{-1}

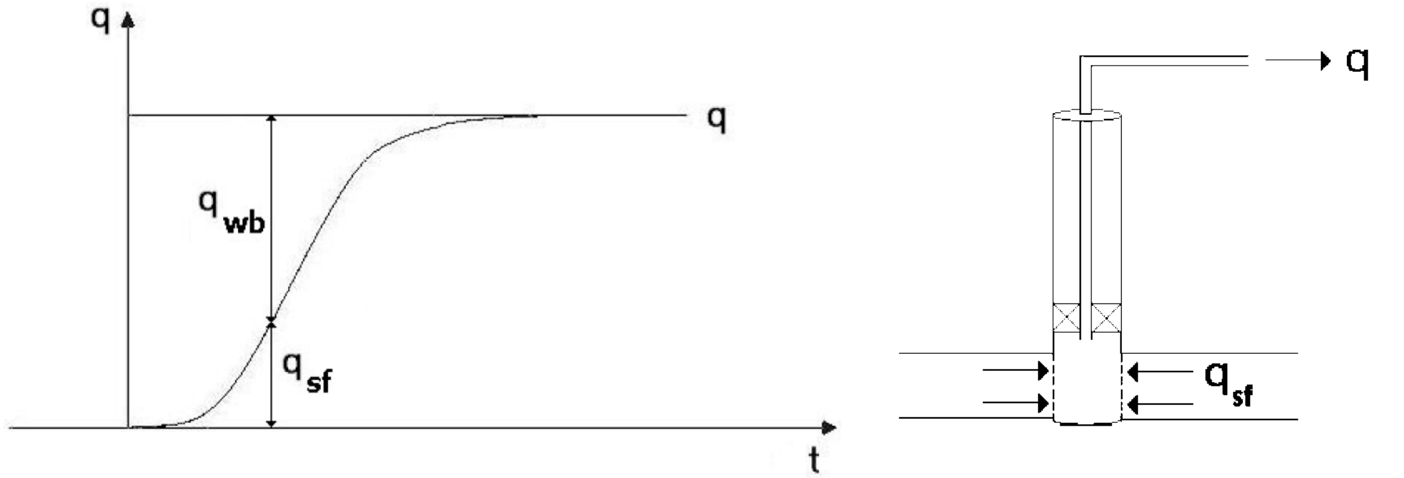
➢ Wellbore Storage

Immediately after a rate change, part of the production may be due to expansion or compression of fluid in the wellbore. It could also be due to a moving fluid contact (interface). These phenomena have been called the ***wellbore storage effect***.

- Many well test interpretation techniques imply the assumption of a constant rate.
- Suppose it is possible to keep the surface flow rate, q, constant. Then the first production comes from the ***wellbore*** and ***not the reservoir***.
- The reservoir flow rate, $\mathbf{q_{sf}}$ (sand face flow rate), will slowly build up to become equal to the surface flow rate.
- The fluid produced, **q**, is the sum production of fluid from the wellbore, $\mathbf{q_{wb}}$, and fluid coming from the reservoir, $\mathbf{q_{sf}}$.

$q = q_{wb} + q_{sf}$ -- **(3-41)**

The first production comes from the well. This is illustrated below.



❖ **after flow:** Immediately after shut-in, the wellbore pressure is lower than out in the reservoir. Fluid will continue to flow into the well after shut-in. This is called ***after flow***.
The wellbore pressure will increase as a result of fluid compression. Finally, the pressures will be equalized and the inflow into the well will stop.

After flow causes several problems such as: -

1) Delay in beginning of MTR.
2) MTR recognition is more difficult.
3) Total lack of development of the MTR, with relatively long periods of after flow.
4) Development of several false straight lines.

➢ Wellbore Storage Analysis

Semi-log analysis is possible when $\boldsymbol{q_{wb}} \approx \mathbf{0}$. Then the wellbore storage effect has died out. The production from the wellbore is given by:

$$q_{wb} = -c_{wb}V_{wb}\frac{d\Delta p_{wf}}{dt} \text{ ------------------- } \mathbf{(3\text{-}41)}$$

Where:

$\mathbf{C_{wb}}$ is the compressibility and $\mathbf{V_{wb}}$ is the volume of the compressed fluid. The pressure drawdown, $\mathbf{\Delta p_{wf}}$, is given by:

$$\Delta p_{wf} = p_i - p_{wf} \text{ ------------------- } \mathbf{(3\text{-}42)}$$

For negligible sandface production, i.e. $\boldsymbol{q_{sf}} \approx \mathbf{0}$, all the fluid produced at the surface derives from fluid expansion.

$q = q_{wb}$

Since the surface rate **q**, is constant, the differential equation eq. (3-41) may be integrated to yield:

$$\Delta p_{wf} = \frac{q\,B}{C_s}t \text{ ------------------- } \mathbf{(3\text{-}43)}$$

To identify the end of wellbore storage effect, the data have been plotted on **log-log coordinates paper** as: **Δp vs. Δt_e ,** in build-up test, **Δp vs. t** , in draw down test.

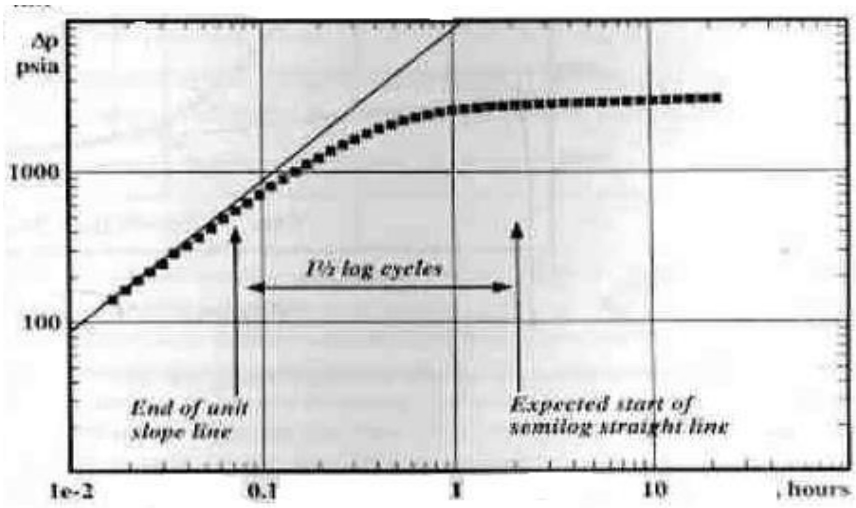
Where:

$\Delta p = (p_{ws} - p_{wf})$, in build-up test

$\Delta p = (p_i - p_{wf})$, in draw down test

$\Delta t_e = \Delta t / (1 + \Delta t / t_p)$

At earliest times, a "**unit-slope line**" (i.e., line with 45° slope) is present on the graph. This line appears and remains as long as all production comes from the well-bore and none comes from the formation.



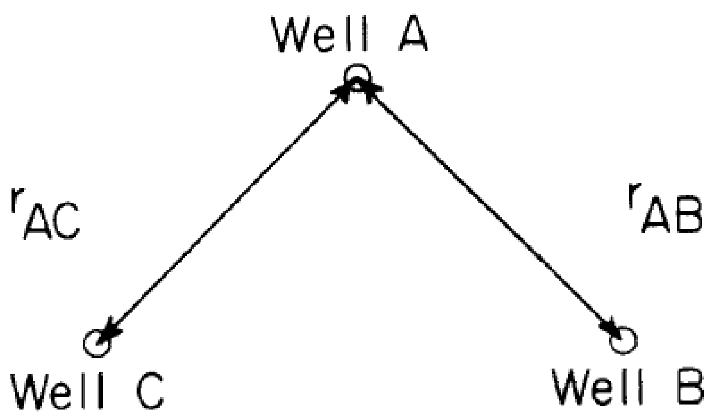
➤ Principle of Superposition

- The solutions of the diffusivity equation are applicable under the following conditions;
  - ✓ Infinite reservoir.
  - ✓ Constant production
  - ✓ Single well.
- But……the real reservoir systems usually have ***several wells*** that are operating at ***variable rates***.
- The principle of superposition is a powerful concept that can be applied to ***remove the restrictions that have been imposed on solutions of the diffusivity equation.***
- The superposition theorem ***states that*** any sum of individual solutions to the diffusivity equation is also a solution to that equation.

