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FUNDAMENTALS OF PERTROLEUM RESERVOIR ENGINEERING (FLUID & ROCK PROPERTIES)

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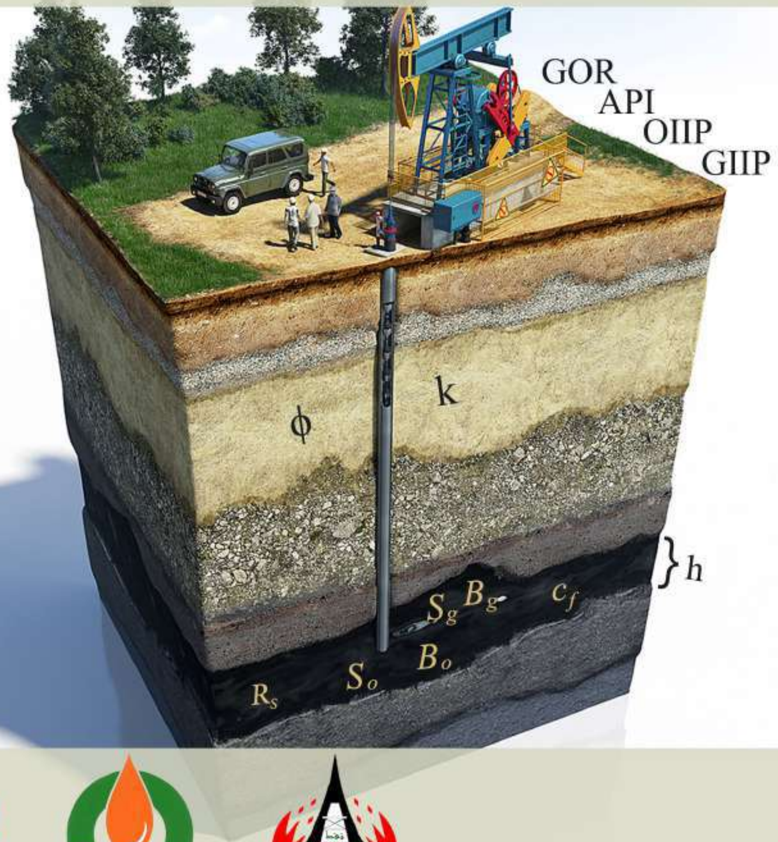
(FLUID & ROCK PROPERTIES)

ISAM M. NAJAR & HAMEED M. AL-NASER



UNIVERSITY OF MOSUL MINISTRY OF OIL NORTH OIL COMPANY

FUNDAMENTALS OF PETROLEUM RESERVOIR ENGINEERING (FLUID & ROCK PROPERTIES)

ISAM MOHAMMED S. NAJAR

&

HAMEED MAHMOOD AL-NASER

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Preface

Because of lacking a simplified textbook about the fundamentals of petroleum reservoir engineering. The authors wanted to collect Najar's lectures as a book to be a reference for teaching this subjects in the universities of Iraq.

Mr. Al-Naser supported Dr. Najar and helped him with great efforts in feeding the lectures using (Word, Excel, Photoshop, and Petrel), adding more details to the subjects, preparing problems to the chapters in addition to adding some sections to the book. Without Al-Naser efforts this book would not have been come out with the current arrangement.

This textbook contains six chapters. The first three chapters deal with petroleum reservoir fluids properties given as one course. The other three chapters deal with petroleum reservoir rock properties given as another course.

The authors hope that this book to be a very helpful source for students during their study.

NAJAR and AL-NASER

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Some Abbreviation of Units Used:

gal	U.S. liquid gallon = 231(inch)^3 British gallon = 277.42 (inch)^3
lb/gal	pound per gallon
lb mole	pound mole
lb/sq in	pound per square inch
lb/sq ft	pound per square foot
lim	Limit
liq	Liquid
ln	natural logarithm (base e)
log	common logarithm (base 10)
M	Meter
meq	milliequivalent weight
mg	Milligram
microsip	Psi$^{-1} \times 10^6$
ml	Milliliter
mm	Millimeter
mm Hg	millimeters of mercury
mol wt	molecular weight
MMscf	million stander cubic feet
MMSTB	million stock-tank barrels
Mscf	thousand stander cubic feet
Mscf/d	thousand stander cubic feet
MSTB	thousand stock-tank barrels
MSTB/d	thousand stock-tank barrels per day
oz/sq in	ounce per square inch
ppm	parts per million
psi	pounds per square inch

psia	puonds per square inch absolute
psig	puonds per square inch gauge
res	Reservoir
res cu ft	cubic feet of reservoir gas reported at reservoir conditions
res bbl	barrels of reservoir liquid or gas (or both) reported at reservoir conditions
^{o}R	degree Rankine
scf	standard cubic feet, volume of gas reported at standard conditions
scf/STB	standard cubic feet of gas per barrel of stock-tank liquid
sec	Second
1/psi	psi^{-1}
sp. gr.	specific gravity
sq	Square
sq cm	square centimeter
sq ft	square foot
sq in	square inch
ST	Stock-tank (at conditions of 14.7psi – 60^{o}F)
STO	Stock-tank oil (used only when necessary to distinguish between quantities in same calculation, such as ρ_{oR} and ρ_{STO}
T	total or two phase
T	temperature or temperature dependent
V	used in p_v to indicate vapor pressure and L_v to indicate vaporization
W	water (oilfield brine)
Wet	wet, with L_c to indicate saturated with water vapor prior to combustion
Wg	wet gas

A	angstrom unit (10^{-8} cm)
AGP	additional gas produced, a parameter in reservoir-gas specific gravity equation
^{o}API	degree (American Petroleum Institute)
atm	atmosphere
bbl	barrel
bbl/d	barrels per day
SP	Separator
SP bbl	barrels of separator liquid
ST	stock tank
STB	barrels of stock-tank oil (stock-tank barrel) reported at standard conditions, also used for barrels of water at standard conditions
STB/d	stock-tank barrels per day
STO	stock-tank oil
Vs	Versus
VEQ	Equivalent gas volume of stock-tank liquid, a parameter in reservoir-gas specific gravity equation
Wt	Weight
lbm	pound mass
lbf	pound force
btu	British thermal unit (steam)
Torr	Pressure exerted by 1mm Hg at 273.15 ^{o}K (0 ^{o}C)

Units Conversion:

	To convert	Into	Multiply by
Length (L)	Feet	inches	12
	Feet	meters	0.3048
	meters	feet	3.28
	inch	cm	2.54
	ft	cm	30.48
	m	inch	39.7
Volume (V)	bbl	m^3	0.159
	bbl	ft^3	5.615
	m^3	bbl	6.29
	lt	ft^3	0.03531
	bbl	lt	159
	m^3	ft^3	35.3
Mass (m)	lb	kg	0.4536
	kg	lb	2.205
Density (ρ)	g/cm^3	lb/ft^3	62.43
	kg/m^3	lb/ft^3	6.243x10^{-2}
	lb/ft^3	g/cm^3	1.602x10^{-2}
	lb/ft^3	kg/m^3	16.02
Pressure (p)	atm	psi	14.7
	psi	atm	6.805x10^{-2}
	bar	atm	0.9869
	bar	Torr	750.061
	atm	pascal	1x10^5
	psi	pascal	47.88

Chapter One

Basic Concepts and Equations

1.1 What is Reservoir Engineering:

Reservoir engineering is a branch of petroleum engineering that applies scientific principles to produce oil and gas from the reservoir with high economic recovery. So, reservoir engineer to do that needs:

1- Applied mathematics.
2- Basic laws of physics and chemistry related to behavior of fluids in reservoir rocks.
3- Geology.
4- Experience and practice.

The goal of reservoir engineering is to set up a **development project** to study the behavior of the reservoir (throughout the life of the field) to derive the information required to reserve estimation, development planning and production operations optimization.

1.2 Some information about hydrocarbons:

Petroleum deposits vary widely in chemical composition and those obtained from different localities may have entirely different physical and chemical properties. In spite of this diversity, the bulk of the chemical compound found in petroleum are hydrocarbons (i.e. consisting of hydrogen and carbon only).

However, carbon compounds are known which have carbon chains consisting of hundreds of carbon atoms (see table1.1 below).

Table1.1: components of typical petroleum gases and liquids.

Hydrocarbon	Natural gas	Gas producing petroleum liquid
Methane	70-98%	45-92%
Ethane	1-10%	4-21%
Propane	trace-5%	1-15%
Butanes	trace-2%	0.5-7%
Pentanes	trace-1%	trace-3%
Hexanes	trace-0.5%	trace-2%
Heptanes +	trace-0.5%	trace-1.5%

1.3 Temperature Scales:

There are three temperature scales in use today, Fahrenheit, Celsius and Kelvin, another scale also used in petroleum industry called as Rankine. Fig. (1.1) shows the total temperature scales. Temperature scales classified as absolute (^oR & ^oK) and relative (^oF & ^oC), the absolute scales start from zero value while the relative scales show negative and positive values.

The relationships between the temperature scales are as follow:

$$T(°C) = T(°K) - 273 \qquad eq(1.1)$$

$$T(°R) = 1.8\,T(°K) \qquad eq(1.2)$$

$$T(°F) = T(°R) - 460 \qquad eq(1.3)$$

$$T(°C) = \frac{T(°F) - 32}{1.8} \qquad eq(1.4)$$

$$T(°R) = T(°F) + 460 \qquad eq(1.5)$$

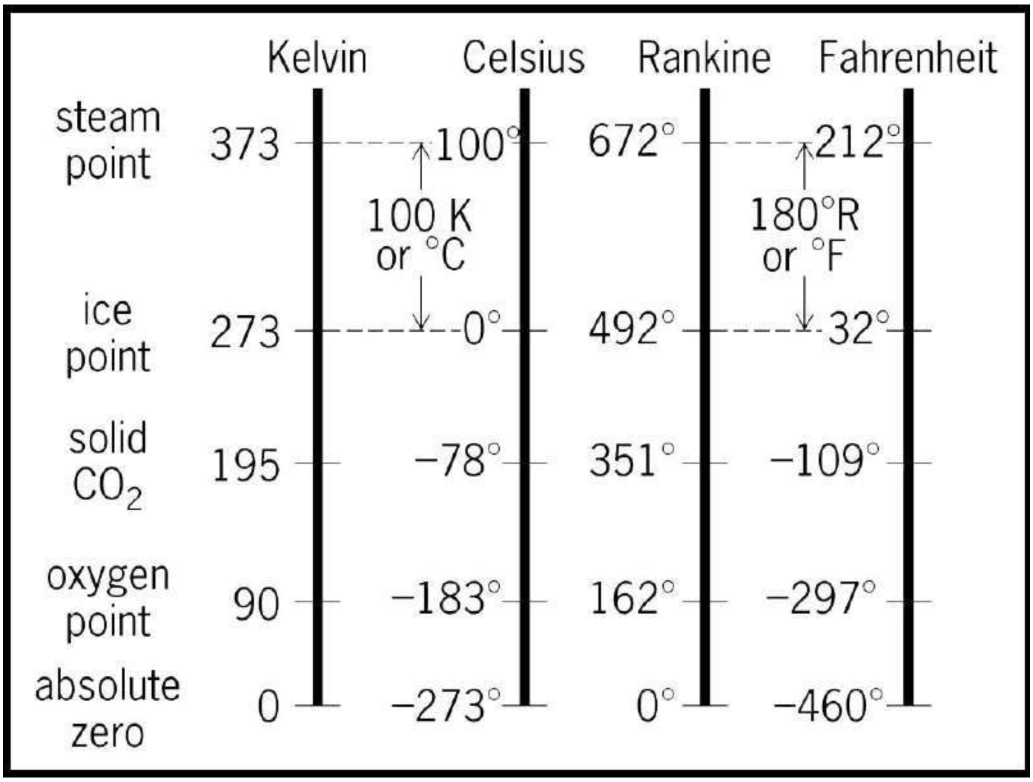


Fig. (1.1): Comparison of temperature scales.

Example 1.1:

A temperature of body is measured to be (26 ^oC). Determine the temperature in ^oR, ^oK and ^oF.

Solution:

$$T(°\mathrm{F}) = 1.8\,°\mathrm{C} + 32 = 1.8(26) + 32 = 78.8\,°\mathrm{F}$$

$$T(°K) = T(°C) + 273 = 26 + 273 = 299\,°K$$

$$T(°R) = T(°\mathrm{F}) + 460 = 78.8\,°\mathrm{F} + 460 = 538.8\,°R$$

1.4 Equation of State:

An analytic expression relating pressure to temperature and volume is called an "Equation of State" (EoS).

1.4.1 Ideal Gas Law:

The ideal gas is defined as the one in which:

1- The Volume occupied by the molecules is small compared to the total volume.

2- All molecular collisions are elastic.

3- There are no attractive or repulsive forces among the molecules.

Ideal gas law is as follow:

$$pV = nRT \qquad eq.(1.6)$$

where, p = absolute pressure, *psia.*
V = volume, *ft*^{3}.
T = absolute temperature, *°R.*
n = number of moles of gas, *lb-mole.*
R = the universal gas constant, which, for the above units, has the value 10.730 *psia.ft*^{3}*/lb-mole.°R.*

The Eq.(1.6) of ideal gas law is a combination of several fundamentals gas laws:

1. Boyle's law.
2. Charles's law.
3. Gay-Lussac's law.
4. Avogadro's law.

1.4.1.1 Boyle's Law:

Boyle's law is based on observations made around 1660 that for a fixed mass of gas at a fixed temperature, the volume of gas is inversely proportional to the pressure exerted as shown in Fig. (1.2).

$$V \propto \frac{1}{p} \qquad ; T = constant \qquad eq. (1.7)$$

$$pV = constant \qquad eq. (1.8)$$

$$p_1V_1 = p_2V_2 \qquad eq. (1.9)$$

$$V_2 = \frac{p_1V_1}{p_2} \qquad eq. (1.10)$$

Example 1.2:

A quantity of gas at a pressure (1 atm) has a volume of (1 L). If the gas is compressed to volume of (0.5 L). Determine the new pressure assuming the temperature is constant.

Solution:

$p_1 = 1\,atm,$ $V_1 = 1\,lt,$ $V_2 = 0.5\,lt,$ $p_{2=??}$

$$p_2 = \frac{p_1V_1}{V_2} = \frac{(1\,atm)(1\,lt)}{(0.5\,lt)} = 2\,atm$$

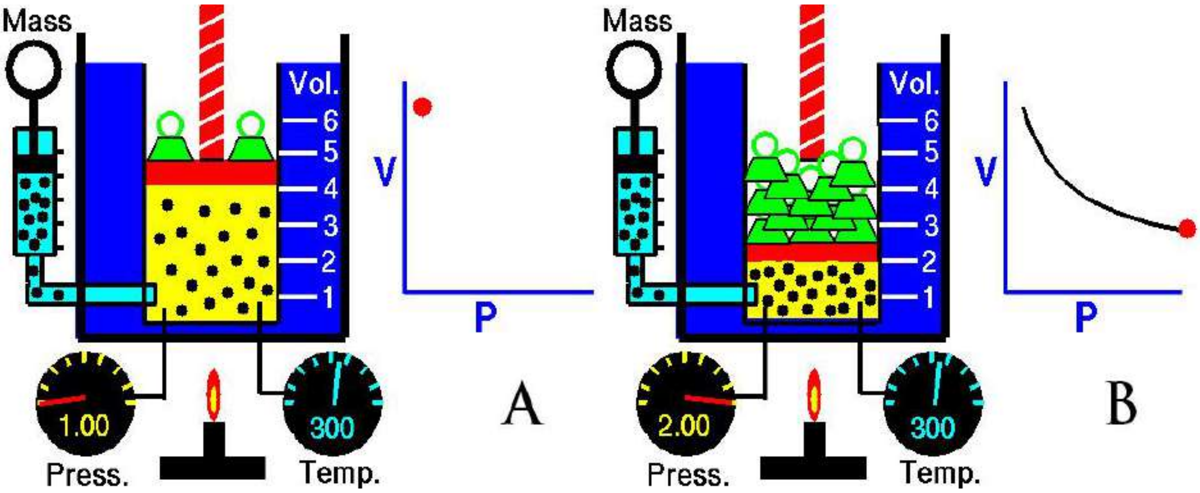


Fig. (1.2): shows Boyle's law principle.

1.4.1.2 Charles's Law:

Over a century later (1787), Charles observed that for a fixed mass of gas at constant pressure, the volume varies linearly with temperature, as shown in Fig. (1.3).

$$V \propto T \qquad ; p = constant \qquad eq.(1.11)$$

$$\frac{V}{T} = constant \qquad eq.(1.12)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad eq.(1.13)$$

$$V_2 = \frac{V_1 T_2}{T_1} \qquad eq.(1.14)$$

Example 1.3:

A given mass of gas has a volume of (500 ft^3) when the temperature is (50 °F) and the pressure is (10 psig). If the pressure remains constant, but the temperature

is changed to (100 °F), what will be the volume of the gas?

Solution:

$$T_1 = 50\ \text{°F} + 460 = 510\ °R$$

$$T_2 = 100\ \text{°F} + 460 = 560\ °R$$

$$V_2 = \frac{V_1 T_2}{T_1} = \frac{(500\ ft^3)(560\ °R)}{(510\ °R)} = 549.02\ ft^3$$

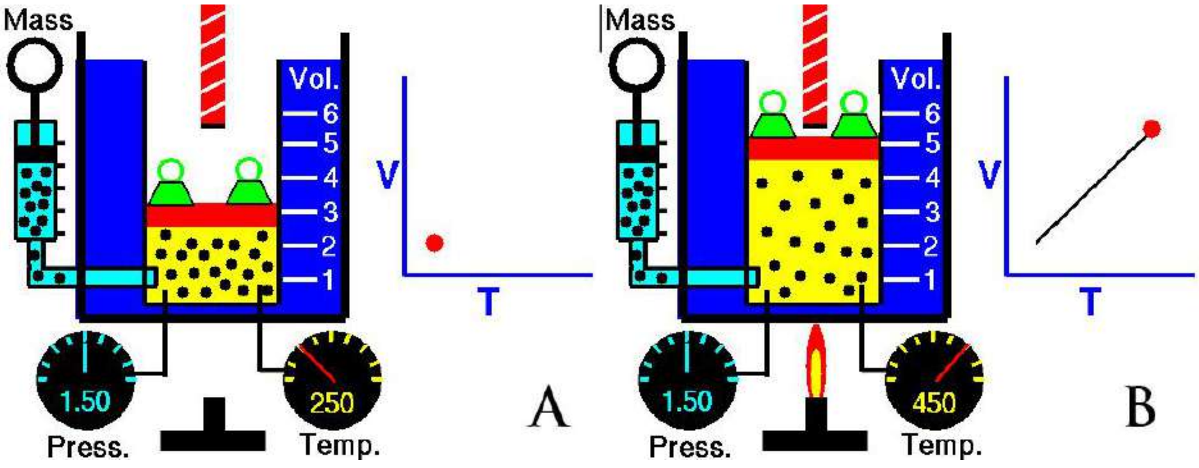


Fig. (1.3): Shows Charles law principle.

1.4.1.3 Gay-Lussac's Law:

This law was found by Joseph Louis Gay-Lussac in 1809. It states that, for a given mass and constant volume of an ideal gas, the pressure exerted on the sides of its container is directly proportional to its absolute temperature.

$$p \propto T \quad ; V = constant \qquad eq.(1.15)$$

$$\frac{p}{T} = constant \qquad eq.(1.16)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad eq.(1.17)$$

$$p_2 = \frac{p_1 T_2}{T_1} \qquad eq.(1.18)$$

Example 1.4:

A given mass of gas has a volume of (500 cu ft) when the temperature is (50 ^{o}F) and the pressure is (10 psig). What would be the new pressure of the gas if the volume remains the same and the temperature is changed from (50 ^{o}F) to (100 ^{o}F)?

Solution:

$$p_1 = 10 + 14.73 = 24.73\, psia$$

$$T_1 = 50\ °\mathrm{F} + 460 = 510\ °R$$

$$T_2 = 100\ °\mathrm{F} + 460 = 560\ °R$$

$$p_2 = \frac{p_1 T_2}{T_1} = \frac{(24.73)(560)}{(510)} = 27.15\, psia$$

1.4.1.4 The Combined Gas Law:

From the combination of eq.(1.8), eq.(1.12) and eq.(1.16) we get the following equations:

$$\frac{pV}{T} = constant \qquad eq.(1.19)$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \qquad eq.(1.20)$$

Example 1.5:

Find the volume of a gas at STP when (2 liters) is collected at (745 mmHg) and (25 °C). (note: STP refers to standard temperature and pressure which is 273 °K and 760 mmHg).

Solution:

p_1=745 mmHg	V_1=2 L	T_1=298 °K
p_2=760 mmHg	V_2=??	T_2=273 °K

$$\frac{p_1V_1}{T_1}=\frac{p_2V_2}{T_2}$$

$$V_2=\frac{p_1V_1T_2}{p_2T_1}=\frac{(745)(2)(273)}{(760)(298)}=1.5\ lt$$

(Note: by SPE standard conditions mean 60°F and 14.7 *psia*).

1.4.1.5 Avogadro's Law:

Under the same conditions of P and T, equal volumes of all ideal gases contain the same number of molecules of ideal gas. Or at a given P & T one molecular weight of any ideal gas occupies the same volume as one molecular weight of another ideal gas. Avogadro's number is (6.023×10^{23} g-mole, 6.023×10^{26} kg-mole or **2.733×10^{26} lb-mole**).

One molecular weight in pounds of any ideal gas at 60° F & 14.7 psia occupies a volume of 379.4 SCF, and so on as shown in Fig. (1.4).

$$V \propto n \qquad eq.(1.21)$$

$$\frac{V}{n}=constant \qquad eq.(1.22)$$

$$\frac{V_1}{n_1}=\frac{V_2}{n_2} \qquad eq.(1.23)$$

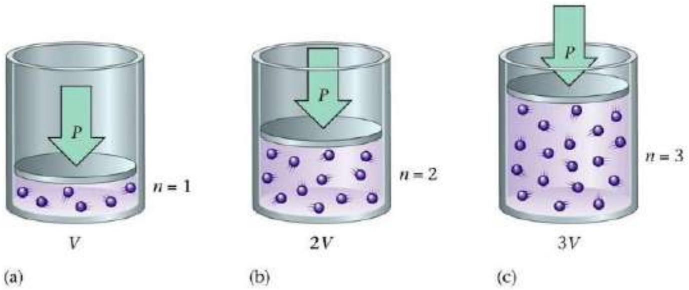


Fig. (1.4): Shows the idea of Avogadro's law.

Example 1.6:

If you have a (5 lt) of a gas which contains (0.965 mole) of molecules. What will be the new volume of the gas if the quantity is increased to (1.80 mole), assuming pressure and temperature are hold constant?

Solution:

$V_1 = 5L,$ $n_1 = 0.965\, mole,$ $n_2 = 1.8\, mole$

$$V_2 = \frac{V_1 n_2}{n_1} = \frac{(5)(1.8)}{(0.965)} = 9.32\, lt$$

1.4.1.6 Molecular Weight (Molar Mass):

It is the summation of all atomic weights of its constituent atoms. And it's also equal to mass divided by number of moles.

$$M_w = \frac{mass}{number\ of\ moles} = \frac{m}{n} \qquad eq. (1.24)$$

$$n = \frac{m}{M_w} \qquad eq. (1.25)$$

Also:

$$n = \frac{Number\ of\ Atoms}{Avogadro\ Number} = \frac{Number\ of\ Atoms}{6.023 \times 10^{23}} \qquad eq. (1.26)$$

Example 1.7:

Find (n) for (9.6 g) of SO_2.

Solution:

m= 9.6 g M_w=?? n=??

$$M_w = (1 \times 32) + (2 \times 16) = 64 \frac{g}{mole}$$

$$n = \frac{m}{M_w} = \frac{9.6\ (g)}{64 \left(\frac{g}{mole}\right)} = 0.15\, mole\ (in\ gram\ case)$$

1.4.1.7 The Universal Gas Constant (*R*):

The ideal gas constant is a universal constant that we use to quantify the relationship between the properties of a gas. From relationships in equations (1.7), (1.11), (1.15) and (1.21), the volume of the ideal gas increases with:

1- Increasing of temperature and mole number of the gas.

2- And, decreasing of pressure.

So:

$$V \propto n\frac{T}{p} \qquad eq.(1.27)$$

$$V = Rn\frac{T}{p} \qquad eq.(1.28)$$

Where (R) is a constant that has the same value for all gases. Due to Avogadro's law any gas occupies (22.4 L) at (273 °K) and (1 atm = 14.73 psia) pressure, Therefor:

$$R = \frac{pV}{nT} = \frac{(1atm)(22.4L)}{(1mole)(273°K)} = 0.082\ \frac{atm.L}{mole.K}$$

Table 1.1 show the different R values at different units of pressure, temperature, volume and mole number.

Table (1.1): Values of the universal gas constant, *R*.

p	V	T	N	R
atm	liters	°K	Mole	0.0821(L.atm / °K.mole)
atm	cc	°K	Mole	82.14 (cm^3.atm / °K.g-mole)
psia	Cu ft	°R	lb-mole	10.73 (ft^3.psia / °R.lb-mole)
g/cm^3	cc	°K	Mole	8.314 (J / °K.mole)

Example 1.8:

Four pounds of methane are placed in a tank at (60 °F). If the pressure on the tank is (100 psia). What is the volume of the tank? ($M_{w\,(CH4)}$ = 16 lb/lb-mole).

Solution:

$$V = \frac{nRT}{p}$$

$$n = \frac{m}{M_w} = \frac{4\ lb}{16\ \left(\frac{lb}{lb-mole}\right)} = \ 0.25\ lb-mole$$

$$T = 60\ °\mathrm{F} + 460 = 520\ °R$$

$$V = \frac{(0.25 lb - mole)(10.73 ft^3 . psia. lb - mole^{-1} °R^{-1})(520\ °R)}{(100\, psia)} = 13.94\, ft^3$$

1.4.1.8 Dalton's Law:

Each gas in a mixture of gases exerts a pressure equal to that which it would exert if it occupied the same volume as the total mixture. This pressure is called the ***partial pressure***. The total pressure is the sum of the partial pressures, as shown in Fig. (1.5). This law is valid only when the mixture and each component of the mixture obey the ideal gas law. It is sometimes called the law of additive pressures.

According to Dalton's law, the total pressure is the sum of the partial pressures:

$$p_T = p_1 + p_2 + p_3 \qquad eq. (1.29)$$

$$p_T = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} + \frac{n_3 RT}{V}$$

In the same container R, V & T are the same. So:

$$p_T = \frac{(n_1 + n_2 + n_3)RT}{V}$$

$$p_T = \frac{n_T RT}{V}$$

$$\frac{p_1}{p_T} = \frac{n_1}{n_T} \qquad eq. (1.30)$$

$$p_j = p_T y_j \qquad eq. (1.31)$$

y_j is defined as the mole fraction of the jth component in the gas mixture.

$$y_j = \frac{n_i}{n_T} \qquad eq. (1.32)$$

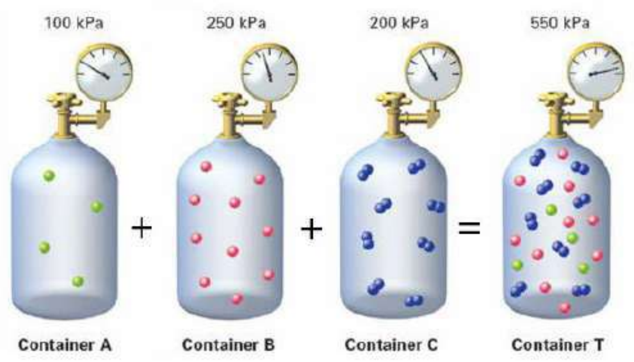


Fig. (1.5): Shows the idea of Dalton's law.

Example 1.9:

Calculate the partial pressure exerted by methane in the following gas, when the gas is at a pressure of (750 psia). Assume that the gas mixture is a mixture of ideal gases.

Component	Composition mole fraction, y_j
Methane	0.85
Ethane	0.10
Propane	0.05
Total	1

Solution:

$$p_{methane} = p_T y_{methane} = (750\, psia)(0.85) = 638\, psia$$

1.4.1.9 Amagat's Law:

The total volume of gaseous mixture is the sum of the volumes that each component would occupy at the given p & T. The volumes occupied by the individual components are known as ***partial volumes***. This law is correct only if the mixture and each component obey the ideal gas law.

$$V_T = V_1 + V_2 + V_3 \qquad eq. (1.33)$$

$$\frac{V_1}{V_T} = \frac{n_1}{n_T} \qquad eq. (1.34)$$

1.4.2 Real Gas Law:

As mentioned earlier, ideal gas law assumes that:

1- Volume of gas molecules is negligible.
2- There is no intermolecular attractive or repulsive forces between gas molecules.

At low pressures and relatively high temperatures, real gases behavior likes ideal gases. But, as the pressure is increased, the ideal gas volume and pressure deduced from the ideal gas law needs correction factor called a compressibility factor (z).

$$pV = znRT \qquad eq.(1.35)$$

As shown in figure (1.6) we can see the following cases:

1. For one mole of gas at SC (V_{ideal} = 22.4 lt = 0.79 ft^3) and for this case (Z =1).
2. If V_{real} is less than 22.4 lt, (at low pressure) then Z<1. In low pressure range, a real volume is less than ideal volume because of attractive forces between molecules and the gas will be easy to liquefy.
3. If V_{real} is larger than 22.4 lt, (at high pressure) then Z>1. In high pressure range, a real volume is greater than ideal volume because of repulsive forces between molecules and the gas will be difficult to liquefy.

Z also called gas deviation factor, correction factor or simply the z-factor. Numerous equations-of-state have been developed in the attempt to correlate the pressure-volume-temperature variables for real gases with experimental data.

The gas compressibility factor z is a dimensionless quantity and is defined as the ratio of the actual (real) volume of n-moles of gas at T and p to the ideal volume of the same number of moles at the same T and p:

$$z = \frac{V_{real}}{V_{ideal}} = \frac{V}{(nRT)\ /\ p}$$

The Z-factor at standard conditions (60 F, 14.7 psia) is equated to (1), this case is for ideal gases also, and can be calculated or measured at other pressures and temperatures. Z-factor is (<1) at low pressures and (>1) at high pressures as shown in Fig. (1.6) for real gases.

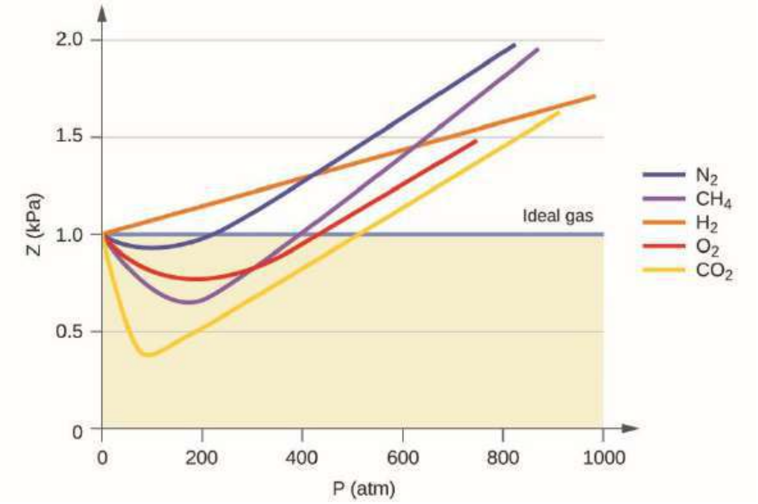


Fig.(1.6): Z-factor vs. pressure for different gas components.
(Note: for H_2 Z>1 over all pressures)

1.4.2.1 Van der Waals Equation:

Various attempts were made in the latter half of the 19th Century to modify the ideal gas law to account for departure from ideal behavior. The first equation of state (EoS) capable of handling the transition from vapour to Liquid states was due to Van der Waales in 1873:

$$\left(p+\frac{an^2}{V^2}\right)(V-nb)=nRT \qquad eq.(1.35)$$

Where:

a= attractive factor.
b= bigness factor.

And when $n=1$,

$$p=\frac{RT}{(V-b)}-\frac{a}{V^2}$$

The parameter “a” was introduced to account for the attractive force between molecules, and the “b” parameters to account for the finite (non-zero) volume of the molecules. The numerical values of the constants “a” and “b” are specific to the gas.

At low pressure attractive force is present so the constant “b” is negligible (b=0), then:

$$\left(p+\frac{a}{V^2}\right)V=RT \qquad eq.(1.36)$$

At high pressure repulsive force is present and the constant “a” is negligible (a=0), so:

$$p(V-b)=RT \qquad eq.(1.37)$$

Example 1.10:

Prove that the value of Z-factor is lower than one at low pressures and larger than one at high pressures.

Solution:

By using the Van der Waals Eq.

1. at low pressures, equation (1.36).

$$\left(p+\frac{a}{V^2}\right)V=RT$$

$$pV + \frac{a}{V} = RT$$

By dividing the above equation by “RT”:

$$\frac{pV}{RT} + \frac{a}{VRT} = \frac{RT}{RT}$$

By substituting (Z= pV/RT) :

$$Z = 1 - \frac{a}{VRT}$$

So:

$$Z < 1$$

2. at high pressures:

By using the Van der Waals equation. Eq. (1.37):

$$p(V - b) = RT$$

$$pV - pb = RT$$

By dividing the above equation by “RT”:

$$\frac{pV}{RT} - \frac{pb}{RT} = \frac{RT}{RT}$$

By substituting (Z= pV/RT):

$$Z = 1 + \frac{pb}{RT}$$

So:

$$Z > 1$$

Note:

There are several other equations of state (EoS) were published at the time of Van der Waal equation:

- Berthelot (1878):

$$\left(p + \frac{a}{TV^2}\right)(V - b) = RT \qquad eq.(1.38)$$

- Redlich-Kwong (1949):

$$p = \frac{RT}{(V - b)} - \frac{a}{V(V + b)} \qquad eq.(1.39)$$

- Clausius (1980):

$$\left(p + \frac{a}{T(V + c)^2}\right)(V - b) = RT \qquad eq.(1.40)$$

Problems

1. A quantity of gas at pressure of (50 psig) has a volume of (1000 ft^3). If the gas is compressed to (100 psig), what volume would it occupy? Assume the barometric pressure is (14.73 psia) and the temperature of the gas remains constant.
2. A (600 ml) sample of Nitrogen is heated from (27 ^oC) to (77 ^oC) at constant pressure. What is the final volume?
3. A (20 L) cylinder containing (6 atm) of gas at (27 ^oC). What would the pressure of the gas be if the gas was heated to (77 ^oC)?
4. Calculate the content of a (500 ft^3) tank of methane at (100 psia) and (100 ^oF) in: moles, pounds, molecules and SCF.
5. How many SCF of an ideal gas are required to fill a (100 ft^3) tank to a pressure of (40 psig) when the temperature of the gas in the tank is (90 ^oF)?
6. An ideal gas exerts (100 psig) in a cylinder at (100 ^oF). What will the pressure be if the temperature is reduced to (32 ^oF)?
7. A container was found to weight (80.0 g) while evacuated. The container then was filled with oxygen and found to weight (81.242 g). When filled with water the container weighed (1000.0 g). The temperature and pressure for the procedure were (14.7 psia) and (60 oF). What is the gas constant in engineering units?

REFERENCES

Bradley, H. B.,1987, "*Petroleum Engineering Handbook*", SPE, USA.

McCain, W. D., 1990. "The Properties of Petroleum Fluids", pennwell books, Tulsa, Oklahoma.

Peng, D. & Robinson, D., 1976. A new two constant equation of state. Ind. Eng. Chem. Fundam. 15 (1), 59–64.

Redlich, O. & Kwong, J., 1949. On the thermodynamics of solutions. An equation of state. Fugacities of gaseous solutions. Chem. Rev. 44, 233–247.

Soave, G., 1972. Equilibrium constants from a modified Redlich-Kwong equation of state. Chem. Eng. Sci. 27, 1197–1203.

Van der Waals, J.D., 1873. On the Continuity of the Liquid and Gaseous State. Sijthoff, Leiden, The Netherlands (Ph.D. dissertation).

Chapter Two

Natural Gas & Crude Oil Properties

2.1 Natural Gas Properties:

Gas is defined as a homogeneous fluid of low viscosity and density, which has no definite volume but expands to completely fill the vessel in which it is placed. Generally, the natural gas is a mixture of hydrocarbon and nonhydrocarbon gases. The hydrocarbon gases normally found in a natural gas are methane, ethane (light gases), propane, butanes, pentanes, and small amounts of hexanes (intermediate gases) and heavier components (heptane, octane, ...etc). The nonhydrocarbon gases, that is, impurities, include carbon dioxide, hydrogen sulfide, and nitrogen.

Knowledge of pressure-volume-temperature (PVT) relationships and other physical and chemical properties of gases are essential for solving problems in natural gas reservoir engineering. The properties of interest include:

- Apparent molecular weight, M_a.
- Gas density, ρ_g.
- Specific gravity, γ_g.
- Specific volume ,ν.
- Compressibility factor, Z.
- Isothermal gas compressibility coefficient, c_g.
- Gas formation volume factor, B_g.
- Gas expansion factor, E_g.
- Viscosity, μ_g.

2.1.1 Apparent Molecular Weight (Ma) and Mole Fraction (y_i):

One of the main gas properties frequently of interest to engineers is the apparent molecular weight. If y_i represents the **mole fraction** of the i^{th} component in a gas mixture, the apparent molecular weight is defined mathematically by the following equation:

$$M_a = \sum_{i=1} y_i M_i \qquad eq.(2.1)$$

where:

M_a = apparent molecular weight of a gas mixture.
M_i = molecular weight of the i^{th} component in the mixture.
y_i = mole fraction of component i in the mixture.

Conventionally, natural gas compositions can be expressed in three different forms: mole fraction, y_i, weight fraction, w_i, and volume fraction, υ_i.

The mole fraction of a particular component, i, is defined as the number of moles of that component, n_i, divided by the total number of moles, n, of all the components in the mixture:

$$y_i = \frac{n_i}{n} = \frac{n_i}{\sum_i n_i} \qquad eq.(2.2)$$

The weight fraction of a particular component, i, is defined as the weight of that component, m_i, divided by the total weight of m, the mixture:

$$w_i = \frac{m_i}{m} = \frac{m_i}{\sum_i m_i} \qquad eq.(2.3)$$

Similarly, the volume fraction of a particular component, v_i, is defined as the volume of that component, V_i, divided by the total volume of V, the mixture:

$$v_i = \frac{V_i}{V} = \frac{V_i}{\sum_i V_i} \qquad eq.(2.4)$$

It is convenient in many engineering calculations to convert from mole fraction to weight fraction and vice versa. The procedure is given in the following steps:

1. As the composition is one of the intensive properties and independent of the quantity of the system, assume that the total number of gas is 1; that is, n=1.
2. From the definitions of mole fraction and number of moles

$$y_i = \frac{n_i}{n} = \frac{n_i}{1} = n_i$$
$$m_i = n_i M_i = y_i M_i$$

3. From the above two expressions, calculate the weight fraction to give

$$w_i = \frac{m_i}{m} = \frac{m_i}{\sum_i m_i} = \frac{y_i M_i}{\sum_i y_i M_i} = \frac{y_i M_i}{M_a}$$

4. Similarly,

$$y_i = \frac{w_i/M_i}{\sum_i w_i/M_i}$$

Example 2.1:

Dry air is a gas mixture consisting essentially of Nitrogen, Oxygen, and small amount of other gases. Compute the apparent molecular weight of air given its approximate composition as:

Component	Mole Fraction(y_i)	Molecular Weight
Nitrogen (N_2)	0.78	28.01
Oxygen (O_2)	0.21	32
Argon (A)	0.01	39.94
	$\sum = 1.00$	

Solution:

$$M_a = \left(y_{N_2} \times M_{N_2}\right) + \left(y_{O_2} \times M_{O_2}\right) + (y_A \times M_A)$$

$$M_a = (0.78 \times 28.01) + (0.21 \times 32) + (0.01 \times 39.94) = 28.97 \cong 29$$

2.1.2 Gas Density, ρ_g:

The density of an ideal gas mixture is calculated by simply replacing the molecular weight, *M*, of the pure component with the apparent molecular weight, *Ma*, of the gas mixture to give

$$\rho_g = \frac{pM_a}{RT} \qquad eq.(2.5)$$

Example 2.2:

Calculate the density of Methane at 1000 psia and 68°F in a cylinder with volume of 3.2 cu ft. assume that Methane is an ideal gas.

Solution:

$$\rho_g = \frac{pM_a}{RT}$$

$$\rho_g = \frac{(1000\,psia)\left(16.04\,\frac{lb}{lb.mole}\right)}{\left(10.732\,\frac{psia\ cu\ ft}{lb.mole\ °R}\right)(528\ °R)}$$

$$\rho_g = 2.83\,\frac{lb}{cu\ ft}$$

2.1.3 Specific Gravity, γ_g:

The specific gravity is defined as the ratio of the gas density to that of the air. Both densities are measured or expressed at the same pressure and temperature. Commonly, the standard pressure, p_{sc}, and standard temperature, T_{sc}, are used in defining the gas specific gravity.

$$\gamma_g = \frac{gas\ density\ @\ 14.7\ and\ 60°}{air\ density\ @\ 14.7\ and\ 60°} = \frac{\rho_g}{\rho_{air}} \qquad eq.(2.6)$$

Assuming that the behavior of both the gas mixture and the air is described by the ideal gas equation, the specific gravity can be then expressed as

$$\gamma_g = \frac{\frac{p_{sc}M_a}{RT_{sc}}}{\frac{p_{sc}M_{air}}{RT_{sc}}}$$

or

$$\gamma_g = \frac{M_a}{M_{air}} = \frac{M_a}{28.96} \qquad eq.(2.7)$$

where:

γ_g = gas specific gravity, 60°/60°.

ρ_{air} = density of the air.

M_{air} = apparent molecular weight of the air =28.96.

M_a = apparent molecular weight of the gas.

P_{sc} = standard pressure, psia.

T_{sc} = standard temperature, °R.

Example 2.3:

A gas well is producing gas with a specific gravity of (0.65) at a rate of (1.1 MMscf/day). The average reservoir pressure and temperature are (1500 psi) and (150°F). Calculate:

1. The apparent molecular weight of the gas.
2. The gas density at reservoir conditions.

Solution:

From Eq. (2.7), solve for the apparent molecular weight:

$$M_a = 28.95\gamma_g = (28.96)(0.65) = 18.82$$

Apply Eq. (2.5) to determine gas density:

$$\rho_g = \frac{pM_a}{RT} = \frac{(1500)(18.82)}{(10.73)(610)} = 4.31\ lb/ft^3$$

Example 2.4:

A gas well is producing a natural gas with the following composition:

Component	y_i
CO2	0.05
C1	0.90
C2	0.03
C3	0.02

Assuming an ideal gas behavior, calculate the apparent molecular weight, specific gravity, gas density at (2000 psia) and (150°F), and specific volume at (2000 psia) and (150°F).

Solution:

The table below describes the gas composition.

Component	y_i	M_i	y_iM_i
CO2	0.05	44.01	2.200
C1	0.90	16.04	14.436
C2	0.03	30.07	0.902
C3	0.02	44.11	0.882
			$M_a = 18.42$

Apply Eq. (2.1) to calculate the apparent molecular weight:

$$M_a = \sum_{i=1} y_i M_i \quad = 18.42$$

Calculate the specific gravity by using Eq. (2.7):

$$\gamma_g = \frac{M_a}{28.96} = \frac{18.42}{28.96} = 0.636$$

Solve for the density by applying Eq. (2.5):

$$\rho_g = \frac{(2000)(18.42)}{(10.73)(610)} = 5.628\ lb/ft^3$$

2.1.4 Specific Volume, ν:

The specific volume is defined as the volume occupied by a unit mass of the gas. For an ideal gas, this property can be calculated by applying Equation (2.8):

$$\nu = \frac{V}{m} = \frac{RT}{PM_a} = \frac{1}{\rho} \qquad eq.(2.8)$$

2.1.5 Critical Temperature and Pressure:

Typical PVT relationships for a pure fluid are illustrated in fig. (2.1). the curve segment B-C-A defines the limits of vapor-liquid co-existence. B-C being the bubble point curve of the liquid and C-A, the dew point curve of the vapor. Any combinations of temperature, pressure and volume above that line segment indicates that the fluid exists in a single phase.

2.1.5.1 Single-Component Systems:

Suppose a fixed quantity of a pure component is placed in a cylinder fitted with a frictionless piston at a fixed temperature T_1. Furthermore, consider the initial pressure exerted on the system to be low enough that the entire system is in the vapor state. This initial condition is represented by point *E* on the pressure-volume phase diagram (*p*-*V* diagram) as shown in Fig. (2.1). Consider the following sequential experimental steps taking place on the pure component:

1. The pressure is increased isothermally by forcing the piston into the cylinder. Consequently, the gas volume decreases until it reaches point *F* on the diagram, where the liquid begins to condense. The corresponding pressure is known as the ***dew point pressure***, p_d, and defined as the pressure at which the first droplet of liquid is formed.
2. The piston is moved further into the cylinder as more liquid condenses from the present gas. This condensation process is characterized by a constant pressure and represented by the horizontal line *FG*. At point *G*, traces of gas remain and the corresponding pressure is called the ***bubble point pressure***, p_b, and defined as the pressure at which the first sign of

gas formation is detected. A characteristic of a single-component system is that, at a given temperature, the dew point pressure and the bubble point pressure are equal.

3. As the piston is forced slightly into the cylinder, a sharp increase in the pressure (point *H*) is noted without an appreciable decrease in the liquid volume. That behavior evidently reflects the low compressibility of the liquid phase.

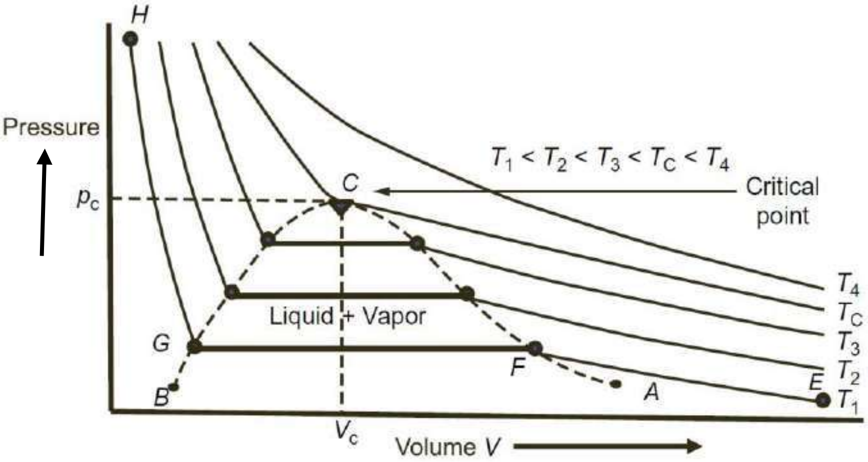


Fig. (2.1): Typical pressure-volume diagram for pure component.

4. By repeating these steps at progressively increasing temperatures, a family of curves of equal temperatures (isotherms) is constructed as shown in Fig. (3.1). Note that, as the temperature increases, the length of the straight-line portion of the isotherm decreases until it eventually vanishes and the isotherm merely has a horizontal tangent and inflection point at the critical point. This isotherm temperature is called the ***critical temperature***, T_c, of that single component.

5. The dashed curve connecting the dew points, called the ***dew point curve*** (line *FC*), represents the states of the "**saturated gas**." The dashed curve connecting the bubble points, called the ***bubble point curve*** (line *GC*), similarly represents the "**saturated liquid**." These two curves meet a point *C*, which is known as the ***critical point***. The corresponding pressure and volume are called the ***critical pressure***, p_c, and ***critical volume***, V_c, respectively. Referring to Fig. (3.1), the area enclosed by the area *AFCGB* is called the ***two-phase region*** or the ***phase envelope***. Within this defined region, vapor and liquid can coexist in equilibrium. Outside the phase envelope, only one phase can exist.

6. The critical point (point *C*) describes the critical state of the pure component and represents the limiting state for the existence of two phases, that is, liquid and gas. In other words, for a single-component system, the critical point is defined as the highest value of pressure and temperature at which two phases can coexist. A more generalized definition of the critical point, which is applicable to a single- or multicomponent system, is that, the ***critical point*** is the point at which all intensive properties of the gas and liquid phases are equal.

   Another means of presenting the results of this experiment is shown in Fig. (2.2), in which the pressure and temperature of the system are the independent parameters. Fig. (2.2) shows a typical pressure-temperature diagram (*p*-*T* diagram) of a single-component system with solids lines that clearly represent three different phase boundaries: vapor-liquid, vapor-solid, and liquid-solid phase separation boundaries. As shown in the illustration, line *AC* terminates at the critical point (point *C*) and can be thought of as the dividing line between the areas where liquid and vapor exist. The curve is commonly called the ***vapor pressure curve*** or the ***boiling point curve***. The corresponding pressure at any point on the curve is called the ***vapor pressure***, p_v, with a corresponding temperature termed the ***boiling point temperature***.

The lower end of the vapor pressure line is limited by the triple point *A*. This point represents the pressure and temperature at which solid, liquid, and vapor coexist under equilibrium conditions. The line *AB* is called the ***sublimation pressure curve*** of the solid phase, and it divides the area where solid exists from the area where vapor exists. Points above *AB* represent solid systems, and those below *AB* represent vapor systems. The line *AD* is called the ***melting curve* or *fusion curve*** and represents the change of melting-point temperature with pressure. The fusion (melting) curve divides the solid phase area from the liquid phase area, with a corresponding temperature at any point on the curve termed the ***fusion* or *melting-point temperature***. Note that the solid-liquid curve (fusion curve) has a steep slope, which indicates that the triple point for most fluids is close to their normal melting-point temperatures. For pure **hydrocarbons**, the melting point generally increases with pressure so the slope of the line *AD* is **positive**. **Water** is the exception in that its melting point decreases with pressure, so in this case, the slope of the line *AD* is **negative**.

Each pure hydrocarbon has a *p*-*T* diagram similar to that shown in Fig. (2.2). In addition, each pure component is characterized by its own vapor pressures, sublimation pressures, and critical values, which are different for each substance, but the general characteristics are similar. If such a diagram is available for a given substance, it is obvious that it could be used to predict the behavior of the substance as the temperature and pressure are changed.

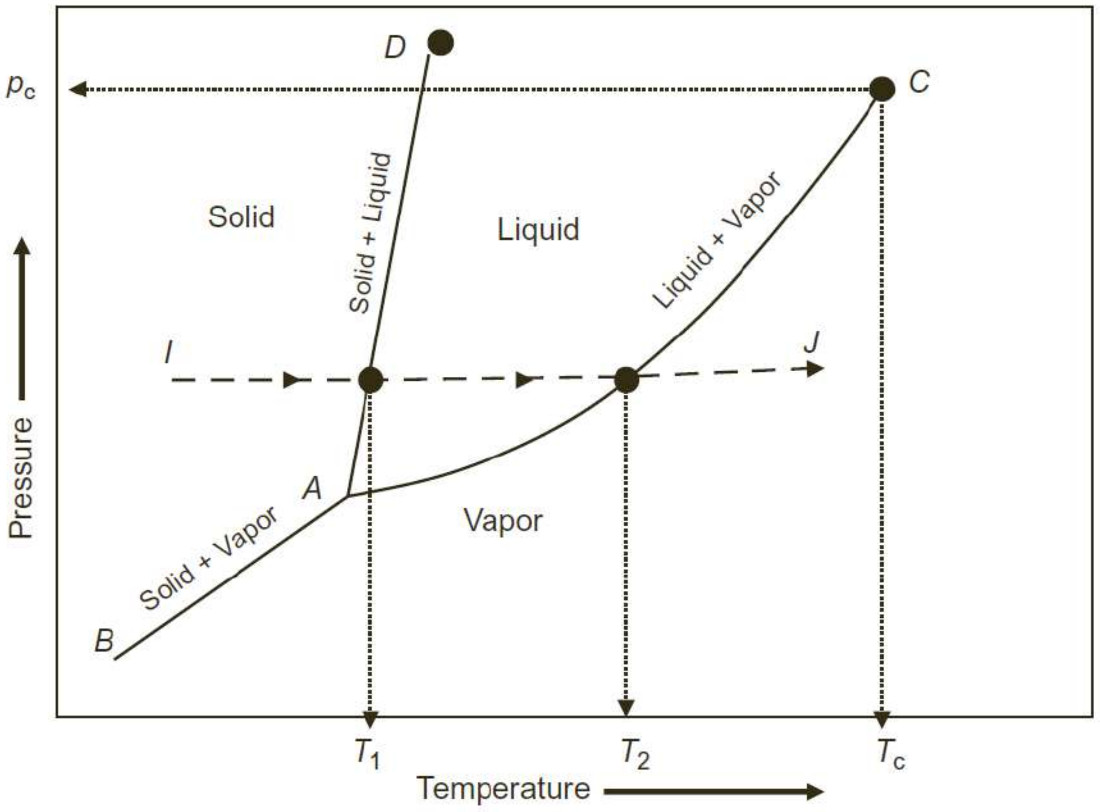


Fig. (2.2): Typical pressure-temperature diagram for a single-component system.

Critical constants for some of the commonly occurring hydrocarbons and other components of natural gas can be found in Table (2.1).

Table (2.1): Physical Properties for Pure Components

Compound	**Formula**	**Molar Mass (Molecular Weight)**	**Critical Constants**		
			Pressure (psia)	**Temperature (°F)**	**Volume (ft3/lb-m)**
Methane	CH_4	16.043	666.4	-116.67	0.0988
Ethane	C_2H_6	30.070	706.5	89.92	0.0783
Propane	C_3H_8	44.097	616	206.06	0.0727
Isobutane	C_4H_{10}	58.123	527.9	274.46	0.0714

n-Butane	C_4H_{10}	58.123	550.6	305.62	0.0703
isopentane	C_5H_{12}	72.150	490.4	369.10	0.0679
n-Pentane	C_5H_{12}	72.150	488.6	385.8	0.0675
Neopentane	C_5H_{12}	72.150	464.0	321.13	0.0673
n-Hexane	C_6H_{14}	86.177	436.9	453.6	0.0688
2-Methylpentane	C_6H_{14}	86.177	436.6	435.83	0.0682
3-Methylpentane	C_6H_{14}	86.177	453.1	448.4	0.0682
Neohexane	C_6H_{14}	86.177	446.8	420.13	0.0667
2.3-Dimethylbutane	C_6H_{14}	86.177	453.5	440.29	0.0665
n-Heptane	C_7H_{16}	100.204	396.8	512.7	0.0691
2-Methylhexane	C_7H_{16}	100.204	396.5	495.00	0.0673
3-Methylhexane	C_7H_{16}	100.204	408.1	503.80	0.0646
3-Ethylpentane	C_7H_{16}	100.204	419.3	513.39	0.0665
2.2-Dimethylpetane	C_7H_{16}	100.204	402.2	477.23	0.0665
2.4-Dimethylpentane	C_7H_{16}	100.204	396.9	475.95	0.0668
3.3-Dimethylpentane	C_7H_{16}	100.204	427.2	505.87	0.0662
Triptane	C_7H_{16}	100.204	428.4	496.44	0.0636
n-Octane	C_8H_{18}	114.231	360.7	564.22	0.0690
Dilsobutyl	C_8H_{18}	114.231	360.6	530.44	0.0676
Isooctane	C_8H_{18}	114.231	372.4	519.46	0.0656
n-Nonane	C_9H_{20}	128.258	331.8	610.68	0.0684
n-Decane	C_8H_{22}	14.285	305.2	652.0	0.0679
Cyclopentane	C_5H_{10}	70.134	653.8	461.2	0.0594
Methylcyclopentane	C_6H_{12}	84.161	548.9	499.35	0.0607
Cyclohexane	C_6H_{12}	84.161	590.8	536.6	0.0586
Methyl cyclohexane	C_7H_{14}	98.188	503.5	570.27	0.0600
Carbon monoxide	CO	28.010	507.5	-220.43	0.0532
Carbon dioxide	CO_2	44.010	1071	87.91	0.0344
Hydrogen sulfide	H_2S	34.08	1300	212.45	0.0461
Sulfur dioxide	SO_2	64.06	1143	315.8	0.0305
Ammonia	NH_3	17.0305	1646	270.2	0.0681
Air	N_2+O_2	2.0159	546.9	-221.31	0.0517
Hydrogen sulfide	H_2	2.0159	188.1	-399.9	0.5165
Oxygen	O_2	31.9988	731.4	-181.43	0.0367
Nitrogen	N_2	28.0134	493.1	-232.51	0.0510
Chlorine	Cl_2	70.906	1157	290.75	0.0280
Water	H_2O	18.0153	3198.8	705.16	0.0497s
Helium	He	4.0026	32.99	-450.31	0.2300
Hydrogen chloride	HCl	36.461	1205	124.77	0.0356
Source: Courtesy of the Gas Processors Suppliers Association. Published in the GPSA Engineering Data Book, 10th edition, 1987.					

2.1.5.2 Two-Component Systems:

A unique feature of the single-component system is that, at a fixed temperature, two phases (vapor and liquid) can exist in equilibrium at only the vapor pressure. For a binary system, two phases can exist in equilibrium at various pressures at the same temperature. The following discussion concerning the description of the phase behavior of a two-component system involves many concepts that apply to the more complex multicomponent mixtures of oils and gases.

An important characteristic of binary systems is the variation of their thermodynamic and physical properties with the composition. Therefore, it is

necessary to specify the composition of the mixture in terms of mole or weight fractions. It is customary to designate one of the components as the more volatile component and the other the less volatile component, depending on their relative vapor pressure at a given temperature.

Suppose that the examples previously described for a pure component are repeated, but this time we introduce into the cylinder a binary mixture of a known overall composition "z_i." Consider that the initial pressure p_1 exerted on the system, at a fixed temperature of T_1, is low enough that the entire system exists in the vapor state. This initial condition of pressure and temperature acting on the mixture is represented by point 1 on the *p/V* diagram of Fig. (2.3). As the pressure is increased isothermally, it reaches point 2, at which point an infinitesimal amount of liquid is condensed. The pressure at this point is called the *dew point pressure*, p_d, of the mixture. It should be noted that, at the dew point pressure, the composition of the vapor phase is equal to the overall composition of the binary mixture. As the total volume is decreased by forcing the piston inside the cylinder, a noticeable increase in the pressure is observed as more and more liquid is condensed. This condensation process is continued until the pressure reaches point 3, at which point traces of gas remain. At point 3, the corresponding pressure is called the *bubble point pressure*, p_b. Because, at the bubble point, the gas phase is only of infinitesimal volume, the composition of the liquid phase therefore is identical with that of the whole system. As the piston is forced further into the cylinder, the pressure rises steeply to point 4 with a corresponding decreasing volume.

Repeating the above process at progressively increasing temperatures, a complete set of isotherms is obtained on the *p/V* diagram of Fig. (2.4) for a binary system consisting of *n*-pentane and *n*-heptane. The bubble point curve, as represented by line *AC*, represents the locus of the points of pressure and volume at which the first bubble of gas is formed. The dew point curve (line *BC*) describes the locus of the points of pressure and volume at which the first droplet of liquid is formed. The two curves meet at the critical point (point *C*). The critical pressure, temperature, and volume are given by p_c, T_c, and V_c, respectively. Any point within the phase envelope (line *ACB*) represents a system consisting of two phases. Outside the phase envelope, only one phase can exist.

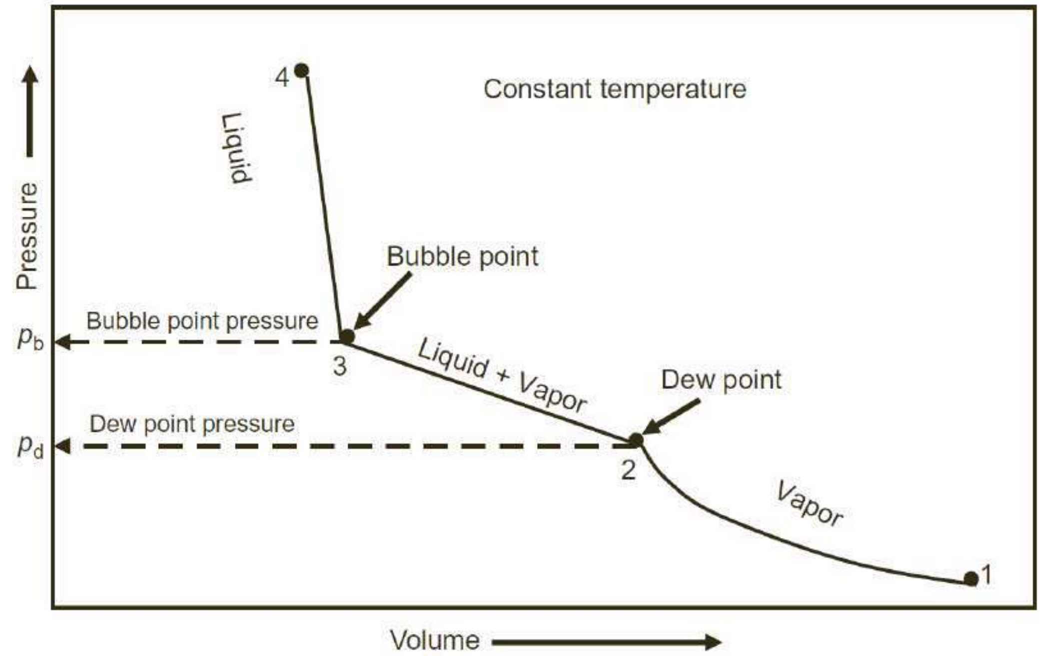


Fig. (2.3): Pressure-volume isotherm for a two-component system.

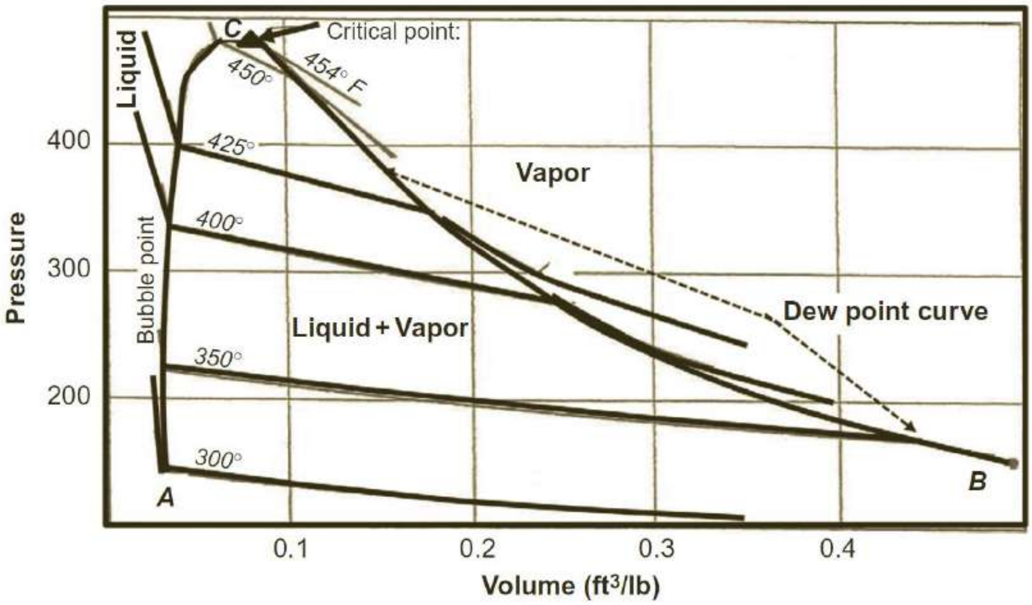


Fig. (2.4): Pressure-volume diagram for a binary system.

Example 2.4:

Consider a mixture 47.6 weight percent of n-pentane and 52.4 weight percent n-heptane. Estimate the specific volume of the liquid at its bubble point at 400°F. Also estimate the specific volume of the gas at its dew point at 400°F.

Solution:

From 400°F isotherm of figure (2.4):

ν of bubble point at 400°F and 332 psia = 0.036 *cu ft/lb*

ν of dew point at 400°F and 275 psia = 0.249 *cu ft/lb*

2.1.5.3 Three-Component Systems:

The phase behavior of mixtures containing three components (ternary systems) is conveniently represented in a triangular diagram, such as that shown in Fig. (2.5). Such diagrams are based on the property of equilateral triangles that the sum of the perpendicular distances from any point to each side of the diagram is a constant and equal to the length on any of the sides. Thus, the composition x_i of the ternary system as represented by point *A* in the interior of the triangle of Fig. (3.5).

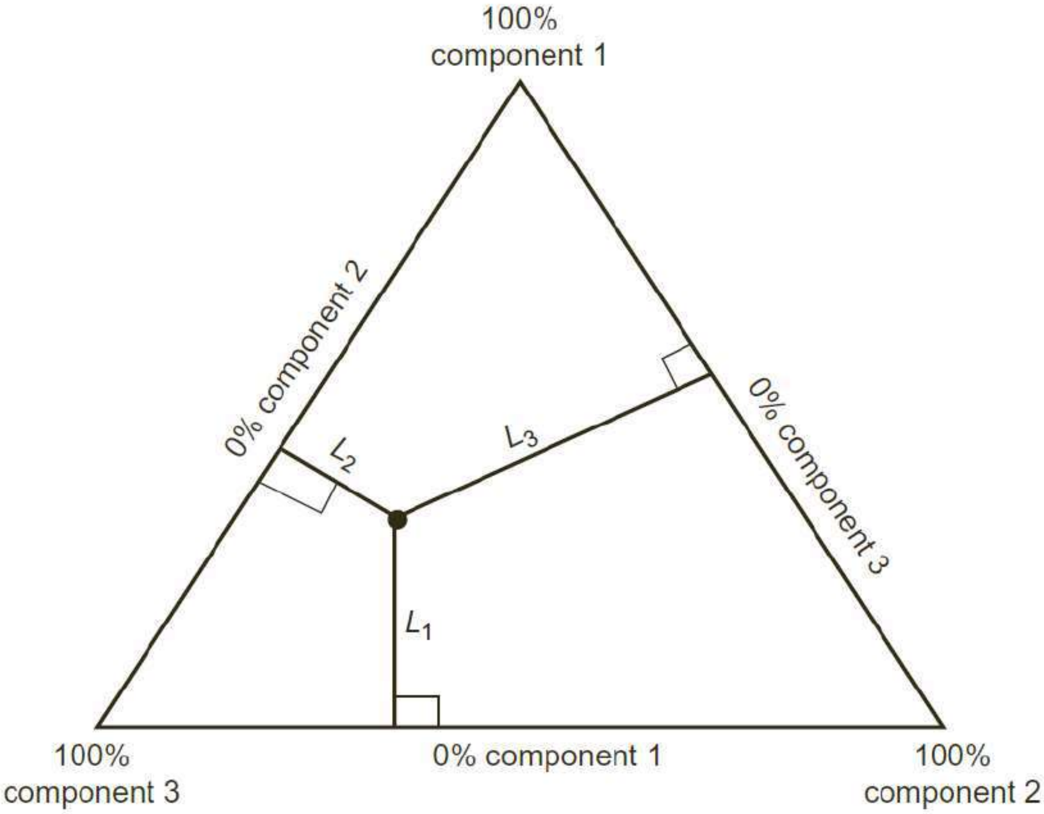


Fig. (2.5): Properties of the ternary diagram.

Typical features of a ternary phase diagram for a system that exists in the two-phase region at fixed pressure and temperature are shown in Fig. (2.6). Any mixture with an overall composition that lies inside the binodal curve (phase envelope) will split into liquid and vapor phases. The line that connects the composition of liquid and vapor phases that are in equilibrium is called the *tie line*. Any other mixture with an overall composition that lies on that tie line will split into the same liquid and vapor compositions. Only the amounts of liquid and gas change as the overall mixture composition changes from the liquid side (bubble point curve) on the binodal curve to the vapor side (dew point curve). The liquid and vapor portions of the binodal curve (phase envelope) meet at the critical point (plait point), where the liquid and vapor phases are identical.

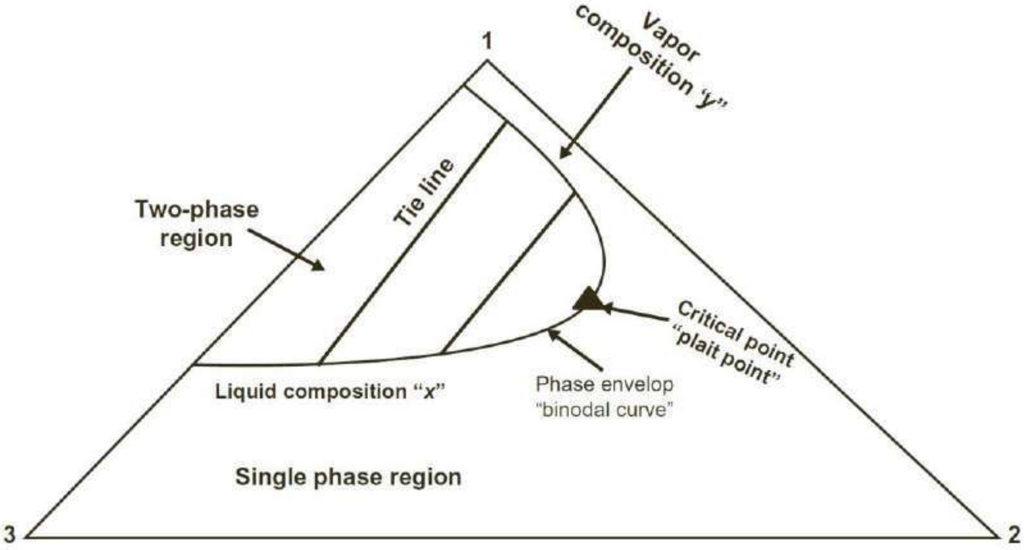


Fig. (2.6): Ternary phase diagram at a constant temperature and pressure for a system that forms a liquid and a vapor.

2.1.5.4 Multicomponent Systems:

The phase behavior of multicomponent hydrocarbon systems in the two-phase region, that is, the liquid-vapor region, is very similar to that of binary systems. However, as the system becomes more complex with a greater number of different components, the pressure and temperature ranges in which two phases lie increase significantly.

The conditions under which these phases exist are a matter of considerable practical importance. The experimental or the mathematical determinations of these conditions are conveniently expressed in different types of diagrams, commonly called ***phase diagrams*** (see chapter three).

2.1.5.5 Principle of Corresponding States:

According to **the law of corresponding states**, the deviation of a real gas from the ideal gas law is the same for different gases at the same corresponding conditions of ***reduced temperature*** (T_r) and ***reduced pressure*** (p_r). This law has been useful in correlating the properties of gases. This principle was developed because observers noticed that the behavior of pure gases was qualitatively similar when compared (on p-V plots, for instance as shown in figure (2.1)) even though the quantitative values of p and V were very dissimilar. The idea was advanced that the properties of substances could be correlated if they were all compared at "corresponding" values of T and p, which could be referenced easily. In the application of the principle of corresponding states to a single-component gas, the critical state of the gas is used as the reference point. The following terms are used.

$$p_r = \frac{p}{p_c} \qquad eq.(2.9)$$

$$T_r = \frac{T}{T_c} \qquad eq.(2.10)$$

$$V_r = \frac{V}{V_c} \qquad eq.(2.11)$$

where:

p_r = reduced pressure.

T_r = reduced temperature.

V_r = reduced volume.

P_c = critical pressure.

T_c = critical temperature.

V_c = critical volume.

2.1.6 Determination of Z-factor Value:

There are two methods of finding Z-factor value:

1- ***Mathematically by using the law of real gases:***

$$pV = ZnRT$$

2- ***by using correlations and charts:***

  a- ***Finding Z –Factor of known composition (A single component):***

Compressibility factors of many pure compounds are available as functions of pressure in most handbooks dealing with gas properties (Standing & Katz, 1942). Compressibility factor charts for *Methane*, *Ethane* and *Propane* are shown in figures (2.7), (2.8) and (2.9). While the principle of corresponding states is not entirely rigorous, its application has been used widely in the determination of gas volumes for engineering purposes. It also has application in the estimation of gas viscosities.

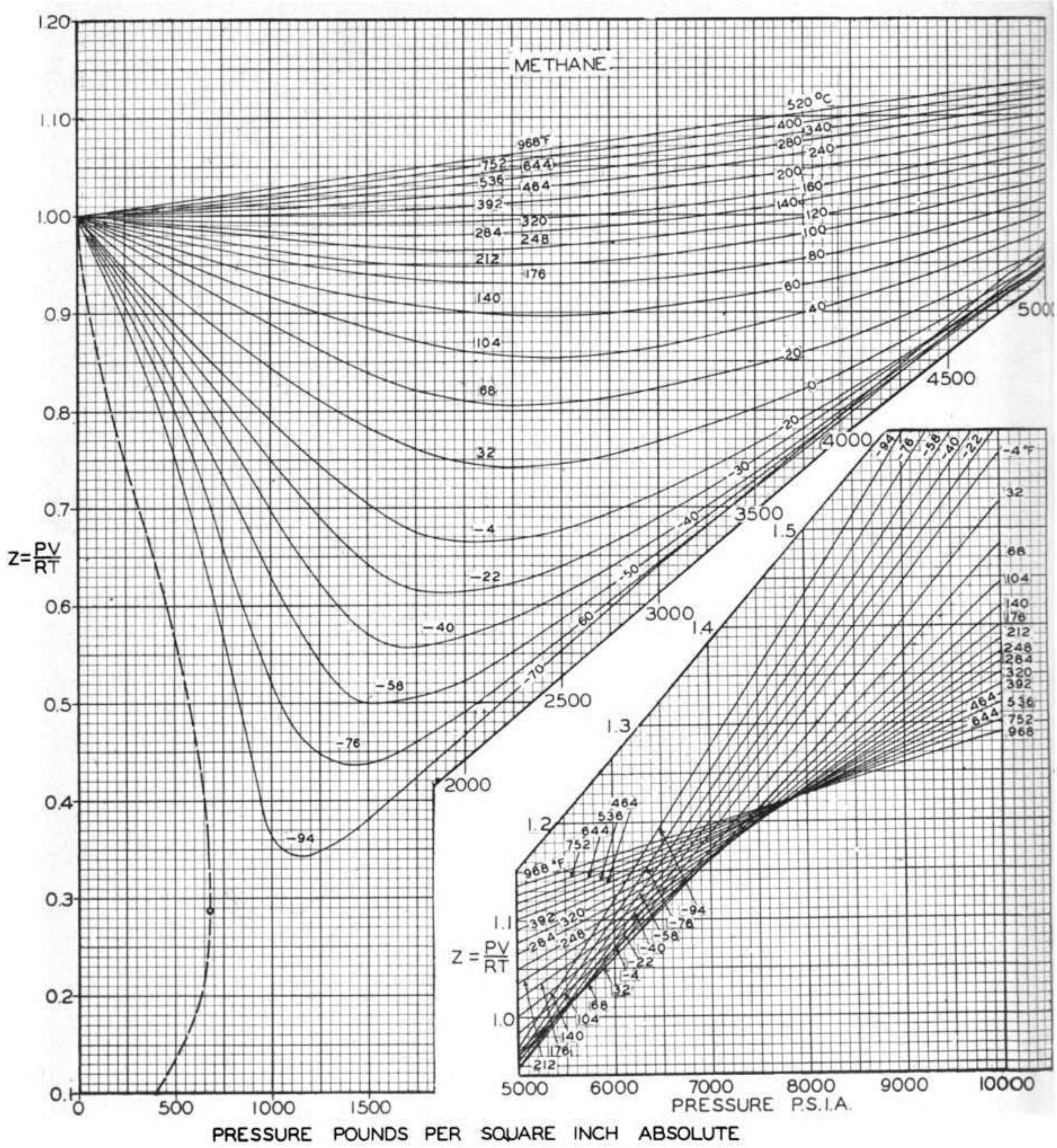


Fig. (2.7): Compressibility factor for methane. (Brown, Katz, Oberfell, and Alden, Natural Gasoline and Volatile Hydrocarbons, 1948, p.24).

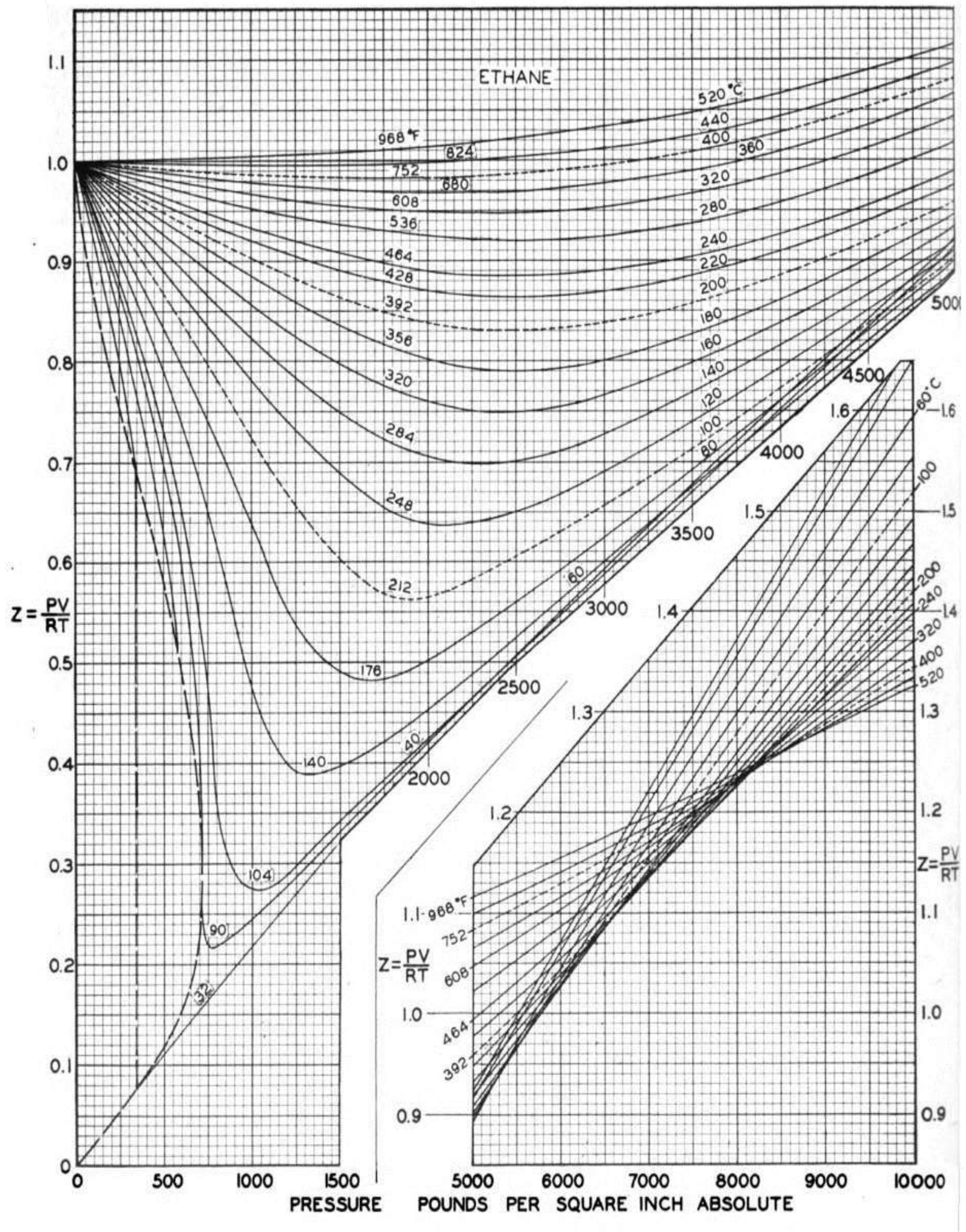


Fig. (2.8): Compressibility factor for ethane (Brown, Katz, Oberfell, and Alden, Natural Gasoline and Volatile Hydrocarbons, 1948, p.26).

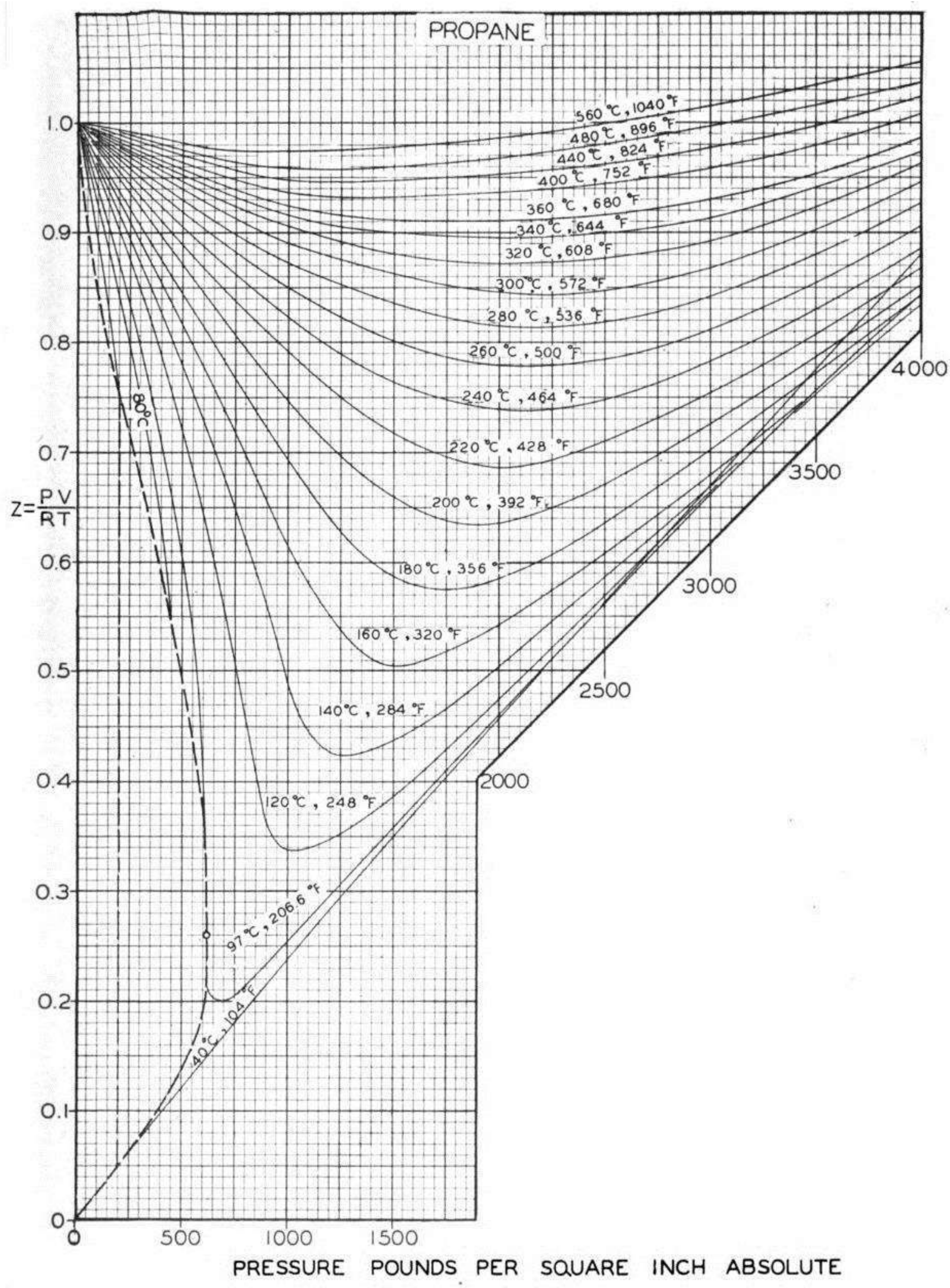


Fig. (2.9): Compressibility factor for propane (Brown, Katz, Oberfell, and Alden, Natural Gasoline and Volatile Hydrocarbons, 1948, p.27).

Example2.5:

Calculate the mass of methane gas contained at 1000 psia and 68ºF in a cylinder with volume of 3.20 cu ft. Do not assume that methane is an ideal gas.

Solution:

$$m = \frac{pMV}{zRT}$$

Z=0.89 from figure (3.7).

$$m = \frac{(1000\,psia)\left(16.04\frac{lb}{lb.mole}\right)(3.20\,cu\,ft)}{(0.89)\left(10.732\frac{psia\,cu\,ft}{lb.mole\,°R}\right)(528\,°R)}$$

$$m = 10.2\,lb$$

Note: that the assumption that methane acts as an ideal gas in **Example2.2** resulted in an error in mass of about 11%.

b- Finding Z –Factor of known composition (Mixture component):

In application of the principle of corresponding states to a mixture of gases (known composition), the true critical temperature and pressure for the gases cannot be used because the paraffinic hydrocarbon series does not strictly follow the principle as stated above. "Pseudocritical" temperature and pressure are defined for use in place of the true critical temperature and pressure to determine the compressibility factor for a mixture. The pseudocritical temperature and pseudocritical pressure normally are defined as the molar average critical temperature and pressure of the mixture components, thus:

$$p_{pc} = \sum y_i p_{ci} \qquad eq.(2.12)$$

and

$$T_{pc} = \sum y_i T_{ci} \qquad eq.(2.13)$$

where:

p_{pc} = pseudocritical pressure of the gas mixture.

T_{pc} = pseudocritical temperature of the gas mixture.

p_{ci} = critical pressure of Component *i* in the gas mixture.

T_{ci} = critical temperature of Component *i* in the gas mixture.

y_i = mole fraction of Component *i* in the gas mixture.

$$y_i = \frac{n_1}{n_1 + n_2} + \frac{n_2}{n_1 + n_2}$$

The pseudocritical pressure and temperature are then used to determine the pseudoreduced conditions:

$$p_{pr} = \frac{p}{p_{pc}} \qquad eq. (2.14)$$

where p_{pr} is the pseudoreduced pressure, and:

$$T_{pr} = \frac{T}{T_{pc}} \qquad eq. (2.15)$$

where T_{pr} is the pseudoreduced temperature. These reduced conditions are used to determine the compressibility factor, z, from Fig. (2.10), which was developed by (Standing & Katz, 1942) from data collected on methane and natural gases. The data used to develop Fig. (2.10) ranged up to 8,200 psia and 250°F. Compressibility factors of high-pressure natural gases (10,000 to 20,000 psia) may be obtained from Fig. (2.11), which was developed by (Standing & Katz, 1942). Figs. (2.12) and (2.13) may be used for low pressure applications after (Brown, et. al, 1948).

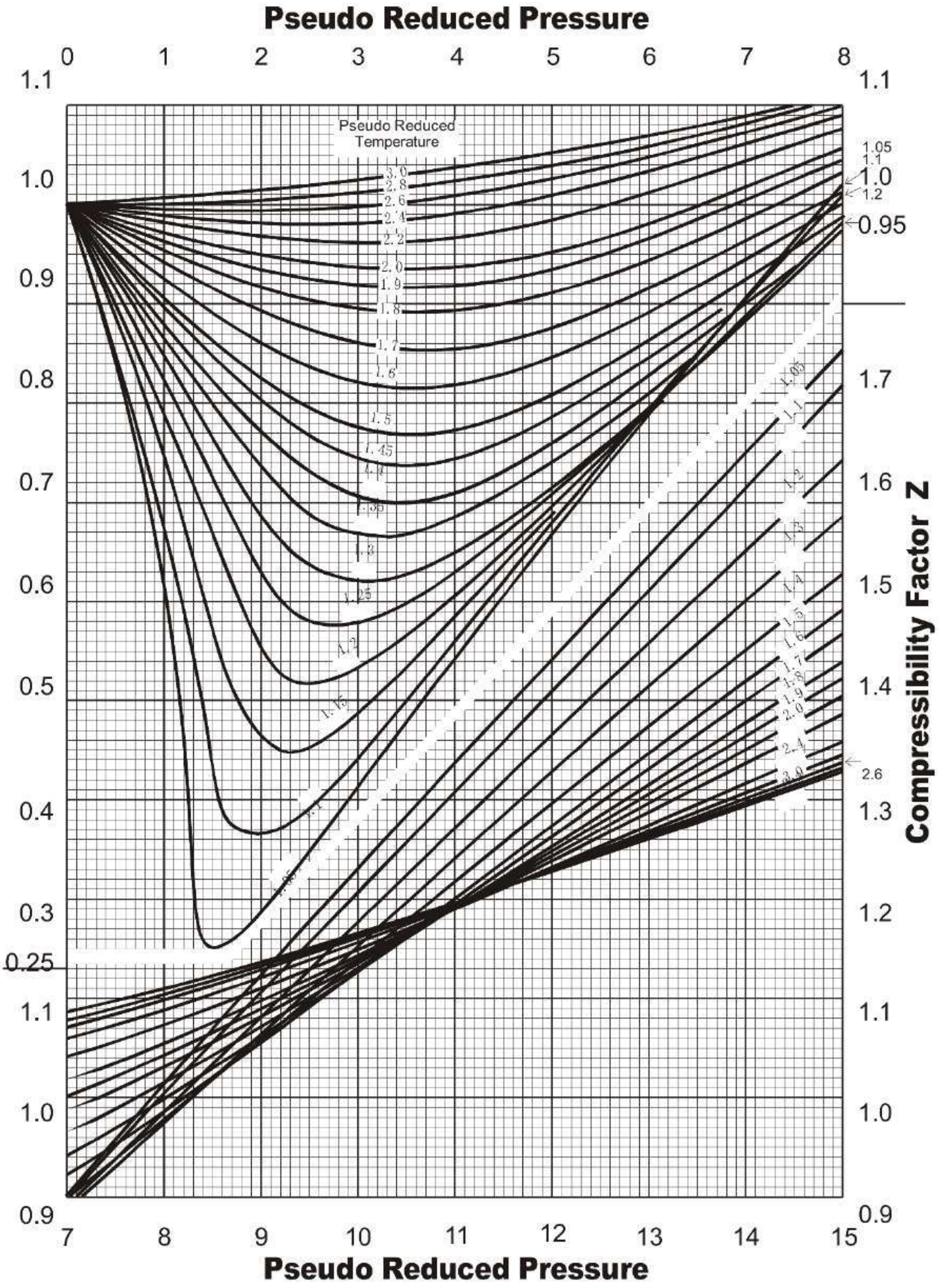


Fig. (2.10): Standing and Katz compressibility factors chart by reduced temperature and pressure.

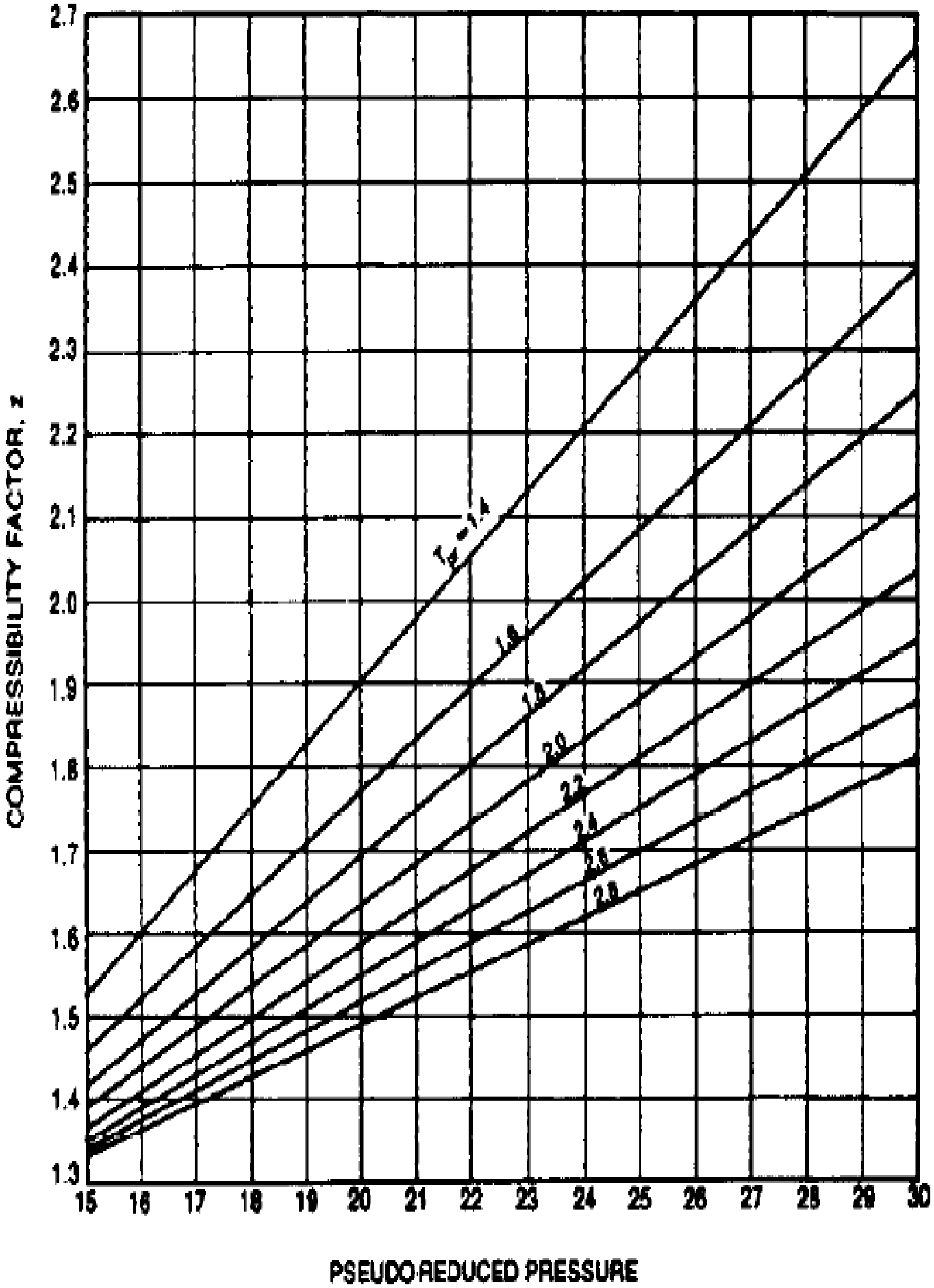


Fig. (2.11): Compressibility factor for natural gases at pressures of 10,000 to 20,000 psia, (from Katz, D.L. et al.: Handbook of Natural Gas Engineering, 1959).

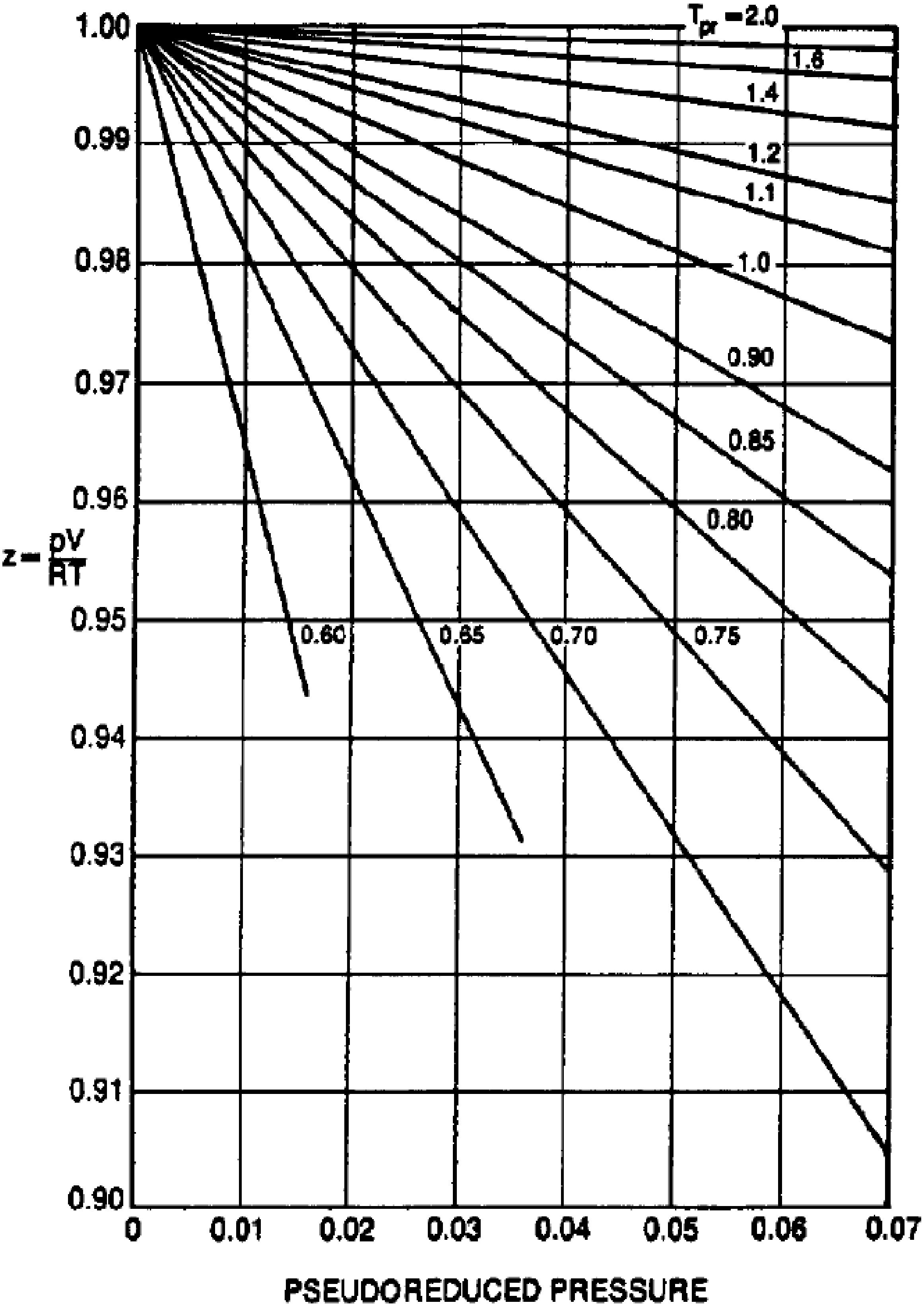


Fig. (2.12): Compressibility factors for natural gases near atmospheric pressures, (from Katz, D.L. et al.: Handbook of Natural Gas Engineering, 1959).

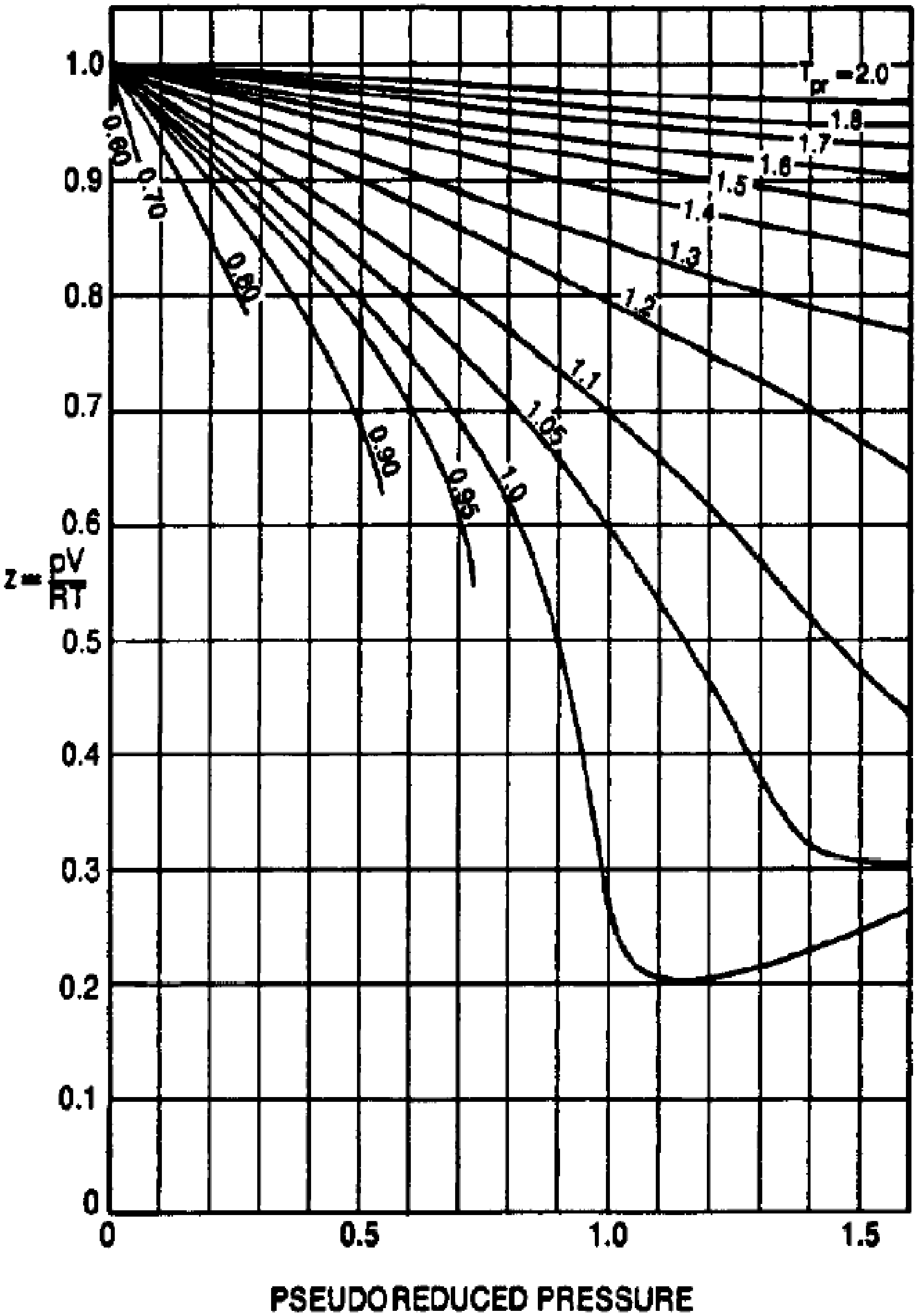


Fig. (2.13) Compressibility factors for natural gases at low reduced pressures, (from Katz, D.L. et al.: Handbook of Natural Gas Engineering, 1959).

Example 2.6:

A gas reservoir has the following gas composition: the initial reservoir pressure and temperature are (3,000 psia) and (180°F), respectively.

Component	y_i
CO_2	0.02
N_2	0.01
C_1	0.85
C_2	0.04
C_3	0.03
i - C_4	0.03
n - C_4	0.02

Calculate the gas compressibility factor under initial reservoir conditions.

Solution:

Step 1. Finding p_{ci} & T_{ci} for each component from (Table 2.1 Physical properties for pure component):

Step 2. Converting T_c unit from (°F) to (°R):

Step 3. Finding p_{pc} & T_{pc} by using the equations below:

$$p_{pc} = \sum_{i=1} y_i \, p_{ci}$$

$$T_{pc} = \sum_{i=1} y_i \, T_{ci}$$

Component	y_i	T_{ci}(°R)	y_iT_{ci}	p_{ci}	y_ip_{ci}
CO_2	0.02	547.91	10.96	1071	21.42
N_2	0.01	227.49	2.27	493.1	4.93
C_1	0.85	343.33	291.83	666.4	566.44
C_2	0.04	549.92	22.00	706.5	28.26
C_3	0.03	666.06	19.98	616.4	18.48
i - C_4	0.03	734.46	22.03	527.9	15.84
n - C_4	0.02	765.62	15.31	550.6	11.01
			T_{pc}=383.38		p_{pc}=666.38

Step 4. Calculate the pseudo-reduced pressure and temperature by applying the following Equations:

$$P_{pr} = \frac{p}{p_{pc}} = \frac{3000}{666.38} = 4.5$$

$$T_{pr} = \frac{T}{T_{pc}} = \frac{640}{383.38} = 1.67$$

Step 5. Determine the z-factor from the chart of (Standing and Katz compressibility factors chart):

$$z = 0.85$$

c- Finding Z –Factor of unknown composition (Mixture component):

In cases where the composition of a natural gas is not available, the pseudocritical properties, p_{pc} and T_{pc}, can be predicted solely from the specific gravity of the gas. Brown (Brown et al. 1948) presented a graphical method for a convenient approximation of the pseudocritical pressure and pseudocritical temperature of gases when only the specific gravity of the gas is available, see Fig. (2.14) (Standing, 1977) expressed this graphical correlation in the following mathematical forms.

For Case 1, ***natural gas systems***,

$$T_{pc} = 168 + 325\gamma_g - 12.5\gamma_g^2 \qquad eq. (2.16)$$

$$p_{pc} = 677 + 15\gamma_g - 37.5\gamma_g^2 \qquad eq. (2.17)$$

For Case 2, ***gas-condensate systems***,

$$T_{pc} = 187 + 330\gamma_g - 71.5\gamma_g^2 \qquad eq. (2.18)$$

$$p_{pc} = 706 - 51.7\gamma_g - 11.1\gamma_g^2 \qquad eq. (2.19)$$

where:

T_{pc} = pseudocritical temperature, °R.
p_{pc} = pseudocritical pressure, psia.
γ_g = specific gravity of the gas mixture.

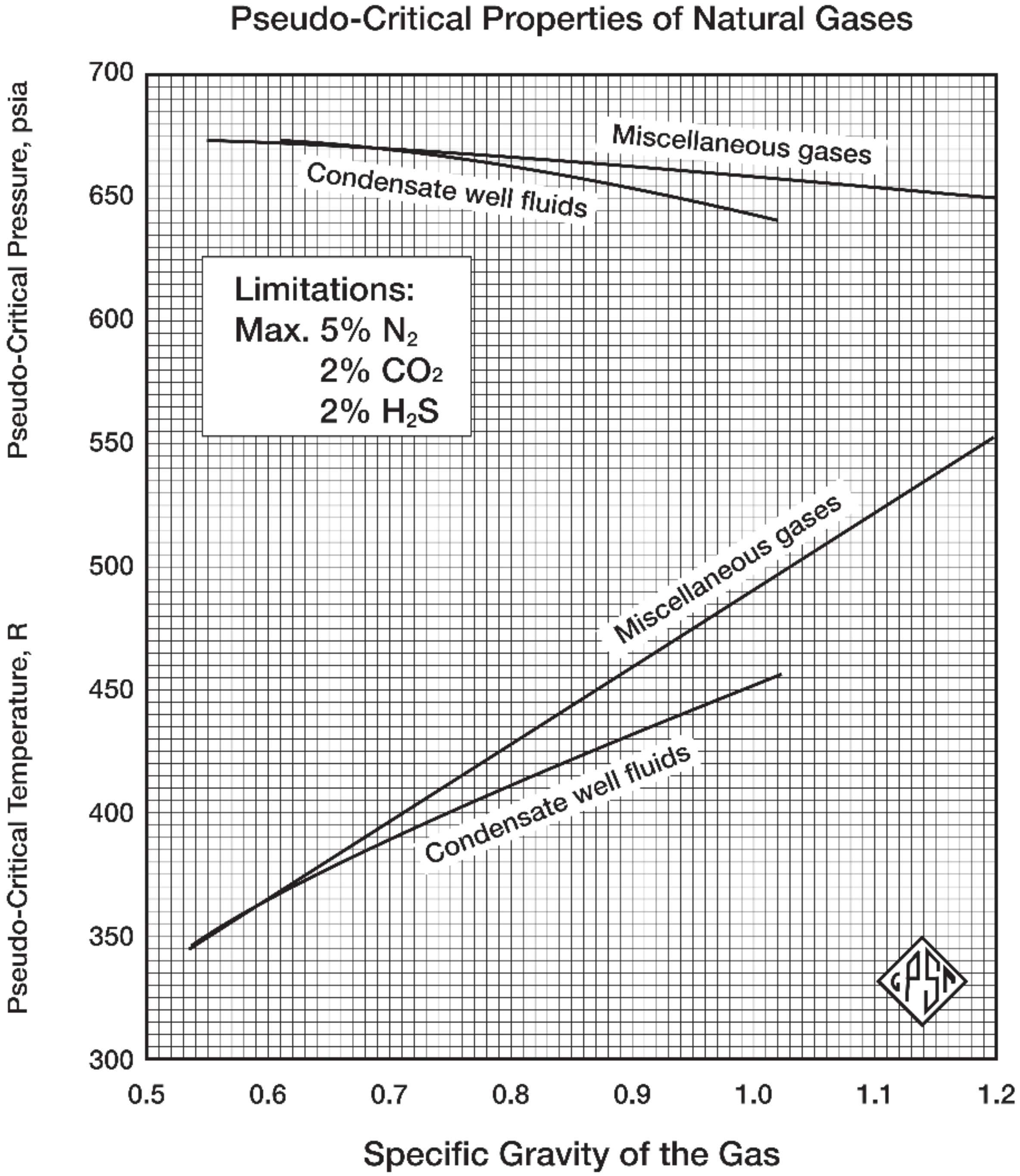


Fig. (2.14): Pseudo-critical properties of natural gases, (Courtesy of GPSA and GPA Engineering Data Book, 10th Edition, 1987).