A. Multiple Wells Reservoirs (Superposition in Space)

- The total pressure drop at any point in a reservoir is the sum of the pressure drop at that point caused by flow in each of the wells in the reservoir.
- As an example, consider three wells, Wells A, B and C, that start to produce at the same time from an infinite reservoir

$$(Pi - Pwf)_{\text{total at Well A}} = (Pi - P)_{\text{due to Well A}} + (Pi - P)_{\text{due to Well B}} + (Pi - P)_{\text{due to Well C}}$$



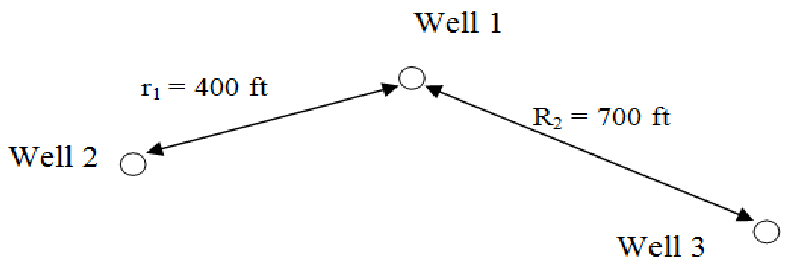
In terms of Ei functions and logarithmic approximations;

$$(Pi - Pwf)_{\text{total at WellA}} = -70.6\left(\frac{q_A\,\mu\,B}{Kh}\right)\left[\ln\left(\frac{1688\,\emptyset\,\mu\,C_t r_{wA}^2}{K\,t}\right) - 2S_A\right] - 70.6\left(\frac{q_B\,\mu\,B}{Kh}\right)E_i\left(\frac{-948\,\emptyset\,\mu\,C_t r_{AB}^2}{K\,t}\right) - 70.6\left(\frac{q_C\,\mu\,B}{Kh}\right)E_i\left(\frac{-948\,\emptyset\,\mu\,C_t r_{AC}^2}{K\,t}\right) \quad \text{-------------------- (3-44)}$$

Problem (3-6)

Assume that the three wells as shown in below figure are producing under a transient flow condition for 15 hours. The following additional data is available:

q_{01} = 100 Stb/day,
q_{02} = 160 Stb/day,
q_{03} = 200 Stb/day,
p_i = 4500 psi,
B_o=1.2 bbl/Stb,
$c_t = 20 * 10^{-6}$ psi^{-1},



s (well1) = -0.5, h = 20 ft, Ø = 15%, k = 40 md, r_w = 0.25 ft, μo = 2 cp,
r_1 = 400 ft, r_2 = 700 ft.

If the three wells are producing at a constant flow rate, calculate the sand face flowing pressure (P_{wf}) at well (1).

Solution:

1- Calculate the pressure drop at well1 caused by its own production by;

$$(\text{Pi} - \text{Pwf})_{\text{total at Well 1}} = -70.6\left(\frac{q_1\,\mu\,B}{Kh}\right)\left[\ln\left(\frac{1688\,Ø\,\mu\,C_t r_w{}^2}{K\,t}\right) - 2S_A\right]$$

$$(\text{Pi} - \text{Pwf})_{\text{total at Well 1}} =$$

$$-70.6\frac{(100)(1.2)(2)}{(40)(20)}\left[\ln\left(\frac{1688(0.15)(2)(20*10^{-6})(0.25)^2}{(40)(15)}\right) - 2(-0.5)\right]$$

$$(\text{Pi} - \text{Pwf})_{\text{total at Well 1}} = 270.2 \text{ psi}$$

2. Calculate the pressure drop at Well 1 due to the production from Well 2.

$$(\text{Pi} - \text{P})_{\text{due to Well 2}} = -70.6\left(\frac{q_2\,\mu\,B}{Kh}\right)E_i\left(\frac{-948\,Ø\,\mu\,C_t r_1{}^2}{K\,t}\right)$$

$$(\Delta p)_{\text{due to well 2}} = -\frac{(70.6)(160)(1.2)(2)}{(40)(20)}$$

$$\times E_i\left[-\frac{(948)(0.15)(2.0)(20\times10^{-6})(400)^2}{(40)(15)}\right]$$

$$= 33.888\,[-E_i\,(-1.5168)]$$

$$= (33.888)\,(0.13) = 4.41 \text{ psi}$$

3. Calculate pressure drop due to production from Well 3.

$$(\text{Pi} - \text{P})_{\text{due to Well 3}} = -70.6\left(\frac{q_3\,\mu\,B}{Kh}\right)E_i\left(\frac{-948\,Ø\,\mu\,C_t r_2{}^2}{K\,t}\right)$$

$$(\Delta p)_{\text{due to well 3}} = -\frac{(70.6)\,(200)\,(1.2)\,(2)}{(40)\,(20)}$$

$$\times E_i\left[-\frac{(948)\,(0.15)(2.0)(20\times 10^{-6})(700)^2}{(40)\,(15)}\right]$$

$$= (42.36)[-E_i\,(-4.645)]$$

$$= (42.36)(1.84\times 10^{-3}) = 0.08\,\text{psi}$$

4. Calculate total pressure drop at Well 1.

$$(\Delta p)_{\text{total at well 1}} = 270.2 + 4.41 + 0.08 = 274.69\text{ psi}$$

5. Calculate p_{wf} at Well 1.

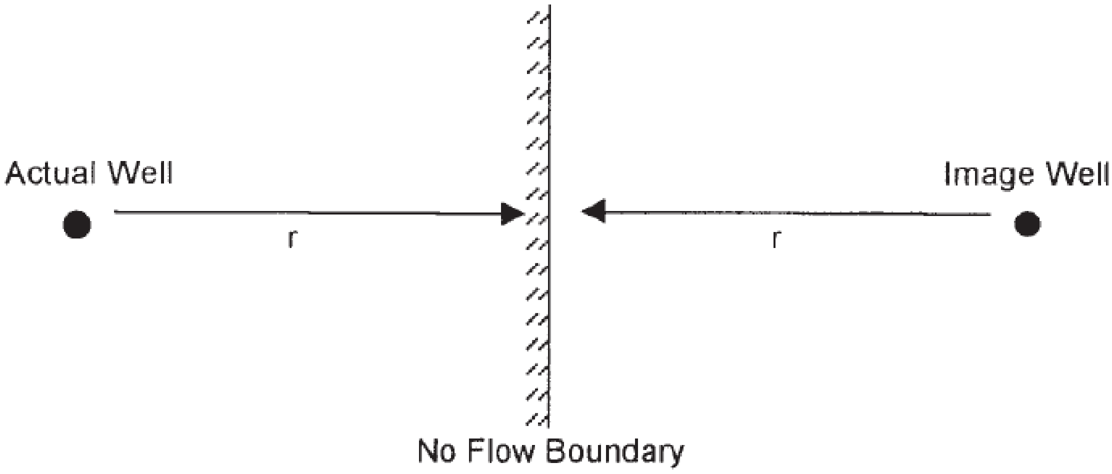
$$\Delta P_{\text{total at Well 1}} = Pi - Pwf$$

$$p_{wf} = 4500 - 274.69 = 4225.31\text{ psi}$$

B. Bounded Reservoir (Superposition in Space)

Consider a well that is located a distance **r** from the no-flow boundary, e.g., sealing fault, which can be represented by zero pressure gradient

$\left(\frac{\partial P}{\partial r}\right)_{\text{boundary}} = 0.$



Assume an ***image* well**, identical to that of the ***actual well***, on the other side of the fault at exactly distance **r**.

The total pressure drops at the actual well will be the pressure drop due to its own production plus the additional pressure drop caused by an identical well at a distance of 2r, or:

$(\Delta p)_{total} = (\Delta p)_{\text{actual well}} + (\Delta p)_{\text{due to image well}}$

Or;

$$(Pi - Pwf)_{total} = -70.6\left(\frac{q\,\mu\,B}{Kh}\right)\left[\ln\left(\frac{1688\,Ø\,\mu\,C_t r_w^{2}}{K\,t}\right) - 2S_A\right] - 70.6\left(\frac{q\,\mu\,B}{Kh}\right)E_i\left(\frac{-948\,Ø\,\mu\,C_t(2r)^2}{K\,t}\right) \quad \textbf{(3-45)}$$

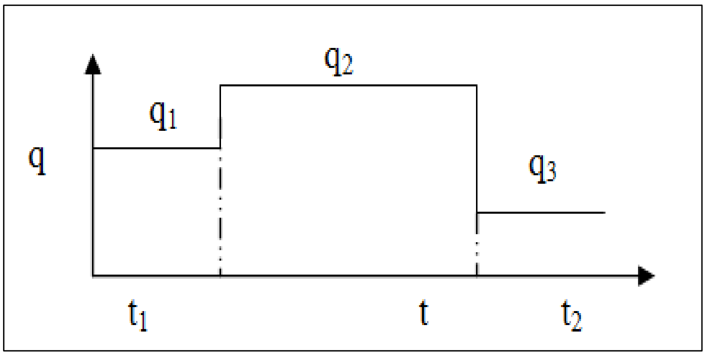
C. Variable Flow Rates (Superposition in Time)