Example 2.7:

A gas reservoir with unknown gas composition, the initial reservoir pressure and temperature are (3,000 psia and 180°F), respectively. Calculate the gas compressibility factor under initial reservoir conditions. Gas Molecular Wight is Ma= (20.23).

Solution:

Step 1. Calculate the specific gravity of the gas:

$$\gamma_g = \frac{M_a}{M_{air}} = \frac{20.23}{28.96} = 0.699$$

Step 2. Solve for the pseudo-critical properties:

$$T_{pc} = 168 + 325\ (0.699) - 12.5\ (0.699)^2 = 389.1°R$$

$$p_{pc} = 677 + 15\ (0.699) - 37.5\ (0.699)^2 = 669.2\text{ psia}$$

Also we can use the chart in Fig. (2.14) to find the pseudo-critical properties.

Step 3. Calculate p_{pr} and T_{pr}:

$$p_{pr} = \frac{p}{p_{pc}} = \frac{3000}{669.2} = 4.48$$

$$T_{pr} = \frac{T}{T_{pc}} = \frac{640}{389.1} = 1.64$$

Step 4. Determine the z-factor from the chart of Fig. (2.10):

$$z = 0.845$$

d-Mixture composition with non-hydrocarbon Components:

In this case we use specific gravity (γ_g) in chart of Fig. (2.14) to find (p_{pc}) and (T_{pc}) or using Standing correlations (2.16 & 2.17). After that we make corrections to these values because of the effect of non-hydrocarbon gases. It's known that natural gases frequently contain materials other than hydrocarbon components, such as nitrogen, carbon dioxide, and hydrogen sulfide. Hydrocarbon gases are classified as sweet or sour depending on the hydrogen sulfide content. Both sweet and sour gases may contain nitrogen, carbon dioxide, or both. A hydrocarbon gas is termed a sour gas if it contains 1 gram of H_2S per 100 ft^3 or 5.7 mg of H_2S / m^3 of natural gas.

The common occurrence of small percentages of nitrogen and carbon dioxide, in part, is considered in the correlations previously cited. Concentrations of up to 5% of these non-hydrocarbon components do not seriously affect accuracy. Errors in compressibility factor calculations as large as 10% may occur in higher concentrations of non-hydrocarbon components in gas mixtures.

Two methods were developed to adjust the pseudocritical properties of the gases to account for the presence of the n-hydrocarbon components: the Wichert-Aziz method and the Carr-Kobayashi-Burrows method.

I- Wichert-Aziz's Correction Method:

Natural gases that contain H_2S and/or CO_2 frequently exhibit different compressibility factors behavior than sweet gases. Wichert and Aziz (1972) developed a simple, easy-to-use calculation to account for these differences. This method permits the use of the Standing-Katz Z-factor chart Fig. (2.10), by using a pseudocritical temperature adjustment factor, ε, which is a function of the concentration of CO_2 and H_2S in the sour gas. This correction factor then is used to adjust the pseudocritical temperature and pressure according to the following expressions:

$$T'_{pc} = T_{pc} - \varepsilon \qquad eq.(2.20)$$

$$p'_{pc} = \frac{p_{pc}T'_{pc}}{T_{pc} + B(1 - B)\varepsilon} \qquad eq.(2.21)$$

Where:

T_{pc} = pseudo-critical temperature, °R.
p_{pc} = pseudo-critical pressure, psia.
T'_{pc} = corrected pseudo-critical temperature, °R.
p'_{pc} = corrected pseudo-critical pressure, psia.
B = mole fraction of H2S in the gas mixture.
ε = pseudo-critical temperature adjustment factor and is defined mathematically by the following expression.

$$\varepsilon = 120[A^{0.9} - A^{1.6}] + 15(B^{0.5} - B^{4.0}) \qquad eq.(2.22)$$

Where the coefficient A is the sum of the mole fraction H_2S and CO_2 in the gas mixture:

$$A = y_{H_2S} + y_{CO_2}$$

The computational steps of incorporating the adjustment factor $\boldsymbol{\varepsilon}$ into the Z-factor calculations are summarized next.

Step 1. Calculate the pseudocritical properties of the whole gas mixture by applying Eqs. (2.16), (2.17) or Eqs. (2.18), (2.19).

Step 2. Calculate the adjustment factor, ε, from Eq. (2.22).

Step 3. Adjust the calculated p_{pc} and T_{pc} (as computed in step 1) by applying Eqs. (2.20), (2.21).

Step 4. Calculate the pseudoreduced properties, p_{pr} and T_{pr}, from Eqs. (2.12), (2.13).

Step 5. Read the compressibility factor from Fig. (2.10).

Example 2.8:

A sour natural gas has a specific gravity of (0.7). The compositional analysis of the gas shows that it contains 5% CO_2 and 10% H_2S. Calculate the density of the gas at (3,500 psia) and (160°F).

Solution:

Step 1. Calculate the uncorrected pseudo-critical properties of the gas:

$$T_{pc} = 168 + 325\ (0.7) - 12.5\ (0.7)^2 = 389.38°R$$

$$p_{pc} = 677 + 15\ (0.7) - 37.5\ (0.7)^2 = 669.1 \text{ psia}$$

Step 2. Calculate the pseudo-critical temperature adjustment factor:

$$\varepsilon = 120\ (0.15^{0.9} - 0.15^{1.6}) + 15\ (0.1^{0.5} - 0.1^4) = 20.735$$

Step 3. Calculate the corrected pseudo-critical temperature:

$$T'_{pc} = 389.38 - 20.735 = 368.64$$

Step 4. Adjust the pseudo-critical pressure p_{pc}:

$$p'_{pc} = \frac{(669.1)(368.64)}{(389.38) + 0.1(1 - 0.1)(20.635)} = 630.44$$

Step 5. Calculate p_{pr} and T_{pr}:

$$p_{pr} = \frac{3500}{630.44} = 5.55$$

$$T_{pr} = \frac{160 + 460}{368\ 64} = 1.68$$

Step 6. Determine the z-factor from chart of Fig. (2.10):

$$z = 0.89$$

Step 7. Calculate the apparent molecular weight of the gas:

$$M_a = 28.96 * (\gamma_g) = 28.96 * 0.7 = 20.27$$

Step 8. Solve for gas density:

$$\rho = \frac{pM_a}{ZTR} = \frac{(3500)(20.27)}{(0.89)(10.73)(620)} = 11.98\, Ib/ft^3$$

II- Carr-Kobayashi-Burrows's Correction Method:

Carr et al. (1954) proposed a simplified procedure to adjust the pseudocritical properties of natural gases when nonhydrocarbon components are present. The method can be used when the composition of the natural gas is not available. The proposed procedure is summarized in the following steps.

Step 1. Knowing the specific gravity of the natural gas, calculate the pseudocritical temperature and pressure by applying Eqs. (2.16), (2.17).

Step 2. Adjust the estimated pseudocritical properties by using the following two expressions:

$$T'_{pc} = T_{pc} - 80y_{CO_2} + 130y_{H_2S} - 250y_{N_2} \qquad eq.(2.23)$$

$$p'_{pc} = p_{pc} + 440y_{CO_2} + 600y_{H_2S} - 170y_{N_2} \qquad eq.(2.24)$$

where:

T'_{pc} = the adjusted pseudocritical temperature, °R.
T_{pc} = the unadjusted pseudocritical temperature, °R.
y_{CO_2} = mole fraction of CO_2.
y_{H_2S} = mole fraction of H_2S in the gas mixture.
y_{N_2} = mole fraction of nitrogen.
p'_{pc} = the adjusted pseudocritical pressure, psia.
p_{pc}= the unadjusted pseudocritical pressure, psia.

Step 3. Use the adjusted pseudocritical temperature and pressure to calculate the pseudoreduced properties.

Step 4. Calculate the Z-factor from Fig. (2.10).

Example 2.9:

Using the data in **Example (2.8)**, calculate the density by employing the preceding correction procedure.

Solution:

Step 1. After calculating the uncorrected T_{pr} and p_{pr} determine the corrected pseudocritical properties from Eqs. (2.23), (2.24):

$$T'_{pc} = 389.38 - 80(0.05) + 130(0.10) - 250(0) = 398.3°R$$

$$p'_{pc} = 669.1 + 440(0.05) + 600(0.01) - 170(0) = 751.1\, psia$$

Step 2. Calculate p_{pr} and T_{pr}:

$$p_{pr} = \frac{3500}{751.1} = 4.56$$

$$T_{pr} = \frac{620}{398.38} = 1.56$$

Step 3. Determine the gas compressibility factor from Fig. (2.10):

$$z = 0.820$$

Step 4. Calculate the gas density:

$$\rho_g = \frac{pM_a}{ZRT} = \frac{(3500)(20.27)}{(0.82)(10.73)(620)} = 13.0\ lb/ft^3$$

Note:

It should be noted that the Standing and Katz Z-factor chart result in unsatisfactory Z-factors for high-molecular weight reservoir gases heptanes-plus fraction, C_{7+}, which their gravity is $\gamma_g > 0.75$. Sutton (1985) and Stewart et al. (1959), developed a method of correction for high-molecular weight gases.

e- Direct Calculation of Z-factor (empirical correlations):

After four decades of existence, the Standing-Katz Z-factor chart is still widely used as a practical source of natural gas compressibility factors. As a result, there was an apparent need for a simple mathematical description of that chart. Several empirical correlations for calculating Z-factors have been developed over the years.

Numerous rigorous mathematical expressions have been proposed to accurately reproduce the Standing and Katz Z-factor chart. Most of these expressions are designed to solve for the gas compressibility factor at any p_{pr} and T_{pr} iteratively. Three of these methods are:

- Hall and Yarborough method (1973).
- Dranchuk, Purvis and Robinson (1974).
- Dranchuk and Abu-Kassem (1975).

For more details see (Ahmed, 2019)

2.1.7 Isothermal Gas Compressibility, c_g:

The coefficient of isothermal compressibility is defined as the fractional change of volume as pressure is changed at constant temperature. The defining equation is:

$$c_g = \frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad eq.(2.25)$$

where, c_g = isothermal gas compressibility, 1/psi.

Negative sign in the equation above refers to volume decreasing with pressure increasing.

This coefficient normally is referred to simply as compressibility or gas compressibility, whereas, the term compressibility factor refers to z-factor.

1- Isothermal compressibility factor for **ideal gas** given by driving the ideal gas law at constant temperature:

$$pV = nRT; \qquad T = constant$$

$$p\partial V + V\partial p = 0$$

$$p\partial V = -V\partial p \implies \frac{\partial V}{\partial p} = \frac{-V}{p} \qquad eq.(2.26)$$

by substituting into Eq. (2.25):

$$c_g = \frac{-1}{V}\left(\frac{-V}{p}\right) \implies c_g = \frac{1}{p}\ (psi^{-1}) \qquad eq.(2.27)$$

2- Isothermal compressibility factor for **real gas** given by driving the real gas law at constant temperature:

$$pV = ZnRT; \qquad T = constant$$

$$p\partial V + V\partial p = nRT\partial Z$$

by dividing the equation above by (∂p):

$$p\frac{\partial V}{\partial p}+V\frac{\partial p}{\partial p}=nRT\frac{\partial Z}{\partial p}$$

$$p\frac{\partial V}{\partial p}=nRT\frac{\partial Z}{\partial p}-V$$

$$\frac{\partial V}{\partial p}=\frac{nRT}{p}\frac{\partial Z}{\partial p}-\frac{V}{p} \qquad eq.(2.28)$$

by substituting (*V=ZnRT/p*) from real gas law and Eq.(2.28) into Eq. (2.25):

$$c_g=\frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

$$c_g=\frac{-p}{ZnRT}\left(\frac{nRT}{p}\frac{\partial Z}{\partial p}-\frac{V}{p}\right)_T$$

$$c_g=\frac{-p}{ZnRT}\frac{nRT}{p}\frac{\partial Z}{\partial p}-\frac{-p}{ZnRT}\frac{V}{p}$$

$$c_g=\frac{V}{ZnRT}-\frac{1}{Z}\frac{\partial Z}{\partial p}$$

By by substituting (*V=ZnRT/p*) from real gas law in the last equation:

$$c_g=\frac{ZnRT}{pZnRT}-\frac{1}{Z}\frac{\partial Z}{\partial p}$$

$$c_g=\frac{1}{p}-\frac{1}{Z}\frac{\partial Z}{\partial p} \qquad eq.(2.29)$$

Equation (2.29) is the isothermal compressibility factor for real gas.

From the law of corresponding state, we know that:

$$p_{pr}=\frac{p}{p_{pc}} \Rightarrow p=p_{pr}p_{pc}$$

By substituting $p=p_{pr}p_{pc}$ from the law of corresponding state into Eq. (2.29):

$$c_g=\frac{1}{p_{pr}p_{pc}}-\frac{1}{z}\left[\frac{\partial Z}{\partial\left(p_{pr}p_{pc}\right)}\right]_{T_{pr}}$$

Multiplying this equation by p_{pc} yields:

$$c_g p_{pc} = c_{pr} = \frac{1}{p_{pr}} - \frac{1}{Z}\left[\frac{\partial Z}{\partial p_{pr}}\right]_{T_{pr}} \qquad eq.(2.30)$$

The term c_{pr} is called the isothermal pseudoreduced compressibility, defined by the relationship:

$$c_{pr} = c_g p_{pc} \qquad eq.(2.31)$$

where:

c_{pr} = isothermal pseudoreduced compressibility.
c_g = isothermal gas compressibility, psi^{-1}.
p_{pc} = pseudoreduced pressure, psi.

Trube (1957a,b) presented graphs from which the isothermal compressibility of natural gases may be obtained. The graphs, Fig. (2.15), (2.16) give the isothermal pseudoreduced compressibility as a function of pseudoreduced pressure and temperature.

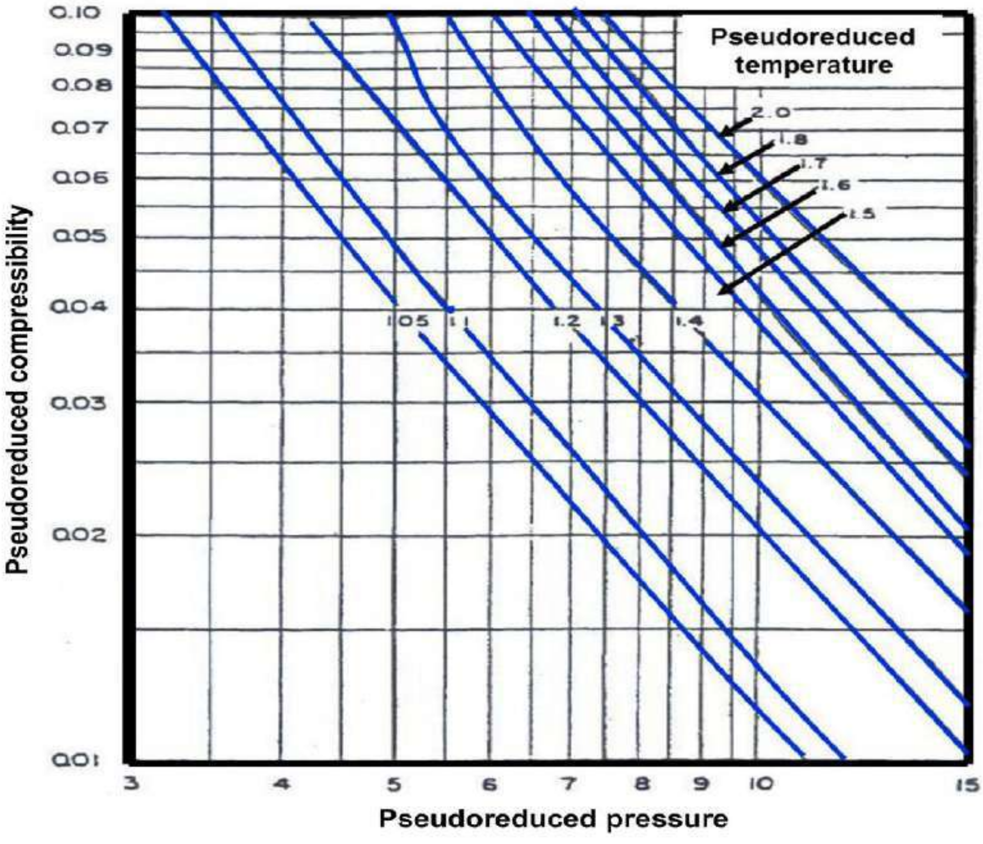


Fig. (2.15): Trube's pseudoreduced compressibility for natural gases. (from Trube, A.S., 1957b. Compressibility of natural gases. Trans. AIME 210, 355–357).

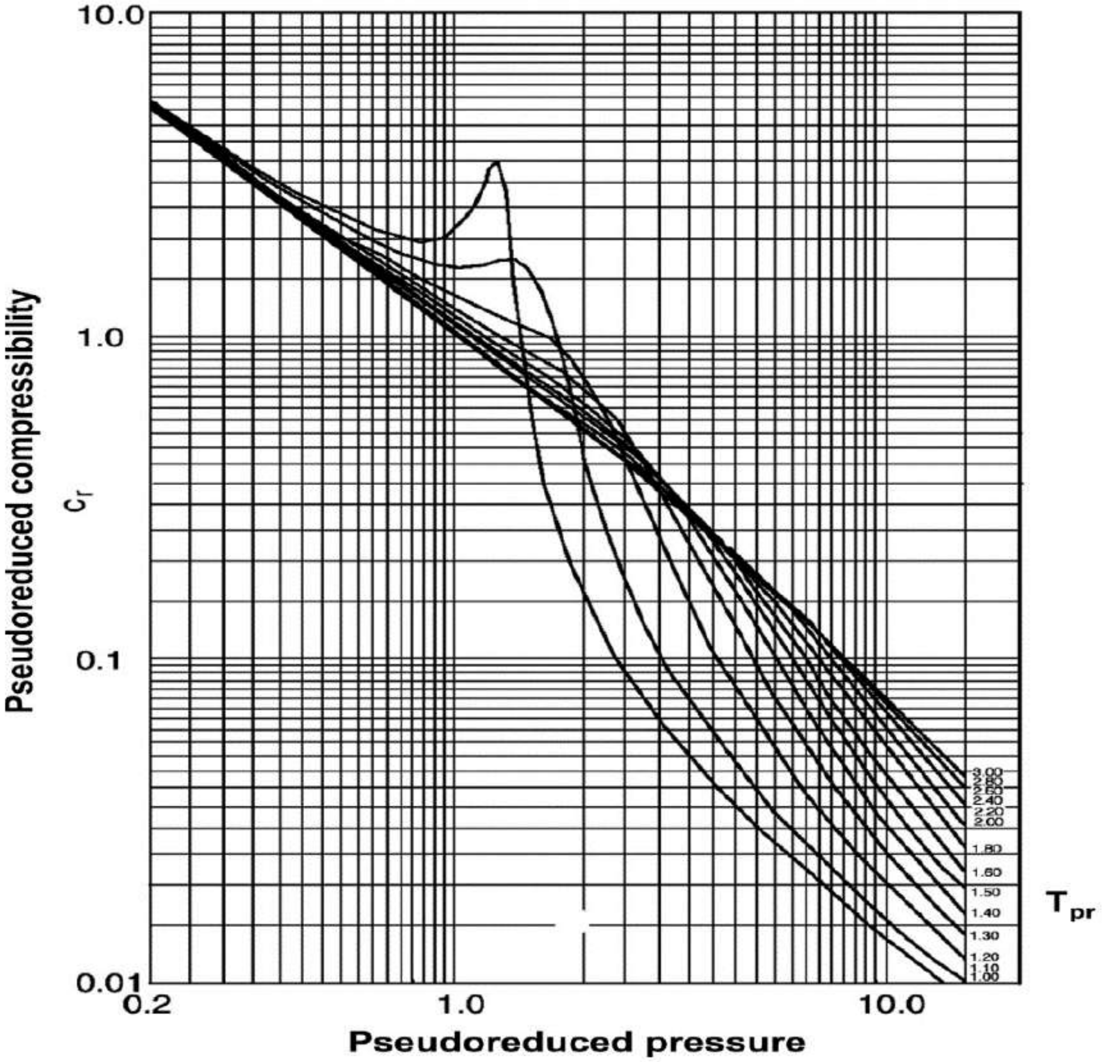


Fig. (2.16): Trube's pseudoreduced compressibility for natural gases, (from Trube, A.S., 1957b. Compressibility of natural gases. Trans. AIME 210, 355–357).

Example 2.10:

A hydrocarbon gas mixture has a specific gravity of (0.72). Calculate the isothermal gas compressibility coefficient at (2000 psia) and (140°F) assuming, first, an ideal gas behavior, then a real gas behavior.

Solution:

Assuming an **ideal gas** behavior, determine c_g by applying Eq. (2.27):

$$c_g = \frac{1}{2000} = 500 \times 10^{-6}\, psi^{-1}$$

Assuming a **real gas** behavior, use the following steps.

Step 1. Calculate T_{pc} and p_{pc} by applying Eqs. (2.16), (2.17):

$$T_{pc} = 168 + 325(0.72) - 12.5(0.72)^2 = 395.5°R$$

$$p_{pc} = 677 + 15(0.72) - 37.5(0.72)^2 = 668.4\,psia$$

Step 2. Compute p_{pr} and T_{pr} from Eqs. (2.14), (2.15):

$$p_{pr} = \frac{2000}{668.4} = 2.99$$

$$T_{pr} = \frac{600}{395.5} = 1.52$$

Step 3. From Fig. (2.15), find c_{pr} to give (c_{pr} = 0.36).

Step 4. Solve for c_g by applying Eq. (2.31):

$$c_g = \frac{c_{pr}}{p_{pc}} = \frac{0.36}{668.4} = 539 \times 10^{-6} psi^{-1}$$

2.1.8 Gas Formation Volume Factor, B_g:

The gas formation volume factor is used to relate the volume of gas, as measured at reservoir conditions, to the volume of the gas as measured at standard conditions; that is, 60°F and 14.7 psia. This gas property is then defined as the actual volume occupied by a certain amount of gas at a specified pressure and temperature, divided by the volume occupied by the same amount of gas at standard conditions. In equation form, the relationship is expressed as:

$$B_g = \frac{(V)_{p,T}}{V_{sc}} \qquad eq.(2.32)$$

where:

B_g = gas formation volume factor, *ft*^{3}*/scf.*
$V_{p,T}$ = volume of gas at pressure p and temperature T, *ft*^{3}.
V_{sc} = volume of gas at standard conditions.

Applying the real gas equation of state, (pV=ZnRT) and substituting for the volume V, gives:

$$B_g = \frac{\frac{ZnRT}{p}}{\frac{Z_{sc}nRT_{sc}}{p_{sc}}} = \frac{p_{sc}ZT}{T_{sc}p}$$

where:

Z_{sc} = Z-factor at standard conditions is equal to (1.0).
p_{sc}, T_{sc} = standard pressure and temperature.

Assuming that the standard conditions are represented by p_{sc}=14.7 psia and T_{sc}=520, the preceding expression can be reduced to the following relationship:

$$B_g = 0.02827\frac{ZT}{p}\ (ft^3/scf) \qquad eq.(2.33)$$

where:

B_g = gas formation volume factor, ft^3/scf.
Z = gas compressibility factor.
T = temperature, °R.

In other field units, the gas formation volume factor can be expressed in *bbl/scf*, by dividing it on 5.614 ft^3/ bbl to give:

$$B_g = 0.005035\frac{ZT}{p}\ (bbl/scf) \qquad eq.(2.34)$$

And also:

$$c_g = \frac{-1}{B_g}\left(\frac{\partial B_g}{\partial p}\right)_T \qquad eq.(2.35)$$

2.1.9 Gas Expansion Factor, E_g:

The reciprocal of the gas formation volume factor, called the *gas expansion factor*, is designated by the symbol E_g:

$$E_g = \frac{1}{B_g}$$

In terms of scf/ft^3, the gas expansion factor is:

$$E_g = 35.37\frac{p}{ZT}, scf/ft^3 \qquad eq.(2.36)$$

In other units:

$$E_g = 198.6\frac{p}{ZT}, scf/bbl \qquad eq.(2.37)$$

Example 2.11:

A gas well is producing from a gas reservoir at an average pressure of (2000 psia) and a temperature of (140°F). The specific gravity is (0.72). Calculate the gas formation volume factor and gas expansion factor in *scf/ft*^{3} and *scf/bbl*.

Solution:

Step 1. Calculate the pseudocritical properties from Eqs. (2.16), (2.17), to give:

$$T_{pc} = 168 + 325(0.72) - 12.5(0.72)^2 = 395.5\ °R$$

$$p_{pc} = 677 + 15(0.72) - 37.5(0.72)^2 = 668.4\, psia$$

Step 2. Calculate the p_{pr} and T_{pr}:

$$p_{pr} = \frac{p}{p_{pc}} = \frac{2000}{668.4} = 2.99$$

$$T_{pr} = \frac{T}{T_{pc}} = \frac{600}{395.5} = 1.52$$

Step 3. Determine the Z-factor from Fig. (2.10):

$$Z = 0.78$$

Step 4. Calculate the gas formation volume factor from Eqs. (2.33), (2.34):

$$B_g = 0.02827\frac{(0.78)(140 + 460)}{(2000)} = 0.006615\ (ft^3/scf)$$

$$B_g = 0.005035\frac{(0.78)(140 + 460)}{(2000)} = 0.001178\ (bbl/scf)$$

Step 5. Calculate the gas expansion factor from Eqs. (2.36), (2.37):

$$E_g = 35.37\frac{(2000)}{(0.78)(600)} = 151.15\ scf/ft^3$$

$$E_g = 198.6\frac{(2000)}{(0.78)(600)} = 848.69\ scf/bbl$$

2.1.10 Gas Viscosity, μ_g:

The viscosity of a fluid is a measure of the internal fluid friction (resistance) to flow. If the friction between layers of the fluid is small, that is, low viscosity, an applied shearing force will result in a large velocity gradient. As the viscosity increases, each fluid layer exerts a larger frictional drag on the adjacent layers and velocity gradient decreases.

The viscosity of a fluid is generally defined as the ratio of the shear force per unit area to the local velocity gradient. Viscosities are expressed in terms of poises, centipoises, or micropoises. One poise equals a viscosity of 1 dyn-s/cm^2 and can be converted to other field units by the following relationships:

1 centipoise = g mass / 100 sec cm

$$\begin{aligned} 1\,poise &= 100\,centipoises \\ &= 1\times 10^{6}\,micropoises \\ &= 6.72\times 10^{-2}\ (lb.mass/ft.sec) \\ &= 20.9\times 10^{-3}\ (lb.sec/ft^{2}) \end{aligned}$$

The gas viscosity is not commonly measured in the laboratory because it can be estimated precisely from empirical correlations. Like all intensive properties, viscosity of a natural gas is completely described by the following function:

$$\mu_g = (p, T, y_i)$$

where, μ_g = the viscosity of the gas phase. This relationship simply states that the viscosity is a function of pressure, temperature, and composition. Many of the widely used gas viscosity correlations may be viewed as modifications of that expression.

There are two popular methods are commonly used to calculate the viscosity of natural gases in the petroleum industry:

1- Carr-Kobayashi-Burrows correlation.
2- Lee-Gonzalez-Eakin method.

1- Carr-Kobayashi-Burrows's Method:

Carr et al. (1954) developed graphical correlations for estimating the viscosity of natural gas as a function of temperature, pressure, and gas gravity. The computational procedure of applying the proposed correlations is summarized in the following steps:

Step 1. Calculate the pseudocritical pressure, pseudocritical temperature, and apparent molecular weight from the specific gravity or the composition of the natural gas. Corrections to these pseudocritical properties for the presence of the nonhydrocarbon gases (CO_2, N_2, and H_2S) should be made if they are present in concentration greater than 5 mol%.

Step 2. Obtain the viscosity of the natural gas at one atmosphere and the temperature of interest from Fig. (2.17). This viscosity, as denoted by μ_1, must be corrected for the presence of nonhydrocarbon components using the inserts of Fig. (2.17). The nonhydrocarbon fractions tend to increase the viscosity of the gas phase. The effect of nonhydrocarbon components on the viscosity of the natural gas can be expressed mathematically by the following relationship:

$$\mu_1 = (\mu_1)_{uncorrected} + (\Delta\mu)_{N_2} + (\Delta\mu)_{CO_2} + (\Delta\mu)_{H_2S} \qquad eq.(2.38)$$

where:

μ_1 = “corrected” gas viscosity at 1 atm and reservoir temperature, cp.
$(\Delta\mu)_{N_2}$ = viscosity corrections due to the presence of N_2.
$(\Delta\mu)_{CO_2}$= viscosity corrections due to the presence of CO_2.
$(\Delta\mu)_{H_2S}$ = viscosity corrections due to the presence of H_2S.
$(\mu_1)_{\text{uncorrected}}$ = uncorrected gas viscosity, cp.

Step 3. Calculate the pseudoreduced pressure and temperature.

Step 4. From the pseudoreduced temperature and pressure, obtain the viscosity ratio (μ_g/μ_1) from Fig. (2.18). The term μ_g represents the viscosity of the gas at the required conditions.

Step 5. The gas viscosity, μ_g, at the pressure and temperature of interest, is calculated by multiplying the viscosity at 1 atm and system temperature, μ_1, by the viscosity ratio. The following examples illustrate the use of the proposed graphical correlations.

Example 2.12:

A gas well is producing from a gas reservoir at an average pressure of (2000 psia) and a temperature of (120°F). The specific gravity is (0.72). Calculate the gas viscosity.

Solution:

Step 1. Calculate the apparent molecular weight of the gas:

$$M_a = (0.72)(28.96) = 20.85$$

Step 2. Determine the viscosity of the gas at 1 atm and 140°F from Fig. (2.17):

$$\mu_1 = 0.0113$$

Step 3. Calculate p_{pr} and T_{pr}:

$$p_{pr} = 2.99$$

$$T_{pr} = 1.52$$

Step 4. Determine the viscosity rates from Fig. (2.18):

$$\frac{\mu_g}{\mu_1} = 1.5$$

Step 5. Solve for the viscosity of the natural gas:

$$\mu_g = \frac{\mu_g}{\mu_1}(\mu_1) = (1.5)(0.0113) = 0.01695cp$$

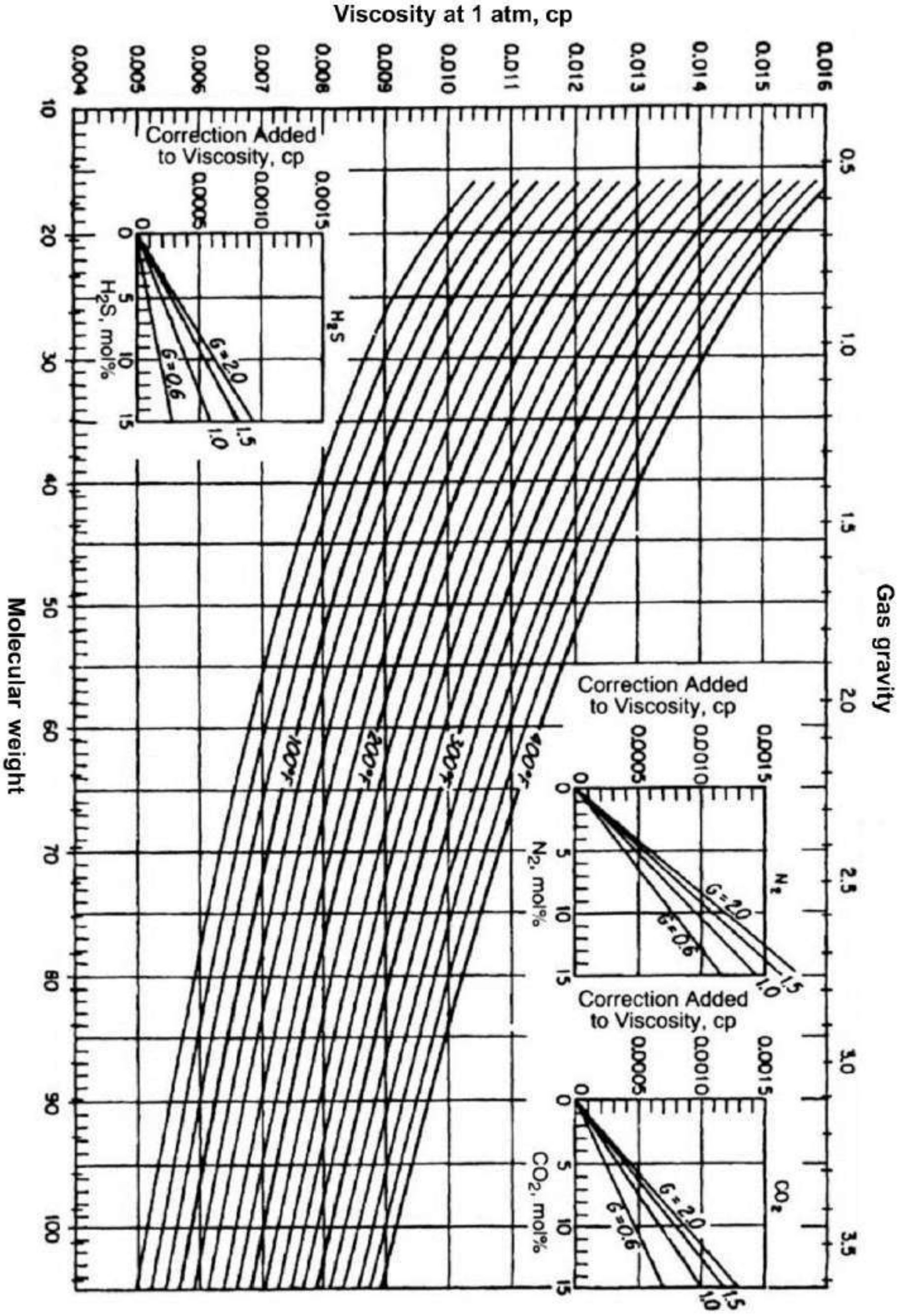


Fig. (2.17): Carr et al.'s atmospheric gas viscosity correlation. (Carr, N., Kobayashi, R., Burrows,D., 1954. Viscosity of hydrocarbon gases under pressure. Trans. AIME 201, 270–275).

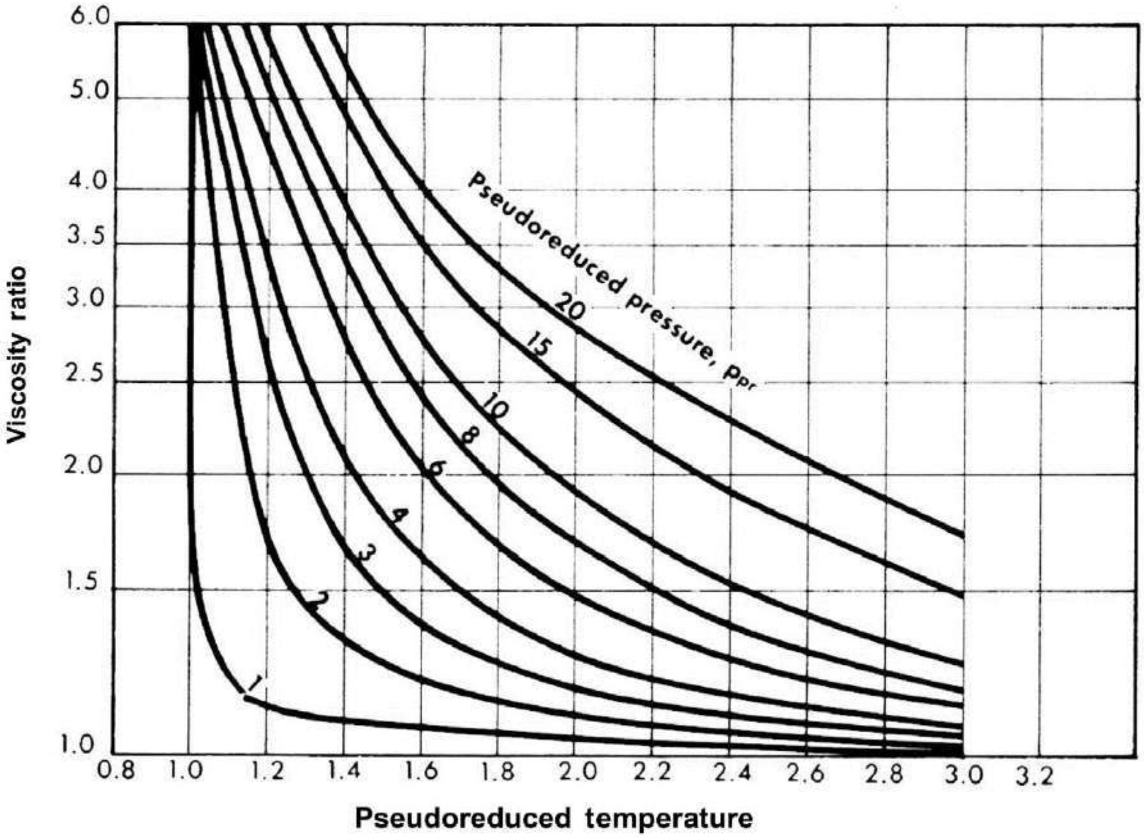


Fig. (2.18): Carr et al.'s viscosity ratio correlation. (Carr, N., Kobayashi, R., Burrows, D., 1954. Viscosity of hydrocarbon gases under pressure, Trans. AIME).

Standing (1977) proposed a convenient mathematical expression for calculating the viscosity of the natural gas at atmospheric pressure and reservoir temperature, μ_1. Standing also presented equations for describing the effects of N_2, CO_2, and H_2S on μ_1. The proposed relationships are:

$$\mu_1 = (\mu_1)_{uncorrected} + (\Delta\mu)_{N_2} + (\Delta\mu)_{CO_2} + (\Delta\mu)_{H_2S}$$

For details see (Ahmed T., 2019, p.66).

2- Lee-Gonzalez-Eakin's Method:

Lee et al. (1966) presented a semiempirical relationship for calculating the viscosity of natural gases. The authors expressed the gas viscosity in terms of the reservoir temperature, gas density, and the molecular weight of the gas. Their proposed equation is given by:

$$\mu_g = 10^{-4} K exp\left[X\left(\frac{\rho_g}{62.4}\right)^Y\right] \qquad eq.(2.39)$$

$$K = \frac{(9.4 + 0.02M_a)T^{1.5}}{209 + 19M_a + T} \qquad eq.(2.40)$$

$$X = 3.5 + \frac{986}{T} + 0.01M_a \qquad eq.(2.41)$$

$$Y = 2.4 - 0.2X \qquad eq.(2.42)$$

where:

ρ = gas density at reservoir pressure and temperature, *lb/ft*^{3}.
T = reservoir temperature, °R.
M_a = apparent molecular weight of the gas mixture.

The proposed correlation can predict viscosity values with a standard deviation of 2.7% and a maximum deviation of 8.99%. The correlation is less accurate for gases with higher specific gravities. The authors pointed out that the method cannot be used for sour gases.

Example 2.11:

Rework **Example (2.10)** and calculate the gas viscosity by using the Lee Gonzales-Eakin method.

Solution:

Step 1. Calculate the gas density from Eq. (2.6):

$$\rho_g = \frac{pM_a}{ZRT} = \frac{(2000)(20.85)}{(10.73)(600)(0.78)} = 8.3\ lb/ft^3$$

Step 2. Solve for the parameters K, X, and Y using Eqs. (2.40), (2.41), (2.42), respectively:

$$K = \frac{(9.4 + 0.02M_a)T^{1.5}}{209 + 19M_a + T}$$

$$K = \frac{[9.4 + (0.02)(20.85)](600)^{1.5}}{209 + (19)(20.85) + 600} = 119.72$$

$$X = 3.5 + \frac{986}{T} + 0.01M_a$$

$$X = 3.5 + \frac{986}{600} + (0.01)(20.85) = 5.35$$

$$Y = 2.4 - 0.2X$$

$$Y = 2.4 - (0.2)(5.35) = 1.33$$

Step 3. Calculate the viscosity from Eq. (2.39):

$$\mu_g = 10^{-4} K exp\left[X\left(\frac{\rho_g}{62.4}\right)^Y\right]$$

$$\mu_g = 10^{-4}(119.72)exp\left[5.35\left(\frac{8.3}{62.4}\right)^{1.33}\right] = 0.0173cp$$

2.2 Crude Oil Properties:

Petroleum (an equivalent term is crude oil) is a complex mixture consisting predominantly of hydrocarbons and containing sulfur, nitrogen, oxygen, and helium as minor constituents. The physical and chemical properties of crude oils vary considerably and depend on the concentration of the various types of hydrocarbons and minor constituents present.

An accurate description of physical properties of crude oils is of a considerable importance in the fields of both applied and theoretical science and especially in the solution of petroleum reservoir engineering problems. Physical properties of primary interest in petroleum engineering studies include:

1. Crude oil gravity "γ_o".
2. Specific gravity of solution gas "γ_g".
3. Crude oil density "ρ_o".
4. Gas solubility “R_s”.
5. Bubble point pressure “p_b”.
6. Oil formation volume factor “B_o”.
7. Isothermal compressibility coefficient of crude oil “c_o”.
8. Total formation volume factor “B_t”.
9. Crude oil viscosity.

Data on most of these fluid properties is usually determined by laboratory experiments performed on samples of actual reservoir fluids. In the absence of experimentally measured properties of crude oils, it is necessary for the petroleum engineer to determine the properties from empirically derived correlations.

2.2.1 Crude Oil API Gravity:

The crude oil density is defined as the mass of a unit volume of the crude at a specified pressure and temperature. It is usually expressed in pounds per cubic foot. The specific gravity of a crude oil is defined as the ratio of the density of the oil to that of water. Both densities are measured at 60°F and atmospheric pressure:

$$\gamma_o = \frac{\rho_o}{\rho_w} \qquad eq. (2.43)$$

where:

γ_o = specific gravity of the oil.
ρ_o = density of the crude oil, *lb/ft³*.
ρ_w = density of the water, *lb/ft³*.

It should be pointed out that the liquid specific gravity is dimensionless but traditionally is given the units 60°/60° to emphasize that both densities are measured at standard conditions (SC). The density of the water is approximately (62.4 *lb/ft³)*, or:

$$\gamma_o = \frac{\rho_o}{62.4}, 60°/60°$$

Although the density and specific gravity are used extensively in the petroleum industry, the API gravity is the preferred gravity scale. This gravity scale is precisely related to the specific gravity by the following expression:

$$API = \frac{141.5}{\gamma_o} - 131.5 \qquad eq. (2.44)$$

The API gravities of crude oils usually range from 47°API for the lighter crude oils to 10°API for the heavier asphaltic crude oils.

Example 2.12:

Calculate the specific gravity and the API gravity of a crude oil system with a measured density of *(53 lb/ft³)* at SC.

Solution:

Step 1. Calculate the specific gravity from Eq. (2.43):

$$\gamma_o = \frac{\rho_o}{\rho_w} = \frac{53}{62.4} = 0.849$$

Step 2. Solve for the API gravity from Eq. (2.44):

$$API = \frac{141.5}{0.849} - 131.5 = 35.2°API$$

2.2.2 Specific Gravity of the Solution Gas, "γ_g":

The specific gravity of the solution gas, γ_g, is described by the weighted average of the specific gravities of the separated gas from each separator. This weighted average approach is based on the separator gas-oil ratio (GOR):

$$\gamma_g = \frac{\sum_{i=1}^{n}(R_{sep})_i(\gamma_{sep})_i + R_{st}\gamma_{st}}{\sum_{i=1}^{n}(R_{sep})_i + R_{st}} \qquad eq. (2.45)$$

where:

n = number of separators.
R_{sep} = separator GOR, scf/STB.
γ_{sep} = separator gas gravity.
R_{st} = GOR from the stock-tank, scf/STB.
γ_{st} = gas gravity from the stock-tank.

Example 2.13:

Separator tests were conducted on a crude oil sample. Results of the test in terms of the separator GOR and specific gravity of the separated gas are given in the table below. Calculate the specific gravity of the separated gas.

Separator	Pressure (psig)	Temperature (°F)	Gas-Oil Ratio (scf/STB)	Gas Specific Gravity
Primary	660	150	724	0.743
Intermediate	75	110	202	0.956
Stock-tank	0	60	58	1.296

Solution:

Calculate the solution specific gravity by using Eq. (2.45):

$$\gamma_g = \frac{\sum_{i=1}^{n}\left(R_{sep}\right)_i\left(\gamma_{sep}\right)_i + R_{st}\gamma_{st}}{\sum_{i=1}^{n}\left(R_{sep}\right)_i + R_{st}}$$

$$\gamma_g = \frac{(724)(0.743) + (202)(0.965) + (58)(1.296)}{724 + 202 + 58} = 0.819$$

2.2.3 Crude Oil Density, "ρ_o":

The crude oil density is defined as the mass of a unit volume of the crude at a specified pressure and temperature, mass/volume. The density usually is expressed in pounds per cubic foot and it varies from 30 lb/ft^3 for light volatile oil to 60 lb/ft^3 for heavy crude oil with little or no gas solubility. It is one of the most important oil properties, because its value substantially affects crude oil volume calculations. This vital oil property is traditionally measured in the laboratory as part of routine pressure-volume-temperature (PVT) tests. When laboratory crude oil density measurement is not available, correlations can be used to generate the required density data under reservoir pressure and temperature.

Numerous empirical correlations for calculating the density of liquids have been proposed over the years. Based on the available limited measured data on the crude, the correlations can be divided into the following two categories:

- Correlations that use the crude oil composition to determine the saturated oil density and temperature.
- Correlations that use limited PVT data, such as gas gravity, oil gravity, and GOR, as correlating parameters.

I. Density Correlations Based on the Oil Composition:

Several reliable methods are available for determining the density of saturated crude oil mixtures from their compositions. The best known and most widely used calculation methods are the following two:

- Standing-Katz (1942).
- Alani-Kennedy (1960).

These two methods were developed to calculate the saturated oil density.

II. Density Correlations Based on Limited PVT Data:

Several empirical correlations for calculating the density of liquids of unknown compositional analysis have been proposed. The correlations employ limited PVT data such as gas gravity, oil gravity, and gas solubility as correlating parameters to estimate liquid density at the prevailing reservoir pressure and temperature. Two methods are presented as representatives of this category:

- Katz’s method (1942).
- Standing’s method (1981).

III. Standing's Method for Determining "ρ_o":

Standing (1981) proposed an empirical correlation for estimating the oil FVF as a function of the gas solubility, R_s; the specific gravity of stock-tank oil, γ_o; the specific gravity of solution gas, γ_g; and the system temperature, T. By coupling the mathematical definition of the oil FVF (as discussed in a later section) with Standing's correlation, the density of a crude oil at a specified pressure and temperature can be calculated from the following expression:

$$\rho_o = \frac{62.4\gamma_o + 0.0136 R_s \gamma_g}{Bo}$$

$$\rho_o = \frac{62.4\gamma_o + 0.0136 R_s \gamma_g}{0.9759 + 0.000120\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25(T - 460)\right]^{1.2}} \qquad eq.(2.46)$$

where:

T = system temperature, °R.
γ_o = specific gravity of the stock-tank oil, 60°/60°.
γ_g = specific gravity of the gas.
R_s = gas solubility, scf/STB.
ρ_o = oil density, lb/ft^3.

Bo=oil formation volume factor, bbl/STB

2.2.4 Gas Solubility "R_s":

The gas solubility, R_s, is defined as the number of standard cubic feet of gas that dissolved in one stock-tank barrel of crude oil at certain pressure and temperature. The solubility of a natural gas in a crude oil is a strong function of the pressure, the temperature, the API gravity, and the gas gravity.

For a particular gas and crude oil to exist at a constant temperature, the solubility increases with pressure until the saturation pressure is reached. At the saturation pressure (bubble point pressure), all the available gases are dissolved in the oil and the gas solubility reaches its maximum value. Rather than measuring the amount of gas that dissolves in a given stock-tank crude oil as the pressure is increased, it is customary to determine the amount of gas that comes out of a sample of reservoir crude oil as pressure decreases.

A typical gas solubility curve, as a function of pressure for an undersaturated crude oil, is shown in Fig. (2.19). As the pressure is reduced from the initial reservoir pressure, p_i, to the bubble point pressure, p_b, no gas evolves from the oil and consequently the gas solubility remains constant at its maximum value of R_{sb}. Below the bubble point pressure, the solution gas is liberated and the value of R_s decreases with pressure.

In the absence of experimentally measured gas solubility on a crude oil system, it is necessary to determine this property from empirically derived correlations. Five empirical correlations for estimating the gas solubility are:

- Standing's correlation.
- Vasquez-Beggs's correlation.
- Glaso's correlation.
- Marhoun's correlation.
- Petrosky-Farshad's correlation.

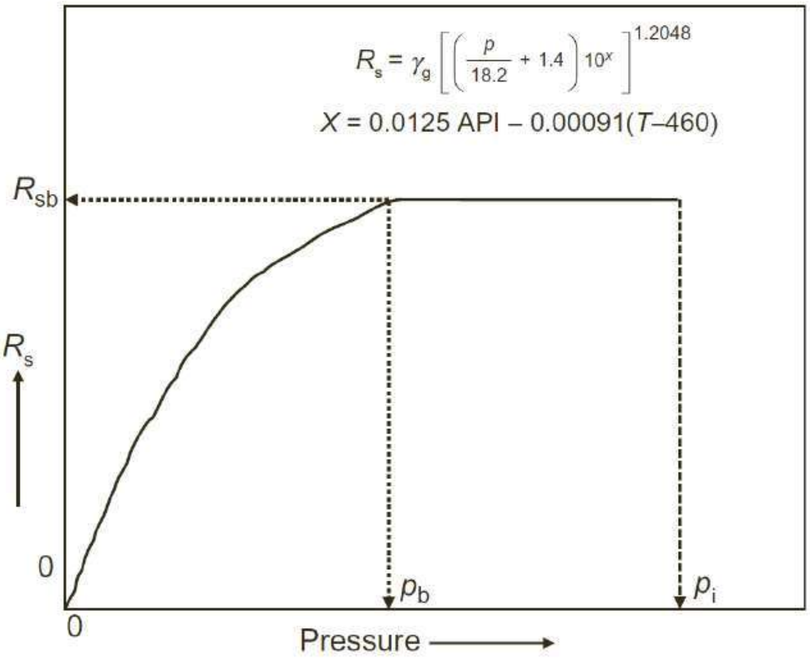


Fig. (2.19): Typical gas solubility-pressure relationship.

I. Standing's Correlation for Determining "R_s":

Standing (1947) proposed a graphical correlation for determining the gas solubility as a function of pressure, gas specific gravity, API gravity, and system temperature. The correlation was developed from a total of 105 experimentally determined data points on 22 hydrocarbon mixtures from California crude oils and natural gases. The proposed correlation has an average error of 4.8%. In a mathematical form, Standing (1981) expressed his proposed graphical correlation in the following more convenient mathematical form:

$$R_s = \gamma_g \left[\left(\frac{p}{18.2} + 1.4\right) 10^x\right]^{1.2048} \qquad eq.(2.47)$$

with:

$$x = 0.0125API - 0.00091(T - 460) \qquad eq.(2.48)$$

where:

R_s = gas solubility, scf/STB.
T = temperature, °R.
p = system pressure, psia.
γ_g = solution gas specific gravity.
API = oil gravity, °API.

It should be noted that Standing's equation is valid for applications at and below the bubble point pressure of the crude oil.

2.2.5 Bubble Point Pressure "p_b":

The bubble point pressure, p_b, of a hydrocarbon system is defined as the highest pressure at which a bubble of gas is first liberated from the oil. This important property can be measured experimentally for a crude oil system by conducting a constant composition expansion (CCE) test.

In the absence of the experimentally measured bubble point pressure, it is necessary for the engineer to make an estimate of this crude oil property from the readily available measured producing parameters. Several graphical and mathematical correlations for determining p_b have been proposed during the last four decades. These correlations are essentially based on the assumption that the bubble point pressure is a strong function of gas solubility, R_s; gas gravity, γ_g; oil gravity, API; and temperature "T":

$$p_b = f(R_s, \gamma_g, API, T)$$

Techniques of combining these parameters in a graphical form or a mathematical expression were proposed by several authors, including:

- Standing.
- Vasquez and Beggs.
- Glaso.
- Marhoun.
- Petrosky and Farshad.

I. Standing’s Correlation for Determining “p_b”:

Based on 105 experimentally measured bubble point pressures on 22 hydrocarbon systems from California oil fields, Standing (1947) proposed a graphical correlation for determining the bubble point pressure of crude oil systems. The correlating parameters in the proposed correlation are the gas solubility, R_s; gas gravity, γ_g; oil API gravity; and the system temperature. The reported average error is 4.8%.

In a mathematical form, Standing (1981) expressed the graphical correlation by the following expression:

$$p_b = 18.2\left[\left(R_s/\gamma_g\right)^{0.83}(10)^a - 1.4\right] \qquad eq.(2.49)$$

with:

$$a = 0.00091(T-460) - 0.0125(API) \qquad eq.(2.50)$$

where:

R_s = gas solubility, scf/STB.
p_b = bubble point pressure, psia.
T = system temperature, °R.

Standing’s correlation should be used with caution if nonhydrocarbon components are known to be present in the system.

2.2.6 Oil Formation Volume Factor “B_o”:

The oil FVF, B_o, is defined as the ratio of the volume of oil (plus the gas in solution) at the reservoir temperature and pressure to the volume of oil at SC. Evidently, Bo always is greater than or equal to unity. The oil FVF can be expressed mathematically as:

$$B_o = \frac{(V_o)_{p,T}}{(V_o)_{sc}} \qquad eq.(2.51)$$

where:

B_o = oil FVF, bbl/STB.

$(V_o)_{p,T}$ = volume of oil, in *bbl*, under reservoir pressure, *p*, and temperature, *T*.

$(V_o)_{sc}$ = volume of oil is measured under SC, STB.

A typical oil formation factor curve, as a function of pressure for an undersaturated crude oil ($p_i > p_b$), is shown in Fig. (2.20). As the pressure is reduced below the initial reservoir pressure, p_i, the oil volume increases due to the oil expansion. This behavior results in an increase in the oil FVF and continues until the bubble point pressure is reached. At p_b, the oil reaches its maximum expansion and consequently attains a maximum value of B_{ob} for the oil FVF. As the pressure is reduced below p_b, volume of the oil and B_o are decreased as the solution gas is liberated. When the pressure is reduced to atmospheric pressure and the temperature to 60°F, the value of B_o is equal to 1.