A well is produces at three different rates…

q_1 from t= 0 to time t_1

q_2 from t_1 to t_2

at time t_2 the rate is changed to q_3.



$$(\Delta p)_{total} = (\Delta p)_{due\ to\ (q1-0)} + (\Delta p)_{due\ to\ (q2-q1)} + (\Delta p)_{due\ to\ (q3-q2)}$$

$$(\Delta \mathbf{p})_{1\ (q1-0)} = (\mathbf{Pi} - \mathbf{Pwf})_1 =$$

$$-70.6\left(\frac{(q_1-0)\ \mu\ B}{Kh}\right)\left[\ln\left(\frac{1688\ \emptyset\ \mu\ C_t r_w^2}{K\ t}\right) - 2S\right]$$

$$(\Delta \mathbf{p})_{2\ (q2-q1)} = (\mathbf{Pi} - \mathbf{Pwf})_2 =$$

$$-70.6\left(\frac{(q_2-q_1)\ \mu\ B}{Kh}\right)\left[\ln\left(\frac{1688\ \emptyset\ \mu\ C_t r_w^2}{K\ (t-t_1)}\right) - 2S\right]$$

$$(\Delta \mathbf{p})_{3\ (q3-q2)} = (\mathbf{Pi} - \mathbf{Pwf})_3 =$$

$$-70.6\left(\frac{(q_3-q_2)\ \mu\ B}{Kh}\right)\left[\ln\left(\frac{1688\ \emptyset\ \mu\ C_t r_w^2}{K\ (t-t_2)}\right) - 2S\right]$$

Thus, the total pressure drawdown for the well with changes in rates is:

$$(Pi - Pwf) = (\Delta p)_1 + (\Delta p)_2 + (\Delta p)_3 =$$

$$-70.6\left(\frac{(q_1-0)\ \mu\ B}{Kh}\right)\left[\ln\left(\frac{1688\ \emptyset\ \mu\ C_t r_w^2}{K\ t}\right) - 2S\right] - 70.6\left(\frac{(q_2-q_1)\ \mu\ B}{Kh}\right)\left[\ln\left(\frac{1688\ \emptyset\ \mu\ C_t r_w^2}{K\ (t-t_1)}\right) - 2S\right] - 70.6\left(\frac{(q_3-q_2)\ \mu\ B}{Kh}\right)\left[\ln\left(\frac{1688\ \emptyset\ \mu\ C_t r_w^2}{K\ (t-t_2)}\right) - 2S\right]$$ -- **(3-46)**

- **Horner's approximation**

- In 1951, Horner reported an approximation that can be used in many cases to avoid the use of superposition in modeling the production history of a variable-rate well.
- With this approximation, we can replace the sequence of **Ei** function, reflecting rate changes, with a single **Ei** function that contains a single producing time and a single producing rate.
- The single rate is the most recent rate at which the well was produced (q_{last}).
- The single producing time is found by dividing cumulative production from the well by the most recent rate.

$$t_p\ (\text{hours}) = \frac{\text{cumulative production from well,Np (STB)}}{\text{most recent rate,} q_{last}\ \text{(STB/D)}}$$ --------------------------------------- **(3-47)**

t_p = single producing time = *pseudo-producing time*

Then, to model pressure behavior at any point in a reservoir, we can use the simple equation;

$$Pi - P = -70.6\ \frac{q_{last}\ \mu\ B}{k\ h}\ E_i\left(\frac{-948\ \emptyset\ \mu\ C_t r^2}{K\ t_p}\right)$$ --- **(3-48)**

Horner's approximation will be adequate(acceptable) in the case when;

$$\frac{\text{time of } q_{last}}{\text{time of q before } q_{last}} \geq 2$$

Problem (3-7)

Following completion, a well is produced for a short time and then shut-in for a buildup test. The production history was as follows.

Production time (hrs)	Total production (STB)
25	52
12	0
26	46
72	68

1) Calculate the pseudo producing time, t_p.
2) Is **Horner's approximation** adequate for this case? If not, how should the production history for this well be simulated?

Solution:

$$q_{last} = \frac{68\ \text{STB}}{72\ \text{hrs}}\left(24\ \frac{\text{hrs}}{\text{day}}\right) = 22.7\ \text{STB/D}$$

Then;

$$t_p = \frac{\text{cumulative production from well,Np (STB)}}{\text{most recent rate,}q_{last}\ \text{(STB/D)}}$$

$$t_p = \frac{24*166}{22.7} = 176\ \text{hrs}$$

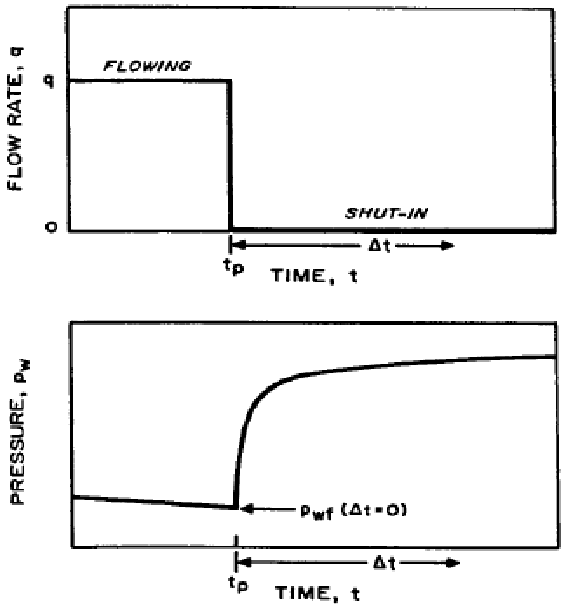
In this case,

$$\frac{\text{time of } q_{last}}{\text{time of q before } q_{last}} = \frac{72}{26} = 2.77 > 2$$

Thus, **Horner's approximation** is probably adequate for this case. It should not be necessary to use superposition, which is required when **Horner's approximation** is not adequate.

➢ Pressure Buildup Test

❖ Pressure buildup analysis describes the buildup in wellbore pressure with time after a well has been shut-in.



Idealized rate schedule and pressure response for buildup testing.

❖ One of the principal objectives of this analysis is to determine the static reservoir pressure without waiting weeks or months for the pressure in the entire(whole) reservoir to stabilize.

❖ pressure buildup analysis used to determine:

- initial reservoir pressure or average drainage area pressure.
- Permeabilty in the drainage area of the well.
- Skin factor in the region immediately adjacent to the well bore.

❖ The equation describing a pressure buildup test is derive bassed on this assummptions;

- Radial, steady state ***flow***.
- An inifinite acting, homogeneous, isotropic ***reservoir***.
- A slightly compressible, single-phase, constant μ_o and B_o ***fluid***.

Then an equation modeling a pressure buildup test can be developed by ues of superposition in time.

❖ pressure buildup analysis based on **Horner Equation, which is a solution of Diffusivity Equation, for fluid flow in porous media.**

❖ The pressure buildup equation was introduced by Horner (1951) and is commonly referred to as the **Horner equation**.

Horner equation or pressure buildup equation is;

$$P_{ws} = P_i - \frac{162.6\ q\ B\ \mu}{k\ h} \log\left(\frac{t_p + \Delta t}{\Delta t}\right) \quad \text{--} \quad \mathbf{(3\text{-}49)}$$

Or:

$$P_{ws} = P_i - \frac{70.6\ q\ B\ \mu}{k\ h} \ln\left(\frac{t_p + \Delta t}{\Delta t}\right) \quad \text{--} \quad \mathbf{(3\text{-}50)}$$

Where p_i = initial reservoir pressure, psi
p_{ws} = bottom hole shut-in pressure, psi
t_p = flowing time before shut-in, hr
Δt = shut-in time, hr

Eq. (3-49) suggests that a plot of $\mathbf{p_{ws}}$ versus $\mathbf{(t_p + \Delta t)/\Delta t}$ would produce a straight-line relationship with intercept pi and slope of **–m**.

Comparing Eq. (3-49) with the equation of a straight line,

$$y = mx + b$$

$$y = p_{ws}$$

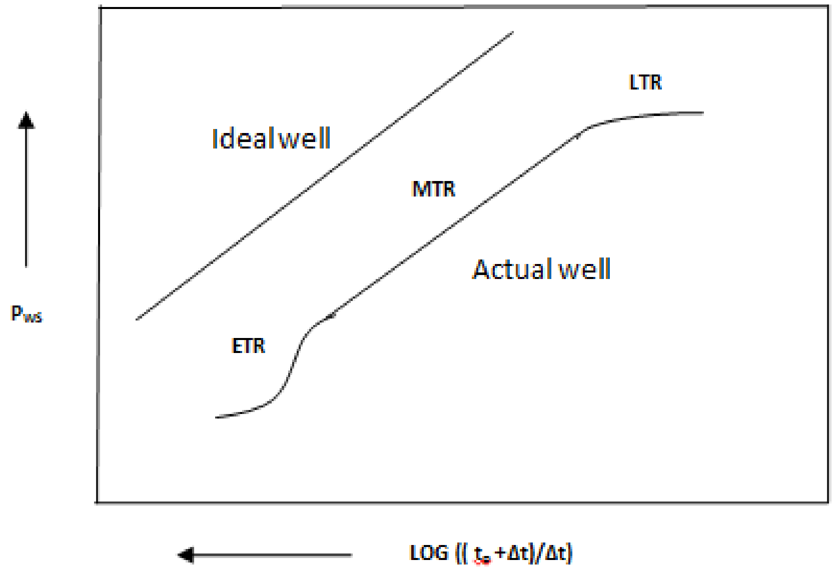
$$b = p_i$$

$$m = -\frac{162.6\ q\ B\ \mu}{k\ h} \quad \text{or} \quad k = -\frac{162.6\ q\ B\ \mu}{m\ h} \quad \text{--} \quad \mathbf{(3\text{-}51)}$$

$$x = \log\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

- plot of $\mathbf{p_{ws}}$ versus $\mathbf{(t_p + \Delta t)/\Delta t}$, is commonly referred to as the Horner plot.
- For **ideal buildup** pressure test, we obtain a single straight line for all times. For **actual buildup test**, obtain a curve with a complicated shape. Buildup curve can divide an into three regions;

1. **An early-time region (ETR):** during which a pressure transient is moving through the formation nearest the wellbore, (include wellbore effects → **wellbore storage, skin factor** and **non-Darcy effects).**
2. **A middle-time region (MTR):** during which the pressure transient has moved away from the wellbore. The data are taken from this region to estimate formation properties because its consider reservoir behavior.
3. **A late-time region (LTR):** in which the radius of investigation has reached the well's drainage boundaries.



Steps of determining reservoir properties by using Horner plot to analysis pressure buildup test

❖ pressure buildup test analysis includes the following steps:

1. Plot **p_{ws} vs. time ratio ((t_p + Δt) / Δt)** on semi-logarithmic paper.
2. Identify the correct straight line from **MTR**, and determine the slope m.
   Note: *the straight-line slope (m) is found by simply subtracting the pressure at any two points on the straight line that are* ***one cycle*** *apart on the semi-log paper.*
3. **Estimate effective formation permeability** (k)

$$k = -\frac{162.6\ q\ B\ \mu}{m\ h} \quad \text{(3-52)}$$

4. **Estimate original reservoir pressure (p_i)** from semi-log graph by extending the straight line to intercept y-axis at time ratio [(t_p + Δt) / Δt] = 1.
5. Determine p_{wf} after 1 hour (P_{1hr}) from the straight-line by setting or choose shut-in time Δt equal to1 hour.
   Δt = 1, i.e. $\frac{t_p + 1}{1}$
6. **Calculate the skin factor** s

$$s = 1.151\left[\left(\frac{p_{1hr} - p_{wf(\Delta t=0)}}{m}\right) - \log\left(\frac{k}{\emptyset\ \mu\ c_t\ r_w^2}\right) + 3.23\right] \quad \text{(3-53)}$$

Where, p_{wf} (Δt = 0) = flowing bottom-hole pressure immediately before shut-in.

- In summary, if we plot **p_{ws} vs. log ((t_p + Δt) / Δt)** with information obtained from a pressure buildup test, we can estimate effective **permeability**, k, **original reservoir pressure**, p_i, and the **skin factor**, s.

Problem (3-8)

A new oil well produced 123 STB/Day for 97.6 hours; it then was shut-in for a pressure buildup test, during which the data in table below were recorded. Δt = 12 hours, p_{wf} (Δt=0) = 4506 psi μ = 1 cp, B = 1.22 RB/STB, Net pay thickness = 20 ft Ø = 20%, $c_t = 20*10^{-6}$ psi^{-1}, r_w = 0.3 ft.

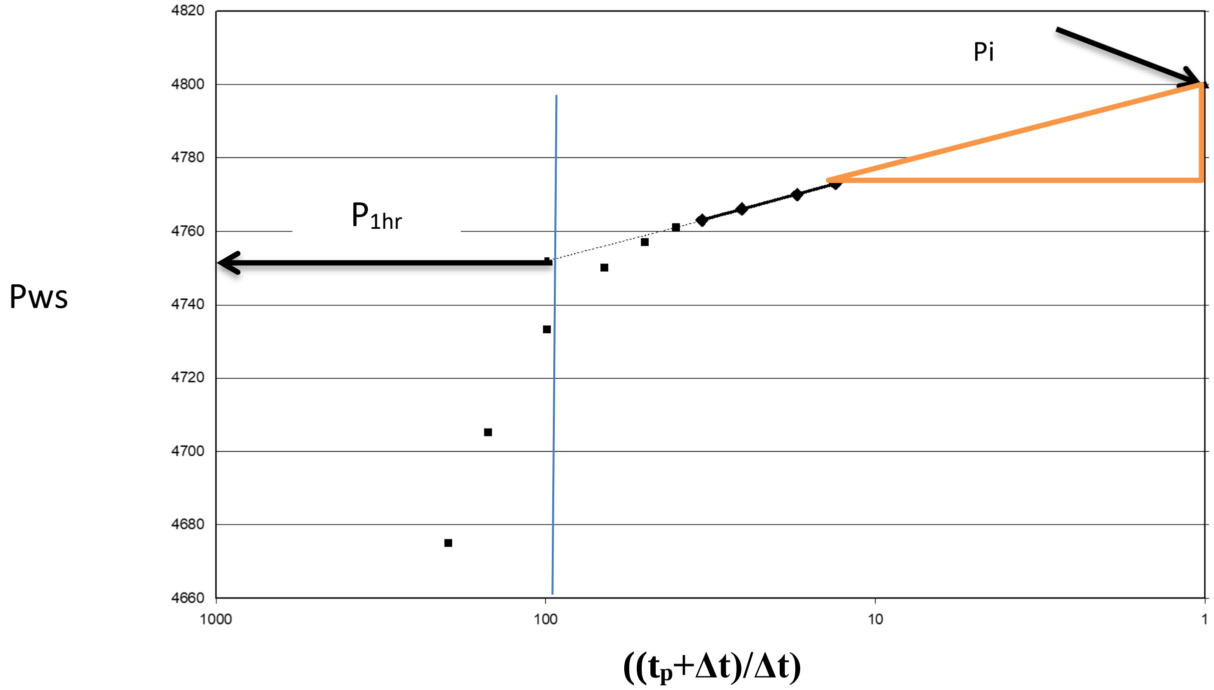
Determine the following:

1. Initial pressure p_i
2. Permeability k
3. Skin factor S
4. Flow efficiency FE
5. Radius of investigations r_i

p_{ws} (psi)	4506	4675	4705	3733	4750	4757	4761	4763	4766	4770	4773	3775	4777
Δt (hrs)	0	0.5	0.66	1	1.5	2	2.5	3	4	6	8	10	12

Solution:

1. plot pws vs. ((tp+Δt)/Δt) on semilog paper, draw the straight line from the transient region MTR (, extend the line to intercept y-axis at ((tp+Δt)/Δt) = 1, to determine **pi = 4800 psi**.



2. By slope of straight line, determine k

$$m = \frac{(4800 - 4775.5)}{(\log 1 - \log 10)} = -24.5 \text{ psi / cycle.}$$

$$k = -\frac{162.6 * 123 * 1 * 1.22}{(-24.5) * 20} = 50 \text{ md}$$

3. at $\Delta t = 1$ hr, $\left(\frac{t_p + \Delta t}{\Delta t}\right) = \frac{97.6 + 1}{1} = 98.6$ hr

from figure, at $\left(\frac{t_p + \Delta t}{\Delta t}\right) = 98.6$ hr, $p_{1hr} = 4752$ psi

$$s = 1.151\left[\frac{4752 - 4506}{24.5} - \log\left(\frac{50}{0.2 * 1 * 20 * 10 - 6\ 0.3^2}\right) + 3.23\right] = 6$$

4.

$$\Delta Pskin = 141.2\left(\frac{q\,\mu\,B}{k\,h}\right)s = 141.2\left(\frac{123 * 1 * 1.22}{50 * 20}\right)(6) = 128 \text{ psi}$$

$$FE = \frac{\Delta P - \Delta Pskin}{\Delta P} = \frac{P_i - P_{wf} - \Delta Pskin}{P_i - P_{wf}}$$

$$= \left(\frac{4800 - 4506 - 128}{4800 - 4506} = 0.56\right.$$

5.

$$r_i = \left(\frac{kt}{948\ \text{Ø}\mu\ ct}\right)^{1/2} = \left[\frac{50 * 97.6}{948 * 0.2 * 1 * 20 * 10^{-6}}\right]^{\frac{1}{2}} = 1134 \text{ ft}$$

➢ Effect and duration of after flow (wellbore storage)

After flow causes several problems such as:-

1. Delay in beginning of MTR.
2. MTR recognition is more difficult.
3. Total lack of development of the MTR, with relatively long periods of after flow.
4. Development of several false straight lines.