Most of the published empirical B_o correlations utilize the following generalized relationship:

$$B_o = f\left(R_s, \gamma_g, \gamma_o, T\right)$$

Six methods of predicting the oil FVF are presented, including:

- Standing's correlation
- Vasquez and Beggs's correlation
- Glaso's correlation
- Marhoun's correlation
- Petrosky and Farshad's correlation
- The material balance equation

It should be noted that all the correlations could be used for any pressure equal to or below the bubble point pressure.

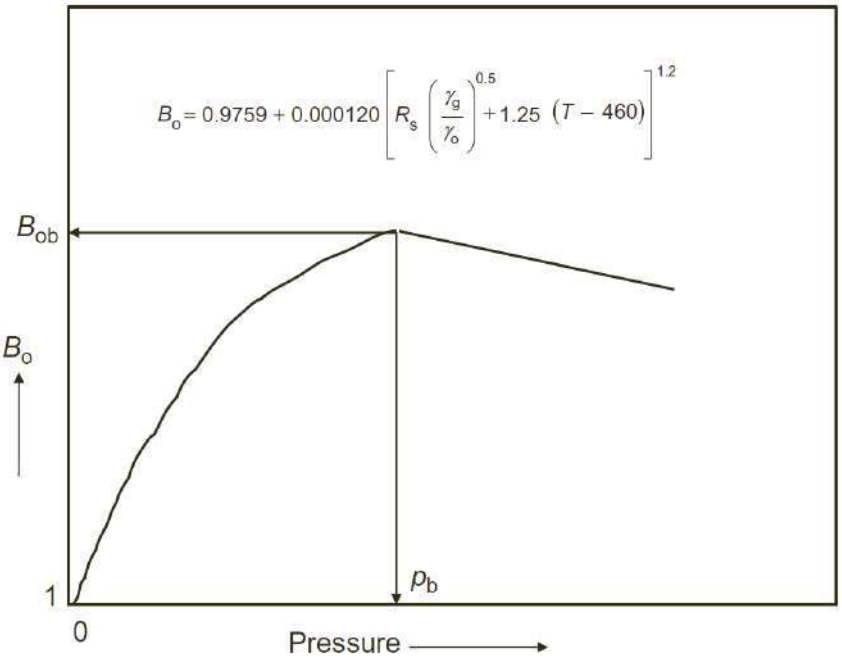


Fig. (2.20): Typical oil formation volume factor-pressure relationship.

I. Standing's Correlation for Determining "B_o":

Standing (1947) presented a graphical correlation for estimating the oil FVF with the gas solubility, gas gravity, oil gravity, and reservoir temperature as the correlating parameters. This graphical correlation originated from examining 105 experimental data points on 22 California hydrocarbon systems. An average error of 1.2% was reported for the correlation.

Standing (1981) showed that the oil FVF can be expressed more conveniently in a mathematical form by the following equation:

$$B_o = 0.9759 + 0.000120\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25(T - 460)\right]^{1.2} \qquad eq.(2.52)$$

where:

T = temperature, °R.
γ_o = specific gravity of the stock-tank oil, 60°/60°.
γ_g = specific gravity of the solution gas.

2.2.7 Isothermal Compressibility Coefficient of Crude Oil "c_o":

The isothermal compressibility coefficient is defined as the rate of change in volume with respect to pressure increase per unit volume, all variables other than pressure being constant, including temperature. Mathematically, the isothermal compressibility, c, of a substance is defined by the following expression:

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

$$c_o = -\frac{1}{V}\left(\frac{V_1 - V_2}{p_1 - p_2}\right)_T$$

For example, table (2.2) shows the variation in the volume of a reservoir fluid relative to the volume at the bubble point 2695 *psig* (V_r) as measured in the laboratory.

Table2.2: Relative volume data

Pressure (*psig*)	Relative volume factor (V_r)
5000	0.9739
4700	0.9768
4400	0.9799

4100	0.9829
3800	0.9862
3000	0.996
2800	0.9985
2695	1.000

Between 5000 *psig* and 4100 *psig*, c_o value will be:

$$c_o = \frac{0.9829 - 0.9739}{0.9739(5000 - 4100)} = 10.27 \times 10^{-6}\, psi^{-1}$$

A compressibility of $10.27 \times 10^{-6}\, psi^{-1}$ means that the volume of 1 million barrels of reservoir fluid will increase by 10.27 bbls for a reduction of 1 psi in pressure.

Isothermal compressibility coefficients are required in solving many reservoir engineering problems, including transient fluid flow problems; also, they are required in the determination of the physical properties of the undersaturated crude oil.

For a crude oil system, the isothermal compressibility coefficient of the oil phase, c_o, is categorized into the following two types based on reservoir pressure:

- At reservoir pressures that are greater than or equal to the bubble point pressure ($p \geq p_b$), the crude oil exists as a single phase with all its dissolved gas still in solution. The isothermal compressibility coefficient of the oil phase, c_o, above the bubble point reflects the changes in the volume associated with oil expansion or compression of the single-phase oil with changing the reservoir pressure. The oil compressibility in this case is termed ***undersaturated isothermal compressibility coefficient***.

- Below the bubble point pressure, the solution gas is liberated with decreasing reservoir pressure or redissolved with increasing the pressure. The changes of the oil volume as the result of changing the gas solubility must be considered when determining the isothermal compressibility coefficient. The oil compressibility in this case is termed ***saturated isothermal compressibility coefficient***.

The following four correlations for estimating the undersaturated oil compressibility are presented below:

- Trube's correlation (1957).
- Vasquez-Beggs's correlation (1980).
- Petrosky-Farshad's correlation (1993).
- Standing's correlation (1974).

I. Standing's Correlation for Determining "c_o":

Standing (1974) proposed a graphical correlation for determining the oil compressibility for undersaturated hydrocarbon systems. Whitson and Brule expressed his relationship in the following mathematical form:

$$c_o = 10^{-6} exp\left[\frac{\rho_{ob} + 0.004347(p - p_b) - 79.1}{0.0007141(p - p_b) - 12.938}\right] \qquad eq.(2.53)$$

where:

ρ_{ob} = oil density at the bubble point pressure, *lb/ft*^{3}.
p_b = bubble point pressure, psia.
c_o = oil compressibility, psia^{-1}.

II. **The Vasquez-Beggs Correlation**

From a total of 4,036 experimental data points used in a linear regression model, Vasquez and Beggs (1980) correlated the isothermal oil compressibility coefficients with Rs, T, °API, γg, and p. They proposed the following expression:

$$c_o = \frac{-1433 + 5R_{sb}17.2(T - 460) - 1180\gamma_{gs} + 12.61°API}{10^5 p} \qquad eq.(2.54)$$

where:

T = temperature, °R
p = pressure above the bubble-point pressure, psia.
R_{sb} = gas solubility at the bubble-point pressure.
γ_{gs} = corrected gas gravity.

See (Ahmed T., 2019) for more details.

2.2.8 Total Formation Volume Factor "B_t":

To describe the pressure-volume relationship of hydrocarbon systems below their bubble point pressure, it is convenient to express this relationship in terms of the total FVF as a function of pressure. This property defines the total volume of a system regardless of the number of phases present.

The total FVF, denoted B_t, is defined as the ratio of the total volume of the hydrocarbon mixture; that is, oil and gas, if present, at the prevailing pressure and temperature per unit volume of the stock-tank oil. Because naturally occurring hydrocarbon systems usually exist in either one or two phases, the term two-phase FVF has become synonymous with the total formation volume.

Mathematically, B_t is defined by the following relationship:

$$B_t = \frac{(V_o)_{p,T} + \left(V_g\right)_{p,T}}{(V_o)_{sc}} \qquad eq.(2.55)$$

where:

B_t = total FVF, bbl/STB.
$(V_o)_{p,T}$ = volume of the oil at p and T, bbl.
$(V_g)_{p,T}$ = volume of the liberated gas at p and T, bbl.
$(V_o)_{sc}$ = volume of the oil at SC, STB.

A typical plot of B_t as a function of pressure for an undersaturated crude oil is shown in Fig. (2.21). The oil FVF curve is also included in the illustration. As pointed out, B_o and B_t are identical at pressures above or equal to the bubble point pressure because only one phase, the oil phase, exists at these pressures. It should also be noted that at pressures below the bubble point pressure, the difference in the values of the two oil properties represents the volume of the evolved solution gas as measured at system conditions per stock-tank barrel of oil.

From the definition of the two-phase FVF,

$$B_t = B_o + (R_{sb} - R_s)B_g \qquad eq.(2.56)$$

where:

R_{sb} = gas solubility at the bubble point pressure, scf/STB.
R_s = gas solubility at any pressure, scf/STB.
B_o = oil FVF at any pressure, bbl/STB.
B_g = gas FVF, bbl/scf.

Several correlations can be used to estimate the two-phase FVF when the experimental data are not available; three of these methods are detailed below:

- Standing's correlation (1947).
- Glaso's correlation (1980).
- Marhoun's correlation (1988).

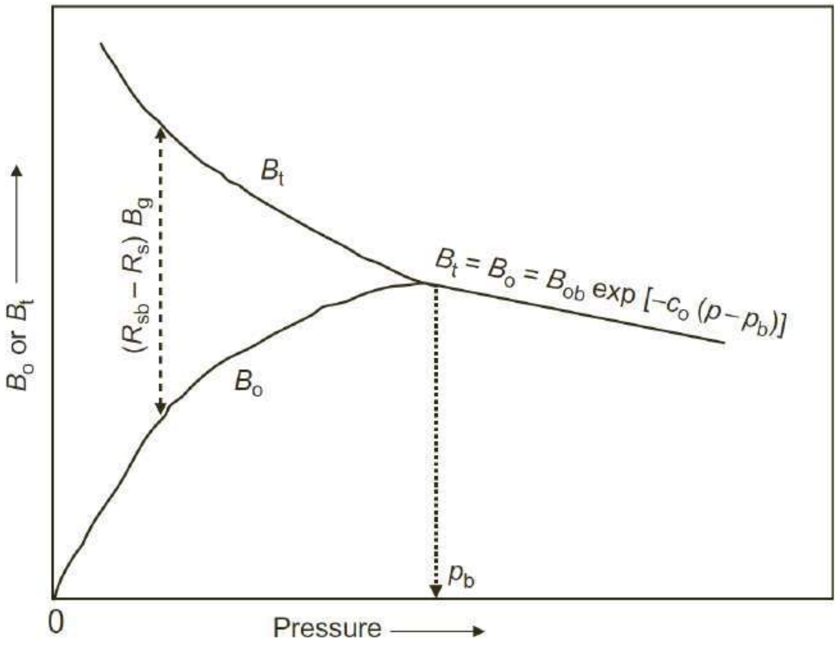


Fig. (2.21): B_o and B_t vs. pressure.

2.2.9 Crude Oil Viscosity "μ_o":

Crude oil viscosity is an important physical property that controls the flow of oil through porous media and pipes. The viscosity, in general, is defined as the internal resistance of the fluid to flow. It ranges from 0.1 cp for near critical to over 100 cp for heavy oil. It is considered the most difficult oil property to calculate with a reasonable accuracy from correlations.

Oil's viscosity is a strong function of the temperature, pressure, oil gravity, gas gravity, gas solubility, and composition of the crude oil. Whenever possible, oil viscosity should be determined by laboratory measurements at reservoir temperature and pressure. The viscosity usually is reported in standard PVT analyses. If such laboratory data are not available, engineers may refer to published correlations, which usually vary in complexity and accuracy, depending on the available data on the crude oil. Based on the available data on the oil mixture, correlations can be divided into the following two types: correlations based on other measured PVT data, such as API or R_s, and correlations based on oil composition. Depending on the pressure, p, the viscosity of crude oils can be classified into three categories:

- ***Dead oil viscosity, μ_{od}.*** The dead oil viscosity (oil with no gas in the solution) is defined as the viscosity of crude oil at *atmospheric pressure* and system temperature, *T*.
- ***Saturated oil viscosity, μ_{ob}.*** The saturated (bubble point) oil viscosity is defined as the viscosity of the crude oil at any pressure less than or equal to the bubble point pressure.
- ***Undersaturated oil viscosity, μ_o.*** The undersaturated oil viscosity is defined as the viscosity of the crude oil at a pressure above the bubble point and reservoir temperature.

The definition of the three categories of oil viscosity is illustrated conceptually in Fig. (2.22).

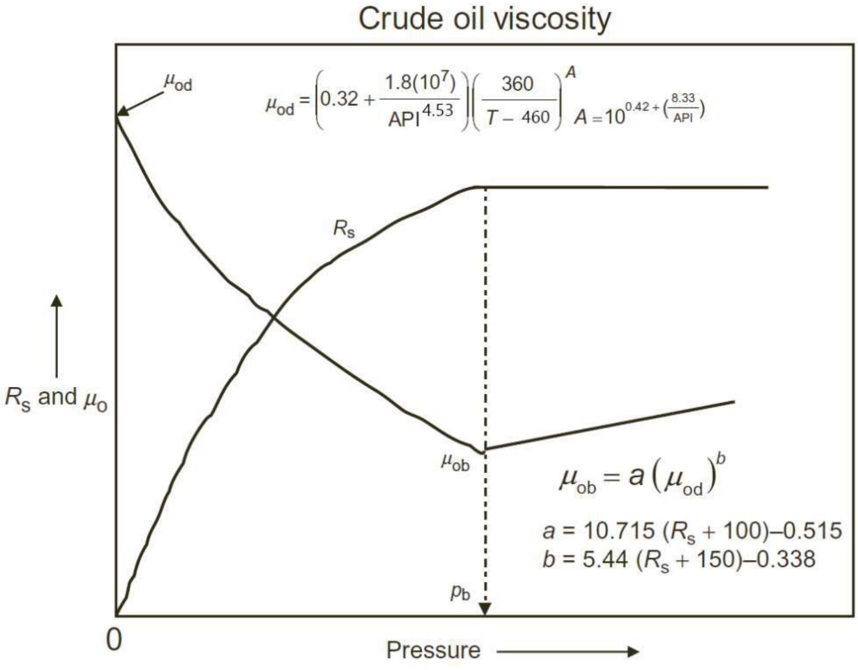


Fig. (2.22) Oil viscosity as a function of R_s and p.

I. Dead Oil Viscosity "μ_{od}":

Several empirical methods are proposed to estimate the viscosity of the dead oil that are based on the API gravity of the crude oil and system temperature. These correlations, as listed below, calculate the viscosity of the dead oil (ie, no dissolved gas) at reservoir temperature "*T* ":

- Beal's correlation (1946).
- Beggs-Robinson's correlation (1975).
- Glaso's correlation (1980).

II. Glaso’s Correlation for Determining "μ_{od}":

Glaso (1980) proposed a generalized mathematical relationship for computing the dead oil viscosity. The relationship was developed from experimental measurements on 26 crude oil samples. The correlation has the following form:

$$\mu_{od} = [3.141(10^{10})](T - 460)^{-3.444}[log(API)]^{A} \qquad eq.(2.57)$$

The temperature T is expressed in °R and the coefficient A is given by:

$$A = 10.313[log(T - 460)] - 36.447 \qquad eq.(2.58)$$

This expression can be used within the range of 50–300°F for the system temperature and 20–48° for the API gravity of the crude. Sutton and Farshad (1984) concluded that Glaso’s correlation showed the best accuracy of the three previous correlations.

III. Saturated Oil Viscosity, "μ_{ob}":

Having calculated the viscosity of the dead crude oil at reservoir temperature using one of the three above correlations, the dead oil viscosity must be adjusted to account for the solution gas (ie, “Rs”). The following three empirical methods are proposed to estimate the viscosity of the saturated oil:

- Chew-Connally correlation (1959).
- Beggs-Robinson correlation (1975).
- Abu-Khamsin and Al-Marhoun (1991).

IV. Abu-Khamsin and Al-Marhoun Correlation for Determining "μ_{ob}"

An observation by Abu-Khamsin and Al-Marhoun (1991) suggests that the saturated oil viscosity, μ_{ob}, correlates very well with the saturated oil density, ρ_{ob}, as given by:

$$\ln(\mu_{ob}) = 8.484462\rho_{ob}^{4} - 2.652294 \qquad eq.(2.59)$$

where, the saturated oil density, ρ_{ob}, is expressed in g/cm^3; that is, $\rho_{ob}/62.4$.

V. Undersaturated Oil Viscosity "μ_{o}":

Oil viscosity at pressures above the bubble point is estimated by first calculating the oil viscosity at its bubble point pressure and adjusted to higher pressures. Three methods for estimating the oil viscosity at pressures above saturation pressure are presented next:

- Beal's correlation (1946).
- Khan's correlation (1987).
- Vasquez-Beggs correlation (1980).

IV. Khan's Correlation for Determining "μ_o":

From 1500 experimental viscosity data points on Saudi Arabian crude oil systems, Khan et al. (1987) developed the following equation with a reported absolute average relative error of 2%:

$$\mu_o = \mu_{ob} exp[9.6(10^{-6})(p - p_b)] \qquad eq. (2.60)$$

Problems

1. Assuming an ideal gas behavior, calculate the density of n-butane at (220°F) and (50 psia).

2. Given the following gas:

Component	Weight Fraction
C_1	0.65
C_2	0.15
C_3	0.10
n-C_4	0.06
n-C_5	0.04

Calculate:

a. Mole fraction of the gas.
b. Apparent molecular weight.
c. Specific gravity.

3. An ideal gas mixture has a density of (1.92 lb/ft$^{3)}$ at (500 psia) and (100°F). Calculate the apparent molecular weight of the gas mixture.

4. Using the gas composition as given in Problem 2, and assuming real gas behavior, calculate:
   a. Gas density at (2,000 psia and 150°F).
   b. Gas formation volume factor in scf/ft^3.

5. Compute the composition in weight fraction of a gas with the following composition:

Component	Composition, mole fraction
Methane	0.870
Ethane	0.049
Propane	0.036
i-Butane	0.025
n-Butane	0.020
	1.00

6. A (20 cu-ft) tank at at (100 oF) is pressured to (200 psia) with a pure paraffin gas. Ten pounds of ethane are added, and the specific gravity of the gas mixture is measured to be (1.68). Assume that the gases act as ideal gases. What was the gas originally in the tank?

7. Compute the apparent molecular weight and specific gravity of the gas of problem 5.

8. A gas composed of methane and ethane has a specific gravity of (0.75). What is the weight percent and volume percent of the methane in the gas mixture?

9. Determine the partial pressure of each component of the gas in problem 5 if the total pressure on the gas is (350 psia) and the temperature is (90 °F).

10. What is the specific gravity of the following gas?

Component	Partial pressure, psia
Methane	17.8
Ethane	1.0
Propane	0.4
i-Butane	0.2
n-Butane	0.6
	20.0

11. (2.4 cu-ft) cylinder of ethane shows a pressure of (1600 psia) at (90 °F). What is the mass in pounds of ethane contained in the cylinder? Do not assume ethane is an ideal gas.

12. The gas in problem 5 was produced from a reservoir at (4650 psia) and (180 °F). What was the density of the gas at reservoir conditions?

13. A tank contains methane at (1000 psia) and (140 °F). Another tank of equal volume contains ethane at (500 psia) and (140 °F). The two tanks are connected, the gases are allowed to mix, and the temperature is restored to (140 °F). Calculate the final pressure, the composition of the

mixture, and the partial pressure of the components at final conditions. Do not assume that ideal gas equation apply.

14. A cylinder has a volume of (0.5 cu-ft) and contains a gas at a pressure of (2000 psia) and (120 °F). The pressure drops to (1000 psia) after (0.0923 lb moles) of gas are removed. The temperature is constant. The z-factor was (0.90) at (2000 psia). What is the z-factor at (1000 psia)?

15. A tank with volume of (2.4 cu-ft) is filled with methane to a pressure of (1500 psia) at (104 °F). Determine the molecular weight and specific gravity of the gas and the pound moles, pounds, and density of the gas in the tank.

16. A cylinder with an initial pressure of (14.7 psia) and volume of (75000 cc) is held at a constant temperature of (200 oF). The volume of the cylinder is reduced by insertion of mercury. The corresponding volumes (cc) and pressures (psia) of the gas inside the cylinder are recorded as follows:

Press.	400	800	1200	2000	2500	3000	4000	5000
Vol.	2448	1080	648.6	350.6	292.9	266.4	234.6	206.2

Calculate the ideal volumes for the gas at each pressure and use them to calculate the z-factor. Plot the z-factor against pressure.

17. Determine the value of z-factor for the gas given below at (5420 psig) and (257 °F):

Component	Composition, mole fraction
Hydrogen sulfide	0.100
Carbon dioxide	0.050
Nitrogen	0.021
Methane	0.703
Ethane	0.062
Propane	0.037
n-Butane	0.027
	1.000

18. The reservoir gas in a wet gas reservoir has a specific gravity of (1.295), a hydrogen sulfide concentration of (20.9 mole percent), and a carbon dioxide concentration of (44.7 mole percent). Determine a value of z-factor for use at reservoir conditions of (5720 psig) and (268 °F).

19. A wet gas has composition as given below. Determine a value of z-factor for this gas at reservoir conditions of (5720 psig) and (268 °F):

Component	**Composition, mole percent**
Hydrogen sulfide	20.90
Carbon dioxide	44.69
Nitrogen	1.22
Methane	21.80
Ethane	3.68
Propane	2.05
i-Butane	0.58
n-Butane	1.09
i-Pentane	0.46
n-Pentane	0.56
Hexanes	0.72
Heptanes plus	2.25
	100.00
Properties of heptanes plus	
Specific gravity	0.844
Molecular weight	115 $lb/lb\ mole$

Compare your answer with laboratory measured z-factor of (0.914) at (5720 psig) and (268 °F) for this gas.

20. A natural gas with a specific gravity of (0.75) has a gas formation volume factor of (0.00529 ft^3/scf) at the prevailing reservoir pressure and temperature. Calculate the density of the gas.

21. A natural gas has the following composition:

Component	**Mole Fraction, y_i**
C_1	0.75
C_2	0.10
C_3	0.05

$i-C_4$	0.04
$n-C_4$	0.03
$i-C_5$	0.02
$n-C_5$	0.01

Reservoir conditions are (3500 psia) and (200 °F). Calculate:

a. Isothermal gas compressibility coefficient

b. Gas viscosity by using the

1. Carr-Kobayashi-Burrows method.
2. Lee-Gonzales-Eakin method.

22. Given the following gas composition:

Component	**Mole Fraction, y_i**
CO_2	0.06
N_2	0.03
C_1	0.75
C_2	0.07
C_3	0.04
$n-C_4$	0.03
$n-C_5$	0.02

If the reservoir pressure and temperature are (2500 psia) and (175°F), respectively, calculate:

a. Isothermal gas compressibility coefficient.

b. Gas viscosity by using the:

1. Carr-Kobayashi-Burrows method.
2. Lee-Gonzales-Eakin method.

23. A crude oil system exists at its bubble-point pressure of (1708.7 psia) and a temperature of (131°F). Given the following data:

API = 40°

Average specific gravity of separator gas = 0.85

Separator pressure = 100 psig

a. Calculate R_{sb} by using Standing correlation.

b. Calculate B_{ob} by using Standing correlation.

24. Estimate the bubble-point pressure of a crude oil system by using Standing correlation with the following limited PVT data:
    API = 35° T = 160°F R_{sb} = 700 scf/STB γ_g = 0.75

25. A crude oil system exists at an initial reservoir pressure of (4500 psi) and (85°F). The bubble-point pressure is estimated at (2109 psi). The oil properties at the bubble-point pressure are as follows:
    B_{ob} = 1.406 bbl/STB R_{sb} = 692 scf/STB
    γ_g = 0.876 API = 41.9°
    Calculate:
    a. Oil density at the bubble-point pressure.
    b. Oil density at 4500 psi.
    c. B_o at 4500 psi.

26. A hydrocarbon gas mixture with a specific gravity of (0.7) has a density of (9 lb/ft^3) at the prevailing reservoir pressure and temperature. Calculate the gas formation volume factor in bbl/scf.

27. A high-pressure cell has a volume of (0.33 ft^3) and contains gas at (2,500 psia) and (130°F), at which conditions its z-factor is (0.75). When (43.6 scf) of the gas are bled from the cell, the pressure dropped to (1,000 psia), the temperature remaining at (130°F). What is the gas deviation factor at (1,000 psia) and (130°F)?

28. A gas reservoir exists at a (150°F). The gas has the following composition:

Component	**Mole %**
C_1	89
C_2	7
C_3	4

The gas expansion factor E_g was calculated as (204.648 scf/ft^3) at the existing reservoir pressure and temperature. Calculate the viscosity of the gas.

29. A (20 ft^3) tank at a pressure of (2500 psia) and (212°F) contains ethane gas. How many pounds of ethane are in the tank?

REFERENCES

Ahmed, T.,2019, “*Reservoir Engineering Handbook”*, Gulf Professional Publishing.

Ahmed, T., 2016, “*Equation of State and PVT Analysis Application for Improved Reservoir Modelling”*. Elsevier Inc, USA.

Alani, G.H., Kennedy, H.T., 1960. Volume of liquid hydrocarbons at high temperatures and pressures. Trans. AIME 219, 288–292.

Brown, G.G., et al., 1948. *Natural Gasoline and The Volatile Hydrocarbons*. National Gas Association of America, Tulsa, OK.

Carr, N., Kobayashi, R., Burrows, D., 1954. Viscosity of hydrocarbon gases under pressure. Trans. AIME 201, 270–275.

Craft, B. C., and Hawkins, M. (Ronald E. Terry, J. Brandon Rogers.), 2015, “*Applied Petroleum Reservoir Engineering”*, Third edition. Englewood Cliffs, NJ: Prentice Hall.

Dranchuk, P.M., Abu-Kassem, J.H., 1975. Calculate of Z-factors for natural gases using equations-of-state. J. Can. Pet. Technol. 34–36.

Dranchuk, P.M., Purvis, R.A., Robinson, D.B., 1974. Computer Calculations of Natural Gas Compressibility Factors Using the Standing and Katz Correlation. Technical Series, no. IP 74–008. Institute of Petroleum, Alberta, Canada.

Glaso, O., 1980. Generalized pressure-volume-temperature correlations. J. Pet. Technol. 32 (5), 785–795.

Hall, K.R., Yarborough, L., 1973. A new equation of state for Z-factor calculations. Oil Gas J. 82–92.

Katz, D.L. et al, 1959, “*Handbook of Natural Gas Engineering”*, McGraw-Hill Book Co. Inc., New York City.

Katz, D., 1942, “*Drilling and Production Practice”*. American Petroleum Institute, Dallas, TX.

Lee, A.L., Gonzalez, M.H., Eakin, B.E., 1966. The viscosity of natural gases. J. Petrol. Technol. 997–1000.

Mattar, L.G., Brar, S., Aziz, K., 1975. Compressibility of natural gases. J. Can. Pet. Technol. 77–80.

Petrosky, G.E., Farshad, F., 1993. Pressure-volume-temperature correlations for Gulf of Mexico crude oils. SPE paper 26644, In: Presented at the 68th Annual Technical Conference of the Society of Petroleum Engineers, Houston, 3–6 October 1993.

Standing, M., 1974. Petroleum Engineering Data Book. Norwegian Institute of Technology, Trondheim.

Standing, M.B., 1977. Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems. Society of Petroleum Engineers, Dallas. pp. 125–126.

Standing, M.B., Katz, D.L., 1942. Density of natural gases. Trans. AIME 146, 140–149.

Standing, M.B., 1981. Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems, ninth ed. Society of Petroleum Engineers, Dallas, TX.

Standing, M.B., 1947. A pressure-volume-temperature correlation for mixtures of California oils and gases. Drilling and Production Practice. API, Washington, D.C, pp. 275–287.

Stewart, W.F., Burkhard, S.F., Voo, D., 1959. Prediction of Pseudo-Critical Parameters for Mixtures. Paper Presented at the AIChE Meeting, Kansas City, MO.

Sutton, R.P.M., 1985. Compressibility factors for high-molecular-weight reservoir gases. In: Paper SPE 14265, Presented at the 60th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Las Vegas.

Sutton, R.P., Farshad, F.F., 1984. Evaluation of empirically derived PVT properties for Gulf of Mexico crude oils. Paper SPE 13172, In: Presented at the 59th Annual Technical Conference, Houston.

Trube, A.S., 1957a. Compressibility of undersaturated hydrocarbon reservoir fluids. Trans. AIME 210, 341–344.

Trube, A.S., 1957b. Compressibility of natural gases. Trans. AIME 210, 355–357.

Vasquez, M., Beggs, H.D., 1980. Correlations for fluid physical property prediction. J. Pet. Technol. 32, 968–970.

Wichert, E., Aziz, K., 1972. Calculation of Z’s for sour gases. Hydrocarb. Process. 51 (5), 119–122.

Chapter Three

Phase Behavior and Laboratory Analysis of Reservoir Fluids

3.1 Phase Behavior and Pressure-Temperature Diagram:

A ***PHASE*** is defined as any homogeneous part of a system that is physically distinct and separated from other parts of the system by definite boundaries. For example, ice, liquid water, and water vapor constitute three separate phases of the pure substance H_2O, because each is homogeneous and physically distinct from the others; moreover, each is clearly defined by the boundaries existing between them. Whether a substance exists in a solid, liquid, or gas phase is determined by the temperature and pressure acting on the substance. It is known that ice (solid phase) can be changed to water (liquid phase) by increasing its temperature and, by further increasing the temperature, water changes to steam (vapor phase). This change in phases is termed ***phase behavior***.

Hydrocarbon systems found in petroleum reservoirs are known to display multiphase behavior over wide ranges of pressures and temperatures. The most important phases that occur in petroleum reservoirs are a **liquid phase**, such as crude oils or condensates, and a **gas phase**, such as natural gases, depending on the nature of crude oil composition (light (C_1 & C_2), intermediate (C_3 – C_6) or heavy components (C_7+)).

The conditions under which these phases exist are a matter of considerable practical importance. The experimental or the mathematical determinations of these conditions are conveniently expressed in different types of diagrams, commonly called ***phase diagrams***. Phase diagram can be defined as a map of the phase of a substance as a function of temperature (X-axis) and pressure (Y-axis). Phase diagram is also called the *pressure-temperature diagram*.

Fig. (3.1) shows a typical pressure-temperature diagram (p-T diagram) of a multicomponent system with a specific overall composition. Although a different hydrocarbon systems would have a different phase diagrams, the general configuration is similar.

These multicomponent *p-T* diagrams are essentially used to:

- Classify reservoirs.
- Specify the naturally occurring hydrocarbon systems.
- Describe the phase behavior of the reservoir fluid.

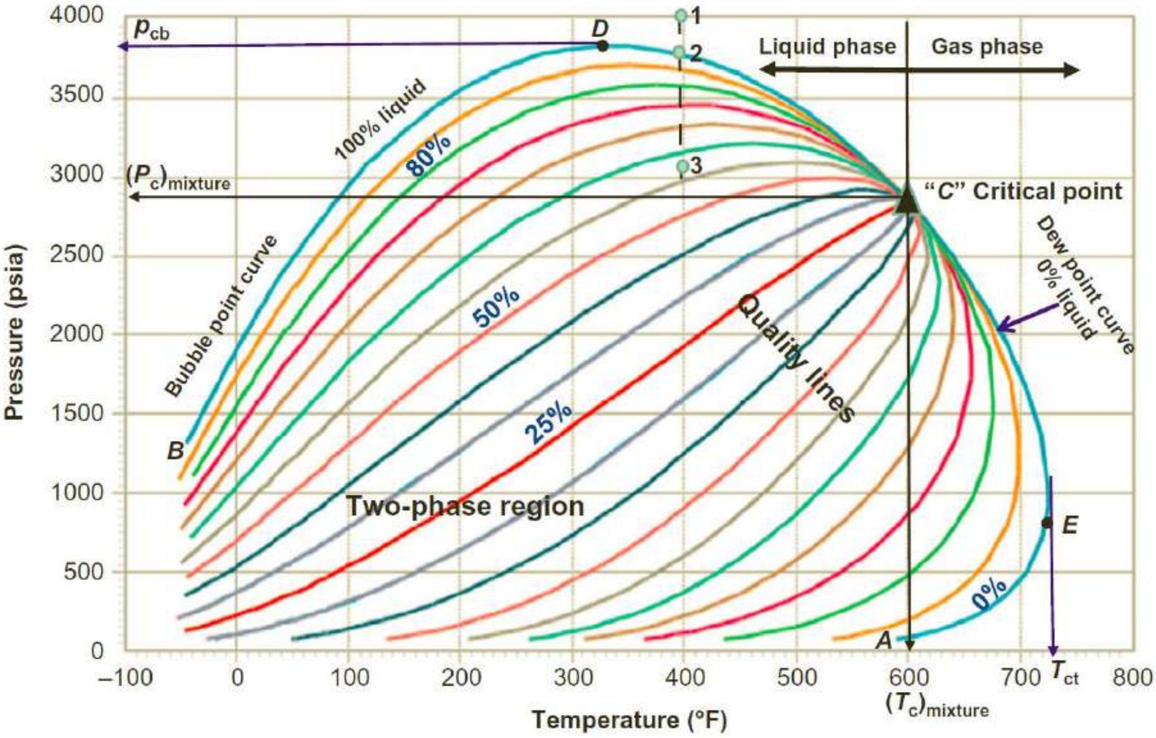


Fig. (3.1): Typical p-T diagram for a multicomponent system.

To fully understand the significance of the *p-T* diagrams, it is necessary to identify and define the following key points on the p-T diagram:

- ❖ ***Cricondentherm* (T_{ct}):** The cricondentherm is the maximum temperature above which liquid cannot be formed regardless of pressure (point *E*), or simply it is the maximum temperature of the tow phase region. The corresponding pressure is termed the *cricondentherm pressure*, p_{ct}.
- ❖ ***Cricondenbar* (p_{cb}):** The cricondenbar is the maximum pressure above which no gas can be formed regardless of temperature (point *D*), or simply it is the maximum pressure of the tow phase region. The corresponding temperature is called the *cricondenbar temperature*, T_{cb}.
- ❖ ***Critical point*:** The critical point for a multicomponent mixture is referred to as the state of pressure and temperature at which all intensive properties of the gas and liquid phases are equal (point *C*). At the critical

point, the corresponding pressure and temperature are called the critical pressure, p_c, and critical temperature, T_c, of the mixture.

- ***Phase envelope (two-phase region):*** The region enclosed by the bubble point curve and the dew point curve (line *BCA*), where gas and liquid coexist in equilibrium, is identified as the phase envelope of the hydrocarbon system.
- ***Quality lines*:** The lines within the phase diagram are called quality lines. They describe the pressure and temperature conditions for equal liquid percentage by volumes (% of liquid). Note that the quality lines converge at the critical point (point *C*).
- ***Bubble point curve*:** The bubble point curve (line *BC*) is defined as the line separating the liquid phase region from the two-phase region.
- ***Dew point curve*:** The dew point curve (line *AC*) is defined as the line separating the vapor phase region from the two-phase region.

3.2 Classification of Reservoirs:

Petroleum reservoirs are broadly classified as **oil** or **gas reservoirs**. As shown in Fig. (3.2) these broad classifications are further subdivided depending on:

1. The composition of the reservoir hydrocarbon mixture.
2. Initial reservoir pressure and temperature.
3. Pressure and temperature of the surface production.
4. Location of the reservoir temperature with respect to the critical temperature and the cricondentherm.

In general, reservoirs are conveniently classified on the basis of the reservoir temperature “T” as compared with the critical temperature of the hydrocarbon mixture “$(T_c)_{mix}$.” It should be noted that perhaps the most important components are the Methane and the Plus-Fraction, e.g., C7+. In general, these two components impact the size of the phase envelope as well as the critical temperature of the mixture. Accordingly, reservoirs can be classified into basically three types:

1. ***Oil reservoirs*:** If the reservoir temperature, T, is less than the critical temperature, T_c, of the reservoir fluid, the reservoir is classified as an oil reservoir.

2. ***Gas reservoirs*****:** If the reservoir temperature is greater than the critical temperature of the hydrocarbon fluid, the reservoir is considered a gas reservoir.
3. ***Near-critical reservoirs*****:** If the reservoir temperature is very close to the critical temperature of the hydrocarbon fluid, the reservoir is classified as near-critical reservoir.

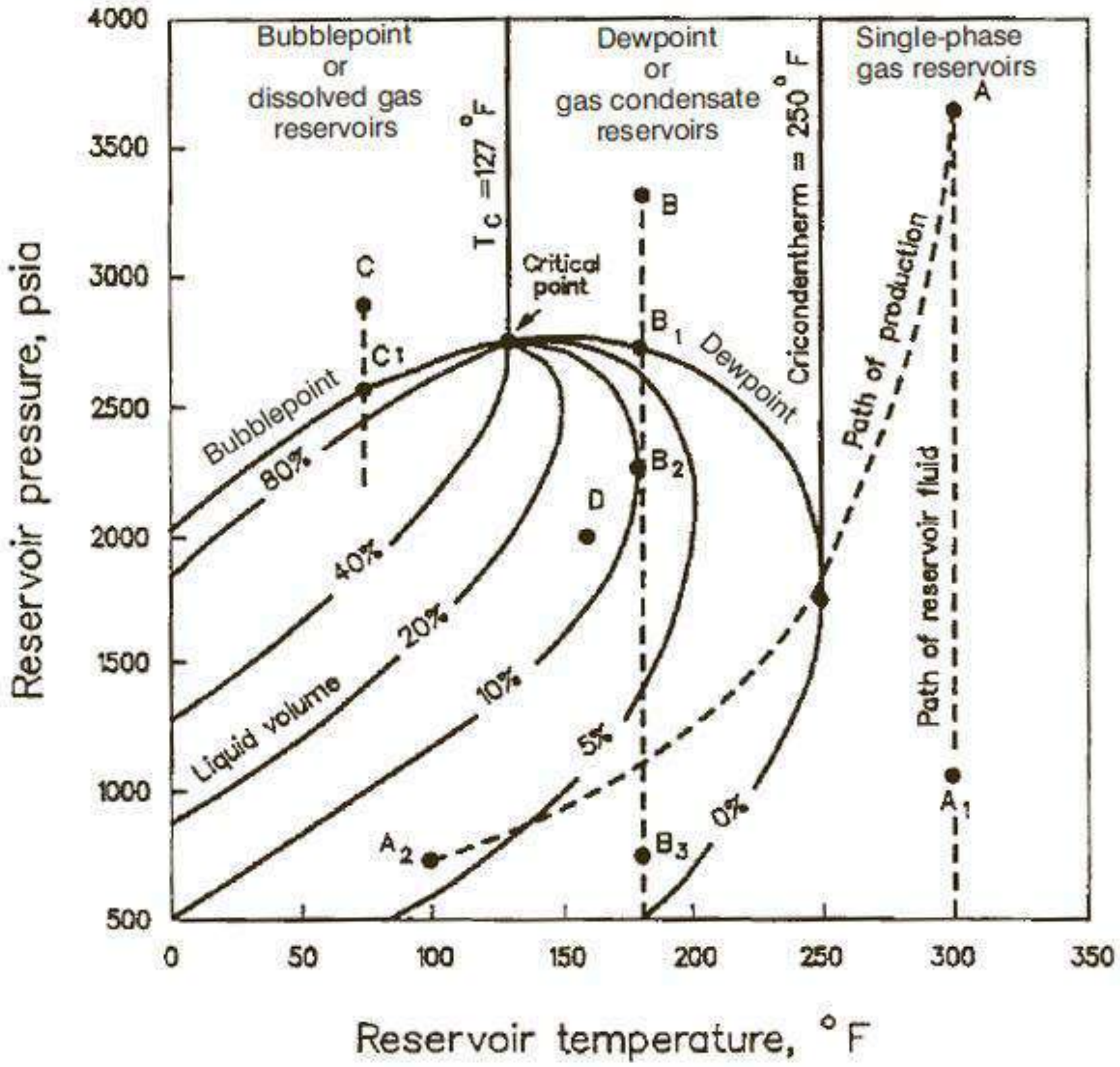
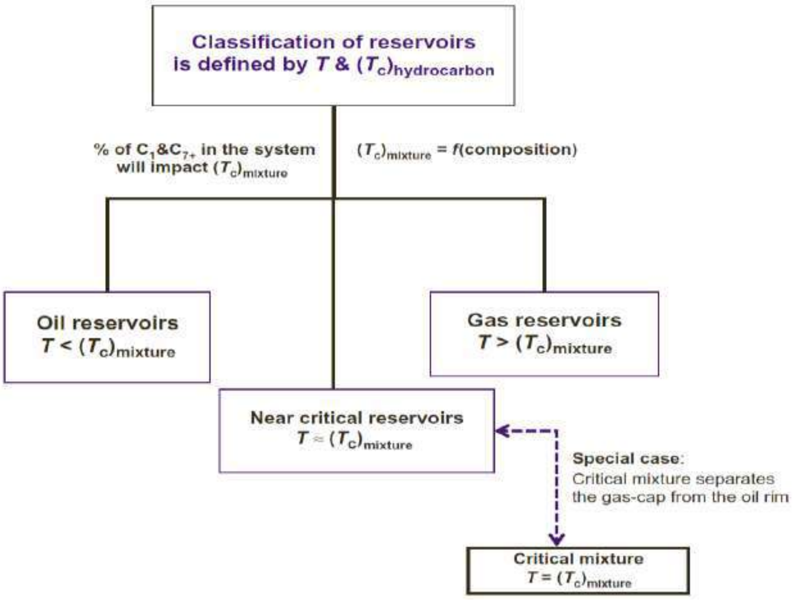


Fig. (3.2): Main reservoir classifications.

Below the flow diagram summarizes the three reservoir classifications. The impact of fluid composition on the shape and size of the phase envelope as well as the reservoir type identification are illustrated in Fig. (3.3).



Composition defines the size of phase envelop

Comp.	Mix 2	P_c, psia	T_c, °R
CO_2	0.000	1071	547.9
N_2	0.000	493	227.6
C_1	0.350	667.8	343.37
C_2	0.100	707.8	550.09
C_3	0.060	616.3	666.01
i-C_4	0.020	529.1	734.98
n-C_4	0.020	550.7	765.65
i-C_5	0.001	490.4	829.1
n-C_5	0.002	488.6	845.7
C_6	0.030	436.9	913.7
C_{7+}	0.417	320.3	1139.4

Comp.	Mix 1	P_c, psia	T_c, °R
CO_2	0.009	1071	547.9
N_2	0.003	493	227.6
C_1	0.535	667.8	343.37
C_2	0.115	707.8	550.09
C_3	0.088	616.3	666.01
i-C_4	0.023	529.1	734.98
n-C_4	0.023	550.7	765.65
i-C_5	0.015	490.4	829.1
n-C_5	0.015	488.6	845.7
C_6	0.015	436.9	913.7
C_{7+}	0.159	320.3	1139.4

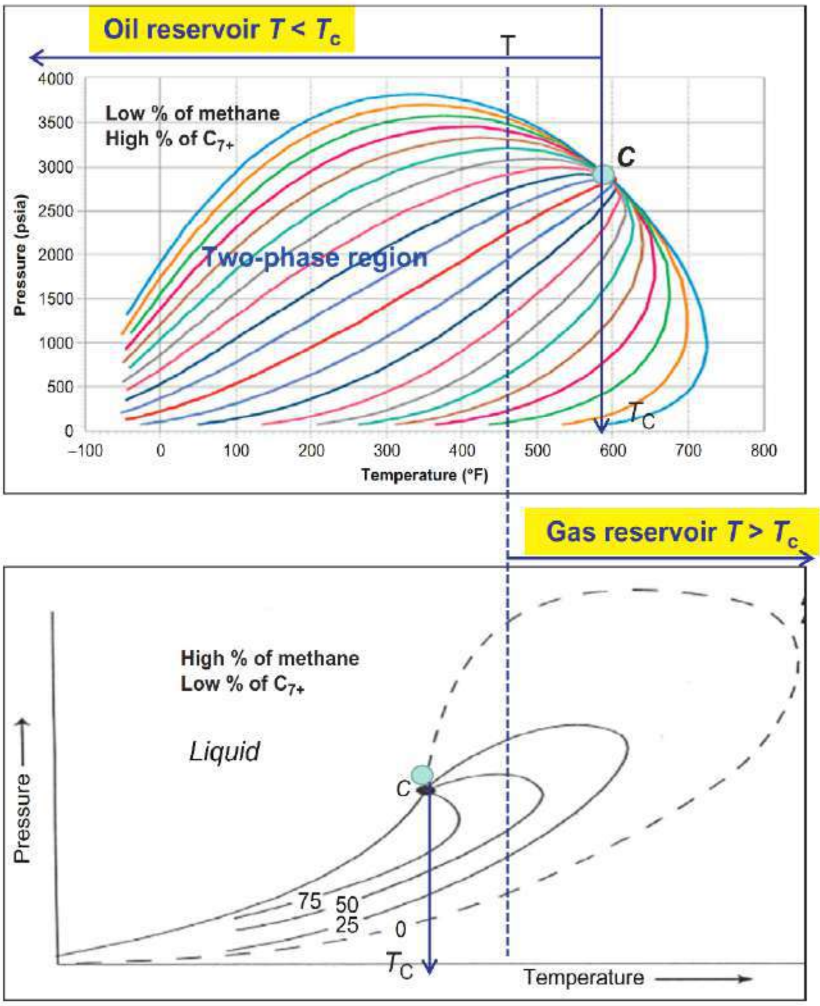


Fig. (3.3): Impact of composition on the size of phase envelope.

3.2.1 Classification of Oil Reservoirs:

Depending on **initial reservoir pressure**, p_i, oil reservoirs can be subclassified into the following categories:

1. ***Undersaturated oil reservoir*****:** If the initial reservoir pressure, p_i (as represented by point 1 in Fig. (3.1)), is greater than the bubble point pressure, p_b, of the reservoir fluid, the reservoir is an undersaturated oil reservoir.
2. ***Saturated oil reservoir*****:** When the initial reservoir pressure is equal to the bubble point pressure of the reservoir fluid, as shown in Fig. (3.1) by point 2, the reservoir is a saturated oil reservoir.
3. ***Gas-cap reservoir*****:** When two phases (gas and oil) initially exit in equilibrium in a reservoir; the reservoir is defined as a gas-cap reservoir. (as represented by point 3 in Fig. (3.1)), in which the gas or vapor phase is underlain by an oil phase.

3.2.2 Classification of Gas Reservoirs:

On the basis of their phase diagrams and the prevailing reservoir conditions, natural gases can be classified into four categories:

1. ***Retrograde gas-condensate reservoir*****:** If the reservoir temperature T lies between the critical temperature T_c and cricondentherm T_{ct} of the reservoir fluid, the reservoir is classified as a retrograde gas-condensate reservoir. This category of gas reservoir is a unique type of hydrocarbon accumulation in that the special thermodynamic behavior of the reservoir fluid is the controlling factor in the development and the depletion process of the reservoir. When the pressure is decreased on these mixtures, instead of expanding (if a gas) or vaporizing (if a liquid) as might be expected, they vaporize instead of condensing. So retrograde condensation is defined as the formation of liquid by an isothermal decrease in pressure or an isobaric increase in temperature. Consider that the initial condition of a retrograde gas reservoir is represented by point 1 on the pressure-temperature phase diagram of Fig. (3.4) because the reservoir pressure is above the upper dew point pressure, the hydrocarbon system exists as a single phase (i.e., vapor phase) in the reservoir. As the reservoir pressure declines isothermally during production from the initial pressure (point 1) to the upper dew point pressure (point 2), the attraction between the molecules of the light and heavy components move farther

apart. As this occurs, attraction between the heavy component molecules becomes more effective; therefore, liquid begins to condense. This retrograde condensation process continues with decreasing pressure until the liquid dropout (LDO) reaches its maximum at (point 3). Further reduction in pressure permits the heavy molecules to commence the normal vaporization process. This is the process whereby fewer gas molecules strike the liquid surface and more molecules leave than enter the liquid phase. The vaporization process continues until the reservoir pressure reaches the lower dew point pressure. This means that all the liquid that formed must vaporize because the system essentially is all vapor at the lower dew point.

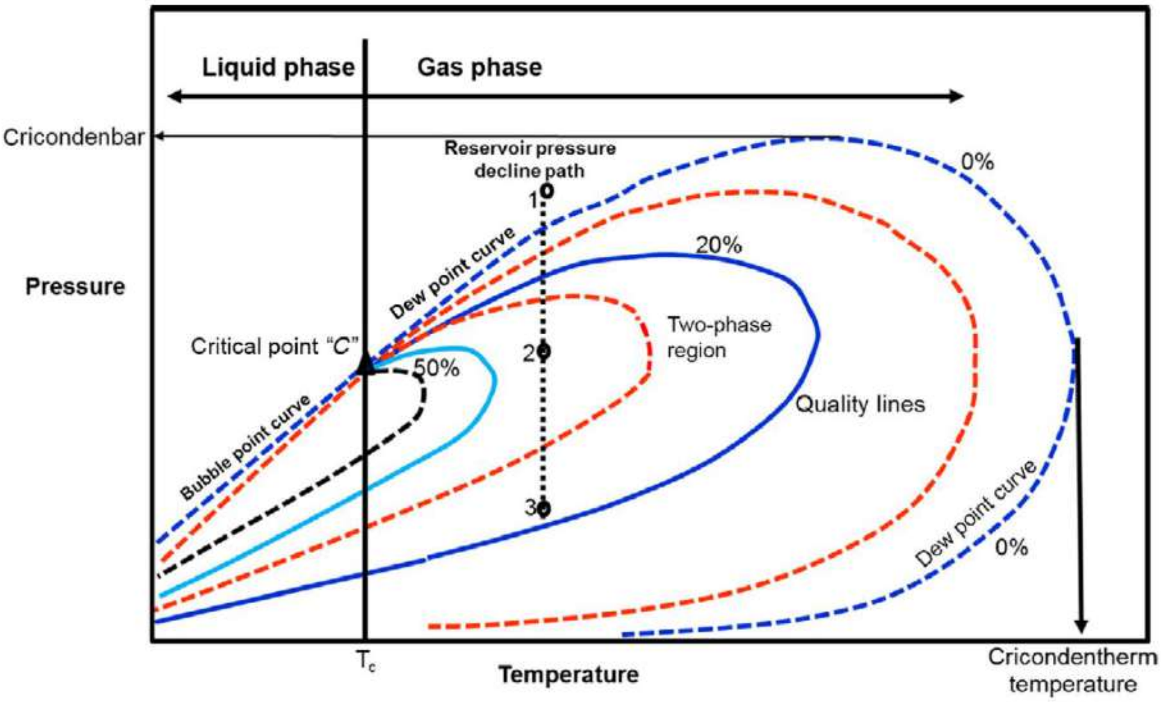


Fig. (3.4): A typical phase diagram of a retrograde system.

2. ***Near-Critical Gas-Condensate Reservoirs:*** If the reservoir temperature is near the critical temperature, as shown in Fig. (3.5), the hydrocarbon mixture is classified as a near-critical gas-condensate. The volumetric behavior of this category of natural gas is described through the isothermal pressure declines, as shown by the vertical line 1–3 in Fig. (3.5) and the corresponding LDO curve of Fig. (3.6) because all the quality lines converge at the critical point as shown in Fig. (3.6), a rapid

liquid buildup immediately occurs below the dew point as the pressure is reduced to (point 2).

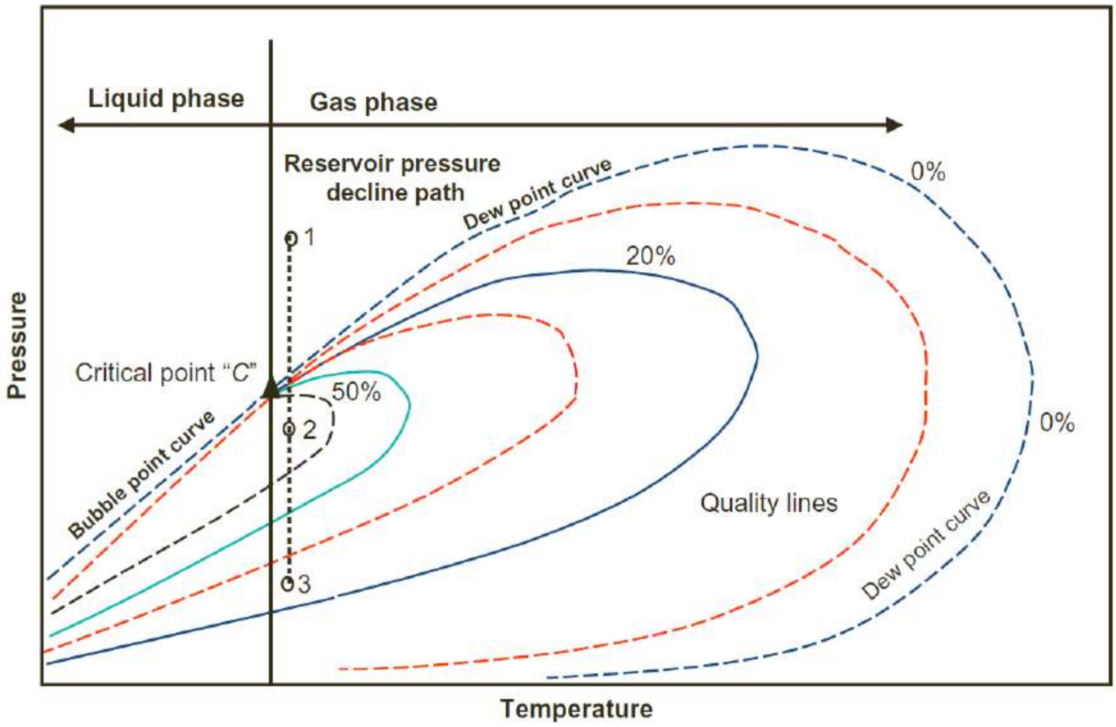


Fig. (3.5): A typical phase diagram for a near-critical gas-condensate reservoir.