To identify the end of wellbore storage effect, the data have been plotted on **log-log coordinates paper** as;**Δp vs. Δt_e ,** in build-up test

Δp vs. t, in draw down test

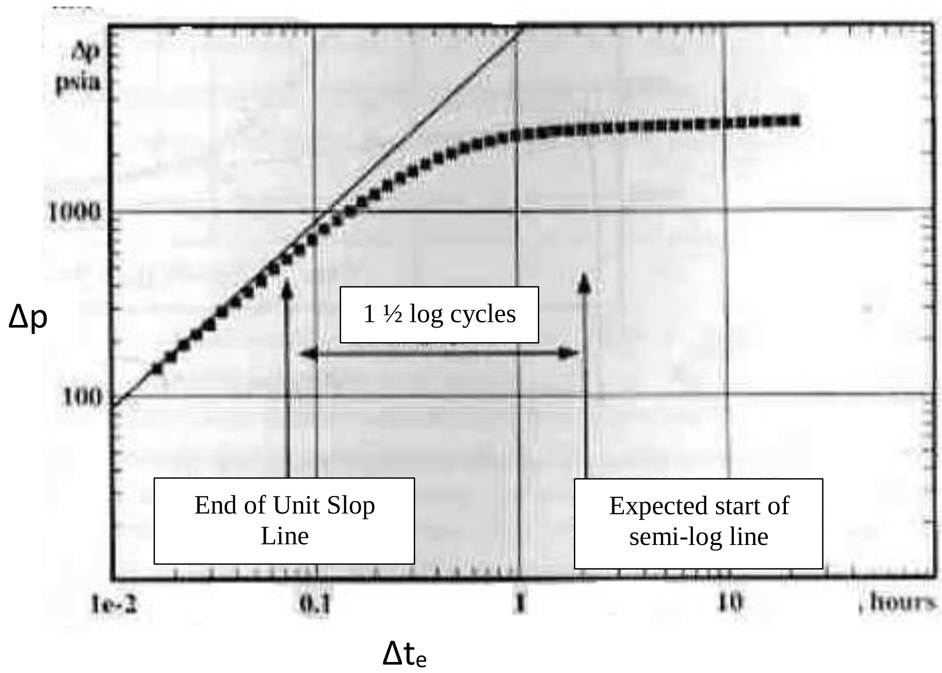
Where:

$\Delta p = (p_{ws} - p_{wf})$, in build-up test

$\Delta p = (p_i - p_{wf})$, in draw down test

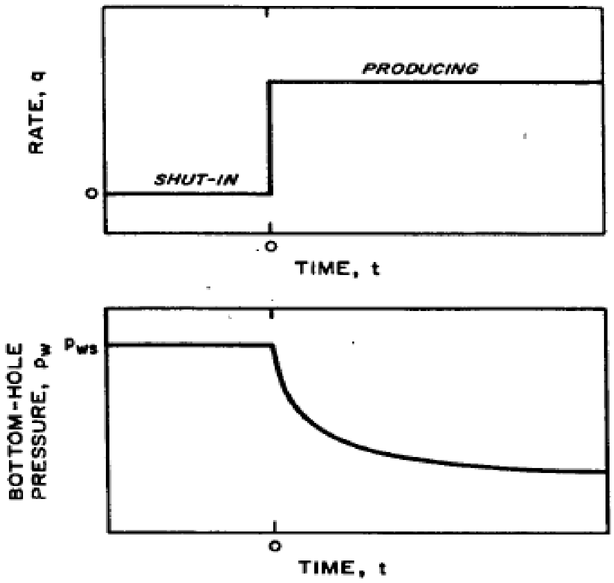
$\Delta t_e = \Delta t / (1 + \Delta t / t_p$

At earliest times, a "**unit-slope line**" (i.e., line with 45° slope) is present on the graph. This line appears and remains as long as all production comes from the well-bore and none comes from the formation.



➢ Drawdown Test

❖ A pressure drawdown test is conducted by producing a well at a known rate or rates while measuring changes in bottom-hole pressure (BHP) as a function of time.



Idealized rate schedule and pressure response for drawdown testing.

❖ The **objective of a pressure drawdown** test usually includes:

- ✓ Estimate of permeability (k), skin factor (s).
- ✓ Estimate the reservoir volume (hydrocarbon volume).

○ when the pressure transient is affected by outer reservoir boundaries, drawdown tests can be used to estimate the reservoir volume these flow tests are called **reservoir-limit tests**.

❖ An idealized constant-rate drawdown test in an infinite-acting reservoir (i.e., during the unsteady-state flow period) is given by the following equation:

$$P_{wf} = P_i - \frac{162.6\, q\, B\, \mu}{k\, h}\left[\log\left(\frac{k\, t}{\varphi\, \mu\, c_t\, r_w^2}\right) - 3.23 + 0.87s\right] \text{ ------------------------------------ } \mathbf{(3\text{-}54)}$$

Eq. (3-54) can be written as:

$$P_{wf} = P_i - \frac{162.6\, q\, B\, \mu}{k\, h}\left[\log(t) + \log\left(\frac{k}{\varphi\, \mu\, c_t\, r_w^2}\right) - 3.23 + 0.87s\right] \text{ -------------------------- } \mathbf{(3\text{-}55)}$$

Re-arrange eq. (3-55)

$$P_{wf} = P_i - \frac{162.6\, q\, B\, \mu}{k\, h}\left[\log\left(\frac{k}{\varphi\, \mu\, c_t\, r_w^2}\right) - 3.23 + 0.87s\right] - \frac{162.6\, q\, B\, \mu}{k\, h}\log(t) \text{ --------------- } \mathbf{(3\text{-}56)}$$

This equation is straight line equation expressed as:

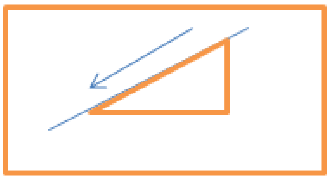
$Y = a + m\,X$

Where:

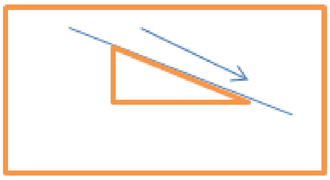
$Y = P_{wf}$ $\qquad\qquad$ $X = \log(t)$

$$a = P_i - \frac{162.6\,q\,B\,\mu}{k\,h}\left[\log\left(\frac{k}{\varphi\,\mu\,c_t\,r_w^2}\right) - 3.23 + 0.87s\right]$$

$$m\ (\text{slop}) = \frac{-162.6\,q\,B\,\mu}{k\,h}$$



$$Slope = \frac{Y2 - Y1}{X2 - X1} = +ve$$



$$Slope = \frac{Y2 - Y1}{X1 - X2} = -ve$$

$$k\,h = \text{formation capacity} = \frac{-162.6\,q\,B\,\mu}{m}$$

If the thickness is known, then the average permeability is given by:

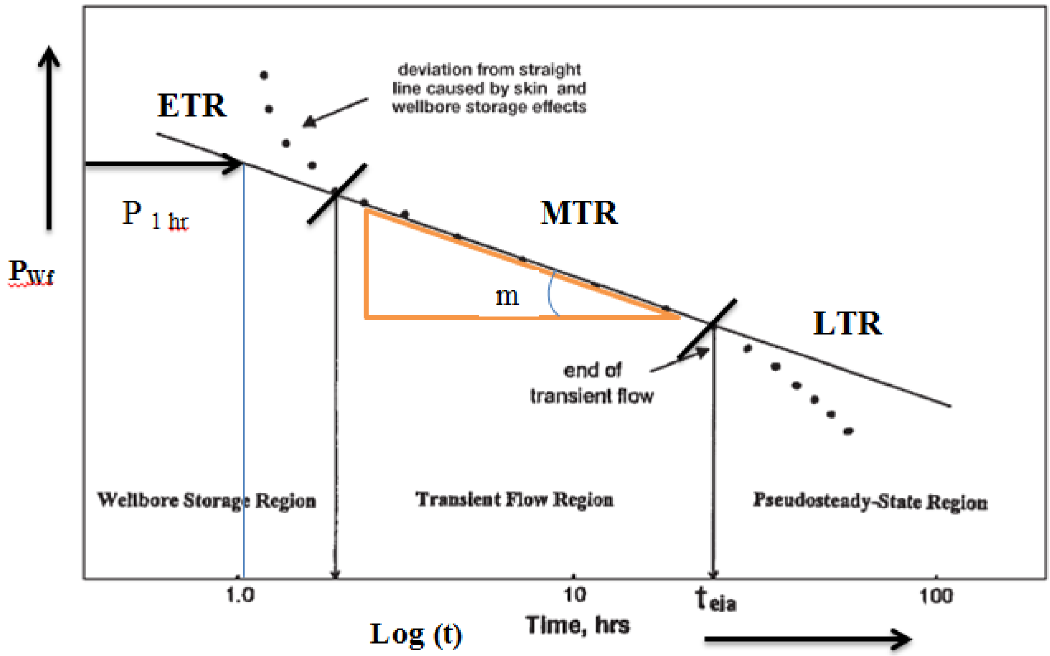
$$k = \frac{-162.6\,q\,B\,\mu}{m\,h}$$

where k = average permeability, md
m = slope, psi/cycle. Notice, slope m is negative.

- The skin effect can be obtained by rearranging eq. (3-56), as:

$$s = 1.151\left(\left(\frac{P_{wf} - P_i}{m}\right) - \log(t) - \log\left(\frac{k}{\varphi\,\mu\,c_t\,r_w^2}\right) + 3.23\right)$$

or, more conveniently, if $t = 1$, $P_{wf} = P_{1hr}$



$$s = 1.151\left(\left(\frac{P_{1\,hr}-P_i}{m}\right) - \log\left(\frac{k}{\varphi\,\mu\,c_t\,r_w^2}\right) + 3.23\right)$$ -- **(3-57)**

P_{1hr} found on the extension of the straight line at log t = 1 hour

- A plot of $\mathbf{P_{wf}}$ **versus time (t)** on **semi log graph** paper would yield a straight line with a slope m in psi/cycle.

1. Early-time region (ETR) (unsteady state flow region)
2. Middle-time region (MTR)
3. Late-time region (LTR) (pseudo-steady state region)

➢ Pressure Drawdown Test Analysis Steps

1- Plot **flowing BHP, p_{wf} vs. flowing time, t**, on semi-log paper.

2- Determine the slop **m** of the most probable MTR, and estimate formation permeability by:

$$k = \frac{-162.6\ q\ B\ \mu}{m\ h}$$

3- Estimate the skin factor, s by:

$$s = 1.151\left(\left(\frac{P_{1\,hr} - P_i}{m}\right) - \log\left(\frac{k}{\varphi\ \mu\ c_t\ r_w^2}\right) + 3.23\right)$$

Problem (3-9)

A new oil well is tested by producing it at a constant rate of 1500 STB/Day for 100 hours. The reservoir data and flowing bottom-hole pressure recorded during the test are detailed below.

p_{wf} (psi)	3500 (p_i)	2917	2900	2888	2879	2869	2848	2830	2794	2762
Δt (hrs)	0	1	2	3	4	5	7.5	10	15	20

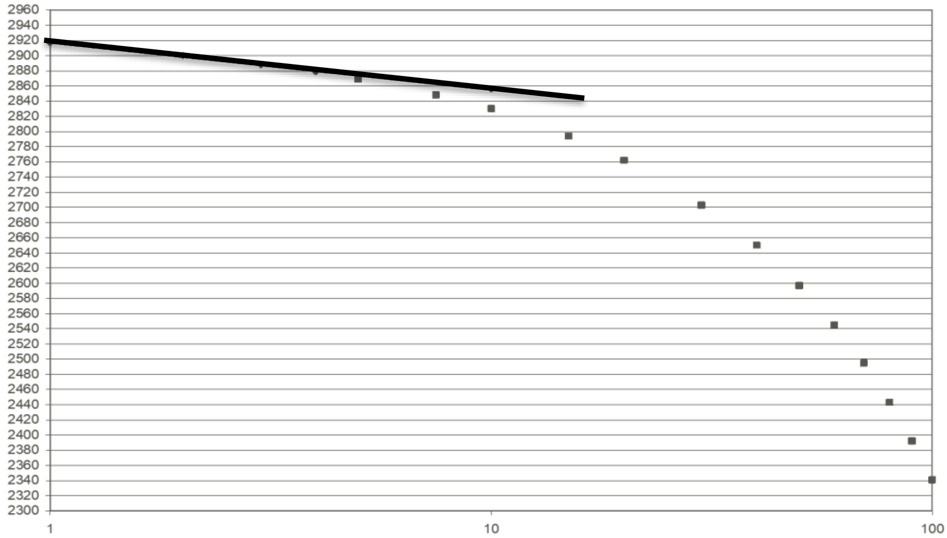
μ = 1 cp, B = 1.2 RB/STB, Net pay thickness = 20 ft

Ø = 18%, $c_t = 15*10^{-6}\ psi^{-1}$, r_w = 0.33 ft

1. Determine the permeability(k) and skin factor (s)
2. Estimate the pore volume of the reservoir?

Solution:

1. plot p_{wf} vs. t on semi-log paper, draw best straight line from the MTR region, determine the slop (m), and estimate formation permeability.



$$m = \frac{(2917 - 2856)}{(\log 1 - \log 10)} = -61 \ \frac{psi}{cycle}$$

$$k = \frac{-162.6 * 1500 * 1 * 1.2}{(-61) * 20} = 240 \text{ md}$$

2. Determine skin factor, s.

$$s = 1.151\left(\left(\frac{2917-3500}{-61}\right) - \log\left(\frac{240}{0.18*1*15*10^{-6}*0.33^2}\right) + 3.23\right) = 4.5$$

3. Estimate reservoir pore volume by using the following steps:

A. Plot $\mathbf{p_{wf}}$ **vs. t** on Cartesian graph paper.

B. Draw best straight line.

C. Determine the slop (m).

D. Estimate reservoir pore volume by using the following equation;

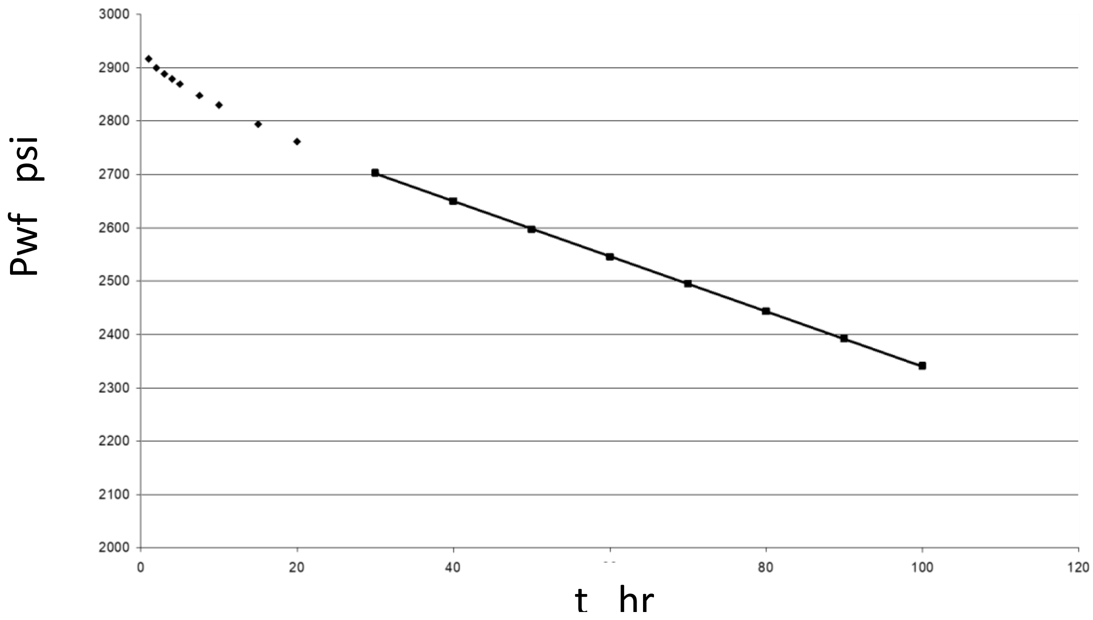
$$v_p = \frac{-0.234 \ qB}{c_t\left(\frac{\partial P_{wf}}{\partial t}\right)}$$

Where;

$$\left(\frac{\partial P_{wf}}{\partial t}\right) = \text{slop (m)}$$

From below figure, $\partial P_{wf} / \partial t$ = slope = - 5.16

$$v_p = \frac{-0.234\, qB}{c_t\left(\frac{\partial P_{wf}}{\partial t}\right)} = \frac{-0.234\,(1500)(1.2)}{15*10-6\,(-5.16)} = 544.18 \; * 10^4 \; ft^3$$



➢ Injectivity Test

Introduction

In many reservoirs, the number of injection wells approaches the number of producing wells, so the topic of testing those wells is important. That is particularly true when tertiary recovery projects are being considered or are in progress. When an input well receives an expensive fluid, its ability to accept that fluid uniformly for a long time is important to the economics of the tertiary recovery project. In particular, increasing wellbore damage must be detected and corrected promptly.