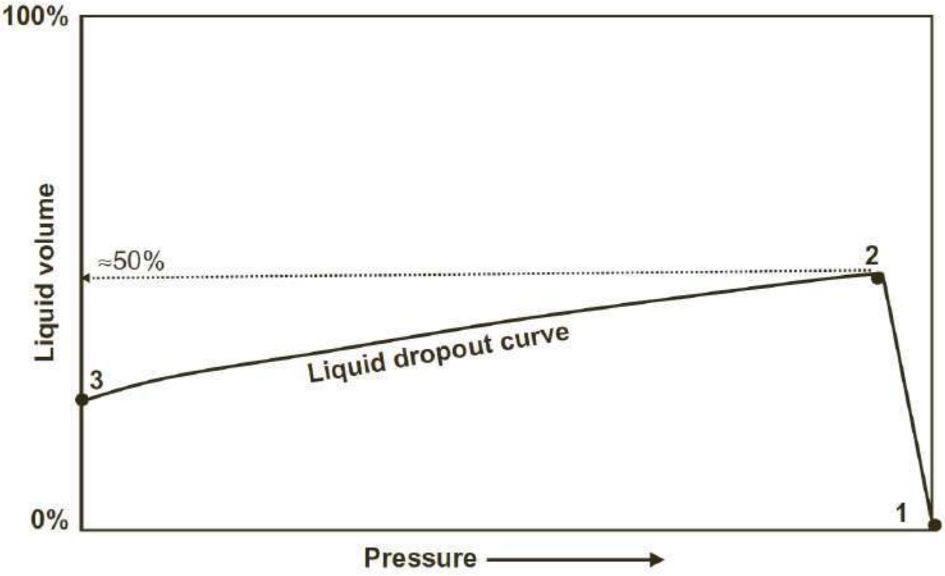


Fig. (3.6): Liquid-shrinkage (dropout) curve for a near-critical gas-condensate system.

3. ***Wet Gas Reservoirs*:** A typical phase diagram of a wet gas is shown in Fig. (3.7), where the reservoir temperature is above the cricondentherm of the hydrocarbon mixture. Because the reservoir temperature exceeds the

cricondentherm of the hydrocarbon system, the reservoir fluid always remains in the vapor phase region as the reservoir is depleted isothermally, along the vertical line *AB*. However, as the produced gas flows to the surface, the pressure and temperature of the gas decline. The gas in this case is composed of intermediate and heavy molecules (C_3 and more) i.e. the attractive force between molecules will be strong if the gas enters the two-phase region, a liquid phase condenses out of the gas and is produced from the surface separators.

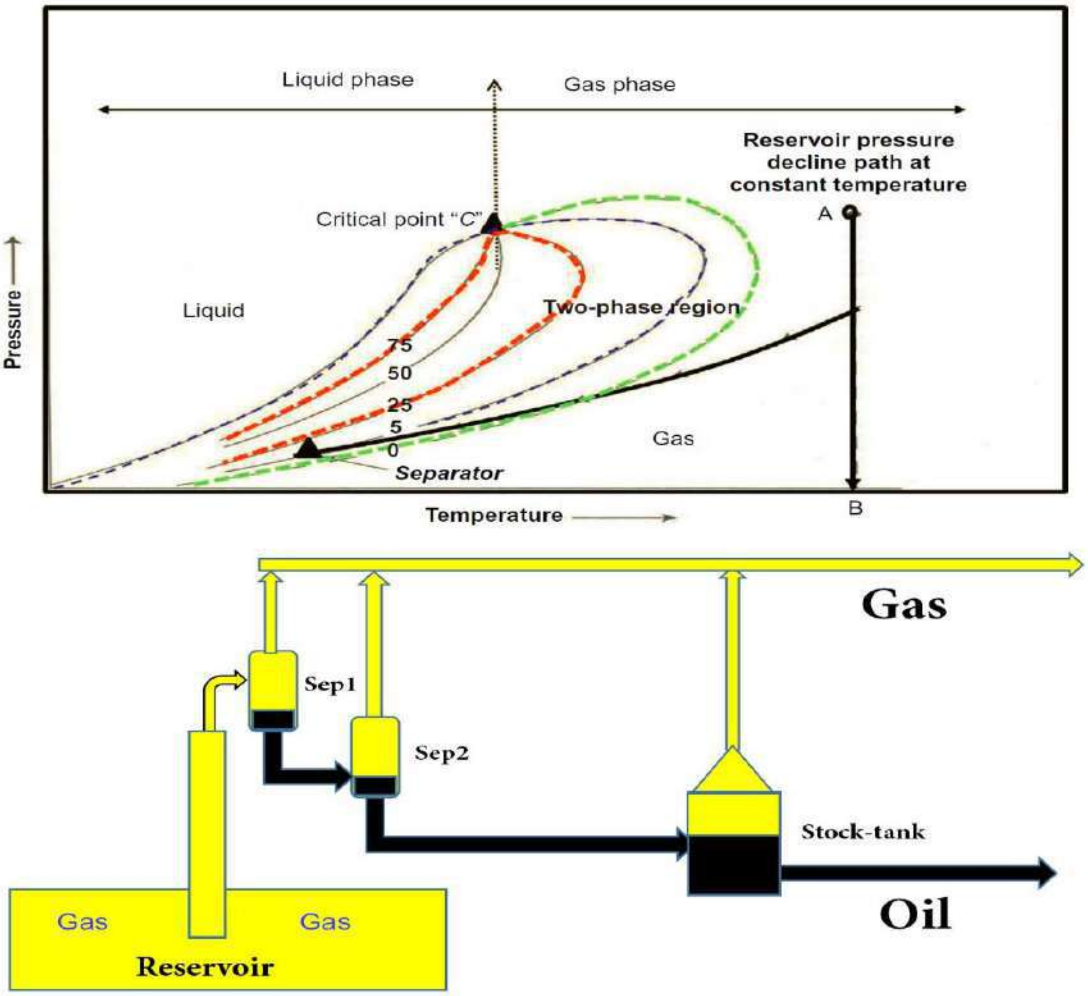


Fig. (3.7): Phase diagram for a wet gas and production process.

4. ***Dry Gas Reservoirs*****:** The hydrocarbon mixture is composed of methane and ethane (light components) which exists as a gas both in the reservoir and the surface because of weak attractive force between these small molecules. The only liquid associated with the gas from a dry gas reservoir is water. Fig. (3.8) is a phase diagram of a dry gas reservoir.

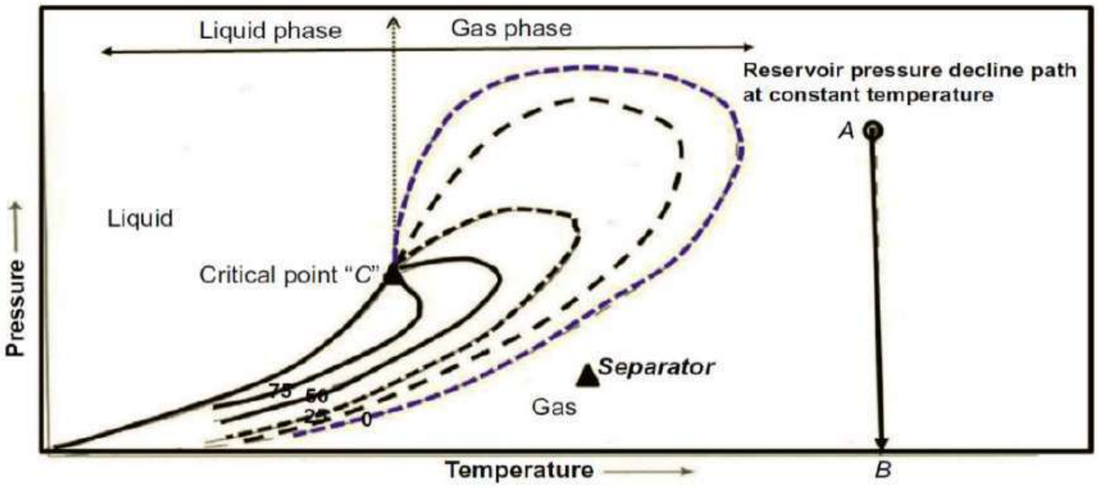


Fig. (3.8): Phase diagram for a dry gas.

3.3 Classification of Crude Oils:

Crude oils cover a wide range in physical properties and chemical compositions, and it is often important to be able to group them into broad categories of related oils. The classification of crude oils essentially is based on the physical and PVT properties exhibited by the crude oil, including:

- Physical properties, such as API gravity of the stock-tank liquid.
- Composition.
- Initial producing gas-oil ratio (GOR).
- Appearance, such as the color of the stock-tank liquid.
- Pressure-temperature phase diagram.

In general, crude oils are commonly classified into the following types:

1. ***Ordinary black oil***: usually yield GORs between 200 and 700 scf/STB and oil gravities of 15–40 API. The stock-tank oil usually is brown to dark green in color.
2. ***Low-shrinkage oil***: The associated properties of this type of crude oil are:
   - Oil formation volume factor less than 1.2 bbl/STB.
   - GOR less than 200 scf/STB.
   - Oil gravity less than 35° API.
   - Black or deeply colored.
3. ***Volatile crude oil***: The other characteristic properties of this oil include:
   - Oil formation volume factor greater than 1.5 bbl/STB.
   - GORs between 2000 and 3000 scf/STB.
   - Oil gravities between 45° and 55° API.

  - Lower liquid recovery of separator conditions.
  - Greenish to orange in color.

4. ***Near-critical crude oil*: ** The near-critical crude oil is characterized by a high GOR, in excess of 3000 scf/STB, with an oil formation volume factor of 2.0 bbl/STB or higher. The compositions of near-critical oils usually are characterized by 12.5–20 mol% heptanes-plus, 35% or more of ethane through hexanes, and the remainder methane. It should be pointed out that nearcritical oil systems essentially are considered the borderline to very rich gas-condensates on the phase diagram.

The liquid-shrinkage curves, given in Fig. (3.9), shows the shrinkage characteristics of all categories of crude oils.

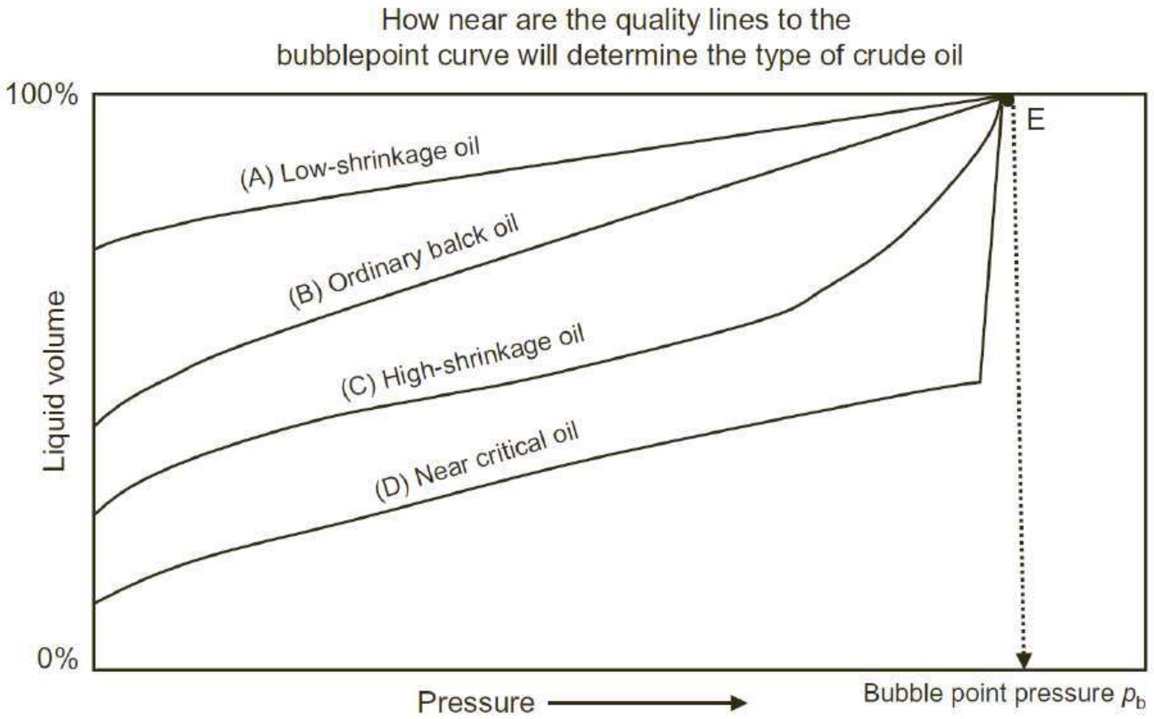


Fig. (3.9): Liquid shrinkage curves for crude oil systems.

The ternary diagram shown in Fig. (3.10) with equilateral triangles can be conveniently used to roughly define the compositional phase boundaries that separate different types of hydrocarbon systems.

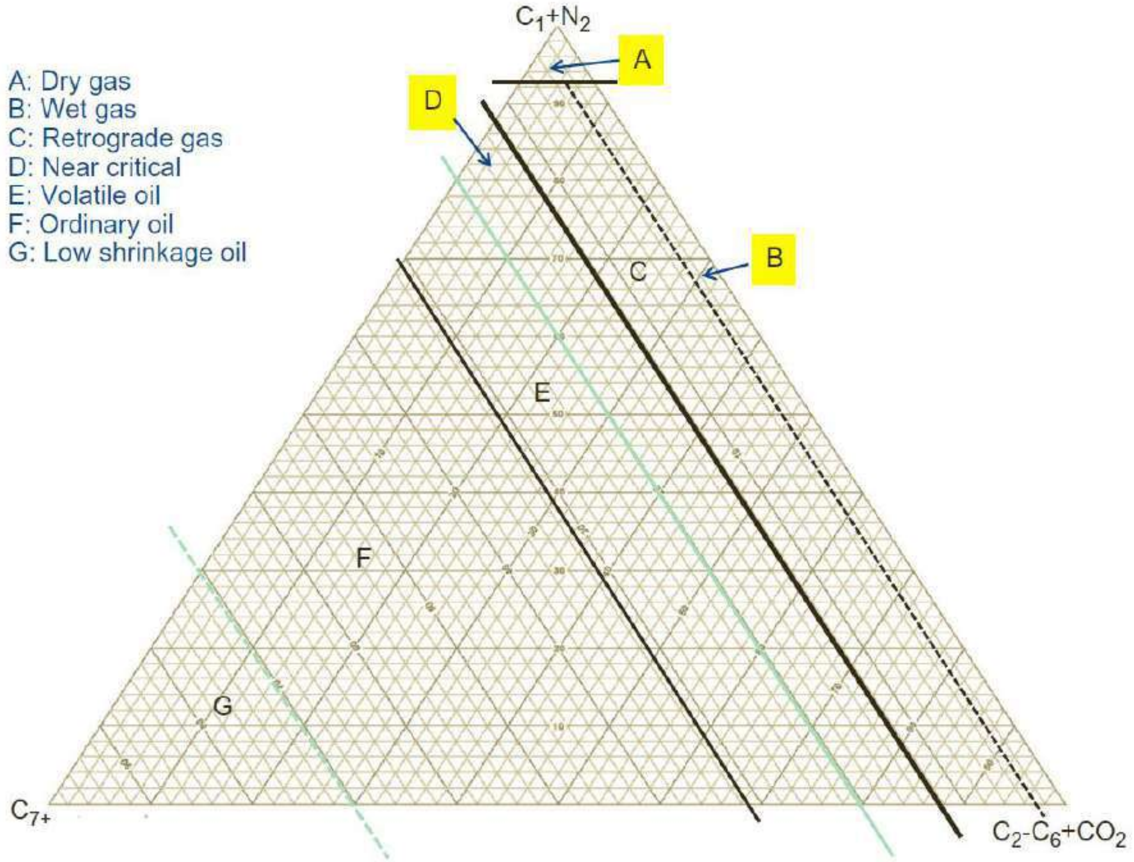


Fig. (3.10) Compositions of various reservoir fluid systems.

3.4 Laboratory Analysis of Reservoir Fluids:

Accurate laboratory studies of PVT and phase-equilibria behavior of reservoir fluids are necessary for characterizing these fluids and evaluating their volumetric performance at various pressure levels by conducting laboratory tests on a reservoir fluid sample. The amount of data desired determines the number of tests performed in the laboratory; however, for a successful PVT analysis it requires fluid samples that represent the original hydrocarbon in the reservoir. Fluid sampling for laboratory PVT analysis must be collected immediately after the exploration wells are drilled to properly characterize the original hydrocarbon in place. Sampling techniques depend on the reservoir.

3.4.1 Well Conditioning:

Proper well conditioning is essential to obtain representative samples from the reservoir. The following procedure is recommended in conditioning of an oil well for subsurface sampling. Before collecting fluid samples, the following guidelines and well condition consecutive steps must be considered:

1. The tested well should be allowed to produce for a sufficient amount of time to remove the drilling fluids, acids, and other well stimulation materials.
2. After the cleaning period, the flow rate should be reduced to one-half the flow rate used during the cleaning period.
3. The well should be allowed to flow at this reduced rate for at least 24 h. However, for tight formation or viscous oil, the reduced flow-rate period must be extended to 48 h or higher to stabilize the various parameters monitored.
4. The pressure drawdown should be controlled and remain low to ensure that bottom-hole pressure does not fall below the saturation pressure.
5. During the reduced flow-rate period, the well should be closely monitored to establish when the wellhead pressure, production rate, and GOR have stabilized.

3.4.2 Fluid Sampling:

The main objective of a successful sampling process is to obtain representative fluid samples for determining PVT properties. For proper definition of the type of reservoir fluid and to perform a proper fluid study, the collection of reservoir fluid samples must occur before the reservoir pressure is allowed to deplete below the saturation pressure of the reservoir fluid. Therefore, it is essential that the reservoir fluid samples be collected immediately after the hydrocarbon discovery, and the only production before sampling be as a result of well cleanout and to remove contamination. The critical steps in any successful sampling program include:

- Avoiding two-phase flow in the reservoir; that is, bottom-hole pressure below the saturation pressure.
- Minimizing fluid contamination introduced by drilling and completion fluids.
- Obtaining adequate volumes.
- Preserving sample integrity.

There are three basic methods of sample collection:

- Subsurface (bottom-hole) sampling.
- Surface (separator) sampling.
- Wellhead sampling.

1. Subsurface sampling uses a sampler tool (eg, Schlumberger Repeat Formation Testing “RFT” or Modular Dynamic Testing tool “MDT”) that is run on a wireline to the reservoir depth. After lowering the sampling chamber to the top of the producing zone, the chamber is opened hydraulically or via an electronic signal from the surface. The crude oil is allowed to flow at a very low flow rate into the tool at constant pressure to avoid the liberation of the solution gas. A piston seals the chamber and the tool is brought to the surface.

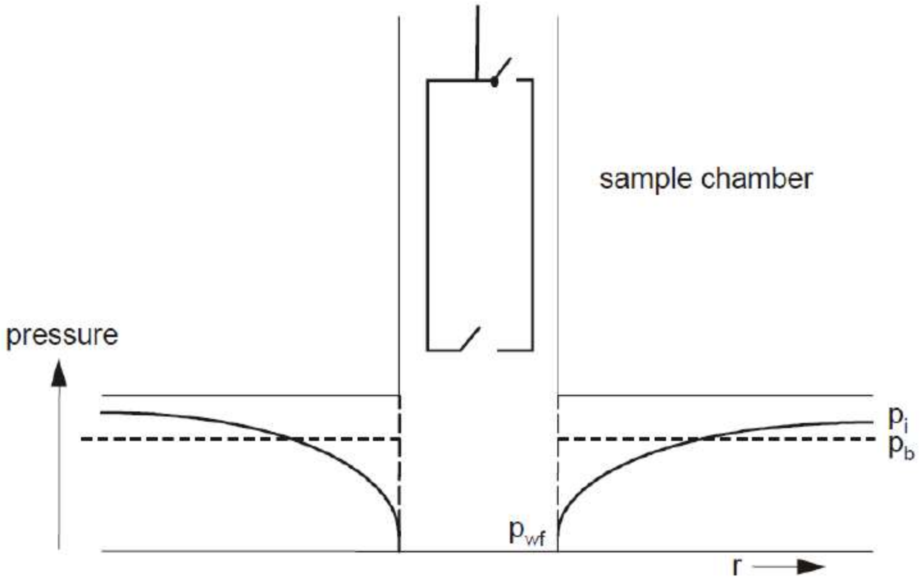


Fig. (3.11): Subsurface collection tool of PVT sample.

2. Surface sampling involves taking samples of the gas and liquid flowing through the surface separator, as conceptually shown in Fig. (3.12), and recombining the two fluids in an appropriate ratio in such a way that the recombined sample is representative of the overall reservoir fluid stream. The oil and gas samples are sent separately to the laboratory, where they are recombined before the PVT analysis is performed. Recombining separator samples requires thorough understanding of the relationship between gas solubility "R_S" and GOR.
3. Wellhead sampling involves the collection of a fluid sample at the surface from the wellhead itself or from the flow line or the upstream side of the choke manifold, provided that the fluid is still in one-phase condition. This option is restricted to wells producing dry gas, very low GOR oils (ie, heavy oils), and highly undersaturated reservoir fluids.

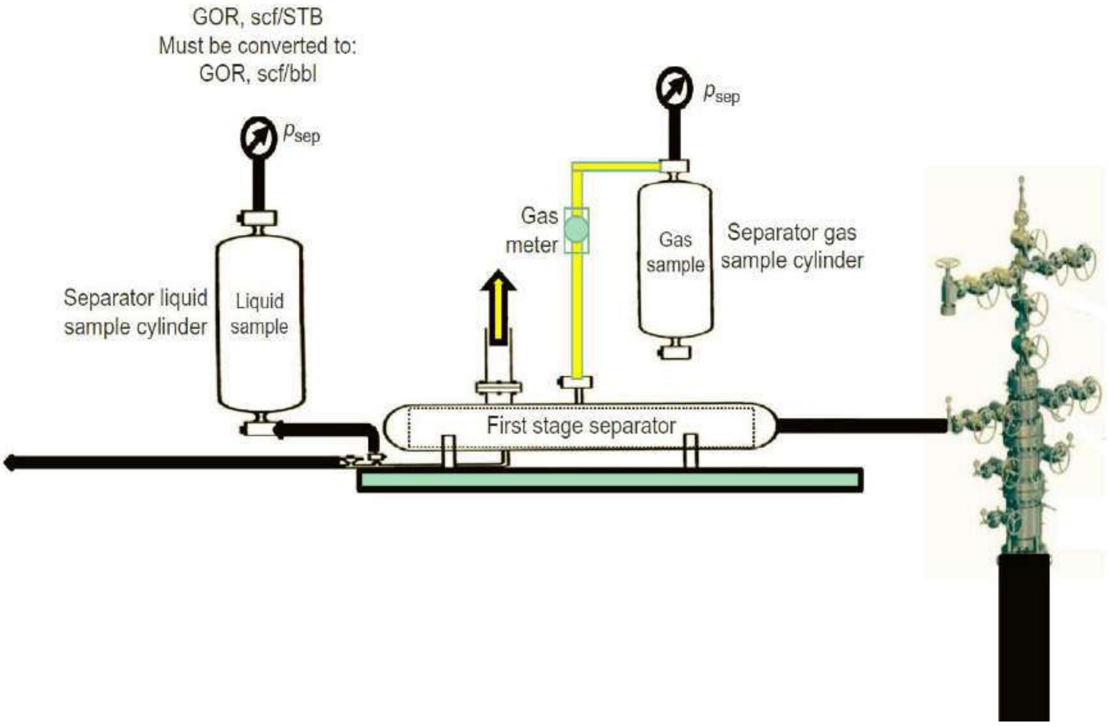


Fig. (3.12): Separator samples.

3.4.3 PVT Laboratory Tests:

PVT tests are conducted in a "PVT CELL" and designed to study and quantify the phase behavior and properties of a reservoir fluid for the use in a numerical simulator or performing material balance applications. In general, there are two types of laboratory tests that are traditionally performed on hydrocarbon samples:

1. ***Routine laboratory tests*:** Several laboratory tests are routinely conducted to characterize the reservoir hydrocarbon fluid, including:
   a. Compositional analysis of the hydrocarbon system (chromatographic test), (see section 3.4.4).
   b. Constant composition expansion (equilibrium test), (see section 3.4.5).
   c. Differential liberation test, (see section 3.4.6).
   d. Separator tests, (see section 3.4.7).
   e. Constant volume depletion.

The b and c tests are very important in distinguishing between the high shrinkage oil from the low shrinkage oil and retrograde condensation oil as it will be shown later (see sections 3.4.6 & 3.4.7).

2. ***Special laboratory PVT tests*:** These types of tests are performed for very specific applications. If a reservoir is to be depleted under miscible gas injection or gas cycling scheme, the following tests may be performed:
   a. Slim-tube test.
   b. Swelling test.
   c. Multiple forward contact experiment.
   d. Multiple backward contact experiment.
   e. Flow assurance tests.

3.4.4 Compositional Analysis of the Reservoir Fluid (chromatographic Test):

An important test on all reservoir samples is the determination of the fluid composition. Compositional analysis generally refers to the measurement of the distribution of hydrocarbons and other components present in oil and gas samples. The components of petroleum and petroleum products number in the tens of thousands. They range in molecular weight from methane (16) to very large uncharacterized components with molecular weight in the thousands. Analysis of the hydrocarbon mixture is complicated not only by the enormous number of components, but also by the large variation in compound concentrations. Using modern chromatography techniques (as shown in figure (3.13)), samples are analyzed to determine the breakdown of the components in the sample. Gas chromatography (GC) is a technique for separating and identifying components of hydrocarbon mixtures.

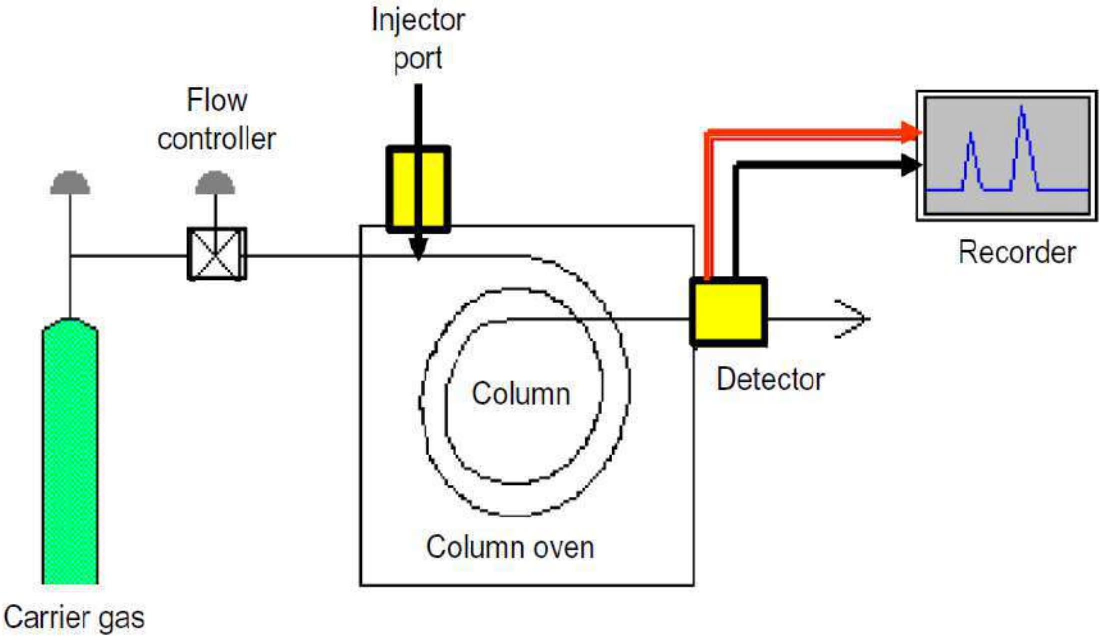


Fig. (3.13): Schematic of GC main components.

3.4.5 Constant-Composition Expansion Tests (Equilibrium Test):

Equilibrium tests are conventionally performed on all types of reservoir fluids to obtain a better understanding of the pressure-volume relationship of these hydrocarbon systems. The equilibrium test can also be referred to by different names, including *pressure-volume relations*, *constant mass expansion*, *flash liberation*, *flash vaporization*, or *flash expansion*. The equilibrium test is performed on crude oil systems for the purposes of determining:

- Saturation pressure.
- Relative volume "V_{rel}".
- Isothermal compressibility coefficient "C_o" above p_b.
- Oil density "ρ_o" at and above p_b.
- The traditional Y-function.
- The extended Y-function "Y_{EXT}".
- Oil FVF "B_o" above p_b.
- Two-phase formation factor B_t below p_b.

The experimental procedure involves placing a hydrocarbon fluid sample in a visual PVT cell at reservoir temperature that is held constant during the experiment. To ensure that the hydrocarbon sample exits in a single phase, the sample is pressurized to a much higher pressure than the initial reservoir pressure. The pressure is reduced isothermally in steps by removing mercury from the PVT cell and the total hydrocarbon volume "V_t" is measured at each pressure (see Fig. (3.14)). The combination of successive reductions in the pressure and the measurement of total fluid volume "V_t" are performed at a equilibrium; that is, no gas or liquid is removed from the PVT cell at any time throughout the experiment. As the pressure approaches the saturation pressure, the total volume continuously increases due to the expansion of the single-phase crude oil.

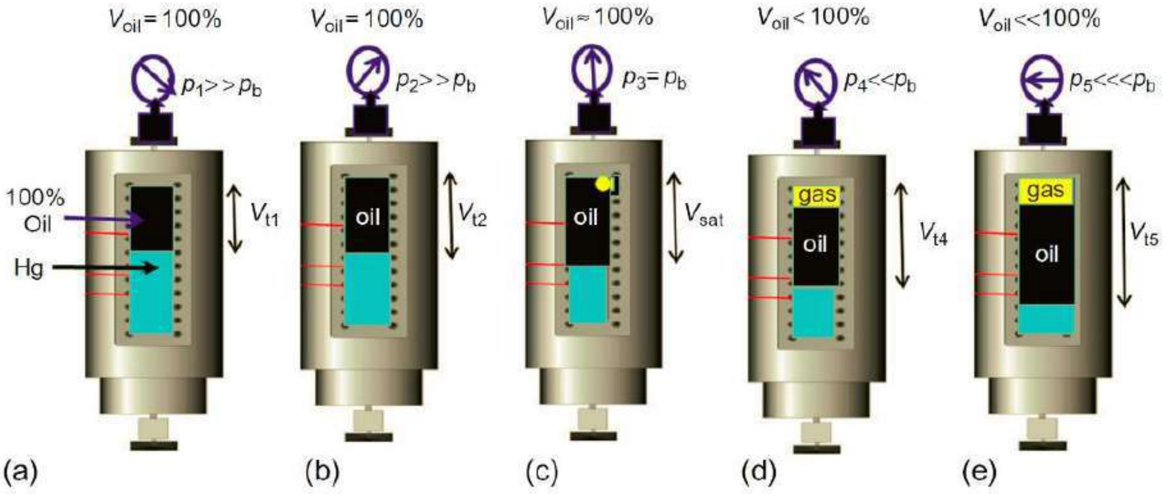


Fig. (3.14): Constant composition expansion test.

Reaching the bubble point pressure marks the appearance and start of liberation of the solution gas. The saturation pressure value is identified by visual inspection and by the point of discontinuity on the plot "pressure-volume" curve, as shown in Fig. (3.15). The saturation pressure and corresponding volume are recorded and designated as "p_{sat}" and "V_{sat}", respectively. The volume at the saturation pressure "V_{sat}" is used as a reference volume and the total volume "V_t" as a function of pressure is reported relative to V_{sat} as a ratio of "V_t/V_{sat}." This volume is termed the relative volume and is expressed mathematically by the following equation:

$$V_{rel} = \frac{V_t}{V_{sat}} \qquad eq.(3.1)$$

where:

V_{rel} = relative volume.
V_t = total hydrocarbon volume.
V_{sat} = volume at the saturation pressure.

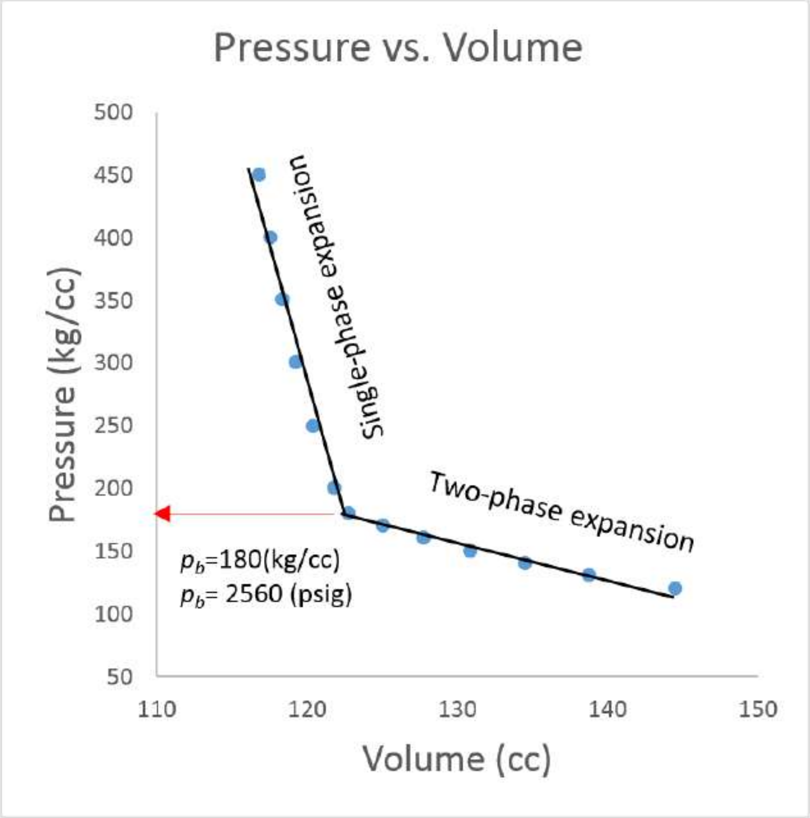


Fig. (3.15): Equilibrium test pressure-volume relationship.

It should be pointed out the change in the slope is related and attributed to the fluid compressibility resulting from the transition from the single phase to the two-phase state. In the single liquid phase region, the liquid compressibility is relatively small resulting in a steep decline in the pressure with a small increase in the liquid volume. As the pressure approaches the bubble point pressure and initiation of liberation solution gas, the evolved gas is characterized by a high compressibility that causes less reduction in the pressure resulting in a sharp break in the slope between the single- and two-phase regions.

- ***Oil density from the equilibrium test:***

  The density is determined directly from mass "m" of the fluid sample and fluid volume "*V*" measurements. Designated ρ_{sat} for the fluid density at the saturation pressure, it can be calculated from:

$$\rho_{sat} = \frac{m}{V_{sat}} \qquad eq. (3.2)$$

  Above the bubble point pressure, the density of the oil can be calculated by using the recorded relative volume at any pressure, to give:

$$\rho = \frac{(m/V_{sat})}{\left(V_{p,T}/V_{sat}\right)} = \frac{\rho_{sat}}{V_{rel}} \qquad eq.(3.3)$$

where:

ρ = density at any pressure above the saturation pressure.
ρ_{sat} = density at the saturation pressure.
V_{rel} = relative volume at the pressure of interest.
$V_{p,T}$ = volume at the pressure "p" and reservoir (cell) temperature.

- ***Isothermal compressibility coefficient from equilibrium test:***
  The instantaneous isothermal compressibility coefficient "c_o" is defined by the following expression:

$$c_o = -\frac{1}{V}\frac{\Delta V}{\Delta p}$$

$$c_o = \frac{-1}{(V_{rel})_2}\frac{(V_{rel})_1 - (V_{rel})_2}{p_1 - p_2} \qquad eq.(3.4)$$

Example 3.1:

Using the following data of CCE test to:

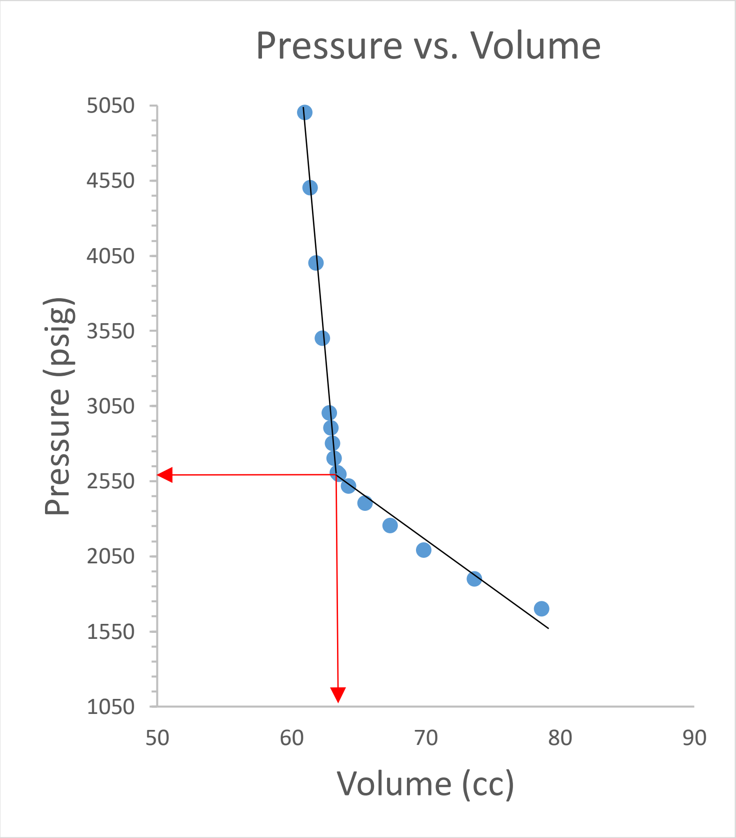
Pressure (psig)	Total Volume (cc)
5000	61.030
4500	61.435
4000	61.866
3500	62.341
3000	62.866
2900	62.974
2800	63.088
2700	63.208
2605	63.455
2591	63.576
2516	64.291
2401	65.532
2253	67.400
2090	69.901
1897	73.655
1698	78.676

1. Find the saturation pressure.
2. Find the relative volume.

3. Calculate ρ_{sat} if m=0.094 lb.

Solution:

1. Plot pressure vs. volume to find the saturation pressure:



From the pressure volume relationship, the saturation pressure is (2570 *psig*) and the saturation volume is (63.776 cc).

2. The values of relative volume are calculated using Eq. (3.1):

$$V_{rel} = \frac{V_t}{V_{sat}} = \frac{V_t}{63.776}$$

the results are shown in the table below:

Pressure (psig)	Total Volume (cc)	V_{rel}
5000	61.030	0.95694305
4500	61.435	0.963293402
4000	61.866	0.97005143
3500	62.341	0.977499373
3000	62.866	0.98573131
2900	62.974	0.987424737
2800	63.088	0.989212243
2700	63.208	0.991093828

2605	63.455	0.994966759
2591	63.576	0.996864024
2570	63.776	1
2516	64.291	1.008075138
2401	65.532	1.027533869
2253	67.400	1.056823884
2090	69.901	1.096039262
1897	73.655	1.15490153
1698	78.676	1.233630206

3. V_{sat} = 63.776 cc

$$\rho_{sat} = \frac{m}{V_{sat}} = \frac{0.094\ lb}{63.776\ cc\ (35.516 \times 10^{-6})\frac{cu\ ft}{cc}} = 41.78\ lb/cu\ ft$$

- **Traditional Y-Function:**

The relative volume data frequently require smoothing to correct for laboratory inaccuracies in measuring the total hydrocarbon volume, particularly below the saturation pressure and also at lower pressures. A dimensionless compressibility function, commonly called the Y-function, is used to smooth the values of the relative volume "V_{rel}" below the saturation pressure. The Y-function in its mathematical expression is ONLY defined below the saturation pressure; that is, $p<p_b$, and given by the following equation:

$$\boldsymbol{Y = \frac{p_b - p}{p\left[\frac{V_t}{V_b} - 1\right]}}$$

where:

p_b= bubble point or saturation pressure.
p = reservoir pressure which Y is being calculated.
$\frac{V_t}{V_b}$ = relative total volume at the pressure p.

The Y-function is computed and plotted as a function of the pressure p, as shown in the figure (3.16) below. For more details see (Amyx, et al, 1960, p384).

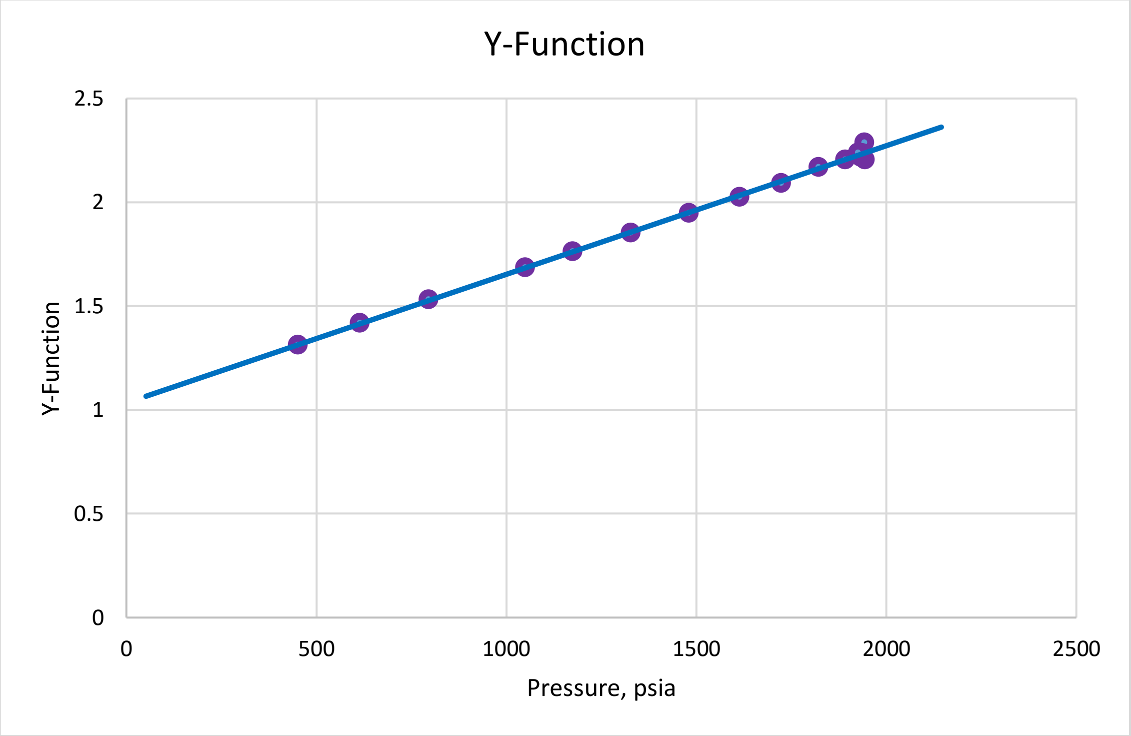


Figure (3.16): smoothing of relative total volume data.

3.4.6 Differential Liberation Test (DL):

The differential liberation (DL) test is perhaps the most common laboratory test that is conducted on crude oil samples. The DL process is summarized in the following steps and is illustrated schematically in Fig. (3.17):

- A crude oil sample is placed in a visual PVT cell at its bubble point pressure and reservoir temperature.
- The cell pressure is reduced in stages, usually 10–15 pressure increments, from the saturation pressure to atmospheric pressure.
- The liberated gas at each pressure stage is allowed to reach equilibrium with the cell remaining oil.
- The volume of the two phases (ie, remaining oil volume "V_L" and liberated gas volume "V_{gas}") are measured and recorded at each pressure level.
- The liberated gas is then displaced at constant cell pressure to a metering device (e.g., gasometer) and the measured volume is corrected to SC and designated to as "$(V_{gas})_{sc}$".
- The above depletion process is repeated at constant reservoir temperature until a pressure close to atmospheric pressure is reached. Having arrived at the last stage, the volume of the residual oil is first measured at last cell

conditions, then corrected to SC (ie, 14.7 psia and 60°F), and designated as “V_{Lsc}”.

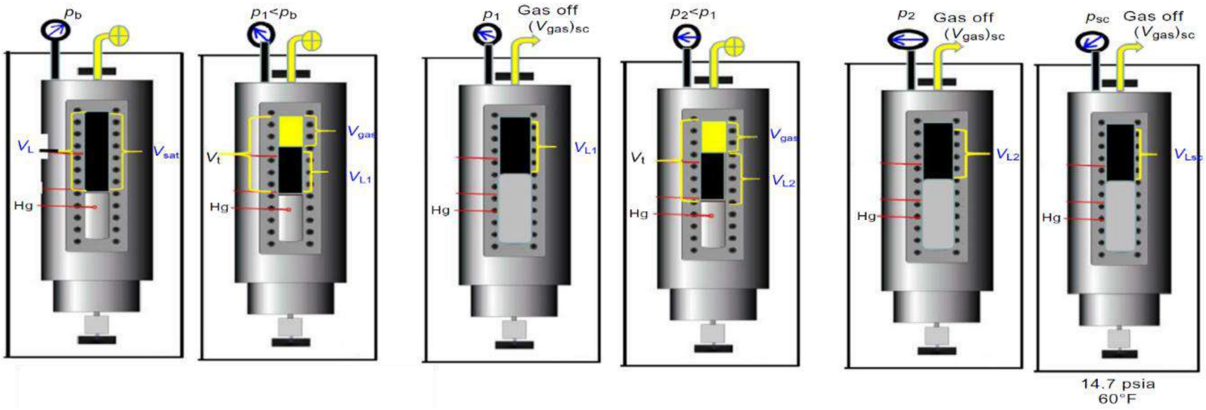


Fig. (3.17): Differential vaporization test at constant temperature “T.”

The following nomenclatures are used to describe the various hydrocarbon volume measurements associated with each pressure-depletion stage during the DL experiment:

P = cell pressure.
T = cell temperature.
V_t = total combined volume of the liberated gas and remaining oil at $p\&T$.
V_L = remaining oil volume at $p\&T$.
V_{Lsc} = remaining oil volume at $p_{sc}\&T_{sc}$.
V_{gas} = liberated gas volume at $p\&T$.
$(V_{gas})_{sc}$ = liberated gas volume at $p_{sc}\&T_{sc}$.

It should be noted that the remaining oil at each depletion stage is subjected to continual compositional changes as it becomes progressively richer in the heavier components. In addition, the above described type of DL is characterized by a varying composition of the total hydrocarbon system; whereas the total system composition is kept unchanged during the equilibrium test.

The experimental data obtained from the DL test include:

- The differential oil FVF “B_{od}”.
- The differential gas solubility; that is, solution GOR, “R_{sd}”.
- Gas FVF “B_g” in ft^3/scf.
- Gas compressibility factor “Z”.
- Total FVF “B_{td}”.
- Composition of the liberated gas.

- Gas specific gravity.
- Density of the remaining oil as a function of pressure.

The combined data from the DL and equilibrium tests are very important to distinguish between low-shrinkage oil and high-shrinkage oil as be shown in the figures (3.18) & (3.19). In Fig. (3.18) under equilibrium conditions, a greater quantity of heavy components are progressively pulled into gas by the light gas components; thus, oil shrinkage is greater. Lower shrinkage by DL under these conditions is caused by gas being removed as it’s formed, thereby preventing further attraction of light gas molecules to heavy liquid molecules and minimizing the quantity of liquid pulled into the gas phase.

A comparison between equilibrium and DL of a very high shrinkage oil is presented in Fig. (3.19). In this case the difference is reversed from that of low shrinkage oil. More gas is formed in the high pressure range than with low shrinkage oil. This is caused by a large quantity of intermediates being pulled into the gas phase (through great attraction to the densely spaced light gas molecules and through high kinetic energy of the intermediates). Once the large quantity of intermediates is removed from the system by DL, they have no further influence on gas liberation and oil shrinkage; therefore, oil shrinkage remains high through lower pressure ranges. If the gas is not removed, as in flash or equilibrium liberation, some intermediate gas molecules may condense because there attraction to light gas molecules is reduced as pressure drops and the light gas molecules become farther apart; thus, vaporizing tendency of high shrinkage oil may be reversed until shrinkage is less under equilibrium conditions than under the differential process.

Shrinkage curves of both low and high shrinkage oil illustrate that, near atmospheric temperature and low pressure conditions, shrinkage of oil greatly increases and presents a problem if maximum liquid volume is to be produced to the stock tank as saleable material.

Oil of all degrees of shrinkage will shrink less to the stock tank if the material is first passed to a separator at some elevated pressure and free gas, formed down to that pressure and temperature, is separated. Also, there is an optimum separator pressure condition where minimum shrinkage of oil is obtained.

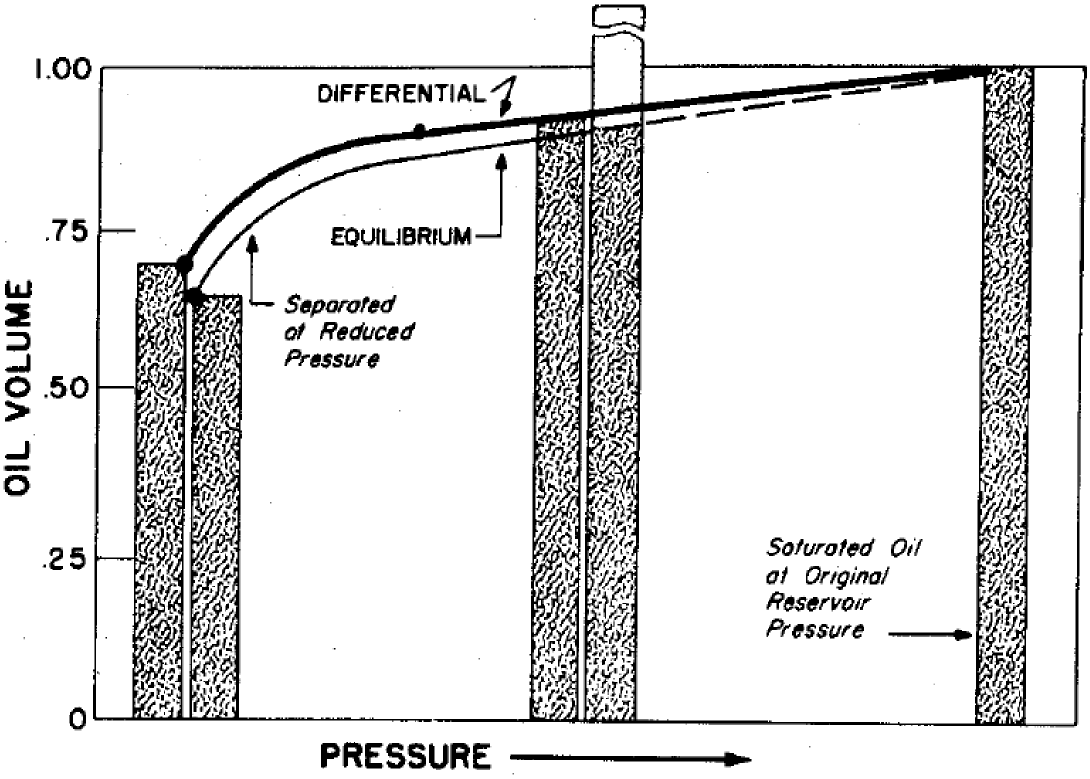


Fig. (3.18): Differential and equilibrium shrinkage of low shrinkage oil, (Clark, 1969).

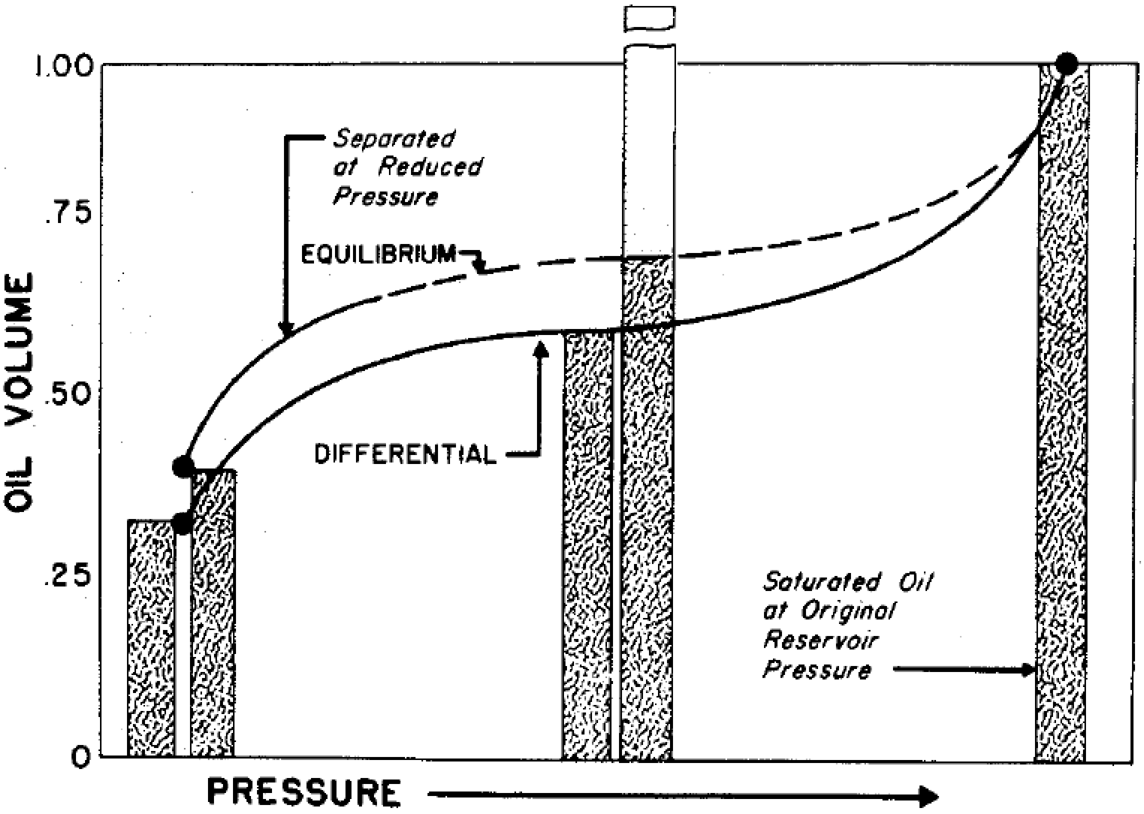


Fig. (3.19): Differential and equilibrium shrinkage of high shrinkage oil, (Clark, 1969).