The information available about injection well testing is much less abundant than information about production well testing. Matthews and Russell[1] summarize injection well testing, but emphasize falloff testing. Injectivity testing is rarely discussed in the literature, but it can be important.[2] Falloff testing is treated[3-7] rather thoroughly, particularly for systems with unit mobility ratio. Gas-well falloff testing, especially in association with in-situ combustion, also has been discussed.[8,9]

Injection-well transient testing and analysis are basically simple — as long as the *mobility ratio* between the injected and the in-situ fluids is about unity. Fortunately, that is a reasonable approximation for many waterfloods. It also is a reasonable approximation in watered-out waterfloods that initially had mobility ratios significantly different from unity, and early in the life of tertiary recovery projects when so little fluid has been injected that it appears only as a skin effect. When the unit-mobility-ratio condition is satisfied, injection well testing for liquid-filled systems is analogous to production well testing. Injection is analogous to production (but the rate, q, used in equations is negative for injection while it is positive for production), so an injectivity test (Section 7.2) parallels a drawdown test (Chapter 3). Shutting in an injection well results in a pressure falloff (Section 7.3) that is analogous to a pressure buildup (Chapter 5). The equations for production well testing in Chapters 3 through 5 apply to injection well testing as long as sign conventions are observed. The analogy should become clear in the next two sections.

When the unit-mobility-ratio assumption is not satisfied, the analogy between production well testing and injection well testing is not so complete. In that situation, analysis depends on the relative sizes of the water bank and the oil bank; generally, analysis is possible only when $r_{ob} > 10r_{wb}$ (see Section 7.5). Fracturing effects, which can have a significant effect on analysis, are discussed in Section 11.3.

Reservoirs with injection wells can reach true steady-state conditions when total injection rate equals total production rate. In that situation, or when the situation is approached, the steady-state analysis techniques of Section 7.7 may be useful.

7.2 Injectivity Test Analysis in Liquid-Filled, Unit-Mobility-Ratio Reservoirs

Injectivity testing is pressure transient testing during injection into a well. It is analogous to drawdown testing, for both constant and variable injection rates. Although sometimes called "injection pressure buildup" or simply "pressure buildup," we prefer to use the term "injectivity testing" to avoid confusion with production-well pressure buildup testing. This section applies to liquid-filled reservoirs with mobility of the injected fluid essentially equal to the mobility of the in-situ fluid. If the unit-mobility-ratio condition is not satisfied, results of analysis by techniques in this section may not be valid. Even in that situation, if the radius of investigation is not beyond the water (injected-fluid) bank, valid analysis can be made for permeability and skin, but not necessarily for static reservoir pressure.

Fig. 7.1 shows an ideal rate schedule and pressure response for injectivity testing. The well is initially shut in and pressure is stabilized at the initial reservoir pressure, p_i. At time zero, injection starts at constant rate, q. Fig. 7.1 illustrates the convention that $q < 0$ for injection. It is advisable to monitor the injection rate carefully so the methods of Chapter 4 (variable-rate analysis) may be applied if the rate varies significantly.

Since unit-mobility-ratio injection well testing is analogous to production well testing, the analysis methods in Chapters 3 and 4 for drawdown and multiple-rate testing may be applied directly to injection well testing. Of course, while pressure at a production well declines during drawdown, pressure at an injection well increases during injection. That difference is accounted for in the analysis methods by using $q < 0$ for injection and $q > 0$ for production.

For the constant-rate injectivity test illustrated in Fig. 7.1, the bottom-hole injection pressure is given by Eq. 3.5:

$$p_{wf} = p_{1hr} + m \log t . \quad \text{(7.1)}$$

Eq. 7.1 indicates that a plot of bottom-hole injection pressure vs the logarithm of injection time should have a straight-line section, as shown in Fig. 7.2. The intercept, p_{1hr}, is given by Eq. 3.7; the slope is m and is given by Eq. 3.6:

$$m = \frac{-162.6\, qB\mu}{kh} \quad \text{(7.2)}$$

As in drawdown testing, wellbore storage may be an important factor in injection well testing. Often, reservoir pressure is low enough so that there is a free liquid surface in the shut-in well. In that case, the wellbore storage coefficient is given by Eq. 2.16 and can be expected to be relatively large. Therefore, we recommend that all injectivity test analyses start with the $\log(p_{wf} - p_i)$ vs $\log t$ plot so the duration of wellbore storage effects may be estimated as explained in Sections 2.6 and 3.2. As indicated in Fig. 7.2, wellbore effects may appear as a semilog straight line on the p_{wf} vs $\log t$ plot; if such a line is analyzed, low values of permeability will be obtained and calculated skin factor will be shifted in the negative direction. Eq. 3.8 may be used to estimate the beginning of the semilog straight line shown in Fig. 7.2:

$$t > \frac{(200{,}000 + 12{,}000\, s)\, C}{(kh/\mu)} \quad \text{(7.3)}$$

Once the semilog straight line is determined, reservoir permeability is estimated from Eq. 3.9:

$$k = \frac{-162.6\, qB\mu}{mh} \quad \text{(7.4)}$$

Skin factor is estimated with Eq. 3.10:

$$s = 1.1513 \left[\frac{p_{1hr} - p_i}{m} - \log\left(\frac{k}{\phi \mu c_t r_w^2}\right) + 3.2275 \right] . \quad \text{(7.5)}$$

Example 7.1 Injectivity Test Analysis in an Infinite-Acting Reservoir

Figs. 7.3 and 7.4 show pressure response data for an injectivity test in a waterflooded reservoir. Before the test, all wells in the reservoir had been shut in for several weeks and pressure had stabilized. Known reservoir data are

depth = 1,002 ft	$h = 16$ ft
$c_t = 6.67 \times 10^{-6}$ psi^{-1}	$\mu = 1.0$ cp
$\phi = 0.15$	$B = 1.0$ RB/STB
$\rho_w = 62.4$ lb$_m$/cu ft	$q = -100$ STB/D
$p_i = 194$ psig	$r_w = 0.25$ ft.

The well is completed with 2-in. tubing set on a packer. The reservoir had been under waterflood for several years. We can safely assume that the unit-mobility-ratio assumption is satisfied, since the test radius of investigation is less than the distance to the water bank, as shown by calculations later in this example.

The log-log data plot, Fig. 7.3, indicates that wellbore storage is important for about 2 to 3 hours. The deviation of the data above the unit-slope line suggests that the wellbore storage coefficient decreased at about 0.55 hour. Sections 2.6 and 11.2 and Figs. 2.12 and 11.5 through 11.7 discuss such changing wellbore storage conditions. The data in Fig. 7.3 start deviating upward from the unit-slope straight line when $\Delta p = 230$ psi and $p_{wf} = 424$ psig. Since the column of water in the well is equivalent to about 434 psi, it appears that the apparent decrease in storage coefficient corresponds to fillup of the tubing.

From the unit-slope portion of Fig. 7.3, $\Delta p = 408$ psig when $\Delta t = 1$ hour. Using Eq. 2.20, we estimate the apparent wellbore storage coefficient:

$$C = \frac{(100)(1.0)}{24} \frac{(1.0)}{(408)} = 0.0102 \text{ bbl/psi.}$$

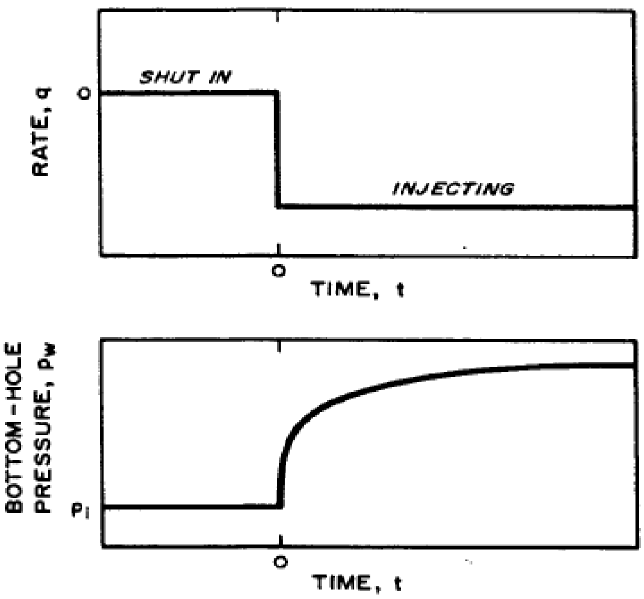


Fig. 7.1 Idealized rate schedule and pressure response for injectivity testing.