Differential Gas Solubility “R_{sd}”:

The gas solubility (i.e., differential solution GOR, “R_{sd}”) at any pressure is defined as the volume of gas that is dissolved (remaining) in solution at the pressure per stock-tank barrel of the corrected residual oil. Mathematically, the differential solution GOR, R_{sd}, is calculated by dividing the volume of gas remaining (dissolved) in solution at the specified pressure by the residual oil volume.

$$R_{sd} = \frac{volume\ of\ remaining\ solution\ gas}{volume\ of\ residual\ oil}$$

$$R_{sd} = \frac{vol.of\ total\ gas\ removed - vol.of\ gas\ removed\ till\ the\ cell\ wanted}{vol.of\ residual\ oil}$$

Differential Oil FVFs “B_{od}”:

The differential oil FVFs, Bod (commonly called the relative oil volume factors), are calculated at all pressure levels by dividing the recorded oil volumes, V_L, by the volume of residual oil, V_{Lsc}:

$$B_{od} = \frac{volume\ of\ oil\ at\ any\ (p,T)}{volume\ of\ residual\ oil\ (at\ p_{sc},T_{sc})} = \frac{V_L}{V_{Lsc}} \qquad eq.(3.5)$$

Gas Deviation Factor “Z”:

Using the volume of the free gas at any pressure and the corresponding volume at SC, the gas deviation factor can be calculated by applying the following equation:

$$Z = \frac{p\left(V_{gas}\right)_{p,T}}{T}\frac{T_{sc}}{p_{sc}\left(V_{gas}\right)_{sc}} = \frac{V_R p_R T_{sc}}{V_{sc} p_{sc} T_R} \qquad eq.(3.6)$$

Gas FVF “B_g”:

The gas FVF is the defined ratio of the volume “$(V_{gas})_{p,T}$” occupied by n moles of gas at a specified pressure and temperature to the volume “$(V_{gas})_{sc}$” occupied by the same number of moles (i.e., n moles) at SC. Mathematically, this is defined by

$$B_g = \frac{\left(V_{gas}\right)_{p,T}}{\left(V_{gas}\right)_{sc}} = \frac{0.02827\ ZT}{p}; ft^3/scf \qquad eq.(3.7)$$

Differential gas oil ratio at the bubble point pressure “R_{sdb}”:

R_{sdb} value is constant in all steps of calculations.

$$R_{sdb} = \frac{vol.\,of\ total\ gas\ removed}{vol.\,of\ residual\ oil}$$

Total FVF “B_{td}” From DL Test:

The two-phase (total) FVF from DL test is determined by applying the definition of this property as expressed mathematically by Eq. (3.8). In terms of differential data, B_{td} is given by

$$B_{td} = B_{od} + (R_{sdb} - R_{sd})B_g \qquad eq.(3.8)$$

where:

R_{sdb} = gas solubility at the bubble point pressure, scf/STB.
R_{sd} = gas solubility at any pressure, scf/STB.
B_{od} = oil FVF at any pressure, bbl/STB.
B_g = gas FVF, bbl/scf.

Example 3.2:

The data from a differential vaporization on a black oil at (220^{O}F) are given below. Prepare a table of solution gas- oil ratios, relative oil volumes, and relative total volumes by this differential process. Also include Z- factors and formation volume factors of the increments of gas removed.

Pressure (psig)	Gas removed,* (cc)	Gas removed,** (scf)	Oil volume, (cc)	Incremental gas gravity
2620	-	-	63.316	-
2350	4.396	0.02265	61.496	0.825
2100	4.292	0.01966	59.952	0.818
1850	4.478	0.01792	58.528	0.797
1600	4.960	0.01693	57.182	0.791
1350	5.705	0.01618	55.876	0.794
1100	6.891	0.01568	54.689	0.809
850	8.925	0.01543	53.462	0.831
600	12.814	0.01543	52.236	0.881
350	24.646	0.01717	50.771	0.988
159	50.492	0.01643	49.228	1.213
0		0.03908	42.540	2.039
0		0.21256	39.572 at60°F	

* at 220°F and cell pressure.

**at 60°F and 14.65 psia (≅ 14.7).

Solution:

Calculations will be at (2100 and 1600 psig) as samples.

Step 1. calculation of differential solution GOR “R_{sd}” at (2100 psig):

$$R_{sd} = \frac{(0.21256 - 0.02265 - 0.019966)scf}{(39.572\, cc,\ residual\ oil)(6.29 \times 10^{-6}\ bbl/cc)} = 684\ scf/bbl$$

Step 2. calculation of differential solution GOR “R_{sd}” at (1600 psig):

$$R_{sd} = \frac{(0.21256 - 0.02265 - 0.01966 - 0.01792 - 0.01693)scf}{(39.572\, cc,\ residual\ oil)(6.29 \times 10^{-6}\ bbl/cc)} = 544\ scf/bbl$$

Step 3. calculation of "B_{od}" at (2100 psig):

$$B_{od}=\frac{59.952\ (cc, at\ reservoir\ p, T)}{39.572\ (cc, residual\ oil\ at\ standard\ conditions)}=1.515\ bbl/STB$$

Step 4. calculation of "B_{od}" at (1600 psig):

$$B_{od}=\frac{57.182\ (cc, at\ reservoir\ p, T)}{39.572\ (cc, residual\ oil\ at\ standard\ conditions)}=1.445\ bbl/STB$$

Step 5. calculation of Z-factor at (2100 psig):

$$z=\frac{V_{res}p_{res}T_{sc}}{V_{sc}p_{sc}T_{res}}$$

$$Z=\frac{(4.292\ cc\ \times 35.315\times 10^{-6}\ ft^3/cc)(2114.7psia)(520°R)}{(0.01966\ scf)(14.65\ psia)(680°R)}=0.851$$

Step 6. calculation of Z-factor at (1600 psig):

$$Z=\frac{(4.960\ cc\ \times 35.315\times 10^{-6}\ ft^3/cc)(1614.7psia)(520°R)}{(0.01693\ scf)(14.65\ psia)(680°R)}=0.872$$

Step 7. calculation of "B_g" at (2100 psig):

$$B_g=0.0282\frac{ZT}{p}$$

$$B_g=\frac{(0.0282)(0.851)(680)}{(2114.7)}=0.00771\ ft^3/scf$$

Step 8. calculation of "B_g" at (1600 psig):

$$B_g=\frac{(0.0282)(0.872)(680)}{(1614.7)}=0.01035\ ft^3/scf$$

Step 9. calculation of solution GOR at bubble point pressure "R_{sdb}":

$$R_{sdb}=\frac{vol.of\ total\ gas\ removed}{vol.of\ residual\ oil}$$

$$R_{sdb}=\frac{(0.21256\ scf)(5.615\ ft^3/bbl)}{(39.572\ cc)(35.315\times 10^{-6}\ ft^3/cc)}=854\ scf/bbl$$

Step 10. calculation of "B_t" at (2100 psig):

$$B_t = B_{od} + B_g(R_{sdb} - R_{sd})$$

$$B_t = 1.515\frac{bbl}{STB} + \frac{0.00771\,ft^3/scf}{5.615\ ft^3/bbl}\,(854 - 684)\frac{scf}{bbl} = 1.748\ bbl/STB$$

Step 11. calculation of "B_t" at (1600 psig):

$$B_t = 1.445\frac{bbl}{STB} + \frac{0.01035\,ft^3/scf}{5.615\ ft^3/bbl}(854 - 544)\frac{scf}{bbl} = 2.0164\ bbl/STB$$

The table below show the total calculations of all pressure values:

Pressure (psig)	***R$_{sd}$*** **(scf/bbl)**	**B_{od} (bbl/STB)**	**Z-factor**	**B_g (ft^3/scf)**	**B_t (bbl/STB)**
2620	854 = R_{sdb}	1.600			1.60002
2350	763	1.554	0.846	0.00686	1.66523
2100	684	1.515	0.851	0.00772	1.748665
1850	612	1.479	0.859	0.00883	1.859753
1600	544	1.445	0.872	0.01036	2.016848
1350	479	1.412	0.887	0.01246	2.244538
1100	416	1.382	0.903	0.01553	2.593997
850	354	1.351	0.922	0.02045	3.17193
600	292	1.320	0.941	0.02936	4.258547
350	223	1.283	0.965	0.05074	6.985549
159	157	1.244	0.984	0.10863	14.7302
0	0	1.075			
0 @ 60°F		1.000			

Example 3.3:

A DL test was conducted on a crude oil sample at (180°F). The sample exhibited a bubble point pressure of (3565 psig). The residual (remaining) oil volume was corrected to SC of 14.7 psia and 60°F to give (60 cm^3). The following measurements were obtained from a differential liberation (DL) test:

Pressure (psig)	**Total Volume (cc)**	**Liquid volume (cc)**	**Gas removed (scf)**
3565	99.29514779	99.29514779	0
3000	105.1180611	92.66229773	0.086792948
2400	100.8688861	87.02573049	0.07660421
1800	98.43652432	82.03645568	0.067170194
1200	100.8219751	77.70667102	0.061887145
600	118.801706	73.45739134	0.059245621
200	168.9280437	70.05558226	0.043396474
0	63.85313132	63.85313132	0.061509785
0 psig, 60 °F		60	∑0.456606

Using the above reported DL measurements data to calculate:

1. Oil FVF "B_{od}".
2. Solution GOR "R_{sd}".
3. Gas FVF "B_g".
4. Z-factor.

Solution:

1. As a sample, calculating B_{od} at 3000 psig:

$$B_{od} = \frac{92.66229773}{60} = 1.544371629$$

2. Calculate the differential solution GOR "R_{sd}.":

   Step 1. Convert the volume of the corrected residual oil volume as expressed in cubic centimeter to stock-tank barrel:

$$Stock - tank\ volume = \frac{60\ cm^3}{(30.48\ cm/ft)^3(5.615\ ft^3/bbl)} = 0.000377\,STB$$

   Step 2. Calculate R_{sd} by dividing the volume of the dissolved gas at each pressure by the volume of residual oil (0.000377 STB), as a sample calculating R_{sd} at 3000 psig:

$$R_{sd} = \frac{(0.456606 - 0.086792948)scf}{0.000377\ \text{STB}} = 980\ scf/STB$$

3. As a sample, calculating B_g at 3000 psig and 180°F:

Step 1. volume of the free gas at 3000 psig = total volume - volume of the liquid:

$$\left(V_{gas}\right)_{3000,180} = \frac{(105.1180611) - (92.66229773)}{(30.48)^3} = 0.00044\, ft^3$$

Step 2. Calculate B_g at 3000 psig and 180°F by applying the definition of this gas property:

$$B_g = \frac{0.00044}{0.086792948} = 0.005068\ {ft^3}\big/{scf}$$

4. As a sample calculating Z-factor at 3000 psig:

$$Z = \frac{V_{res} p_{res} T_{sc}}{V_{sc} p_{sc} T_{res}}$$

$$Z = \frac{(3000 + 14.7)(0.00044)(520)}{(180 + 460)(14.7)(0.08679)} = 0.8445$$

The table below show the total calculations of all pressure values:

Pressure (psig)	R_{sd} (scf/bbl)	B_{od} (bbl/STB)	**Z-factor**	B_g (ft^3/scf)	B_t (bbl/STB)
3565	1211	1.6549			1.655
3000	980	1.5443	0.84448	0.005068	1.752
2400	777	1.4504	0.85174	0.006382	1.943
1800	599	1.3672	0.864839	0.008622	2.306
1200	435	1.2951	0.885583	0.01319	3.116
600	278	1.2242	0.918313	0.027028	5.711
200	163	1.1675	0.954804	0.080459	16.17
0	0	1.0642			
0 @ 60 °F		1.000			

3.4.7 Separator Tests:

Separator tests are conducted to determine the changes in the volumetric behavior of the reservoir fluid as the fluid passes through the separator (or separators) and then into the stock tank (as shown in Fig. (3.20)). The resulting volumetric behavior is influenced to a large extent by the operating conditions, i.e., pressures and temperatures, of the surface separation facilities. The primary objective of conducting separator tests, therefore, is to provide the essential laboratory information necessary for determining the optimum surface separation conditions, which in turn will maximize the stock-tank oil production. In addition, the results of the test, when appropriately combined with the differential liberation test data, provide a means of obtaining the PVT parameters (B_o, R_s, and B_t) required for petroleum engineering calculations. These separator tests are performed only on the original oil at the bubble point.

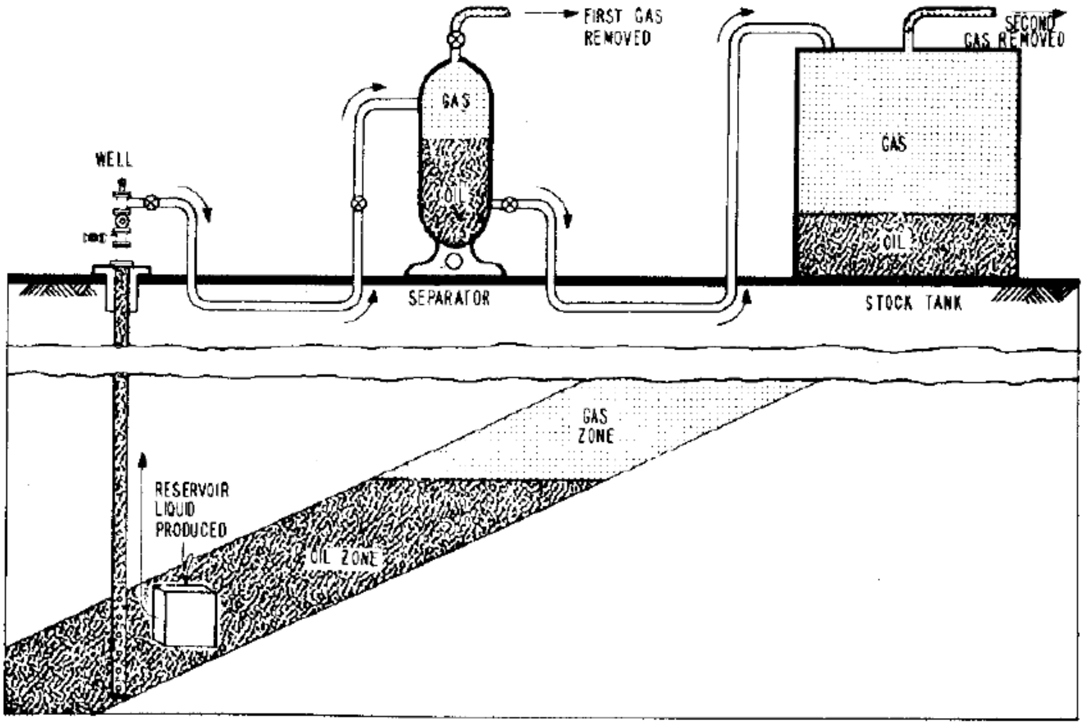


Fig. (3.20): Single-stage separation system (one separator), (Clark, 1960).

The test involves placing a hydrocarbon sample at its saturation pressure and reservoir temperature in a PVT cell. The volume of the sample is measured as V_{sat}. The hydrocarbon sample is then displaced and flashed through a laboratory multistage separator system—commonly one to three stages. The pressure and temperature of these stages are set to represent the desired or actual surface separation facilities. The gas liberated from each stage is removed and its specific gravity and volume at standard conditions are measured. The volume of the remaining oil in the last stage (representing the stock-tank condition) is measured and recorded as $(V_o)_{st}$. These experimental measured data can then be used to determine the oil formation volume factor and gas solubility at the bubble-point pressure as follows:

$$B_{ofb} = \frac{V_{sat}}{(V_o)_{st}} \qquad eq.(3.9)$$

$$R_{sfb} = \frac{(V_g)_{sc}}{(V_o)_{st}} \qquad eq.(3.10)$$

where:

B_{ofb} = bubble-point oil formation volume factor, as measured by flash liberation, bbl of the bubble-point oil/STB.

R_{sfb} = bubble-point solution gas-oil ratio as measured by flash liberation, scf/STB.

$(V_g)_{sc}$ = total volume of gas removed from separators, scf.

Almost without exception, oil production is attended by gas that comes from solution in the oil at reservoir conditions. This gas, when liberated from solution in the oil, cause the oil to shrinkage in volume. The term “shrinkage factor” is given to the ratio of the amount of oil recovered in the stock tank from one unit volume of produced reservoir oil. The greater the amount of gas evolving from the oil, the greater will be the shrinkage of the oil and the smaller will be the volume of saleable oil at the surface from a given volume of produced reservoir oil. Both the amount of gas which a given reservoir oil will liberate and the resulting shrinkage factor will depend upon the composition of the reservoir oil and the temperature and pressure under which the oil and gas exist at separation.

3.4.7.1 Differential and Flash Separation:

To best explain the various separation processes, it is convenient to consider the component hydrocarbon as “lights”, “intermediates” and “heavies” (as explained earlier in the section (3.1)). Mechanically, there are two types (or methods) of gas-oil separation: “differential” separation and “flash” or “equilibrium” separation. The light components may be removed little by little from the remaining oil mixture as pressure on the oil is reduced; this is differential separation. If the gas is separated in this manner, a maximum amount of heavy and intermediate components will remain in the liquid and minimum shrinkage will result. This occurs because gas liberated earlier at higher pressures is not present at lower pressures to attract the intermediates and heavies and pull them into the gas phase in excessive quantities. Flash, or equilibrium separation, is accomplished by keeping all liberated gas in contact with the liquid until its instantaneous removal at the final separation pressure. A maximum proportion of intermediate and heavy components are attracted into the gas phase by this process and result in maximum oil shrinkage. Less oil shrinkage and, thus, greater stock tank oil recovery will normally occur by differential separation processes than will occur by flash separation.

3.4.7.2 Single-Stage Separation

It can be seen from the foregoing that the best mechanical separation system for retaining absolute maximum recovery of stock tank liquid would be one which would include a large number of separators. In such a system, the oil would be produced to a separator set at the highest possible producible pressure. The separator liquid obtained therein would be passed to successively lower pressure separators until the remaining liquid finally would emerge from a very low pressure separator into the stock tank. This would result in maximum oil recovery because such a procedure would approach the differential liberation process. Quite obviously, such a large separation system would require an extremely high investment, and for that reason, it is impractical. In practice, however, with most average shrinkage oils, only one separator is utilized in the system. Such a system is termed “single-stage separation” (see Fig. (3.18)).

Single-stage separation of low shrinkage oils generally will recover an amount of oil very near to that which might be expected under conditions of many stages of separation.

In a single-stage separation system, the gas is removed in two batches from the original liquid as pressure drops. The first batch of gas is removed in the separator and the second in the stock tank. By removing some gas in a separator the quantity of intermediate and heavy components attracted to the lights of the gas are minimized and final oil shrinkage is lessened so that more oil is retained in the tank from a given amount of reservoir oil.

It has been found that separator pressure can be adjusted to some particular point which will result in less shrinkage to the oil than would occur at other separator pressures.

1. **If separator pressure is set high**, a small amount of gas will be removed in the first step at the separator, and a large amount will be removed in the second step at the stock tank. Large quantities of intermediates and heavier components will be lost from the liquid to gas in the stock tank where a large quantity of gas is removed as a second “batch”; therefore, high shrinkage with low stock tank oil recovery will be obtained.

2. **If the separator pressure is set very low**, a large amount of gas will be separated in the first step at the separator, with a small amount of gas being separated in the second step from the stock tank. Here again, large quantities of intermediates and heavier components will be lost from the liquid to the gas in the separator where a large quantity of gas is removed as a first batch; therefore, high shrinkage with low stock tank oil recovery again will result.
3. **If the separator pressure is adjusted to more properly balance** the amounts of gas removed, at the separator and at the stock tank, a minimum of intermediates and heavier components will be lost from the liquid to the gas; therefore, lowest shrinkage will occur to the oil and the highest stock tank oil recovery will be obtained from a particular reservoir liquid. The separator pressure found to accomplish this is termed "***optimum separator pressure***" (see Fig. (3.21)).

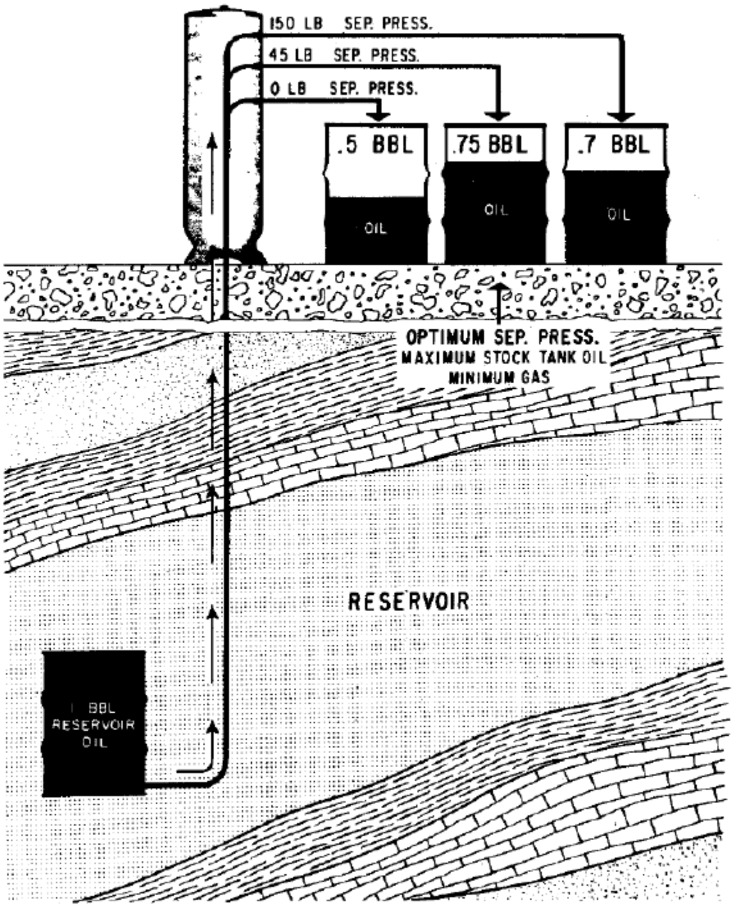


Fig. (3.21): Optimum single-stage oil separation for a given oil, (Clark, 1960).

3.4.7.3 Two-Stage Separation:

Some reservoir oils are classified high shrinkage oils. Although this classification is arbitrary, these oils have higher shrinkage than usual because in the original mixture there are greater quantities of lighter or intermediate components present which vaporize to greater extent when pressure on the mixture is lowered. Therefore, it may be desirable to employ two separator in the system to obtain optimum economic separation. Such a system is known as “two-stage separation” (see Fig. (3.22)). In the case of high shrinkage oil, the two stages of separation merely permit the gas to be removed at three points: first in the high pressure separator, second in the low pressure separator and third in the stock tank. With the gas divided into three batches, there is less tendency for these smaller amounts, separated under their respective conditions of temperature and pressure, to attract and remove the intermediate and heavier (and more valuable) components from the liquid.

In this system, there are optimum pressure conditions for both high and low pressure separators just as there are for one separator system.

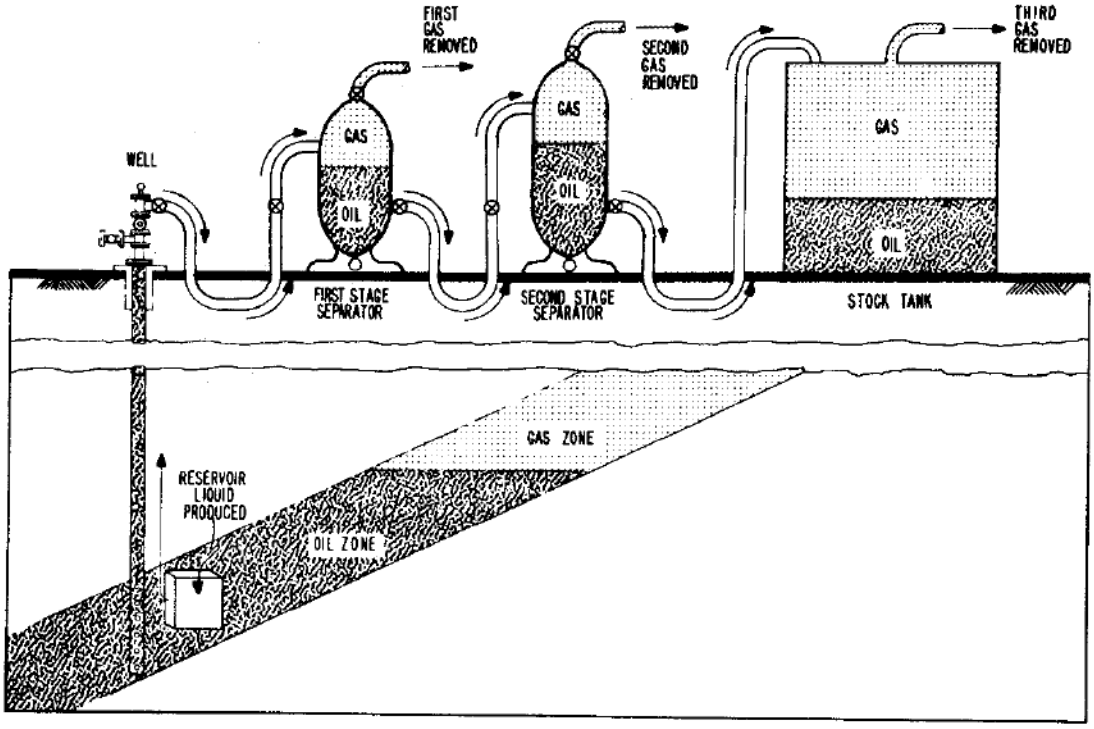


Fig. (3.22): Two-stage separation system (two separators), (Clark, 1960).

3.4.7.4 Relationship of Gasoline Plants in Separation:

It is not always economical to separate oil and gas by optimum separation. In many cases (for example, in high shrinkage oils and especially for oils where large reserves are concerned), it may be most economical to produce the separated gas to a gasoline plant and remove the last remaining quantities of intermediate and heavy materials in the form of natural gasoline (even after considerable investment for gasoline plant equipment has been made). If such is true, it may be economically justifiable to operate the separators under vacuum conditions to further vaporize intermediate and heavy materials into the gas separated at the separator. This results in removing from the liquid at the separator a maximum amount of intermediate and heavier components which would be passed as gas to the gasoline plant, providing higher volumes of gasoline plant products.

3.4.7.5 Economic Summary:

The over-all economic problems involving oil and gas separation is summarized in Fig. (3.23) which schematically shows a producing well connected to two types of separation systems: one to an optimum and the other to a low pressure separation. In the case of **optimum separation**, maximum oil will be recovered in the stock tank; less intermediate and heavier components will be produced to the gasoline plant; and plant product yield will be small. In the case of **low pressure separation system**, a minimum of stock tank oil will be recovered in the stock tank; more intermediate and heavier components will be produced to the gasoline plant; and plant produce yield will be large. Therefore, the over-all economics of project would include such factors as prices of gasoline plant products and stock tank oil, cost of investment in gasoline plant equipment and separators, degree of oil shrinkage, analysis of separated gas, availability and uniformity of markets, and size of oil and gas reserves.

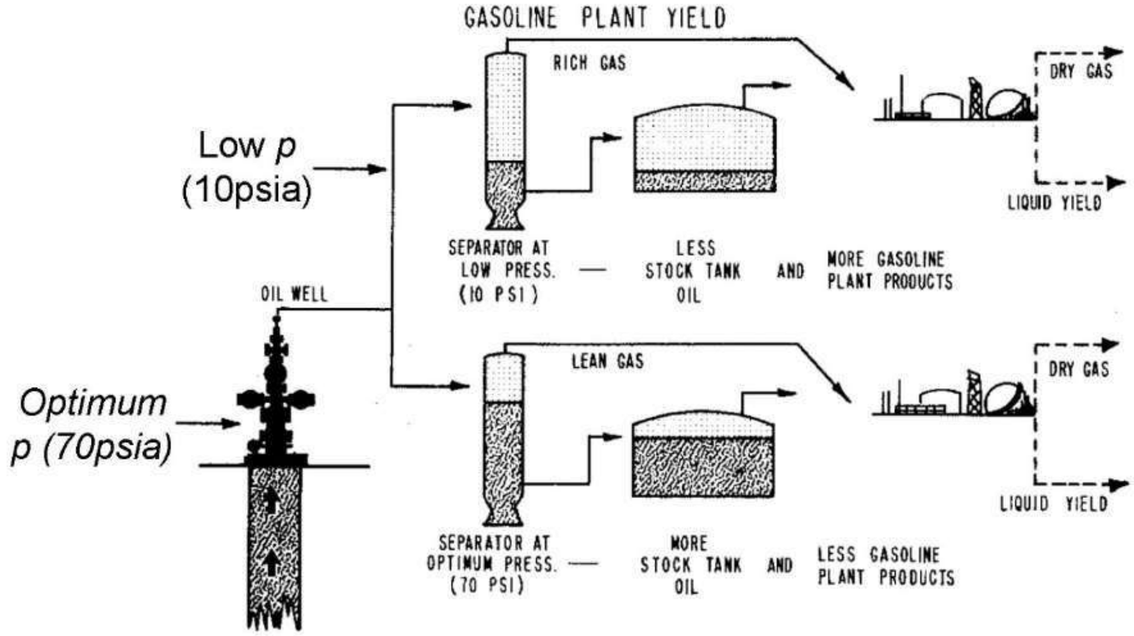


Fig. (3.23): Separation and gasoline plant yield, (Clark, 1960).

Problems

1. The average gas-oil ratio produced from the upper Washita-Fredericksburg formation of the Summerland Field is (275 scf/STB). The gravity of the produced oil is (26 °API). The color of the stock-tank oil is black. What type of reservoir fluid is in this formation?

2. Laborotary analysis of a sample from the Summerland (Upper Washita-Fredericksburg) Field indicates a ratio of volume oil leaving the reservoir to volume of oil arriving at the stock-tank of (1.10 resbbl/STB). Does this information confirm your answer to the problem 1? Why or why not?

3. A field in north Louisiana discovered in 1953 and developed by 1956 had an initial producing gas-oil ratio of (2000 scf/STB). The stock-tank liquid was ‘medium orange’ and had a gravity of (51.2 °API). Classify this reservoir fluid.

4. Answer the following questions:
   a. Cricondentherm of a black oil is always higher than the cricondentherm of a dry gas. **True or False**
   b. Dry gases have the widest phase envelope among all five reservoir fluids. **True or False**
   c. In a saturated oil reservoir, the bubble-point pressure and dew-point pressure of the oil column and gas cap fluids, respectively, are both equal to the reservoir pressure at the reservoir temperature. **True or False**
   d. A retrograde condensate is produced in a dry gas reservoir. **True or False**
   e. Gas-to-condensate (oil) ratio and the API gravity of the produced condensate remain constant throughout the producing life of a wet-gas reservoir. **True or False**
   f. In gas-condensate reservoirs, gas-to-condensate (oil) ratio and the API gravity of the produced condensate remain constant as long as the reservoir pressure stays above the dew-point pressure. **True or False**
   g. The production characteristics of an Alaska North Slope reservoir include a GOR of (548 scf/STB), stock tank oil of (26.9°API), and

a formation volume factor of (1.29 res. Bbl/STB). What type of fluid is in this reservoir?

h. The initial reservoir pressure and temperature in a North Sea reservoir is (5000 psia) and (260°F). The PVT analysis indicated the bubble-point pressure of the oil at (3500 psia). Is the reservoir fluid saturated or undersaturated? How do you know?

5. Compositional analysis of a reservoir fluid from a field in India reported a C_{7+} of (15.0 mol%), while the PVT analysis of this fluid indicated a formation volume factor of (2.5 res. bbl/STB). What type of reservoir fluid exists in this field?

6. A PVT cell contains a natural gas mixture at (1400 psia) and (190°F). The cell volume is (10.0 ft^3) and the total lb-mol of gas are determined to be (2.0). Calculate the gas deviation factor.

7. The following PV data from a CCE test at 220°F are available for a reservoir oil. Determine the bubble-point pressure of this oil:

Pressure (psia)	**Total Cell Volume (cc)**
5000	144.6
4100	146.3
3500	147.7
2900	149.2
2700	149.8
2605	150.3
2516	152.4
2253	159.7
1897	174.4
1477	204.2
1040	267.6
640	414.0

8. The following data are available from a laboratory test carried out on a black oil at (225°F). What is the bubble-point pressure of this oil? Calculate the total formation volume factor and subsequently plot pressure versus B_o and B_t:

Pressure (psia)	$R_s\ (scf/STB)$	$B_o (res.bbl/STB)$	Gas Z-factor
4500	632	1.3474	
4000	632	1.3575	
3500	632	1.3686	
3000	632	1.3811	
2682	632	1.4040	
2500	584	1.3782	0.8140
2200	509	1.3369	0.8165
2000	460	1.3109	0.8208
1800	414	1.2864	0.8269
1600	369	1.2634	0.8347
1400	326	1.2416	0.8440
1200	285	1.2208	0.8548
1000	245	1.2002	0.8670
800	205	1.1791	0.8808
600	163	1.1566	0.8964
400	119	1.1315	0.9140
200	70	1.1024	0.9339

REFERENCES

Ahmed, T., 2019, “*Reservoir Engineering Handbook*”, Gulf Professional Publishing.

Ahmed, T., 2016, “*Equation of State and PVT Analysis Application for Improved Reservoir Modelling”*. Elsevier Inc, USA.

Amyx, J., Bass, D. and Whiting, R., 1960, “*Petroleum Reservoir Engineering Physical Properties”*, McGraw-Hill Book Company. New York.

Burcik, E.J., 1997, “*Properties of Petroleum Reservoir Fluids*'', John Wiley & Sons, USA.

Bradley, H. B., 1987, “*Petroleum Engineering Handbook*”, SPE, USA.

Clark, N.J., 1969, “*Elements of Petroleum Reservoirs*”, Dallas, Texas.

McCain, William D., Jr.,1990, “*The Properties of Petroleum Fluids*”, Tulsa, Oklahoma.

Nnaemeka Ezekwe, 2011, “*Petroleum Reservoir Engineering Practice*”, Pearson Education, Inc., Boston.

Salah Abd., Waleed K., 1978, “Physical and thermodynamical properties of the reservoir oil of Buzurgan field”, NIOC.

Chapter Four

Basic Reservoir Rock Properties

Rock properties are determined by performing laboratory analysis on cores from the reservoir. Basically, two main categories of core analysis tests are performed on core samples regarding physical properties of reservoir rocks. These are:

Routine Core Analysis (RCAL):

- Porosity.
- Permeability.
- Saturation.

Special Core Analysis (SCAL):

- Overburden pressure.
- Capillary pressure.
- Relative permeability.
- Wettability.
- Surface and interfacial tension.

These rock properties data are essential for reservoir engineering calculations as they directly affect both the quantity and the distribution of hydrocarbons and, when combined with fluid properties, control the flow of the existing phases (i.e., gas, oil, and water) within the reservoir.

4.1 Porosity:

The porosity of a rock is a measure of the storage capacity (pore volume) that is capable of holding fluids. Quantitatively, the porosity is the ratio of the pore volume to the total volume (bulk volume).

$$\phi = \frac{pore\ volume}{bulk\ volume} = \frac{V_p}{V_b} = \frac{V_b - V_g}{V_b} \qquad eq. (4.1)$$

where:

$\phi =$ fractional porosity.

V_p = pore volume.

V_b= bulk volume.

V_g = grain volume.

Porosity may be classified as:

- Un-effective porosity
- Effective porosity

— Total porosity

Un-effective porosity accounts for the isolated pores, whereas effective porosity only accounts for the interconnected pores. Total porosity accounts for all the pores in the rock (interconnected and isolated pores) Therefore, effective porosity will be less than or equal to total porosity depending on the amount of isolated pores in the rock, as shown in Fig. (4.1). From a reservoir engineering standpoint, it is the effective porosity that matters, not the total porosity.

Geologically porosity may also be classified as:

- Primary porosity.
- Secondary porosity.

Primary porosity is that which was formed at the time of deposition whereas secondary porosity was developed after deposition and burial of the formation. Sandstone porosity is practically all primary porosity whereas carbonate porosity tends to be secondary porosity.

Porosity can be measured **directly** from laboratory analysis of formation cores (see section 4.1.5) or **indirectly** from the analysis of well logs (as a sample see the example 4.1).

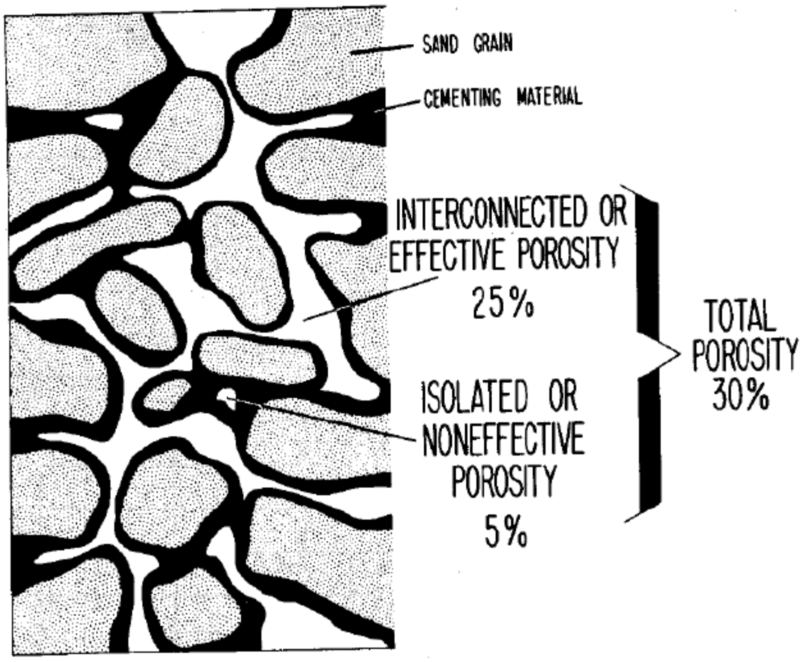


Fig.(4.1): effective, non-effective and total porosity.

4.1.1 Factors Affecting Sandstone Porosity:

These include packing, sorting and cementation. Packing describes the arrangement of the sand grains relative to one another. Fig. (4.3) shows three idealized types of packing for spherical sand grains and their theoretical porosities. The cubic packing has a porosity of 47.6%; the hexagonal packing has a porosity of 39.5% and the rhombohedral packing has a porosity of 25.9%.

Well sorted sandstone consists of grains having approximately the same size whereas poorly sorted sandstone consists of grains having a wide range of different grain sizes, Poor sorting reduce the porosity of the sandstone as may be seen in Fig. (4.4) in which the small grains fit into the pores created by the large grains, reducing the porosity.

Fig. (4.3): Effect of packing on porosity of uniform spheres.

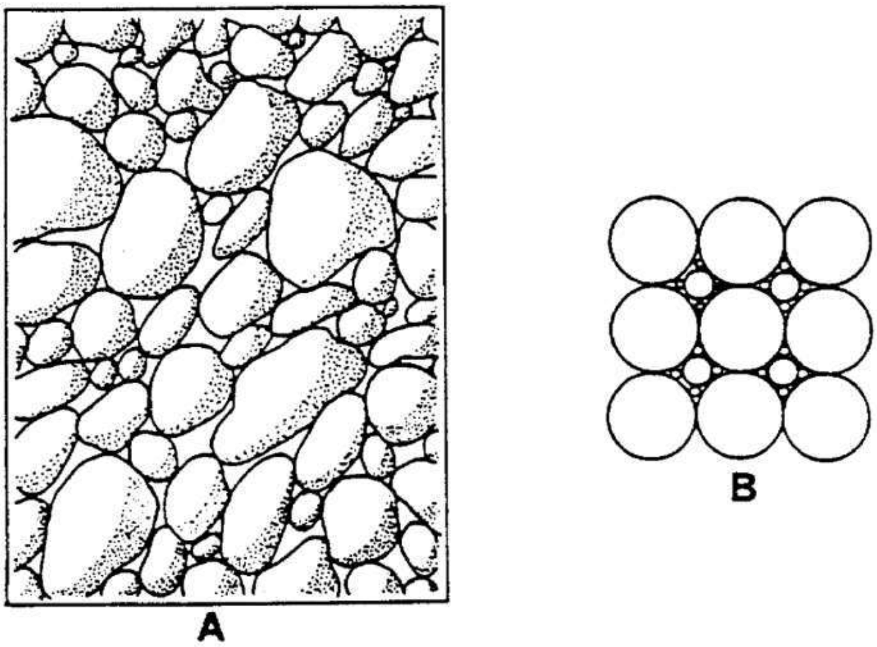


Fig. (4.4): Effect of sorting on porosity. (A) Irregular grains, (B) Idealized spherical grains.

In consolidated rocks, the sand grains are cemented together usually by quartz or carbonates. Cementation reduce the porosity of the sand as shown in Fig.(4.5).

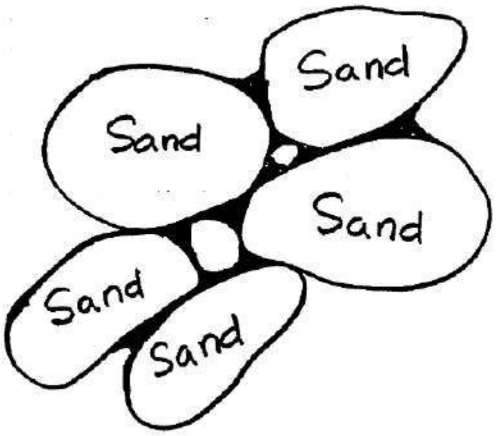


Fig. (4.5): Effect of cementation on porosity.

4.1.2 Factors Affecting Carbonate Porosity:

In carbonates, secondary porosity is usually more important than primary porosity. The major sources of secondary porosity are fracturing, solution and chemical replacement.

Fractures are cracks in the rock. Although fracture porosity is generally small, the fractures are very useful in allowing fluids to flow more easily through the rock. Therefore, they greatly enhance the flow capacity of the rock.

Solution is a chemical reaction in which water with dissolved carbon dioxide reacts with calcium carbonate to form calcium bicarbonate which is soluble. This reaction increases the porosity of the limestone.

$$CO_2 + H_2S = H_2CO_3 \qquad eq.(4.2)$$

$$H_2CO_3 + CaCO_3 = Ca(HCO_3)_2 \qquad eq.(4.3)$$

Chemical Replacement is a chemical reaction in which one type of ion replaces another with a resulting shrinkage in the size of the new compound. An example is *dolomitization* in which some of the calcium ions in calcium carbonate are replaced by magnesium ions to form calcium magnesium carbonate (dolomite). This replacement causes a shrinkage of 12 to 13%, with a corresponding increase in secondary porosity.

$$2CaCO_3 + M_gCl_2 = CaM_g(CO_3)_2 + CaCl_2 \qquad eq.(4.4)$$

4.1.3 Typical Reservoir Porosity Values:

Sandstone have porosities that typically range from 8% to 38%, with an average of 18%. About 95% of sandstone porosity is effective porosity. Sandstone porosity is usually mostly intergranular porosity. Carbonate have porosities that typically range from 3% to 15%, with an average of about 8%. About 90% of carbonate porosity is effective porosity. Carbonate porosities are much more difficult to characterize and may consist of (1) intergranular, (2) intercrystalline, (3) fissures, and (4) vugular porosities.

4.1.4 Porosity Averaging:

The reservoir rock may generally show large variations in porosity vertically (vertical porosity) but does not show very great variations in porosity parallel to the bedding planes (horizontal porosity). In this case, the arithmetic average porosity or the thickness-weighted average porosity is used to describe the average reservoir porosity. A change in sedimentation or depositional conditions, however, can cause the porosity in one portion of the reservoir to be greatly different from that in another area. In such cases, the areal-weighted average or the volume-weighted average porosity is used to characterize the average rock porosity. These averaging techniques are expressed mathematically in the following forms:

$$Arithmetic\ average \qquad \phi = \frac{\sum \phi_i}{n} \qquad eq.(4.5)$$

$$Thickness - weighted\ average \qquad \phi = \frac{\sum \phi_i h_i}{\sum h_i} \qquad eq.(4.6)$$

$$Areal - weighted\ average \qquad \phi = \frac{\sum \phi_i A_i}{\sum A_i} \qquad eq.(4.7)$$

$$Volumetric - weighted\ average \qquad \phi = \frac{\sum \phi_i A_i h_i}{\sum A_i h_i} \qquad eq.(4.8)$$

where:

n = total number of core samples.
h_i = thickness of core sample *i* or reservoir area *i*.
ϕ_i = porosity of core sample *i* or reservoir area *i*.
A_i = reservoir area *i*.

Example 4.1:

Calculate the arithmetic average and thickness-weighted average from the following measurements:

Sample	Thickness, ft	Porosity, %
1	1.0	10
2	1.5	12
3	1.0	11
4	2.0	13
5	2.1	14
6	1.1	10

Solution:

- Arithmetic average:

$$\phi = \frac{10 + 12 + 11 + 13 + 14 + 10}{6} = 11.6\%$$

- Thickness-weighted average:

$$\phi = \frac{(1)(10) + (1.5)(12) + (1)(11) + (2)(13) + (2.1)(14) + (1.1)(10)}{1 + 1.5 + 1 + 2 + 2.1 + 1.1}$$

$$= 12.11\%$$

4.1.5 Laboratory Measurement of Porosity:

The porosity of reservoir rocks can be determined essentially by two different methods: routine core analysis (laboratory measurements on core plugs drilled from whole core samples as shown in Figure (4.6)) and well logging techniques. In addition to the two methods of routine core analysis and well logging, other nonconventional techniques of porosity determination exist, such as x-ray computerized tomography (CT) scanning.

A given reservoir rock sample basically comprises three different volumes: bulk volume (BV), pore volume (PV), and grain volume (GV). These three volumes are related by the following simple relationship:

$$BulkVolume = PoreVolume + Grain\ Volume$$

Therefore, in the laboratory measurement of porosity, it is necessary to determine only two of the three volumes: BV, PV, or GV.

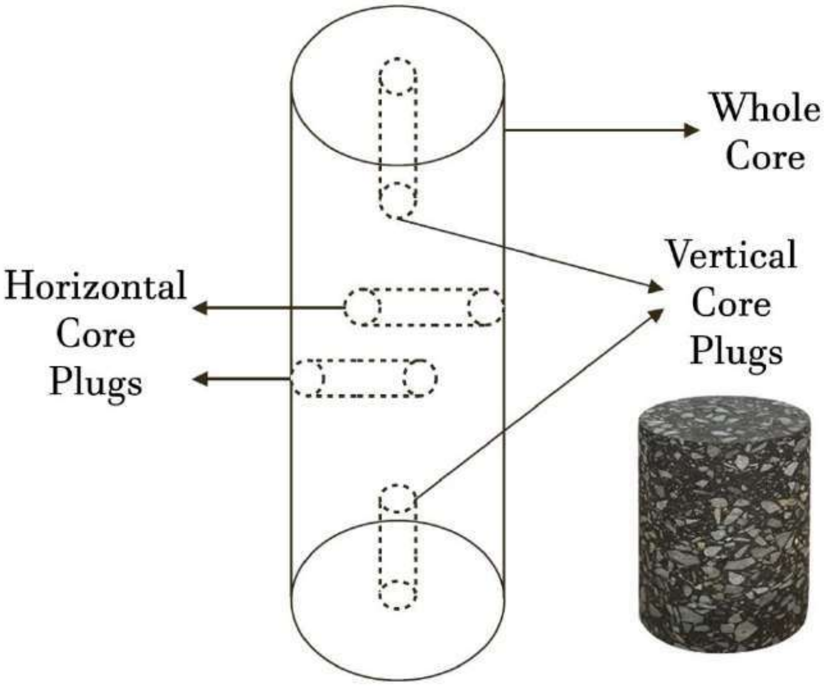


Fig. (4.6): Core plugs drilled from a whole core sample.

4.1.5.1 Bulk Volume Measurement:

The most common types of samples used in routine core analysis are cylindrical core plugs which allow the determination of BV from the dimensions of the sample (L = length and D=diameter; BV=$(\pi/4)D^2L$). This is the easiest and simplest method of determining the BV of a reservoir rock sample. For irregular core samples a method of displacing fluid (Archimedes principle) is used to find the core sample.

Example 4.2:

An irregular sample of sandstone is 35.25 gm in mass, when coated with varnish, its mass increased to 36.55 gm. Compute the porosity if the coated sample displaced 15.7 ml of water when fully submerged if you have ($\rho_g = 2.65\, gm/cm^3$, $\rho_w = 1\ gm/cm^3$, $\rho_{vr} = 1.8\ gm/cm^3$)

Solution:

$$m_{vr} = coated\ mass - sample\ mass$$

$$m_{vr} = 36.55 - 35.25 = 1.3\ gm$$

$$\rho_{vr} = \frac{m}{V} \rightarrow V_{vr} = \frac{m}{\rho} = \frac{1.3}{1.8} = 0.72\ cc$$

$$V_{bulk} = 15.7 - 0.72 = 14.98\,cc$$

$$V_{grain} = \frac{m}{\rho} = \frac{35.25}{2.65} = 13.3\,cc$$

$$V_{pores} = V_{bulk} - V_{grain} = 14.98 - 13.3 = 1.68\,cc$$

$$\phi = \frac{V_{pores}}{V_{bulk}} = \frac{1.68}{14.98} = 0.112 = 11.2\%$$

4.1.5.2 Pore Volume Measurement:

All methods used for determining pore volume are based on either extraction of a fluid from the rock sample or reintroduction of a fluid in the pore spaces of the rock sample. When introducing fluids into the pore spaces of the rock sample, a number of methods are used for the determination of pore volume of reservoir rocks. These methods typically use three different types of fluids: helium, water or synthetic oil, and mercury. The porosity measured is, however, effective porosity because the saturating fluids only penetrate the interconnected and dead-end pore spaces. Although mercury has some distinct advantages, its use in laboratory testing is accompanied with the associated health hazards and, additionally, the rock sample is rendered unusable. The various methods that employ these saturating fluids are described in the following text.

*I. **Helium Porosimeter:***

The use of helium in the determination of porosity has certain obvious advantages over other gases and liquids: Helium is a clean inert gas and does not cause any unwanted rock–fluid interactions that may affect/change the original porosity; molecules are small that can rapidly penetrate the small pores, and it can be considered an ideal gas (compressibility factor = 1) for pressures and temperatures usually employed in the procedure. Additionally, porosity measurements can be completed in a short amount of time. The use of helium in desktop-type porosimeters, commonly available in the market, is by far the most common technique for measuring porosities of plug size core samples. Fig. (4.7) shows a general arrangement of the helium porosimeter. In principle, the apparatus consists of two equal-volume chambers or cells called the reference chamber and the sample chamber. The reference chamber has a volume V_1 at initial pressure P_1 (usually 100 psig), and the sample chamber has an unknown volume V_2 and initial pressure P_2 (normally atmospheric). The system is then brought to equilibrium by opening the valve to the sample chamber, allowing the determination of the unknown volume V_2 by noting the resultant equilibrium pressure P. The application of Boyle's law allows the equalization of pressures (for isothermal conditions) before and after the opening of the valve to the sample chamber, as per the following equation:

$$p_1V_1 = p_2V_2$$

where:

V_1= reference chamber volume
$V_2 = V_1 + V_{tube} + V_{pores}$
p_1 = pressure at reference chamber
p_2 = final pressure after opening the valve to the sample chamber.

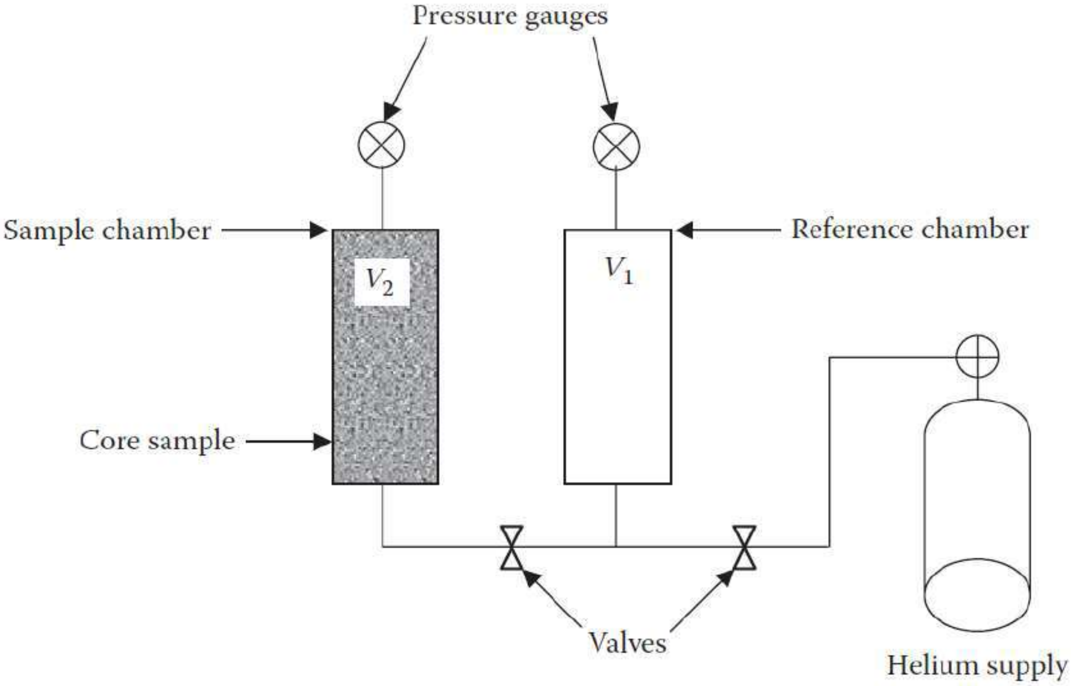


Figure (4.7): Schematic illustration of a helium porosimeter.

Example 4.4:

Neglecting the tube volume in helium porosimeter method find V_p and ϕ from the following data. $V_1 = 20\ cc,\quad p_1 = 25\ psia,\quad p_2 = 21\ psia.$

Solution:

$$p_1V_1 = p_2V_2$$

$$25 \times 20 = 21 \times V_2$$

$$V_2 = 23.81\ cc$$

$$V_2 = V_1 + V_{tube} + V_{pores}$$

$$V_{pores} = 23.81 - 20 - 0 = 3.81\ cc$$

$$\phi = \frac{V_p}{V_b} = \frac{3.81}{20} = 0.19 = 19\%$$

II. Vacuum Saturation:

The vacuum saturation method is in fact one of the very basic methods of obtaining the pore volume of a rock sample. One of the advantages is the fact that pore volumes of multiple samples can be determined in one step. The method uses a large enough vacuum flask or a beaker, filled with a degassed liquid, normally water, in which dry rock samples are placed. Subsequently, as soon as the evacuation of the vacuum flask is initiated, air bubbles are seen in the saturating liquid as it replaces air from the pore spaces of the rock samples. The disappearance of the air bubbles gives an indication that the saturation is complete and at this point the evacuation is terminated, and porosity is calculated as follows:

$$\phi = \frac{(WW - DW)}{BV\rho_w} \times 100\% \qquad eq.(4.9)$$

Where:

WW: is the wet weight of the sample, after vacuum saturation.
DW: is the dry weight of the sample, before vacuum saturation.
ρ_w : is the saturating fluid (water) density.

III. Liquid Saturation by Other Methods:

The other methods of introduction of a liquid into the pore spaces of a rock sample include forced saturation by either water or synthetic oil (liquids as chloroform and tetrachloroethane) also kerosene. The rock sample is held in a special device called a core holder, and a given liquid is injected through the sample by use of a pump. This method, however, requires advanced apparatus called a core flooding rig or a displacement apparatus, compared to the techniques discussed earlier.

4.1.5.3 Grain Volume Measurement:

All methods measuring grain volume usually yield total or absolute porosity, simply because the rock samples are normally crushed for grain volume measurements which actually destroy all pores, thus resulting in total porosity as grain volume is subtracted from the bulk volume. Although only the effective pore space has direct application in most reservoir engineering calculations, knowledge of the magnitude and distribution of the isolated pore spaces can reveal other characteristics of reservoir rocks.

4.1.5.4 Indirect Methods of Porosity Measurement (Logging Method):

These measurements are carried out in the well employing special instrument as part of well logging operations. Therefore, no core samples are needed and the porosity is estimated for the rock as it exists in the reservoir. Two of the most common well logging methods sonic and density logging. (for more details see (Bassiouni, 1994)).

Example 4.5:

The bulk density of a clean, sandy interval saturated with water was measured by the density logging tool to be (2.4 g/cm^3). Assuming that the density of the formation water is (1.04 g/cm^3) and the density of the matrix is (2.67 g/cm^3), calculate the density porosity of the formation. Fig. (4.7) illustrates the solution.

Solution:

$\rho_b = 2.4\ g/cm^3,\qquad \rho_{fw} = 1.04\ g/cm^3,\qquad \rho_{ma} = 2.67\, g/cm^3\qquad \Phi_d = ??$

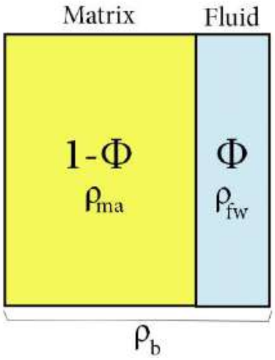


Fig. (4.7): illustrates the given data above.

$$\Phi_d = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_{fw}}$$

$$\Phi_d = \frac{2.67 - 2.4}{2.67 - 1.04} = 0.166 = 16.6\%$$

Example 4.6:

A sonic log measured travel time of 58 μs for a formation. If the formation is primarily limestone (46 μs) and contains oil only (190 μs) compute the rock porosity.

Solution:

$$\Phi_s = \frac{\Delta t_{log} - \Delta t_{ma}}{\Delta t_f - \Delta t_{ma}}$$

$$\Phi_s = \frac{58 - 46}{190 - 46} = 0.083 = 8.3\%$$

Note: This value is the average absolute porosity of the formation.

4.1.6 Pore Volume Compressibility:

Rock at reservoir conditions is subjected to overburden stress, while at the surface, recovered core has been relieved of the overburden stresses. It is not usual to perform routine porosity measurements under stress approaching reservoir conditions. Because of this, laboratory-measured porosities are generally expected to be higher than *in situ* values. Pore volume compressibility can be used to correct Laboratory-measured porosity to an *in situ* value and for other reservoir engineering calculations. Pore volume compressibility is defined as:

$$c_f = \frac{1}{\phi}\left[\frac{\partial \phi}{\partial p}\right]_{T=constant} \qquad eq.(4.10)$$

Note that pore volume compressibility is defined without the negative sign because porosity decreases as pore pressure decreases.

Total system compressibility is a parameter of importance in pressure transient testing and material balance calculations. Total system compressibility is defined as the combined compressibility of pore volume and all fluids saturating the medium:

$$c_t = S_o c_o + S_w c_w + S_g c_g + c_f \qquad eq.(4.11)$$

where:

S_o, S_w, S_g = oil, water and gas saturations, fraction.
c_o, c_w, c_g = oil, water and gas coefficients of compressibility, psi^{-1}.
c_f = pore volume coefficient of compressibility, psi^{-1}.

4.2 Fluid Saturation:

Saturation is defined as that fraction, or percent, of the pore volume occupied by a particular fluid (oil, gas, or water). This property is expressed mathematically by the following relationship:

$$fluid\ saturation = \frac{total\ volume\ of\ fluid}{pore\ volume}$$

Applying this mathematical concept of saturation to each reservoir fluid gives

$$S_o = \frac{volume\ of\ oil}{pore\ volume} \qquad eq. (4.12)$$

$$S_g = \frac{volume\ of\ gas}{pore\ volume} \qquad eq. (4.13)$$

$$S_w = \frac{volume\ of\ water}{pore\ volume} \qquad eq. (4.14)$$

where:

S_o = oil saturation.
S_g = gas saturation.
S_w = water saturation.

Thus, all saturation values are based on pore volume and not on gross reservoir volume.

The saturation of each individual phase ranges between 0 and 100%. By definition, the sum of the saturations is 100%; therefore,

$$S_o + S_g + S_w = 1.0 \qquad eq. (2.15)$$

In addition to the bottom (or edge) water, connate water will be distributed throughout the oil and gas zones. The water in these zones will have been reduced to some irreducible minimum. The forces retaining the water in the oil and gas zones are referred to as ***capillary forces*** because they are important only in pore spaces of capillary size.

Connate (interstitial) water saturation S_{wc} is important primarily because it reduces the amount of space available between oil and gas. It is generally not uniformly distributed throughout the reservoir but varies with permeability, lithology, and height above the free water table.

Another particular phase saturation of interest is called the critical saturation and it is associated with each reservoir fluid. The definition and significance of the critical saturation for each phase is described next.

Critical Oil Saturation, S_{oc}

For the oil phase to flow, the saturation of the oil must exceed a certain value, which is termed critical oil saturation. At this particular saturation, the oil remains in the pores and, for all practical purposes, will not flow.

Residual Oil Saturation, S_{or}

During the displacing process of the crude oil system from the porous media by water or gas injection (or encroachment), some remaining oil will be left that is quantitatively characterized by a saturation value that is larger than the *critical oil saturation*. This saturation value is called the *residual oil saturation*, S_{or}. The term residual saturation is usually associated with the nonwetting phase (oil or gas) when it is being displaced by a wetting phase (water), (see section 4.5).

Movable Oil Saturation, S_{om}

Movable oil saturation S_{om} is another saturation of interest and is defined as the fraction of pore volume occupied by movable oil as expressed by the following equation:

$$S_{om} = 1 - S_{wc} - S_{oc} \qquad eq. (4.16)$$

where:

S_{wc} = connate water saturation.
S_{oc} = critical oil saturation.

Critical Gas Saturation, S_{gc}

As the reservoir pressure declines below the bubble-point pressure, gas evolves from the oil phase and consequently the saturation of the gas increases as the reservoir pressure declines. The gas phase remains immobile until its saturation exceeds a certain saturation, called critical gas saturation, above which gas begins to move. So, S_{gc} is the minimum saturation at which gas flow through a porous medium. If S_{gc} = 5% then gas doesn't flow until it's S_{gc} exceeds 5%.

Critical Water Saturation, S_{wc}

The critical water saturation, connate water saturation, and irreducible water saturation are extensively used interchangeably to define the maximum water saturation at which the water phase will remain immobile.

4.2.1 Saturation Averaging:

Proper averaging of saturation data requires that the saturation values be weighted by both the interval thickness h_i and interval porosity ϕ. The average saturation of each reservoir fluid is calculated from the following equations:

$$S_o = \frac{\sum_{i=1}^{n} \phi_i h_i S_{oi}}{\sum_{i=1}^{n} \phi_i h_i} \qquad eq.(4.17)$$

$$S_w = \frac{\sum_{i=1}^{n} \phi_i h_i S_{wi}}{\sum_{i=1}^{n} \phi_i h_i} \qquad eq.(4.18)$$

$$S_g = \frac{\sum_{i=1}^{n} \phi_i h_i S_{gi}}{\sum_{i=1}^{n} \phi_i h_i} \qquad eq.(4.19)$$

where the subscript i refers to any individual measurement and h_i represents the depth interval to which ϕ_i, S_{oi}, S_{gi}, and S_{wi} apply.

Example 4.7:

Calculate average oil and connate water saturation from the following measurements:

Sample	h_i, ft	ϕ, %	S_o, %	S_{wc}, %
1	1.0	10	75	25
2	1.5	12	77	23
3	1.0	11	79	21
4	2.0	13	74	26
5	2.1	14	78	22
6	1.1	10	75	25

Solution:

Construct the following table and calculate the average saturation for the oil and water phase:

Sample	h_i, ft	ϕ	ϕh	S_o	$S_o\phi h$	S_{wc}	$S_w\phi h$
1	1.0	0.10	0.100	0.75	0.0750	0.25	0.0250
2	1.5	0.12	0.180	0.77	0.1386	0.23	0.0414
3	1.0	0.11	0.110	0.79	0.0869	0.21	0.0231
4	2.0	0.13	0.260	0.74	0.1924	0.26	0.0676
5	2.1	0.14	0.294	0.78	0.2293	0.22	0.0647
6	1.1	0.10	0.110	0.75	0.0825	0.25	0.0275
			1.054		0.8047		0.2493

Calculate average oil saturation by applying Eq.(4.17):

$$S_o = \frac{0.8047}{1.054} = 0.7635$$

Calculate average water saturation by applying Eq.(4.18):

$$S_w = \frac{0.2493}{1.054} = 0.2364$$

4.2.2 Laboratory Measurement of Fluid Saturation:

Fluid saturation in reservoir rocks can be determined by essentially two different approaches: direct and indirect. The direct approach involves using preserved core plug samples or rather plug-end trims of the core plug, that is, a small section of the rock sample removed from a petroleum reservoir, for fluid saturation determination. The indirect method is further divided into two categories: (1) use of some other measurements on core plug samples such as capillary pressures based on which the fluid saturations are determined and (2) use based on traditional well-logging techniques where fluid saturations are not measured on core plugs but are measured *in situ*, for the entire formation itself at various depths.

All the methods for measurement of original reservoir rock saturation are based on the principle of leaching that basically refers to the process of removal of liquids from a solid (rock sample in this case). Based on the principle of leaching, two methods are devised for the determination of fluid saturation. The first method involves using heat to extract the fluids present in the pore spaces of the rock and is termed *retort distillation* (see Figure (4.8)). The second method involves using both heat and an organic solvent to extract the pore fluids and is called *Dean–Stark extraction* (see Figure (4.9)).

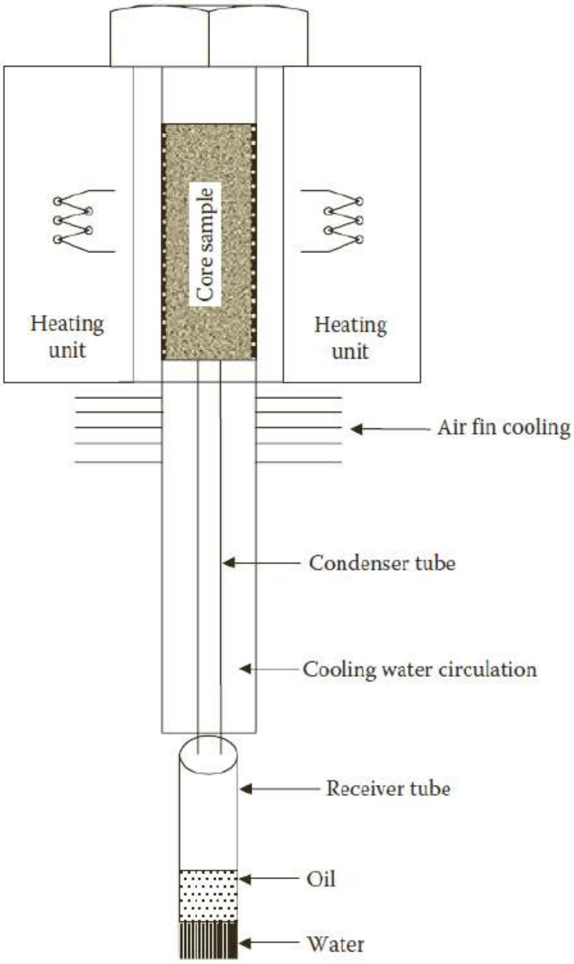


Figure (4.8): Schematic of a retort distillation unit.

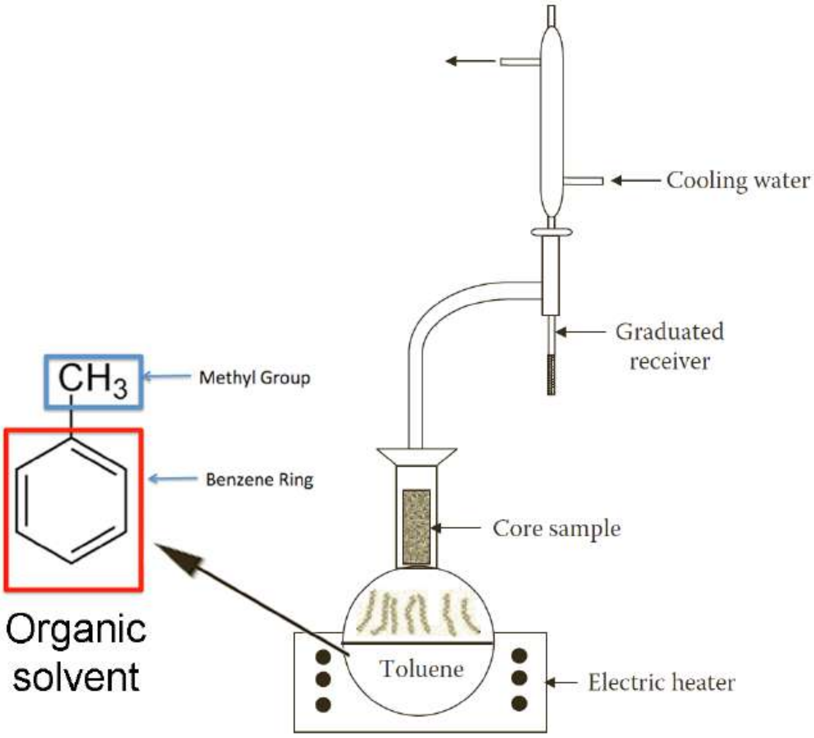


Figure (4.9): Schematic of the Dean–Stark distillation extraction unit.

4.3 Permeability:

Permeability is a property of the porous medium to transmit fluids through it. The rock permeability, k, is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by (Henry Darcy in 1856). In fact, the equation that defines permeability in terms of measurable quantities is called Darcy's Law.

Darcy developed a fluid flow equation that has since become one of the standard mathematical tools of the petroleum engineer. If a horizontal linear flow of an incompressible fluid is established through a core sample of length L and a cross-section of area A (see Fig. (4.10)), then the governing fluid flow equation is defined as:

$$q = -\frac{kA}{\mu}\frac{dp}{dx} \qquad eq.(4.20)$$

where:

q = volumetric flowrate, cm^3/sec.
k = absolute permeability of the rock, darcy.
A = cross sectional area in the flow direction, cm^2.
μ = fluid viscosity, cp. (μ*=mass/length.time =m/L.t =gm/cm.sec (poise)*).
$\frac{dp}{dx}$ = fluid pressure gradient, atm/cm.

One darcy is a relatively high permeability, as the permeabilities of most reservoir rocks are less than 1 darcy. In order to avoid the use of fractions in describing permeabilities, the term millidarcy is used.

$$1\, darcy = 1000\, md$$

The negative sign in equation (4.20) is necessary, as the pressure increases in one direction while the length increases in the opposite direction.

Equation (4.20) can be integrated when the geometry of the system through which fluid flows is known. For the simple linear system shown in Figure (4.10), the integration is performed as follows:

$$q\int_0^L dL = -\frac{kA}{\mu}\int_{p_1}^{p_2} dp$$

Integrating the above expression yields:

$$q = -\frac{kA}{\mu L}(p_2 - p_1)$$

It should be pointed out that the volumetric flow rate, q, is constant for liquids because the density does not change significantly with pressure. Since p_1 is greater than p_2, the pressure terms can be rearranged, which will eliminate the negative term in the equation. The resulting equation is:

$$q = \frac{kA(p_1 - p_2)}{\mu L} \qquad eq.(4.21)$$

Equation (4.21) is the conventional linear flow equation used in fluid flow calculations.

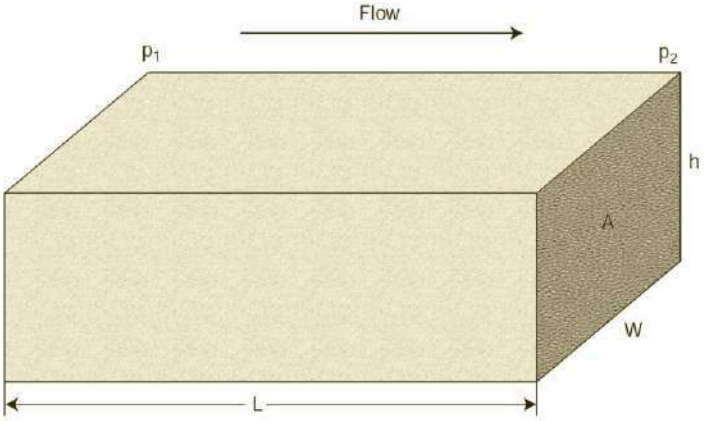


Fig. (4.10): Linear flow model.

Example 4.8:

A brine is used to measure the absolute permeability of a core plug. The rock sample is (4 cm) long and (3 cm^2) in cross section. The brine has a viscosity of (1.0 cp) and is flowing at a constant rate of (0.5 cm^3/sec) under a (2.0 atm) pressure differential. Calculate the absolute permeability.

Solution:

$$q = \frac{kA(p_1 - p_2)}{\mu L}$$

$$0.5 = \frac{k(3)(2)}{(1)(4)}$$

$$k = 0.333\ darcy$$

For steady-state linear flow in oilfield units (see table (4.1) below), Eq. (4.21) becomes:

$$q = \frac{0.001127kA}{\mu B_o}\frac{(p_1 - p_2)}{L} \qquad eq.(4.22)$$

where:

q = volumetric flowrate, STB/Day.
k = absolute permeability of the rock, millidarcy (md).
A = cross sectional area in the flow direction, ft^2.
p_1 = inlet pressure, psig.
p_2 = outlet pressure, psig.
μ = fluid viscosity, cp.
L = length of the rock, ft.
B_o = oil formation volume factor, bbl/STB.

Note:

Field units and Lab. units of the above symbols are as shown in table below:

Table (4.1): Field and Lab. units of darcy's law.

Parameter	**Lab units**	**Field units**	**Conversion factor**
q	cm^3/sec	bbl/day	$\frac{(30.48)^3 ft^3 . 5.615\, bbl/ft^3}{60 \times 60 \times 24\, day}$
k	$darcy$	md	10^{-3}
A	cm^2	ft^2	$(30.48)^2$
Δp	atm	psi	$1/14.7\, psi$
L	cm	ft	30.48
μ	cp	cp	

For radial flow of fluids into a wellbore (Fig. (4.11)), darcy's law may be expressed in radial coordinates in field units as:

$$q = \frac{0.001127.2\pi.kh(p_e - p_w)}{\mu B_o \ln\left(\frac{r_e}{r_w}\right)}$$

$$q = \frac{0.00708kh(p_e - p_w)}{\mu B_o \ln\left(\frac{r_e}{r_w}\right)} \qquad eq.(4.23)$$

where:

q = volumetric flowrate, STB/Day.
k = absolute permeability of the rock, millidarcy (md).
h = pay thickness, *ft*.
p_e= pressure at external radius, psig.
p_w = pressure at wellbore, psig.
µ = fluid viscosity, cp.
L = length of the rock, *ft*.
B_o = oil formation volume factor, bbl/STB.
r_e = external drainage area, *ft*.
r_w = wellbore radius, *ft*.
ln = natural logarithm.

Equation (4.23) can be deduced as follow:

$$q = \frac{kA}{\mu}\frac{dp}{dr}$$

$$q\int_{r_w}^{r_e} dr = \frac{kA}{\mu}\int_{p_w}^{p_e} dp$$

$$q\int_{r_w}^{r_e} \frac{dr}{A} = \frac{k}{\mu}\int_{p_w}^{p_e} dp$$

$$q\int_{r_w}^{r_e} \frac{dr}{2\pi rh} = \frac{k}{\mu}\int_{p_w}^{p_e} dp$$

$$\frac{q}{2\pi h}\int_{r_w}^{r_e} \frac{dr}{r} = \frac{k}{\mu}\int_{p_w}^{p_e} dp$$

$$\frac{q}{2\pi h}[\ln(r_e) - \ln(r_w)] = \frac{k}{\mu}(p_e - p_w)$$

$$\frac{q}{2\pi h}\left[\ln\frac{r_e}{r_w}\right] = \frac{k}{\mu}(p_e - p_w)$$

$$q = \frac{2\pi kh(p_e - p_w)}{\mu\left[\ln\frac{r_e}{r_w}\right]}$$

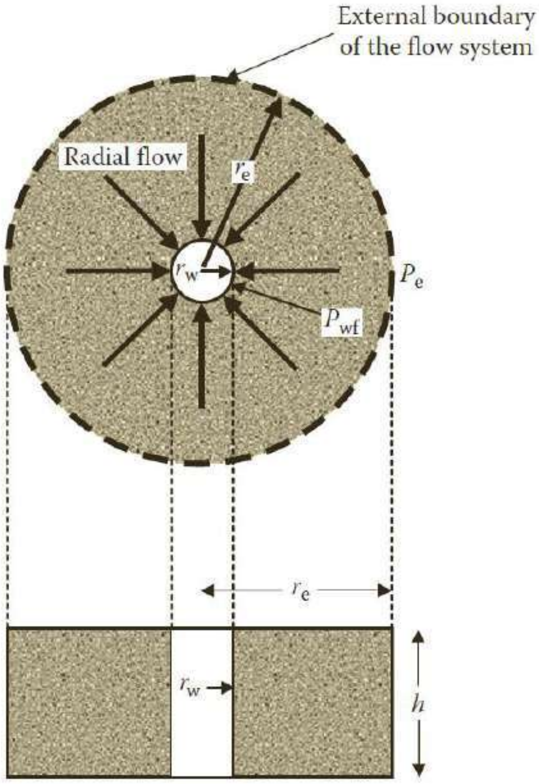


Fig. (4.11): Radial flow system.

Standard laboratory analysis procedures will generally provide reliable data on permeability of core samples. If the rock is not homogeneous, the whole core analysis technique will probably yield more accurate results than the analysis of core plugs (small pieces cut from the core). Procedures that have been used for improving the accuracy of the permeability determination include cutting the core with an oil-base mud, employing a pressure-core barrel, and conducting the permeability tests with reservoir oil.

Permeability is reduced by overburden pressure, and this factor should be considered in estimating permeability of the reservoir rock in deep wells because permeability is an isotropic property of porous rock in some defined regions of the system, that is, in other words, it is directional. Routine core analysis is generally concerned with plug samples drilled parallel to bedding planes and, hence, parallel to direction of flow in the reservoir. These yield **horizontal permeabilities (k_h)**.

The measured permeability on plugs that are drilled perpendicular to bedding planes is referred to as **vertical permeability (k_v)**. Figure (4.12) shows a schematic illustration of the concept of the core plug and the associated permeability.

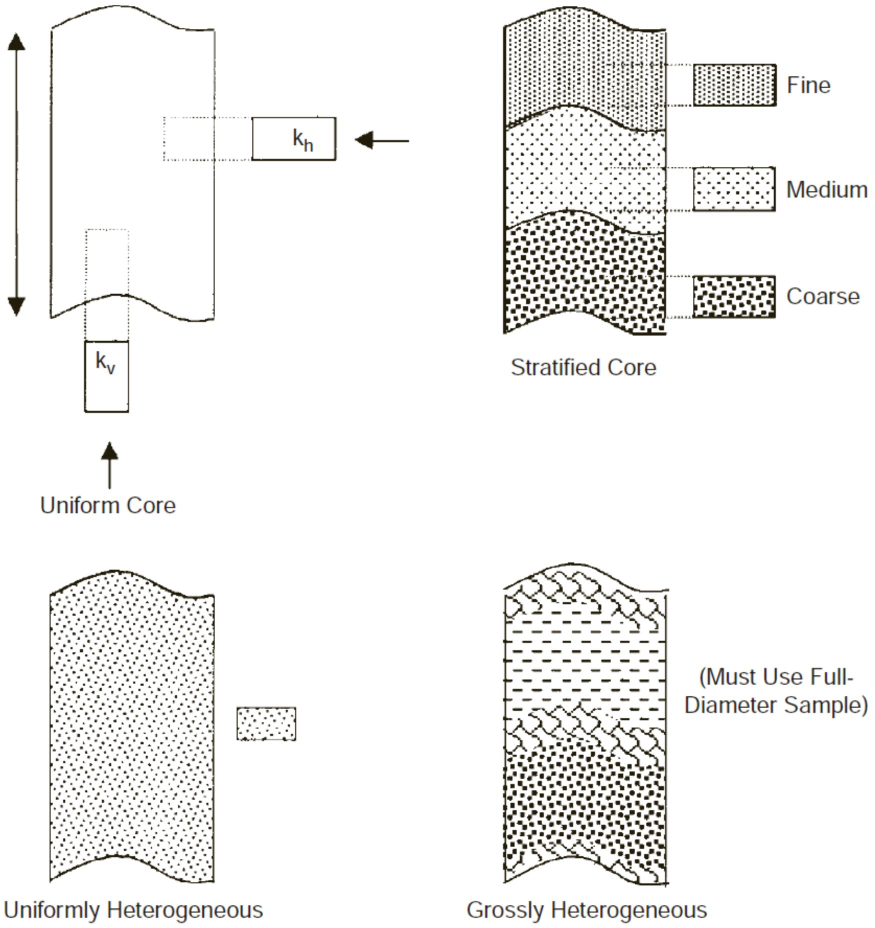


Fig. (4.12): Representative samples of porous media.

It is important to notify that there are three types of permeability which are:

1. **Absolute permeability**: is the permeability of a porous medium saturated with a single fluid.
2. **Effective permeability**: is a measure of the conductance of a porous medium for one fluid phase when the medium is saturated with more than one fluid.
3. **Relative permeability**: is the ratio of the effective permeability of a fluid at a given saturation to the absolute permeability. (see section 5.2).

Example 4.9:

Calculate flow rate in bbl/day for oil flows in a linear porous media with the following properties. L=2000 ft, h=20 ft, w=300 ft, k=100 md, $\phi = 15\%$, μ=2 cp, p_1=2000 psia, p_2=1990 psia.

solution:

$$A = hw = 20 \times 300 = 6000\, ft^2$$

$$q = \frac{0.001127kA}{\mu}\frac{(p_1 - p_2)}{L}$$

$$q = \frac{0.001127(100)(6000)}{(2)}\frac{(2000 - 1990)}{2000} = 1.69\ bbl/day$$

4.3.1 Averaging Absolute Permeabilities:

The absolute permeability expression such as the one in Equation (4.21) is derived based on a fairly uniform or continuous value of permeability between the inflow and outflow faces. However, such uniformity and consistency is rarely seen in reservoir rocks. Most reservoir rocks have space variations of permeability. For example, reservoir rocks may contain distinct layers, blocks, or concentric rings of fixed permeability. In such cases, the permeability values are averaged according to the particular type of flow: parallel or series. The mathematical expressions for averaging permeability for these cases are developed in the following text.

4.3.1.1 Parallel Flow:

As shown in Figure (4.13), consider the case of fluid flow taking place in parallel through different layers of vertically stacked porous media. These individual layers of porous media that have varying permeability and thickness are separated from one another by infinitely thin impermeable barriers that preclude the possibility of cross flow or vertical flow. The average permeability for such a combination can be easily developed by applying Darcy's law to the individual layers. For ***layer 1***,

$$q_1 = \frac{k_1 W h_1 \Delta p}{\mu L} \qquad eq.(4.24)$$

for ***layer 2***,

$$q_2 = \frac{k_2 W h_2 \Delta p}{\mu L} \qquad eq.(4.25)$$

for ***layer 3***,

$$q_3 = \frac{k_3 W h_3 \Delta p}{\mu L} \qquad eq.(4.26)$$

However, since flow is taking place in parallel, the total volumetric flow rate can be equated to the summation of the individual flow rates through the three layers:

$$q_t = q_1 + q_2 + q_3 \qquad eq. (4.27)$$

Similarly, the total height is given by:

$$h_t = h_1 + h_2 + h_3 \qquad eq. (4.28)$$

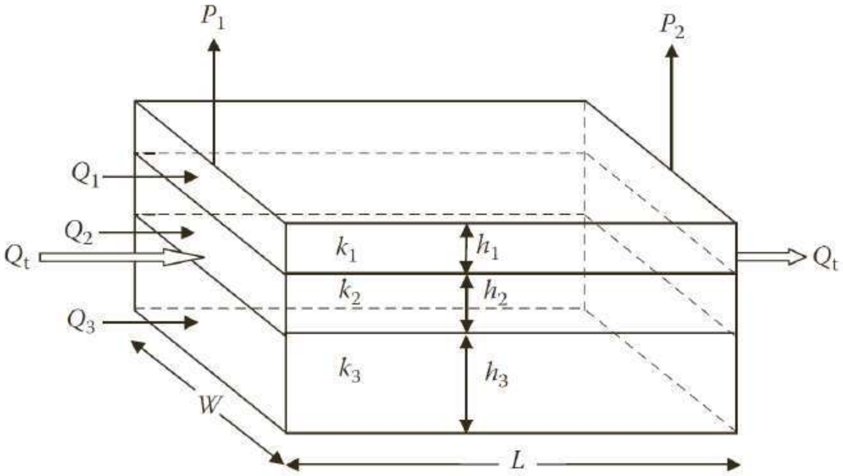


Fig. (4.13): Fluid flow through a parallel combination.

Based on Equations (4.27) and (4.28), Darcy's law can be written for the total flow rate for the entire systems using k_{avg} as the average absolute permeability:

$$q_t = \frac{k_{avg} W h_t \Delta p}{\mu L} = \frac{k_1 W h_1 \Delta p}{\mu L} + \frac{k_2 W h_2 \Delta p}{\mu L} + \frac{k_3 W h_3 \Delta p}{\mu L} \qquad eq. (4.29)$$

or,

$$k_{avg} h_t = k_1 h_1 + k_2 h_2 + k_3 h_3 \qquad eq. (4.30)$$

that subsequently leads to the final generalized expression for calculating the average absolute permeability for a parallel system of n layers:

$$k_{avg} = \frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{n} h_i} \qquad eq. (4.31)$$

Equation (4.22) and the schematic used for deriving this equation demonstrate the practical significance from a petroleum reservoir point of view, that is, in horizontal flow situations, fluids travel through the reservoir strata to production wells and remain in the zone in which they originated. Or in other words, the case of fluid flow in parallel is relevant to conventional wells (vertical) where fluid flow takes place parallel to the bedding planes (horizontal).

The average absolute permeability expressions for a more generalized case of the parallel flow can also be developed using an approach similar to the one described earlier. In such a case, the width of the layers is varied rather than being kept constant. For example, Figure (4.13) can be modified such that *layer 1* has shortest width W_1, *layer 2* has medium width W_2, and *layer 3* has the largest width W_3 as explained in figure (4.14) below:

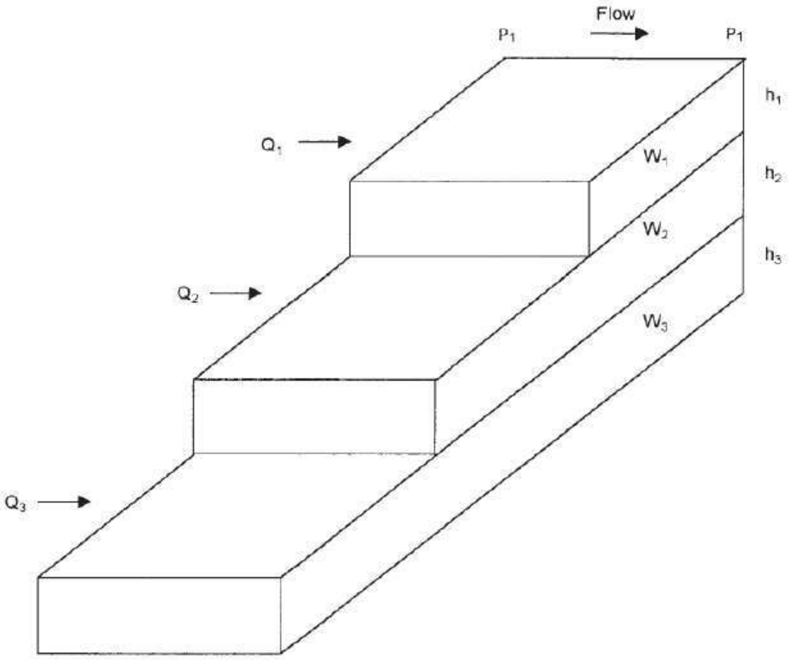


Figure (4.14): Linear flow through layered beds with variable area.

Given this arrangement, the following equations can be set up for the three individual layers and the average for the entire system:

$$q_1 = \frac{k_1 W_1 h_1 \Delta p}{\mu L} \qquad eq. (4.32)$$

$$q_2 = \frac{k_2 W_2 h_2 \Delta p}{\mu L} \qquad eq. (4.33)$$

$$q_3 = \frac{k_3 W_3 h_3 \Delta p}{\mu L} \qquad eq. (4.34)$$

$$q_t = \frac{k_{avg}(W_1 h_1 + W_2 h_2 + W_3 h_3)\Delta p}{\mu L} \qquad eq. (4.35)$$

Using Equation (4.27),

$$q_t = q_1 + q_2 + q_3$$

$$\frac{k_{avg}(W_1h_1 + W_2h_2 + W_3h_3)\Delta p}{\mu L} = \frac{k_1W_1h_1\Delta p}{\mu L} + \frac{k_2W_2h_2\Delta p}{\mu L} + \frac{k_3W_3h_3\Delta p}{\mu L} \qquad eq.(4.36)$$

$$k_{avg} = \frac{k_1W_1h_1 + k_2W_2h_2 + k_3W_3h_3}{W_1h_1 + W_2h_2 + W_3h_3} \qquad eq.(4.37)$$

The earlier equation can be generalized for a system of n layers to give the average permeability:

$$k_{avg} = \frac{\sum_{i=1}^{n} k_iA_i}{\sum_{i=1}^{n} A_i} \qquad eq.(4.38)$$

where A_i is the cross-sectional area of layer i, expressed as:

$$A_i = \sum_{i=1}^{n} W_ih_i \qquad eq.(4.39)$$

Example 4.10:

Given the following permeability data from a core analysis report, calculate the average permeability of the reservoir.

Depth, ft	Permeability, md
3998-4002	200
4002-4004	130
4004-4006	170
4006-4008	180
4008-4010	140

Solution:

h_i, ft	k_i	h_ik_i
4	200	800
2	130	260
2	170	340
2	180	360
2	140	280
h_t=12		$\sum h_ik_i = 2040$

$$k_{avg} = \frac{2040}{12} = 170\ md$$

4.3.1.2 Series Flow (Harmonic Average Permeability):

Permeability variations can occur laterally in a reservoir as well as in the vicinity of a wellbore. Figure (4.15) illustrates a series flow taking place through a stack of porous media of varying absolute permeabilities and lengths. The mathematical expression for calculating the average absolute permeability for a flow system shown in Figure (4.15) is developed in the following text. Again writing Darcy's law for each of the layers or blocks of porous medium stacked in series,

for ***layer 1***,

$$q_1 = \frac{k_1 W h \Delta p_1}{\mu L_1} \qquad eq.(4.40)$$

for ***layer 2***,

$$q_2 = \frac{k_2 W h \Delta p_2}{\mu L_2} \qquad eq.(4.41)$$

for ***layer 3***,

$$q_3 = \frac{k_3 W h \Delta p_3}{\mu L_3} \qquad eq.(4.42)$$

It should be noted that for series flow, each of these layers or blocks has a different differential pressure and the summation of these is equal to the total or overall differential pressure of the entire flow system. Additionally, the total flow rate is also equal to the individual flow rates:

$$\Delta p_1 = p_1 - p_2, \quad \Delta p_2 = p_2 - p_3, \quad \Delta p_3 = p_3 - p_4 \qquad eq.(4.43)$$

or,

$$\Delta \mathrm{p} = p_1 - p_4 = \Delta \mathrm{p}_1 + \Delta \mathrm{p}_2 + \Delta \mathrm{p}_3 \qquad eq.(4.44)$$

and,

$$q_t = q_1 = q_2 = q_3 \qquad eq.(4.45)$$

Now, Darcy's law can be written for the total flow rate as:

$$q_t = \frac{k_{avg} W h \Delta p}{\mu L} \qquad eq.(4.46)$$

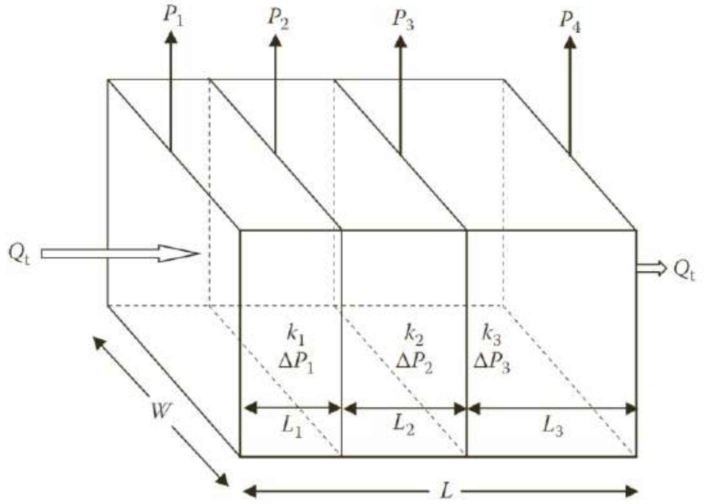


Fig. (4.15): Fluid flow through a series combination.

Subsequently, the differential pressures can be separated from Equations (4.40) through (4.42) and (4.46) and substituted into Equation (4.44):

$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3$$

$$\frac{q_t \mu L}{k_{avg} W h} = \frac{q_1 \mu L_1}{k_1 W h} + \frac{q_2 \mu L_2}{k_2 W h} + \frac{q_3 \mu L_3}{k_3 W h} \qquad eq. (4.47)$$

or,

$$\frac{L}{k_{avg}} = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \qquad eq. (4.48)$$

$$k_{avg} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} (L_i / k_i)} \qquad eq. (4.49)$$

Equation (4.49) is used for calculating the average absolute permeability for the linear flow system.

In the radial system shown in Figure (4.16), the preceding averaging methodology can be applied to produce the following generalized expression:

$$k_{avg} = \frac{\ln(r_e / r_w)}{\sum_{j=1}^{n} \left[\frac{\ln(r_j / r_{j-1})}{k_j} \right]} \qquad eq. (4.50)$$

Equation (4.50) comes from the following expressions:

$$q_t = q_1 = q_2 = q_3$$

$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3$$

$$\Delta p_1 = \frac{q_1 \mu \ln \frac{r_1}{r_w}}{k_1 A}$$

$$\Delta p_2 = \frac{q_2 \mu \ln \frac{r_2}{r_1}}{k_2 A}$$

$$\Delta p_3 = \frac{q_3 \mu \ln \frac{r_3}{r_2}}{k_3 A}$$

$$\Delta p_t = \frac{q_t \mu \ln \frac{r_e}{r_w}}{k_{avg} A}$$

$$\frac{q_t \mu \ln \frac{r_e}{r_w}}{k_{avg} A} = \frac{q_1 \mu \ln \frac{r_1}{r_w}}{k_1 A} + \frac{q_2 \mu \ln \frac{r_2}{r_1}}{k_2 A} + \frac{q_3 \mu \ln \frac{r_3}{r_2}}{k_3 A}$$

$$\frac{\ln \frac{r_e}{r_w}}{k_{avg}} = \frac{\ln \frac{r_1}{r_w}}{k_1} + \frac{\ln \frac{r_2}{r_1}}{k_2} + \frac{\ln \frac{r_3}{r_2}}{k_3}$$

$$k_{avg} = \frac{\ln \frac{r_e}{r_w}}{\frac{\ln \frac{r_1}{r_w}}{k_1} + \frac{\ln \frac{r_2}{r_1}}{k_2} + \frac{\ln \frac{r_3}{r_2}}{k_3}}$$

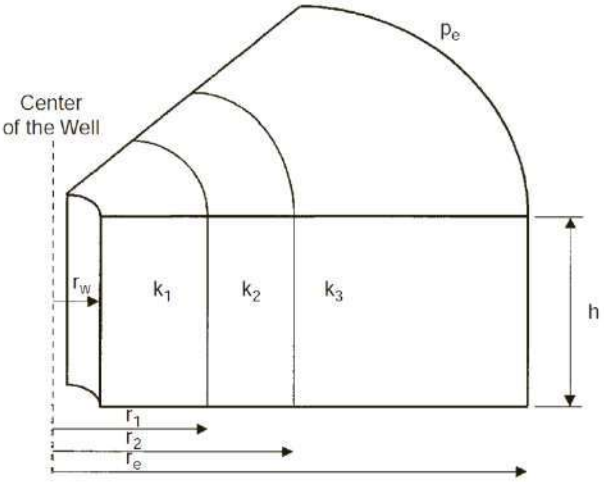


Fig. (4.16): Radial Flow through series beds.

The relationship in Equation (4.50) can be used as a basis for estimating a number of useful quantities in production work. For example, the effects of mud invasion, acidizing, or well shooting can be estimated from it.

Example 4.11:

A hydrocarbon reservoir is characterized by five distinct formation segments that are connected in series. Each segment has the same formation thickness. The length and permeability of each section of the five-bed reservoir follow:

Lenght, ft	Permeability, md
150	80
200	50
300	30
500	20
200	10

Calculate the average permeability of the reservoir by assuming:

a. Linear flow system.
b. Radial flow system.

Solution:

For a ***linear system:***

L_i, ft	k_i, md	L_i/k_i
150	80	1.8750
200	50	4.000
300	30	10.000
500	20	25.000
200	10	20.000
1350		$\sum L_i/k_i = 60.785$

Using Equation (4.49) gives:

$$k_{avg} = \frac{1350}{60.875} = 22.18md$$

For a ***radial system:***

The solution of the radial system can be conveniently expressed in the following tabulated form. The solution is based on Equation (4.50) and assumes a wellbore radius of 0.25 ft:

Segment	r_j, ft	$\ln(r_j/r_{j-1})$	k_j	$[\ln(r_j/r_{j-1})]/k_j$
Wellbore	0.25	-	-	-
1	150	6.397	80	0.080
2	350	0.847	50	0.017
3	650	0.619	30	10.021
4	1150	0.571	20	0.029
5	1350	0.160	10	0.016
				0.163

From Equation (4.50),

$$k_{avg} = \frac{\ln(1350/0.25)}{0.163} = 52.72\, md$$

4.3.1.3 Geometric Average Permeability:

Warren and Price (1961) illustrated experimentally that the most probable behavior of a heterogeneous formation approaches that of a uniform system having a permeability that is equal to the geometric average. The geometric average is defined mathematically by the following relationship:

$$k_{avg} = exp\left[\frac{\sum_{i=1}^{n}(h_i \ln(k_i))}{\sum_{i=1}^{n}(h_i)}\right] \qquad eq. (4.51)$$

where:

k_i= permeability of core sample i.
h_i= thickness of core sample i.
n = total numbers of samples.

Example 4.12:

Given the following core data, calculate the geometric average permeability:

Sample	**h_i, ft**	**k_i, md**
1	1	10
2	1	30
3	0.5	100
4	1.5	40
5	2	80
6	1.5	70

7	1	15
8	1	50
9	1.5	35
10	0.5	20

Solution:

Sample	*h_i, ft*	*k_i, md*	*h_i * ln(k_i)*
1	1	10	2.303
2	1	30	3.401
3	0.5	100	2.303
4	1.5	40	5.533
5	2	80	8.764
6	1.5	70	6.373
7	1	15	2.708
8	1	50	3.912
9	1.5	35	5.333
10	0.5	20	1.498
	sum = 11.5		sum = 42.128

$$k_{avg} = exp\left[\frac{\sum_{i=1}^{n}(h_i \ln(k_i))}{\sum_{i=1}^{n}(h_i)}\right]$$

$$k_{avg} = exp\left[\frac{42.128}{11.5}\right] = 39\, md$$

4.3.2 Absolute Permeability Correlations:

Timur (1968) proposed the following expression for estimating the permeability from connate-water saturation and porosity:

$$k\ (md) = \left(\frac{(93).\phi^{2.2}}{S_{wc}}\right)^2 \qquad eq.(4.52)$$

$$k\ (darcy) = 8.58102\left(\frac{\phi^{2.2}}{S_{wc}}\right)^2 \qquad eq.(4.53)$$

While Morris and Biggs (1967) presented the following two expressions for estimating the permeability if oil and gas reservoirs:

For an oil reservoir:

$$k\ (darcy) = 62.5\left(\frac{\phi^3}{S_{wc}}\right)^2 \qquad eq.(4.54)$$

For a gas reservoir:

$$k\ (darcy) = 2.5\left(\frac{\phi^3}{S_{wc}}\right)^2 \qquad eq.(4.55)$$

where:

k = absolute permeability.
ϕ = porosity, fraction.
S_{wc} = connate water saturation, fraction.

Example 4.13:

Estimate the absolute permeability of an oil zone with a connate-water saturation and average porosity of 25% and 19%, respectively.

Solution:

Applying the Timur equation:

$$k = \left(\frac{(93).(0.19)^{2.2}}{0.25}\right)^2 = 92.8\,md$$

From the Morris and Biggs correlation:

$$k = 62.5\left(\frac{0.19^3}{0.25}\right)^2 = 0.047\ darcy$$

4.3.3 Permeability of Fractures and Channels:

The various aspects of absolute permeability looked at so far were confined to the matrix permeability. However, petroleum reservoir rocks such as sandstones and carbonates frequently contain solution channels and natural or artificial fractures. Therefore, the key to understanding the flow of fluids in petroleum reservoir rocks is to account for flow in all the three permeability elements: the matrix, channel, and fracture.

Matrix permeability refers to the flow in primary pore spaces in a reservoir rock. ***Fracture*** or ***channel permeability*** refers to the flow in cracks or breaks in the rock or in the so-called ***secondary network***. However, these channels and fractures do not change the permeability of the matrix but do change the ***effective permeability*** of the overall flow network, mainly because the latter permeability is saturation dependent and pore space saturation distribution varies in matrix or fracture/channel network.

4.3.4 Factors Affecting Absolute Permeability:

A number of factors affect the absolute permeability of a reservoir rock. The discussion of these factors clearly categorizes and subsequently reviews each factor which can be grouped as rock-related factors, fluid phase-related factors, thermodynamic factors, and mechanical factors:

- ***Rock-related factors*** are basic characteristics, structure, or indigenous properties of reservoir rocks, such as grain size and shape and clay cementing. These can in fact also be termed natural factors.
- ***The type of fluid medium*** (i.e., gas/brine/water) used for permeability measurement as well as the physical and chemical characteristics of these fluids are also major factors that affect the absolute permeability. These factors can be characterized as artificial or laboratory factors that may temporarily affect permeability.
- ***The thermodynamic factors*** affecting absolute permeability basically consist of temperature effects, based on some literature data, these fall under the category of fluid–rock interaction-induced laboratory artifacts that affect permeability.
- ***The mechanical factors*** are related to the effect of mechanical stresses or confining pressures on absolute permeability and also fall under the category of laboratory artifacts.

4.3.5 Laboratory Measurement of Absolute Permeability:

Laboratory measurement of absolute permeability usually involves the direct application of the Darcy equation, based on the measurement of individual variables such as flow rate, pressure drop, sample dimensions, and fluid properties. Most laboratory measurements are carried out on formation samples of well-defined geometry, such as cylindrical core plugs. These core plugs are generally 1 or 1.5 in. in diameter with lengths typically varying from 2 to 4 in.

Prior to using core plug samples for permeability measurements, the residual fluids or in situ formation fluids are removed so that the sample is 100%

saturated by air. Considering the fact that Absolute permeability can only be measured by conducting a flow experiment in a porous media, gases or nonreactive liquids are commonly used as a fluid phase. Several commercial benchtop permeameters or minipermeameters that use gases or nonreactive liquids for permeability measurement are available.

The apparatus used for conducting flow experiments on core plug samples is a core flooding rig or a displacement apparatus. The schematic of a typical displacement apparatus capable of using both liquids and gases for permeability measurement is shown in figure (4.17).

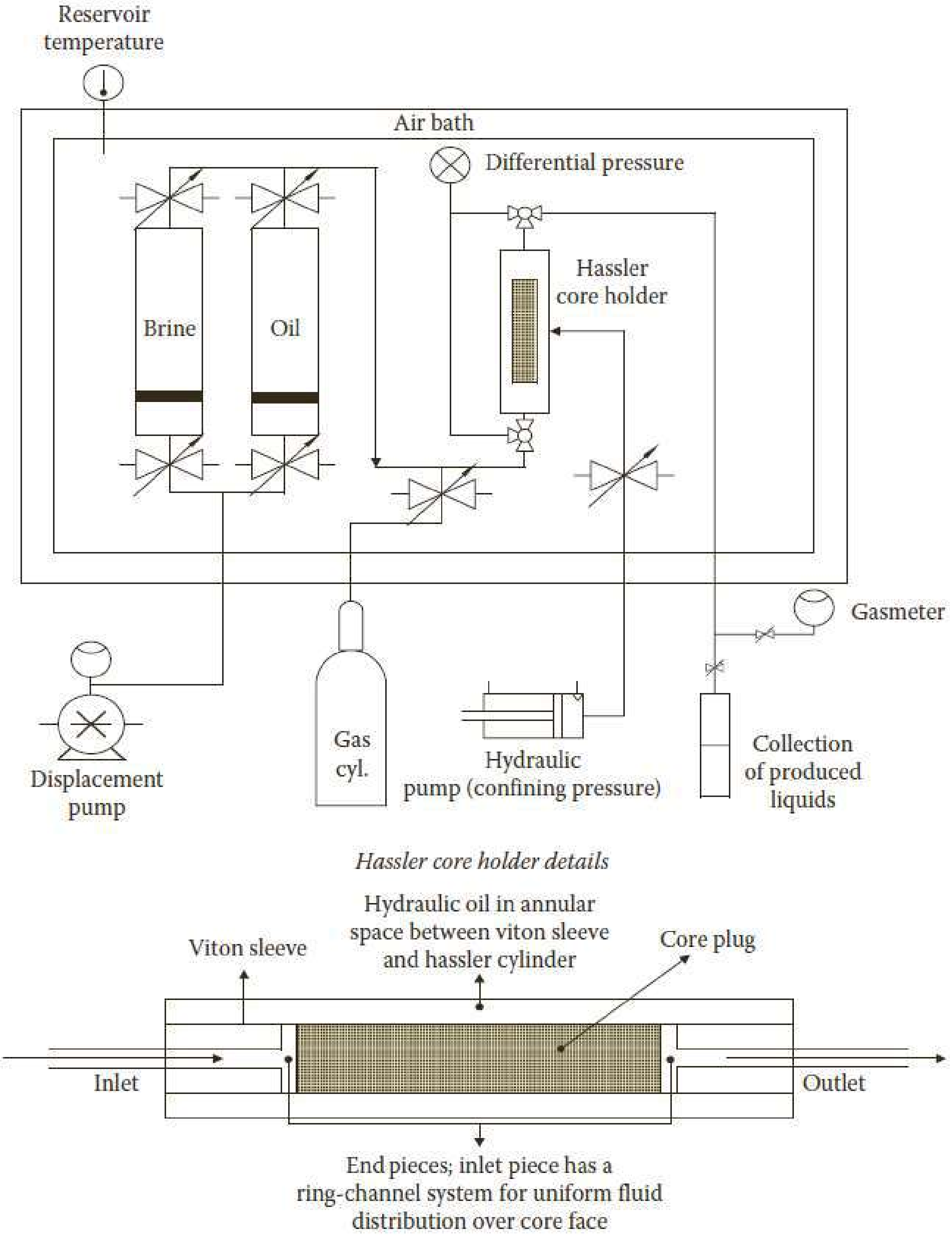


Fig. (4.17): Schematic of a typical displacement apparatus for absolute permeability measurement using gases and liquids, (Dandekar, 2013).

The most common liquids used for the measurement of absolute permeability are formation waters (sometimes called brine) or degassed crude oil. Formation water or crude oil used is generally from the same formation for which the absolute permeability measurement is desired. In some cases, synthetic oil is also used. Absolute permeability measurements of core plug samples are also carried out using gases instead of liquids. Dry gases such as nitrogen, helium, or air are commonly used as the fluid medium in permeability measurements. Choosing a gas is simply convenient and practical because a gas is clean, nonreactive, and does not alter the pore network; in other words, absolute permeability measurements are not influenced by any rock–fluid interactions.

4.3.6 Klinkenberg Effect:

Klinkenberg (1941) discovered that permeability measurements made with air as the flowing fluid showed different results from permeability measurements made with a liquid as the flowing fluid. The permeability of a core sample measured by flowing air is always greater than the permeability obtained when a liquid is the flowing fluid. Klinkenberg postulated, on the basis of his laboratory experiments, that liquids had a zero velocity at the sand grain surface, while gases exhibited some finite velocity at the sand grain surface. In other words, the gases exhibited slippage at the sand grain surface. This slippage resulted in a higher flow rate for the gas at a given pressure differential. Klinkenberg also found that for a given porous medium, as the mean pressure increased the calculated permeability decreased.

Mean pressure is defined as upstream flowing plus downstream flowing pressure divided by 2, [$p_m = (p_1 + p_2)/2$]. If a plot of measured permeability versus 1/pm were extrapolated to the point where $1/p_m = 0$, in other words, where p_m = infinity, this permeability would be approximately equal to the liquid permeability. A graph of this nature is shown in Figure (4.18). The absolute permeability is determined by extrapolation as shown in Figure (4.18).