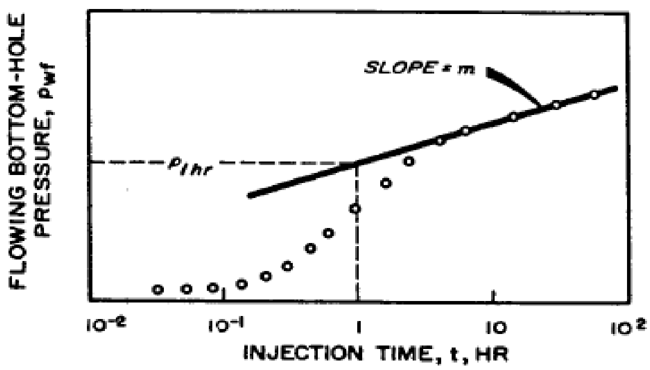


Fig. 7.2 Semilog plot of typical injectivity test data.

(C is always positive.) Wellbore capacity for a rising fluid level can be estimated (from Eq. 2.16) to get $V_u = 0.0044$ bbl/ft. Two-inch tubing has a capacity of about 0.004 bbl/ft, so the unit-slope straight line does correspond to a rising fluid level in the tubing. If we use $C = 0.0102$ in Eq. 7.3, or if we go 1 to 1.5 cycles in Δt after the data start deviating from the unit-slope line (Section 2.6), we would decide that the semilog straight line should not start for 5 to 10 hours of testing. Those rules indicate too long a time for a *decreasing* wellbore storage condition. Figs. 7.3 and 7.4 clearly show that wellbore storage effects have died out after about 2 to 3 hours.

Fig. 7.4 shows a semilog straight line through the data after 3 hours of injection. From this line, $m = 80$ psig/cycle and $p_{1hr} = 770$ psig. Permeability is estimated using Eq. 7.4:

$$k = \frac{-(162.6)(-100)(1.0)(1.0)}{(80)(16)} = 12.7 \text{ md.}$$

We may now determine if the unit-mobility-ratio analysis applies. The estimated permeability is used to estimate a radius of investigation from Eq. 2.41:

$$r_d \simeq 0.029\sqrt{\frac{kt}{\phi\mu c_t}}$$

$$\simeq 0.029\sqrt{\frac{(12.7)(7)}{(0.15)(1.0)(6.67\times10^{-6})}}$$

$$\simeq 273 \text{ ft.}$$

A volumetric balance provides an estimate of the distance to the water bank. The volume injected is

$$W_i = \frac{\pi r_{wb}^2 h\phi\Delta S_w}{5.6146},$$

so

$$r_{wb} = \sqrt{\frac{5.6146\, W_i}{\pi h\phi\Delta S_w}}.$$

Assuming that $\Delta S_w = 0.4$ and that injection has been under way for at least 2 years,

$$W_i \simeq (100 \text{ STB/D})(1.0 \text{ RB/STB})(2 \text{ years})(365 \text{ D/year})$$

$$\simeq 73{,}000 \text{ res bbl}$$

and

$$r_{wb} = \sqrt{\frac{(5.6146)(73{,}000)}{\pi(16)(0.15)(0.4)}} \simeq 369 \text{ ft.}$$

Since $r_d < r_{wb}$, we are justified in using the unit-mobility-ratio analysis.

Eq. 7.5 provides an estimate of the skin factor:

$$s = 1.1513\left\{\frac{770-194}{80} - \log\left[\frac{12.7}{(0.15)(1.0)(6.67\times10^{-6})(0.25)^2}\right] + 3.2275\right\}$$

$$= 2.4.$$

The well is damaged; the pressure drop across the skin may be estimated from Eq. 2.9:

$$\Delta p_s = \frac{(141.2)(-100)(1.0)(1.0)(2.4)}{(12.7)(16)}$$

$$= -167 \text{ psi.}$$

The negative sign here indicates damage since the pressure decreases away from the well (in the positive r direction) for injection while it increases for production. This is seen by computing the flow efficiency from Eq. 2.12. Assume $\bar{p} = p_i = 194$ psi, since the reservoir is stabilized before injection. Using $p_{wf} = 835$ psig from the last available data point, the *flow efficiency* is

$$\frac{194 - 835 - (-167)}{194 - 835} = 0.74.$$

If we had ignored the sign on q when estimating Δp_s, we would have incorrectly computed a flow efficiency of 1.26, indicating improvement instead of damage.

Multiple-rate injection testing, constant-pressure injection testing, injectivity testing after falloff testing, etc., are all performed and analyzed as explained for production well

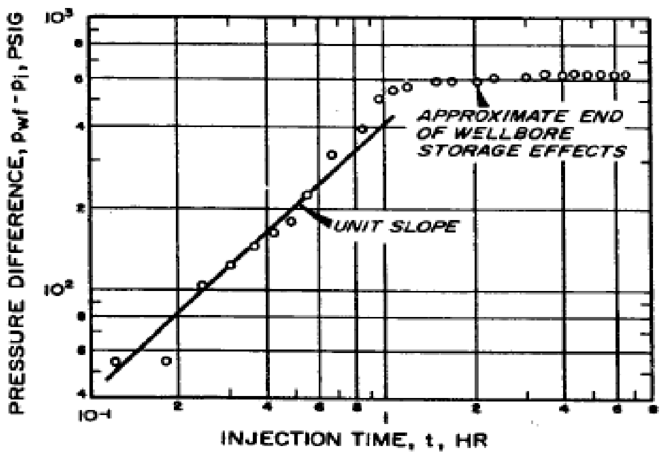


Fig. 7.3 Log-log data plot for the injectivity test of Example 7.1. Water injection into a reservoir at static conditions.

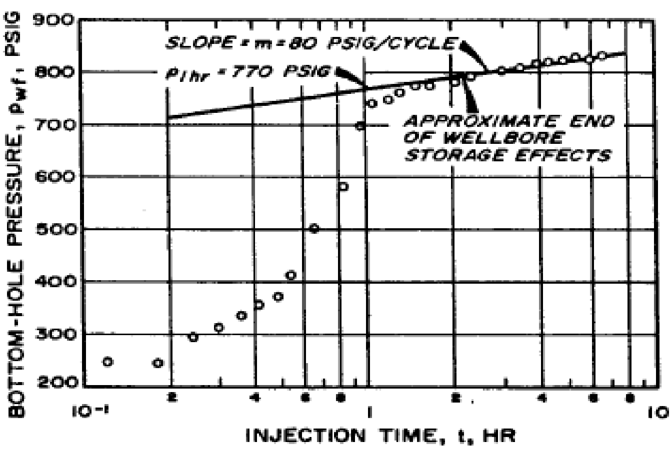


Fig. 7.4 Semilog plot for the injectivity test of Example 7.1. Water injection into a reservoir at static conditions.

testing in Chapters 3 and 4. Type-curve matching for injection well testing is done just as it is for production well testing (Section 3.3); the Δp used must be positive for plotting the log scale, although it is actually a negative number. The signs must be considered in analysis.

Eqs. 7.1 through 7.5 apply to injectivity testing in infinite-acting reservoirs, just as do Eqs. 3.5 through 3.10 for drawdown testing. When an injection well in a developed reservoir shows the effects of interference from other wells, the infinite-acting analysis may not be strictly applicable. In that case, the techniques presented in Section 3.4 should be used.

Table (3-2): Parameters obtained from well testing (Kamal, Freyder, and Murray, 1995)

Type of test	Obtained parameter
DST	Reservoir behavior Permeability, Skin Nearby boundaries Reservoir pressure
Repeat-multiple-formation test	Pressure Profile
Drawdown test	Reservoir behavior Permeability, Skin Fracture length Reservoir limit, Boundaries
Buildup test	Average Reservoir pressure Permeability, Skin Fracture length Reservoir boundaries
Step-rate test	Formation parting pressure Permeability, Skin
Falloff test	Mobility in various banks Skin Average Reservoir pressure Fracture length Location of front, Boundaries
Interference and pulse tests	Communication between wells Transmissivity Porosity, storativity Interwell permeability Vertical permeability
Layered reservoir test	Properties of individual layers Horizontal permeability, Vertical permeability Skin Average layer pressure Outer boundaries