The magnitude of the Klinkenberg effect varies with the core permeability and the type of gas used in the experiment as shown in Figures (4.19) and (4.20). The resulting straight-line relationship can be expressed as:

$$k_g = k_L + c\left[\frac{1}{p_m}\right] \qquad eq.(4.56)$$

where:

k_g= measured gas permeability.

p_m = mean pressure.

k_L = equivalent liquid permeability, i.e., absolute permeability, k.

c = slop of the line.

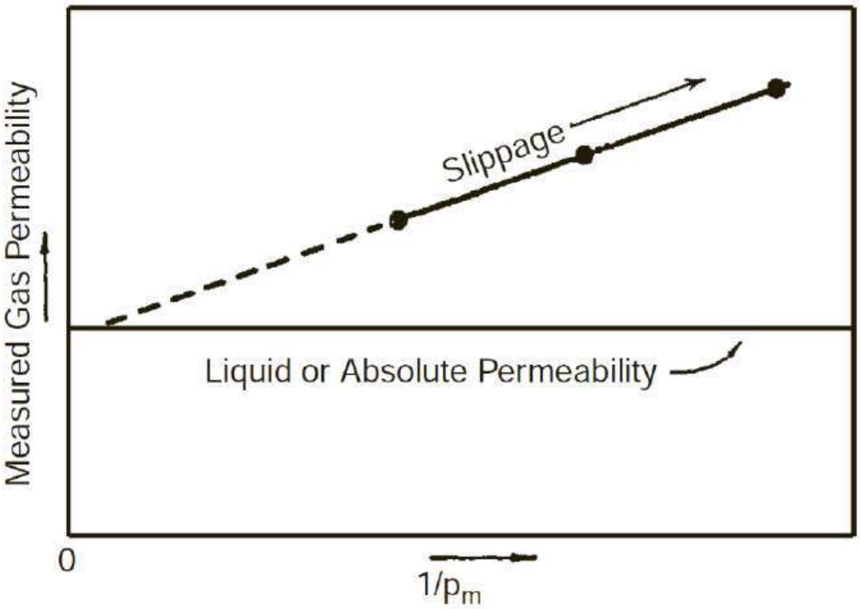


Fig. (4.18): The Klinkenberg effect in gas permeability measurements.

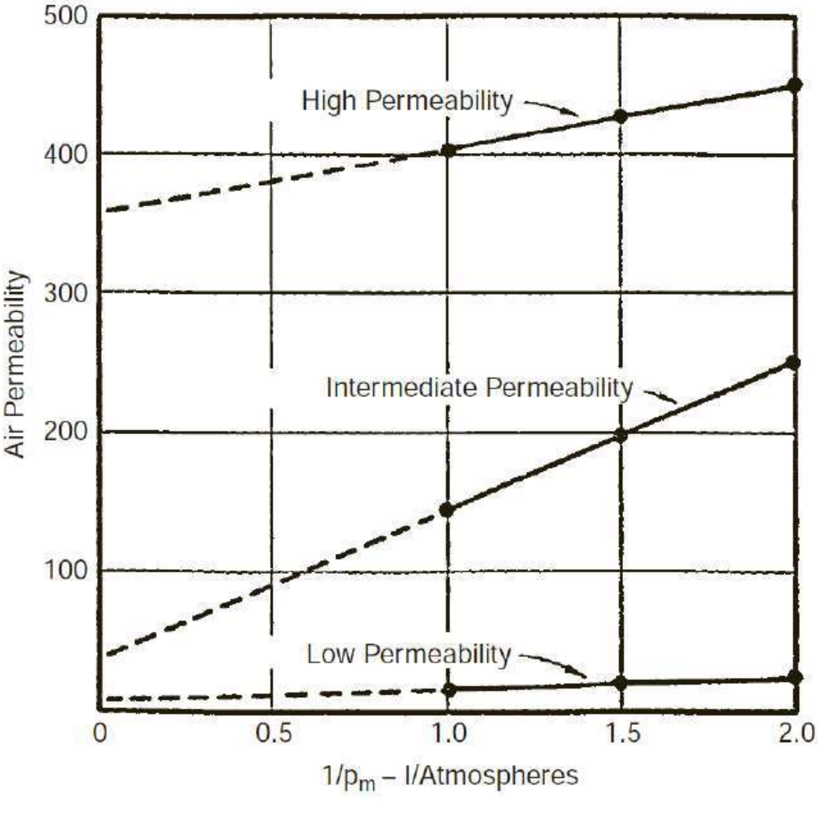


Fig.(4.19): Effect of permeability on the magnitude of the Klinkenberg effect. (After Cole, F., 1969.)

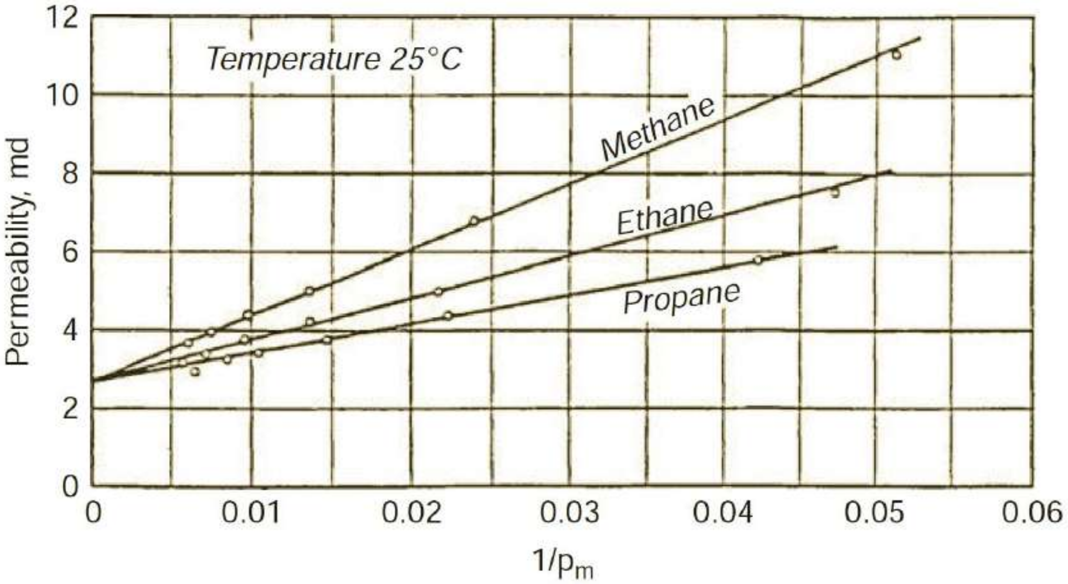


Fig. (4.20): Effect of gas pressure on measured permeability for various gases. (After Calhoun, J., 1976).

4.4 Surface and Interfacial Tensions:

In petroleum reservoirs, up to three fluid phases, gas, oil, and water, may coexist. All these fluid phases are immiscible at the pertinent reservoir conditions. When these immiscible fluid phases in a petroleum reservoir are in contact, these fluids are separated by a well-defined interface between gas–oil, gas–water, and oil–water pairs. This particular interface is extremely small in thickness and is typically of the order of about 10 Å (1 Å = 1.0×10^{-10} m); see (Douillard 2009) for reported values of interface thicknesses for some common organic liquids and water. In dealing with multiphase systems such as those encountered in petroleum reservoirs, it is necessary to consider the effect of the forces that exist at the interface when two immiscible fluids are in contact.

The term ***surface tension*** (ST) is normally used when characterizing the **gas–liquid surface forces**, i.e. at the liquid surface. However, in the case of **two immiscible liquids**, the term ***interfacial tension*** (IFT) is used when describing the liquid–liquid interfacial forces (also between liquid and solid). Technically, in a petroleum reservoir that contains all the three phases— gas, oil, and water—three different IFT or ST values are of significance: **gas–oil ST, gas–water ST, and oil–water IFT.**

To understand the concept of interfacial tension or surface tension, consider a system of two immiscible fluids, oil and water, as shown in Figure (4.21). An oil or water molecule, remote from the interface, is surrounded by other oil or water molecules, thus having a resulting net attractive force on the molecule of zero as it is pulled in all directions. However, a molecule at the interface has a force acting upon it from the oil lying immediately above the interface and water molecules lying below the interface. The resulting forces are not balanced because the magnitude of forces is different (i.e., forces from above and below) and gives rise to interfacial tension. and make the surface to concave to downward.

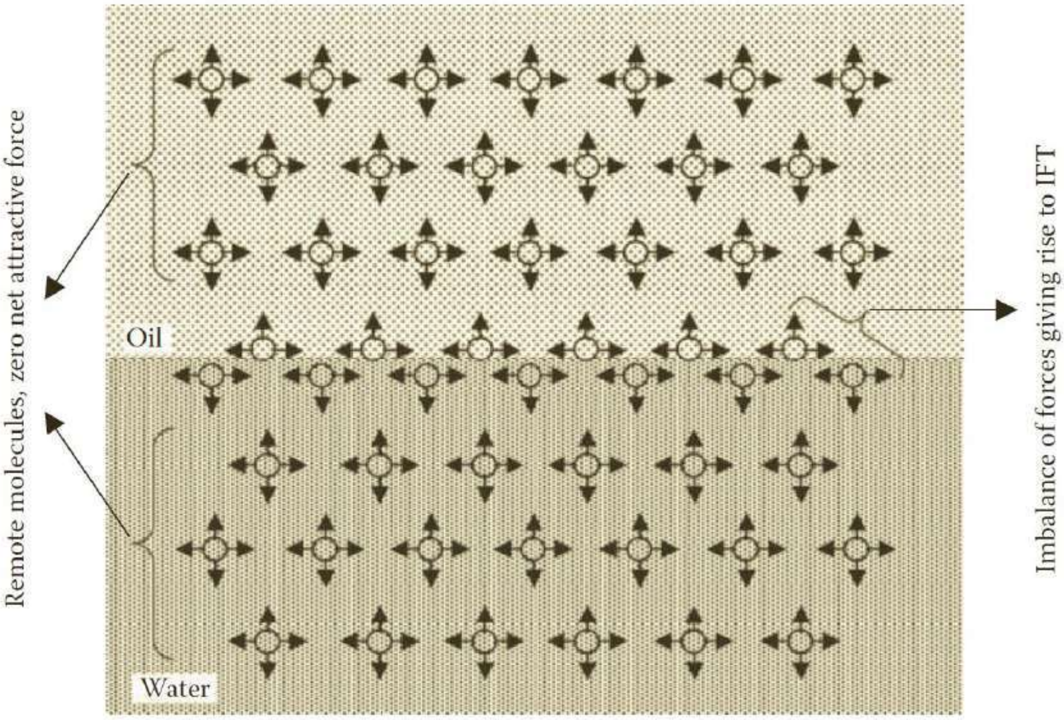


Fig. (4.21): The concept of interfacial tension (IFT) between two immiscible liquids.

Given the earlier definition of surface or interfacial tension, it has the dimensions of force per unit length usually expressed as (mN/m) or (10^{-3} N/m (dyne/cm)) and commonly denoted by the Greek symbol σ, (see Fig. (4.22)).

Despite the fact that interfacial tension or surface tension is an entirely fluid- or interface-related property and not a petroleum reservoir rock property, it significantly influences other important rock properties such as wettability (see section 4.5), capillary pressure (see section 5.1), and relative permeabilities, all of which in turn affect the recovery of hydrocarbon fluids from petroleum reservoirs.

If a glass capillary tube is placed in a large open vessel containing water, the combination of surface tension and wettability of tube to water will cause water to rise in the tube above the water level in the container outside the tube as shown in Figure (4.22).

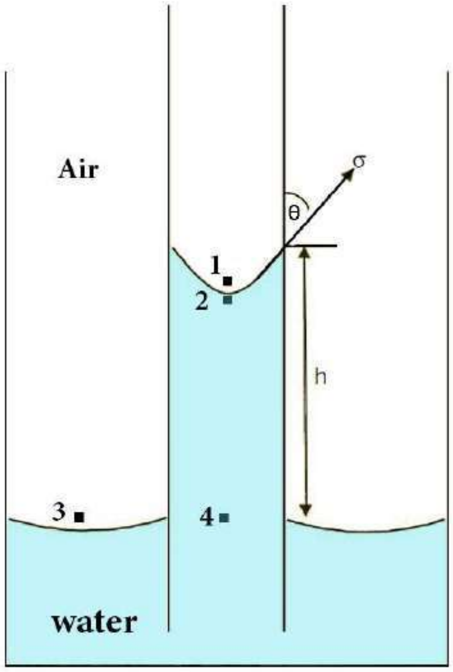


Fig. (4.22): Pressure relations in capillary tubes.

The water will rise in the tube until the total force acting to pull the liquid upward is balanced by the weight of the column (due to gravity force) of liquid being supported in the tube. Assuming the radius of the capillary tube is r, the total upward force F_{up}, which holds the liquid up, is equal to the force per unit length of surface times the total length of surface, or

$$F_{up} = (2\pi r)(\sigma_{gw})(\cos\theta) \qquad eq. (4.57)$$

where:

σ_{gw} = surface tension between air (gas) and water (oil), dynes/cm.
θ = contact angle.
r = radius, cm.

The upward force is counteracted by the weight of the water, which is equivalent to a downward force of mass times acceleration, or:

$$F_{down} = \pi r^2 h(\rho_w - \rho_{air})g \qquad eq. (4.58)$$

where:

h = height to which the liquid is held, cm.
g = acceleration due to gravity, cm/sec^2.
ρ_w = density of water, gm/cm^3.
ρ_{air} = density of gas, gm/cm^3.

Because the density of air is negligible in comparison with the density of water, Equation (4.58) is reduced to:

$$F_{down} = \pi r^2 h \rho_w g \qquad eq.(4.59)$$

At points 1 and 2 in Fig. (4.22):

$$F_{up} = F_{down}$$

$$(2\pi r)(\sigma_{gw})(\cos\theta) = \pi r^2 h \rho_w g$$

Equating Equation (4.57) with (4.59) and solving for the surface tension between water and gas gives:

$$\sigma_{gw} = \frac{rhp_w g}{2\cos\theta} \qquad eq.(4.60)$$

The generality of Equations (4.57) through (4.60) will not be lost by applying them to the behavior of two liquids, i.e., water and oil. Because the density of oil is not negligible, Equation (4.60) becomes:

$$\sigma_{ow} = \frac{rhg(\rho_w - \rho_o)}{2\cos\theta} \qquad eq.(4.61)$$

where:

ρ_o = density of oil, gm/cm^3.
σ_{ow} = interfacial tension between the oil and the water, dynes/cm.

4.4.1 Effect of Pressure and Temperature on Interfacial and Surface Tension:

Katz (1959) presented ST data as a function of temperature, on a number of pure hydrocarbon components showing the decline in ST values with an increase in the temperature. The effect of pressure on ST is also somewhat similar to the temperature effect. When considering the gas–liquid ST at high pressures, in most cases, the high-pressure vapor over the surface of a liquid would result in a low ST by bringing a fairly large number of gaseous molecules within reach of the surface. The attractions of these molecules to the surface molecules of the liquid would neutralize, to some extent, the inward attraction on the surface molecules diminishing the ST, (Danesh, A., 1998). The high-pressure gas phase tends to develop miscibility toward the companion liquid phase, thereby reducing ST as pressure increases. There are many studies in this subject like Kundt's (1881), Dandekar (1994) and Hjelmeland and Larrondo (1986). These studies stated that the effects of pressure and temperature on interfacial tension indicate that observed trends would depend on the type of systems studied. This phenomenon has not been well explained, and a theoretical explanation is still lacking.

4.4.2 Laboratory Measurement of Interfacial Tension:

The experimental techniques that are used for measuring interfacial tension or surface tension are essentially identical, that is, an apparatus or experimental setup used for measuring the interfacial tension between two liquids, generally, can also be employed for conducting surface tension measurements. A variety of experimental techniques are available for the measurement of IFT or ST values and are referred to as ***tensiometers***. These tensiometers are instruments that simply measure force. Some of the commonly used tensiometers are the Wilhelmy plate method, du Noüy ring method, spinning drop method, and the maximum pull force method, (for more details see (Dandekar, 2013)).

4.5 Wettability:

Wettability is the tendency of one fluid to spread on a solid surface in the presence of other immiscible fluids. In dealing with petroleum reservoir fluids in a reservoir system, it is necessary to consider not only the surface and interfacial tension between immiscible fluid phases but also the forces that are active at the interface between the liquids and the solids (reservoir rock surface). The consideration of the interface between the liquids and the solid assumes a significant importance in reservoir engineering simply because the petroleum reservoir fluids are ubiquitously in contact with the solid (reservoir rocks) until they are brought to the surface as part of the production process. It is the combination of all the active forces that determines the wettability of reservoir rocks. The knowledge of reservoir wettability is critical because it influences various important reservoir properties such as the distribution of gas, oil, and water within a reservoir rock, capillary pressure and relative permeability characteristics, and consequently the production of hydrocarbons.

For example, in a system comprising of oil, water (brine), and rock, (sandstone or a carbonate), one of the phases (either the oil phase or the water phase) has a tendency to preferentially wet the rock. The concept of wettability, from the very basic definition, is simple to illustrate and is shown in Figure (4.23).

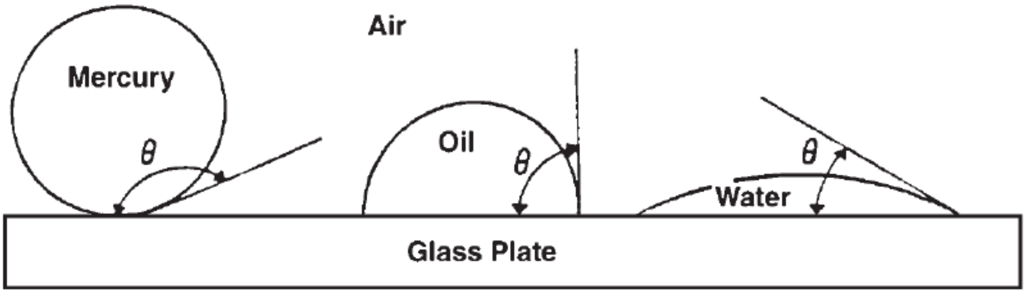


Fig. (4.23): Illustration of wettability

The spreading tendency of a fluid can be expressed more conveniently as ***adhesion tension, A_T***, which was first introduced by (Benner et al. in 1938). For the system shown in Figure (4.24), (oil and water system), the adhesion tension is defined by:

$$A_T = \sigma_{so} - \sigma_{sw} \qquad eq.(4.62)$$

where:

A_T = *adhesion tension.*
σ_{so} = the interfacial tension between the solid and the lighter fluid phase (oil in this case).
σ_{sw} = the interfacial tension between the solid and the denser fluid phase (water in this case).

The angle of contact, θ (measured through the denser liquid phase, water in this case), at the liquid–solid surface is also shown in Figure (4.24). Obviously, the value of θ will range from 0° to 180°. By definition, the cosine of the contact angle θ is:

$$\cos\theta_{ow} = \frac{\sigma_{so} - \sigma_{sw}}{\sigma_{ow}}$$

$$\cos\theta_{ow}\,\sigma_{ow} = \sigma_{so} - \sigma_{sw} \qquad eq.(4.63)$$

Combining Equations (4.62) and (4.63):

$$A_T = \sigma_{ow}\cos\theta_{ow} \qquad eq.(4.64)$$

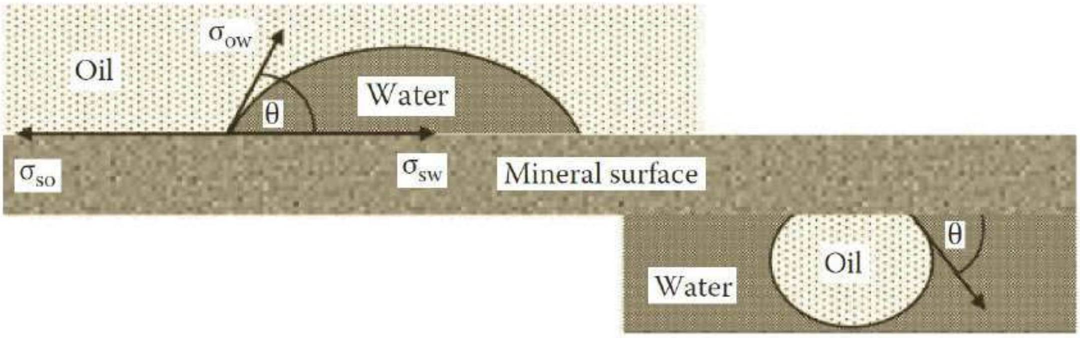


Fig. (4.24): Schematic of a system of two immiscible liquids (oil and water) in contact with a mineral surface.

According to Equation (4.64), ***positive*** adhesion tension indicates that the denser phase (water in this case) preferentially wets the solid surface, whereas a ***negative*** value of adhesion tension indicates a wetting preference by the lighter phase (oil in this case). An adhesion tension of ***zero*** indicates that both phases (oil and water) have equal wettability or affinity for the solid surface. The wetting preferences indicated by the adhesion tension can also be expressed in terms of the contact angle; a 0° contact angle indicates a completely water-wet system, whereas a contact angle of 180° indicates an oil-wet system. A 90° contact angle indicates a neutral-wet system, that is, both phases have equal affinity for the solid surface.

4.5.1 Classification or Types of Wettability:

A variety of wettability states exist for petroleum reservoirs, primarily depending on both reservoir fluid and rock characteristics. On a pore level, wettability in porous media has been classified as either homogeneous or heterogeneous. By definition, homogeneous means the entire rock surface has uniform wetting tendencies, whereas heterogeneous indicates distinct rock surface regions that exhibit different wetting tendencies. (Radke et al., 1992) have stated that strongly water wet, strongly oil wet, and intermediate wet systems fall under the category of homogeneous, whereas fractional and mixed-wet systems fall under the category of heterogeneous.

4.5.1.1 Water Wet:

In this wettability state, all pore surfaces of the rock have preference for the water phase rather than the hydrocarbon phase, and as a result of this condition, the gas and oil are contained in the centers of the pores. See Figure (4.25, A) for a simple pore level illustration.

4.5.1.2 Oil Wet:

This wettability state is exactly the opposite of the water-wet state, that is, the relative positions of the hydrocarbons and water are reversed. It is believed that surface active asphaltenic components of the oil phase cause this wetting state. See Figure (4.25, B) for a simple pore level illustration.

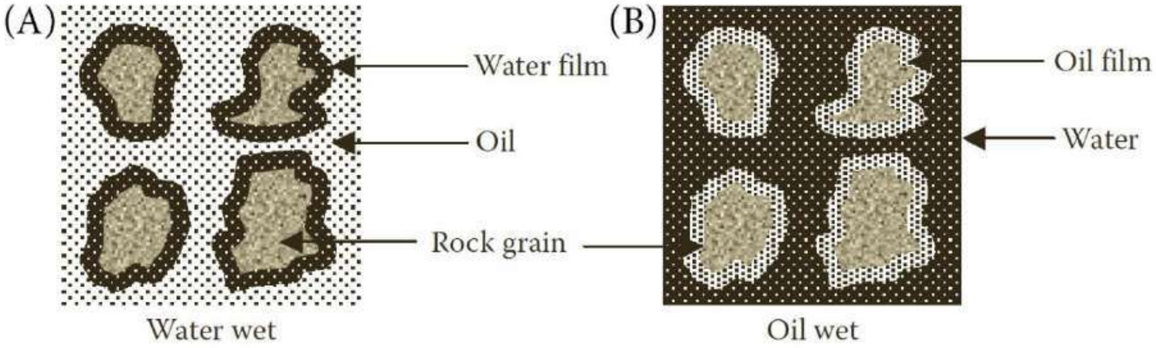


Fig. (4.25): Schematic representation of water-wet and oil-wet pore spaces.

4.5.1.3 Intermediate Wet:

The definition of intermediate wettability state from a pore level standpoint is somewhat vague in that there is some tendency for both phases (oil and water) to have preference for the rock surface; however, if that tendency is equal, then this may be termed as neutral-wetting state or considered as a special category of intermediate wettability.

4.5.1.4 Fractional Wettability:

Fractional wettability has been variously characterized as Dalmatian, speckled, or spotted (Radke, et al, 1992) because some of the pores are water wet, while others are oil wet, or, in other words, a portion of the rock is strongly water wet, while the rest is strongly oil wet. Dalmatian wettability is where there are oil-wet and water-wet regions in the same pore.

4.5.1.5 Mixed Wettability:

Mixed wettability was proposed by (Salathiel in 1973), referring to a special type of fractional wettability in which the oil-wet surfaces form continuous paths through the larger pores. Salathiel, however, states that mixed wettability he introduced should be distinguished from the fractional wettability. (Jerauld and Rathmell) in their paper have introduced Prudhoe Bay oil field in Alaska as a large and prolific mixed-wet reservoir. However, they conclude that the wettability of Prudhoe Bay differs from the mixed wettability proposed by Salathiel because aspects of the wettability they studied are closer to Dalmatian wettability.

4.5.2 Factors Affecting Wettability:

Reservoir wettability is almost entirely dependent on the characteristics of the fluids involved and the lithology of the rock in question. Obviously, these factors primarily affect the reservoir wettability. Additionally, reservoir pressure and temperature, locations (with respect to depths) of fluid contacts in the reservoir, and effect of drilling-mud filtrate invasion are also some of the factors that play a role in dictating the reservoir wettability. However, many uncertainties exist as to the relative importance of these various factors in affecting the wettability in reservoirs.

4.5.3 Measurement of Reservoir Rock Wettability:

Reservoir wettability can be evaluated by two different groups of methods: ***qualitative*** and ***quantitative***. In qualitative methods, wettability is indirectly inferred from other measurements, such as capillary pressure curves or relative permeability curves. Quantitative methods are direct measurement methods, where the wettability is measured on actual rock samples using reservoir fluid samples and wettability is reported in terms of a certain wettability index, signifying the degree of water, oil wetness, or intermediate wetness. These direct quantitative methods include contact angle measurement, the Amott test, and the U.S. Bureau of Mines (USBM) wettability method. for more details see (Dandekar A., 2013).

Problems

1. The following measurements on pay zone are available:

Sample	Thickness, ft	ϕ, %	S_{oi}, %
1	2	12	75
2	3	16	74
3	1	10	73
4	4	14	76
5	2	15	75
6	2	15	72

Calculate:

a. Average porosity.

b. Average oil and water saturation (assuming no gas).

2. Assuming a steady-state laminar flow, calculate the permeability from the following measurement made on a core sample by using air.

flow rate = 2 cm^3/sec T = 65°F

upstream pressure = 2 atm downstream pressure = 1 atm

A = 2 cm^2 L = 3 cm viscosity = 0.018 cp

3. Calculate average permeability from the following core analysis data:

Depth, ft	k, md
4000-4002	50
4002-4005	20
4005-4006	70
4006-4008	100
4008-4010	85

4. Calculate the average permeability of a formation that consists of four beds in series, assuming:

a. Liner System

b. Radial system with r_w=0.3 and r_e =1450 ft

Bed	Length of bed Linear or radial	k, md
1	400	70
2	250	400
3	300	100

4	500	60

5. Calculate the arithmetic average and thickness-weighted average porosity for the following core data:

Sample	**Depth, ft**	**Porosity, %**
1	3705.5	40.1
2	3706.5	35.1
3	3707.5	39.3
4	3708.5	36.5
5	3709.5	29.1

6. Brine flood in a (1.9 in.) long and (1.5 in.) diameter core plug from the North Sea resulted in a stabilized pressure drop of (46.05 psi). The flood was carried out at (0.05 mL/min) with brine viscosity of (0.443 cp). Determine the absolute permeability of this plug in millidarcies.

7. Three beds of equal cross section have permeabilities of (100, 200, and 300 mD) and lengths of (50, 15, and 85 ft), respectively. What is the average permeability of the beds placed in series?

8. Three beds of (50, 110, and 795 mD) and (5, 7, and 15 ft) thick, respectively, are conducting fluid in parallel flow. If all are of equal length and width, what is the average permeability?

9. The daily production from a certain oil well is (718.5 bbls). The drainage area is (160 acre) where the average pressure is at (1850 psi). If the oil viscosity = 2.2 cp, well-bore radius = 0.5 ft, thickness of pay zone = 16 ft, and formation permeability = 180 mD, what is the well flowing pressure? Note: 1 acre = 43560 ft^2.

10. A (1 ft × 1 ft × 1 ft) North Sea chalk gridblock contains three solution channels (0.1 mm in diameter), traversing the entire length. If the absolute permeability of the sample matrix is (5 mD), then what would be the flow rate of a (10 cp) oil through this gridblock under a pressure drop of (5 atm)?

11. For the same sample as in Problem 10, if it now contains three fractures (0.1 mm wide), traversing the entire length, what would be the oil flow rate through this gridblock under a pressure drop of (5 atm)?

12. Answer the following questions:
    a. Injection of gas in a gas cap to increase oil recovery is an imbibition process. **True or False**
    b. Water injection in a water-wet system is a drainage process. **True or False**
    c. Hydrocarbon gas phase is always a nonwetting phase. **True or False**

REFERENCES

Ahmed, T., 2019, "*Reservoir Engineering Handbook*", Gulf Professional Publishing

Ahmed, T., 2006, "*Working Guide to Reservoir Rock Properties and Fluid Flow*, Elsevier Inc.

Anderson,W.G., 1986, Wettability literature survey–Part 2: Wettability measurement, Journal of Petroleum Technology, 38, 1246–1262.

Bassiouni, Z., 1994 "*Theory, Measurement, and Interpretation of Well Logs". SPE. TX.

Benner, F.C., Riches, W.W., and Bartell, F.E., 1938, Nature and importance of surface forces in production of petroleum, API Drilling and Production Practice, 442–448.

Benner, F.C. and Bartell, F.E., 1941, The effect of polar impurities upon capillary and surface phenomena in petroleum production, API Drilling and Production Practice, 341–348.

Clark, N., 1969, "*Elements of Petroleum Reservoirs". Dallas, TX: SPE.

Croft, H.O., 1938 "*Thermodynamics, Fluid Flow and Heat Transmission",* McGraw-Hill, New York, p. 129.

Dandekar, A.Y., 2013, *"Petroleum Reservoir Rock and Fluid Properties",* CRC Press.

Dandekar, A.Y., 1994, Interfacial tension and viscosity of reservoir fluids, PhD thesis, Heriot-Watt University, Edinburgh, U.K.

Danesh, A., 1998, "*PVT and Phase Behavior of Petroleum Reservoir Fluids",* Elsevier Science, Amsterdam, the Netherlands.

Darcy, H., 1856, Les Fontaines Publiques de la Ville de Dyon, Victor Dalmont.

Douillard, J.M., 2009, Experimental approach of the relation between surface tension and interfacial thickness of simple liquids, Journal of Colloid and Interface Science, 337, 307–310.

Hjelmeland, O.S. and Larrondo, L.E., 1986, Experimental investigation of the effects of temperature, pressure, and crude oil composition on interfacial properties, SPE Reservoir Engineering, 321–328.

Jerauld, G.R. and Rathmell, J.J., 1997, Wettability and relative permeability of Prudhoe Bay: A case study in mixed-wet reservoirs, Society of Petroleum Engineers SPE paper number 28576.

Katz, D.L., *1959, "Handbook of Natural Gas Engineering",* McGraw-Hill, New York.

Klinkenberg, L.J., 1941, The permeability of porous media to liquids and gases, Drilling and Production Practices, American Petroleum Institute, p. 200.

Kundt, A., 1881, Annalen der Physik, 12, 538; International Critical Tables, 4, 475.

Peters, E.J., 2012, "*Advanced Petrophysics",* The University of Texas at Austin, Austin, Texas 78712, USA.

Radke, C.J., Kovscek, A.R., and Wong, H., 1992, A pore-level scenario for the development of mixed wettability in oil reservoirs, Society of Petroleum Engineers SPE paper number 24880.

Salathiel, R.A., 1973, Oil recovery by surface film drainage in mixed-wettability rocks, Journal of Petroleum Technology, 25, 1216–1224.

S.C. Jones, 1972, A Rapid Accurate Unsteady State Klinkenberg Parameter, SPEJ 12(5) (1972) 383–397.

Tiab, D. and Donaldson, E.C., 1996, "*Theory and Practice of Measuring Reservoir Rocks and Fluid Transport Properties"*, Gulf Publishing Company, Houston, TX.

Wang, W. and Gupta, A., 1995, Investigation of the effect of temperature and pressure on wettability using modified pendant drop method, Society of Petroleum Engineers SPE paper number 30544.

Chapter Five
Capillary Pressure and Relative Permeability

5.1 Capillary pressure:

When two immiscible fluids are in contact, a discontinuity in pressure exists between the two fluids, which depends upon the curvature of the interface separating the fluids. We call this pressure difference the *capillary pressure* and it is referred to by p_c (as seen previously in section 4.4).

Capillary forces in petroleum reservoirs are an expressing of the combination of IFT, wetting characteristics, and pore sizes of a given system. The presence of capillary forces in a porous medium causes hydrocarbon entrapment. The classic example is the migration of hydrocarbons from a source rock to the water-saturated reservoir rock. Petroleum trapped in a reservoir represents an equilibrium state between gravity that wants to move the petroleum upward (due to its low density), which is resisted by capillary pressures of water (with its high density).

Capillary forces also play a major role in the dynamic problems, involving the flow of immiscible fluid phases in porous media under the influence of capillarity, gravity, and an impressed external pressure differential, (Leverett, M.C., 1941).

Thus, it is clear from the foregoing that it is advantageous to understand the nature of these capillary forces both from a static reservoir structure (in terms of fluid contacts, transition zones, and free water level [FWL]) and the dynamic actual hydrocarbon recovery standpoint.

Denoting the pressure in the wetting fluid by p_w and that in the nonwetting fluid by p_{nw}, the capillary pressure can be expressed as:

Capillary pressure = (pressure of the nonwetting phase) – (pressure of the wetting phase)

$$p_c = p_{nw} - p_w \qquad eq.(5.1)$$

That is, the pressure excess in the nonwetting fluid is the capillary pressure, and this quantity is a function of saturation. This is the defining equation for capillary pressure in a porous medium.

There are three types of capillary pressure:

- Water-oil capillary pressure (denoted P_{cwo}).

- Gas-oil capillary pressure (denoted P_{cgo}).
- Gas-water capillary pressure (denoted P_{cgw}).

Applying the mathematical definition of the capillary pressure as expressed by Equation (5.1), the three types of capillary pressure can be written as:

$$p_{cwo} = p_o - p_w$$

$$p_{cgo} = p_g - p_o$$

$$p_{cgw} = p_g - p_w$$

Where p_g, p_o, and p_w represent the pressure of gas, oil, and water, respectively.

The pressure difference across the interface between ***points 1*** and ***2*** as shown in figure (5.1) is essentially the capillary pressure, i.e.:

$$p_c = p_1 - p_2 \qquad eq.(5.2)$$

The pressure of the water phase at ***point 2*** is equal to the pressure at ***point 4*** minus the head of the water (cohesion force), or:

$$p_2 = p_4 - gh\rho_w \qquad eq.(5.3)$$

The pressure just above the interface at *point 1* represents the pressure of the air and is given by:

$$p_1 = p_3 - gh\rho_{air} \qquad eq.(5.4)$$

It should be noted that the pressure at ***point 4*** within the capillary tube is the same as that at ***point 3*** outside the tube. Subtracting Equation (5.3) from (5.4) gives:

$$p_c = gh(\rho_w - \rho_{air}) = gh\Delta\rho \qquad eq.(5.5)$$

Where $\Delta\rho$ is the density difference between the wetting and nonwetting phase. The density of the air (gas) is negligible in comparison with the water density.

In practical units, Equation (5.5) can be expressed as:

$$p_c = \left(\frac{h}{144}\right)\Delta\rho \qquad eq.(5.6)$$

where:

p_c = capillary pressure, *psi (lb/in^2) = lb/144 ft^2; 1 ft=12 in.*
h = capillary rise, *ft.*
$\Delta\rho$ = density difference, *lb/ft^3*.

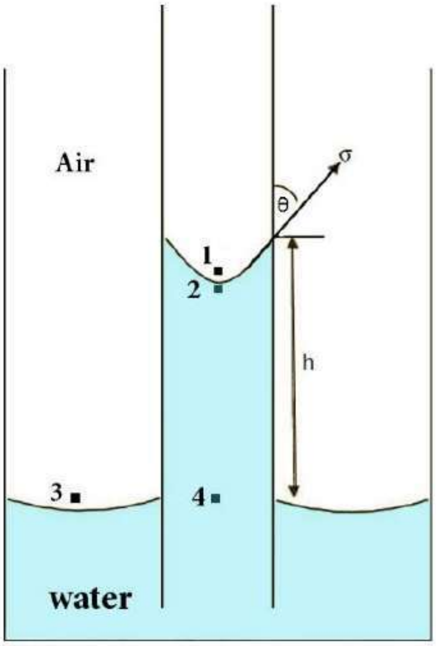


Fig. (5.1): Pressure relations in capillary tubes.

In the case of an oil-water system, Equation (5.6) can be written as:

$$p_c = \left(\frac{h}{144}\right)(\rho_w - \rho_o) \qquad eq.(5.7)$$

The capillary pressure equation can be expressed in terms of the surface and interfacial tension by combining interfacial tension equations with Equation (5.5) to give:

- **Gas-Liquid System.**

$$p_c = gh(\rho_w - \rho_g)$$

from equation (4.61):

$$\sigma = \frac{rhg(\Delta\rho)}{2\cos\theta}$$

$$gh(\Delta\rho) = \frac{2\sigma\cos\theta}{r}$$

$$p_c = \frac{2\sigma_{gw}(\cos\theta)}{r} \qquad eq.(5.8)$$

and,

$$h = \frac{2\sigma_{gw}(\cos\theta)}{rg(\rho_w - \rho_g)} \qquad eq.(5.9)$$

where:

ρ_w = water density, gm/cm^3.
σ_{gw} = gas-water surface tension, dynes/cm.
r = capillary radius, cm.
θ = contact angle.
h = capillary rise, cm.
g = acceleration due to gravity, cm/sec^2.
p_c = capillary pressure, dynes/cm^2.

- **Oil-Water System.**

$$p_c = \frac{2\sigma_{ow}(\cos\theta)}{r} \qquad eq.(5.10)$$

and,

$$h = \frac{2\sigma_{ow}(\cos\theta)}{rg(\rho_w - \rho_o)} \qquad eq.(5.11)$$

where: σ_{ow} is the water-oil interfacial tension.

Example 5.1:

Calculate the pressure difference, i.e., capillary pressure, and capillary rise in an oil-water system from the following data:

$\theta = 30°$ $\quad \rho_w = 1.0\ gm/cm^3$ $\quad \rho_o = 0.75\ gm/cm^3$

$r = 10^{-4} cm$ $\quad \sigma_{ow} = 25\ dynes/cm$

Solution:

Step 1. Apply Equation (5.10) to give:

$$p_c = \frac{2\sigma_{gw}(\cos\theta)}{r}$$

$$p_c = \frac{(2)(25)(\cos 30\,°)}{0.0001} = 4.33 \times 10^5\ dynes/cm^2$$

Since 1 $dyne/cm^2$ = 1.45× 10^{-5}psi

$$p_c = 6.28\,psi$$

This result indicates that the oil-phase pressure is 6.28 psi higher than the water-phase pressure.

Step 2. Calculate the capillary rise by applying Equation (5.11):

$$h = \frac{2\sigma_{ow}(\cos\theta)}{rg(\rho_w - \rho_o)}$$

$$h = \frac{(2)(25)(\cos 30°)}{(0.0001)(980.7)(1.0 - 0.75)} = 1766\ cm = 75.9\ ft$$

5.1.1 Dependence of Capillary Pressure on Rock and Fluid Properties:

Equations (5.8) and (5.10) show the capillary pressure of an immiscible pair of fluids expressed in terms of surface or interface forces, wettability, and capillary size. Capillary pressure is a function of the **adhesion tension** (σ cosθ) and inversely proportional to the radius of the capillary tube. Now we make a qualitative examination of the effect of pore size (capillary radius in this case) and the adhesion tension on capillary pressure. Figure (5.2) shows the effect of varying the wetting characteristics of the system and varying the radius of the capillary tube.

In Figure (5.2):

a. The wetting characteristics are the same, that is, the same IFT and contact angle, but the radius of the capillary tube is different. In this case, the capillary pressure is inversely proportional to the capillary tube radius, while the adhesion tension remains constant. By merely looking at the mathematical expression of capillary pressure, it is easily understood that the higher the capillary tube radius, the lower the capillary pressure, or vice versa.

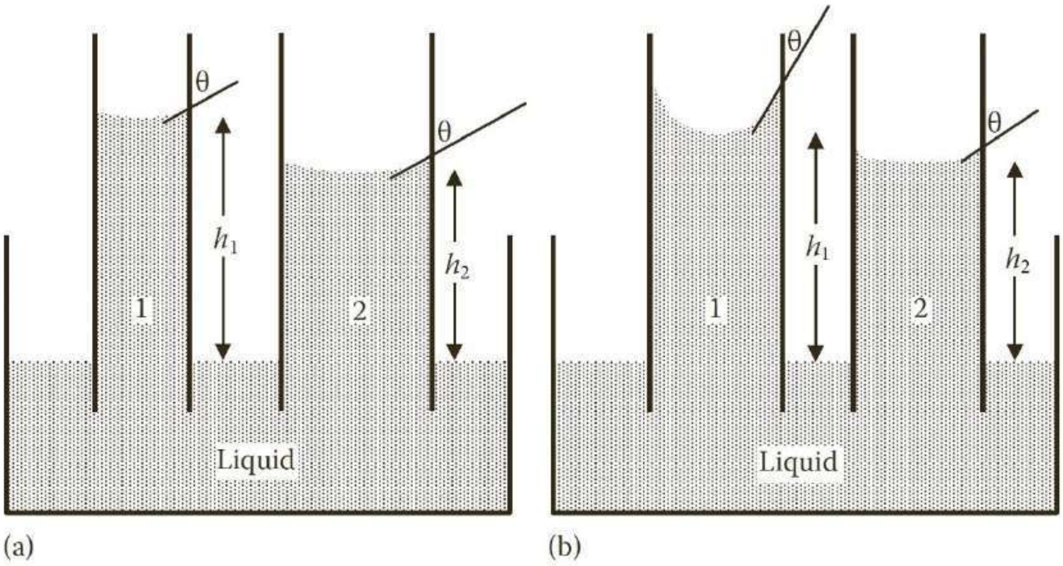


Fig. (5.2): Dependence of capillary pressure on wetting characteristics and pore size (tube radius in this case). Case (a) $\theta_1 = \theta_2$, $h_1 > h_2$ and $r_1 < r_2$, case (b) $\theta_1 < \theta_2$, $h_1 > h_2$ and $r_1 = r_2$.

b. On the other hand, when the capillary tubes of same radius but different wetting characteristics are considered, the denominator in the capillary pressure equation will be a constant, and the value of capillary pressure will be directly proportional to the adhesion tension or the wetting characteristics of the system. Figure (5.2b) shows such a system of same radius and different contact angles. In this case, the smaller the contact angle, the greater the height of liquid rise and stronger the adhesion tension, leading to higher capillary pressure, whereas the opposite is evident from the other tube having weaker wetting characteristics or adhesion tension that obviously results in lower capillary pressure.

5.1.2 Laboratory Measurement of Capillary Pressure:

All laboratory experiments are basically designed to first mimic or simulate the saturation history of the reservoir, that is, migration of hydrocarbons from source rocks into water-filled reservoir rocks. Under the assumption that the rocks are water wet, at least prior to hydrocarbon migration, this is accomplished by the ***drainage process*** by displacing the wetting phase (water) by a nonwetting phase (oil or gas), which primarily establishes the fluid saturations that are found when the reservoir is discovered.

The data from such an experiment are shown in Figure (5.3). Two important phenomena can be observed in Figure (5.3). First, a finite capillary pressure at 100% water saturation is necessary to force the nonwetting phase into a capillary filled with the wetting phase. This minimum capillary pressure is known as the ***displacement pressure,*** p_d**.**

As the wetting phase is displaced, the second phenomenon of any immiscible displacement process is encountered, that is, reaching some finite minimum irreducible saturation. This irreducible water saturation is referred to as ***connate water,*** S_{wc}**.**

The other principal flow process of interest involves reversing the drainage process by displacing the nonwetting phase (oil or gas) with the wetting phase, (e.g., water). This displacing process is termed the ***imbibition process*** and the resulting curve is termed the *capillary pressure imbibition curve*.

Following the two primary displacement processes in the porous medium (drainage and imbibition), laboratory measurement of capillary pressure is confined to imbibition and drainage. In all the laboratory measurement methods, wetting-phase saturation is increased in the imbibition process, while in the drainage process, wetting-phase saturation is decreased from a maximum value (typically 100%) to the irreducible minimum by increasing the capillary pressure from 0 to a large positive value. The plots of capillary pressure data obtained in such a fashion are called *imbibition* and *drainage capillary pressure curves*, respectively. The process of ***saturating*** (see curve (1) in Fig. (5.4)) and

desaturating (see curve (2) in Fig. (5.4)) a core with the nonwetting phase is called ***capillary hysteresis***. Figure (5.4) shows typical *drainage* and *imbibition capillary pressure curves*. The two capillary pressure-saturation curves are not the same.

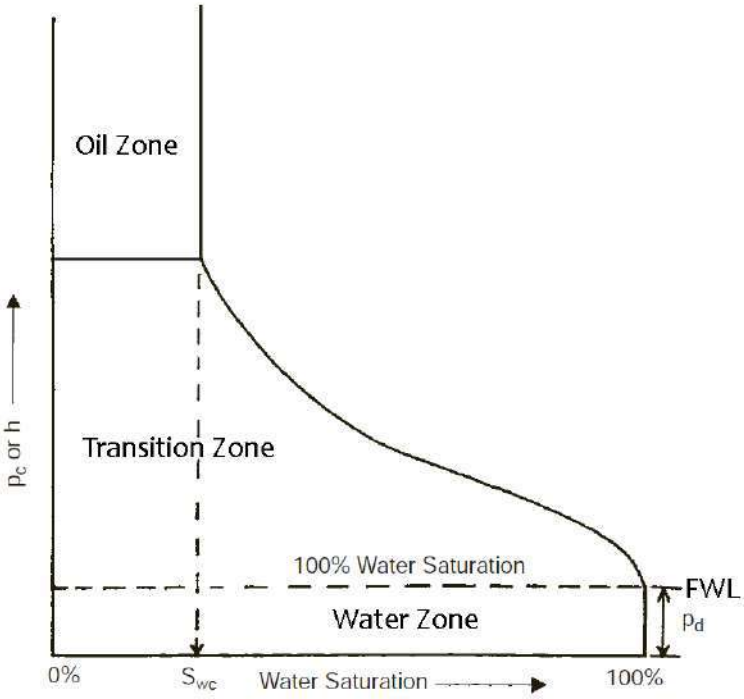


Fig. (5.3): Capillary pressure curve.

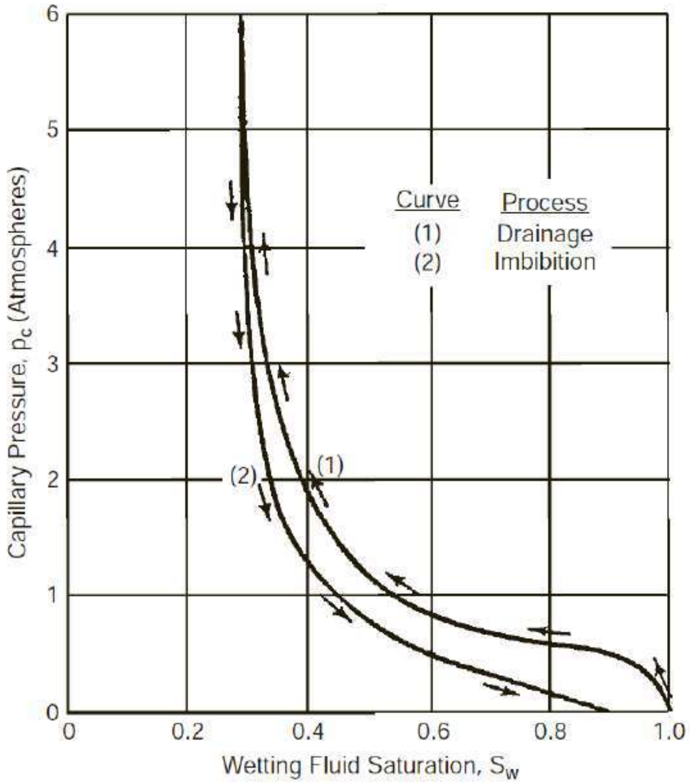


Fig. (5.4): Capillary pressure drainage and imbibition curves.

On the other hand, oil production performance (oil as a nonwetting phase) is generally governed by the imbibition process (water flooding). Therefore, imbibition capillary pressure curves are generally used in reservoir studies, (Ertekin, T. et al, 2001).

In almost all laboratory measurement methods, capillary pressures are measured on cylindrical core plug samples of the representative formation. The laboratory measured imbibition and drainage capillary pressure–saturation relationships can be understood as somewhat equivalent to measuring these relationships for a bundle of several capillaries of varying tortuosities and sizes. The core sample is normally flooded with representative formation water or brine so that a maximum in the wetting-phase saturation is obtained (under the presumption that the sample is water wet). This saturation is then reduced in steps by injecting a nonwetting phase to mimic the original reservoir fluid distribution. Subsequently, the imbibition cycle can be completed by increasing the wetting-phase saturation but without reaching 100% water saturation because of residual oil saturation. Some of the most commonly used laboratory techniques for measuring capillary pressures are listed below:

- Leverett's capillary pressure measurement.
- Porous diagram method.
- Centrifuge method.
- Mercury injection method (MICP).

For more details see (Dandekar, 2013).

Example 5.1:

Plot capillary pressure with saturation curve of the given data and find the p_d and S_{wi} values.

Capillary pressure, (psi)	*Mercury saturation*
0	0
5	0
10	0.1
20	0.3
30	0.6
40	0.8
100	0.9
200	0.9

Solution:

Water saturation = 1 – mercury saturation

Capillary pressure, (psi)	*Mercury saturation*	*Water Saturation*
0	0	1
5	0	1
10	0.1	0.9
20	0.3	0.7
30	0.6	0.4
40	0.8	0.2
100	0.9	0.1
200	0.9	0.1

Figure (5.5) shows the capillary pressure vs. water saturation plot

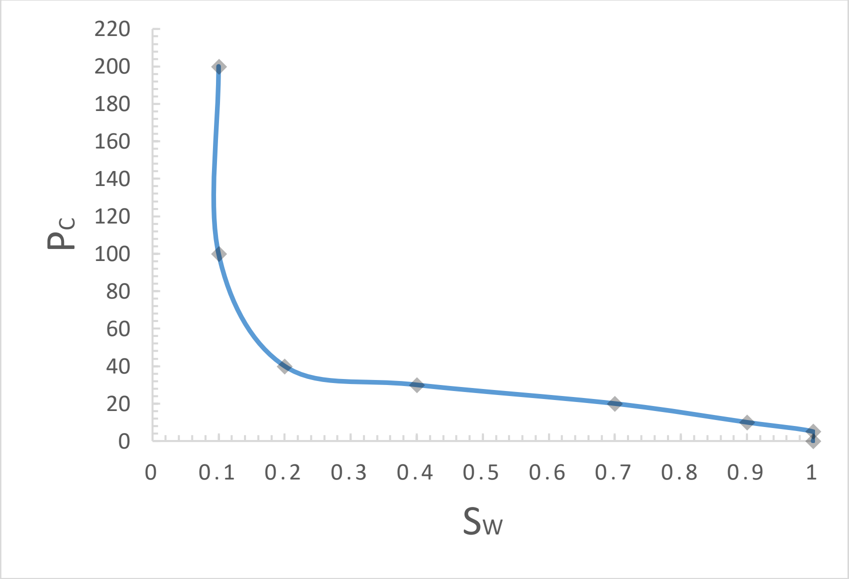


Fig. (5.5): Capillary pressure vs. water saturation.

From figure (5.5) the p_d value is 5 *psi* and S_{wi} is 10%.

5.1.3 Averaging Capillary Pressure Data: J-Function:

Capillary pressure data are obtained on small core samples that represent an extremely small part of the reservoir and, therefore, it is necessary to combine all capillary data to classify a particular reservoir. The fact that the capillary pressure-saturation curves of nearly all naturally porous materials have many features in common has led to attempts to devise some general equation describing all such curves. (Leverett, 1941) approached the problem from the standpoint of dimensional analysis.

Realizing that capillary pressure should depend on: 1- the porosity, 2- interfacial tension, 3- and mean pore radius, Leverett defined the dimensionless function of saturation, which he called the J-function, as:

$$J(S_w) = \frac{p_c}{\sigma \cos\theta}\sqrt{\frac{k}{\phi}}$$

$$J(S_w) = 0.21645\frac{p_c}{\sigma}\sqrt{\frac{k}{\phi}} \qquad eq. (5.12)$$

where:

$J(S_w)$ = Leverett J-function.
p_c = capillary pressure, psi.
σ = interfacial tension, dynes/cm.
k = permeability, md.
ϕ = fractional porosity.

In doing so, Leverett interpreted the ratio of permeability, k, to porosity, ϕ, as being proportional to the square of a mean pore radius.

The J-function was originally proposed as a means of converting all capillary-pressure data to a universal curve. There are significant differences in correlation of the J-function with water saturation from formation to formation, so that no universal curve can be obtained. For the same formation, however, this dimensionless capillary-pressure function serves quite well in many cases to remove discrepancies in the p_c versus S_w curves and reduce them to a common curve. This is shown for various unconsolidated sands in Figure (5.6).

Example 5.4:

A laboratory capillary pressure test was conducted on a core sample taken from the Nameless Field. The core has a porosity and permeability of 16% and 80 md, respectively. The capillary pressure-saturation data are given as follows:

S_w	p_c, psi
1.0	0.50
0.8	0.60
0.6	0.75
0.4	1.05
0.2	1.75

The interfacial tension (σ) is measured at 50 dynes/cm. Further reservoir engineering analysis indicated that the reservoir is better described at a porosity value of 19% and an absolute permeability of 120 md. Generate the capillary pressure data for the reservoir.

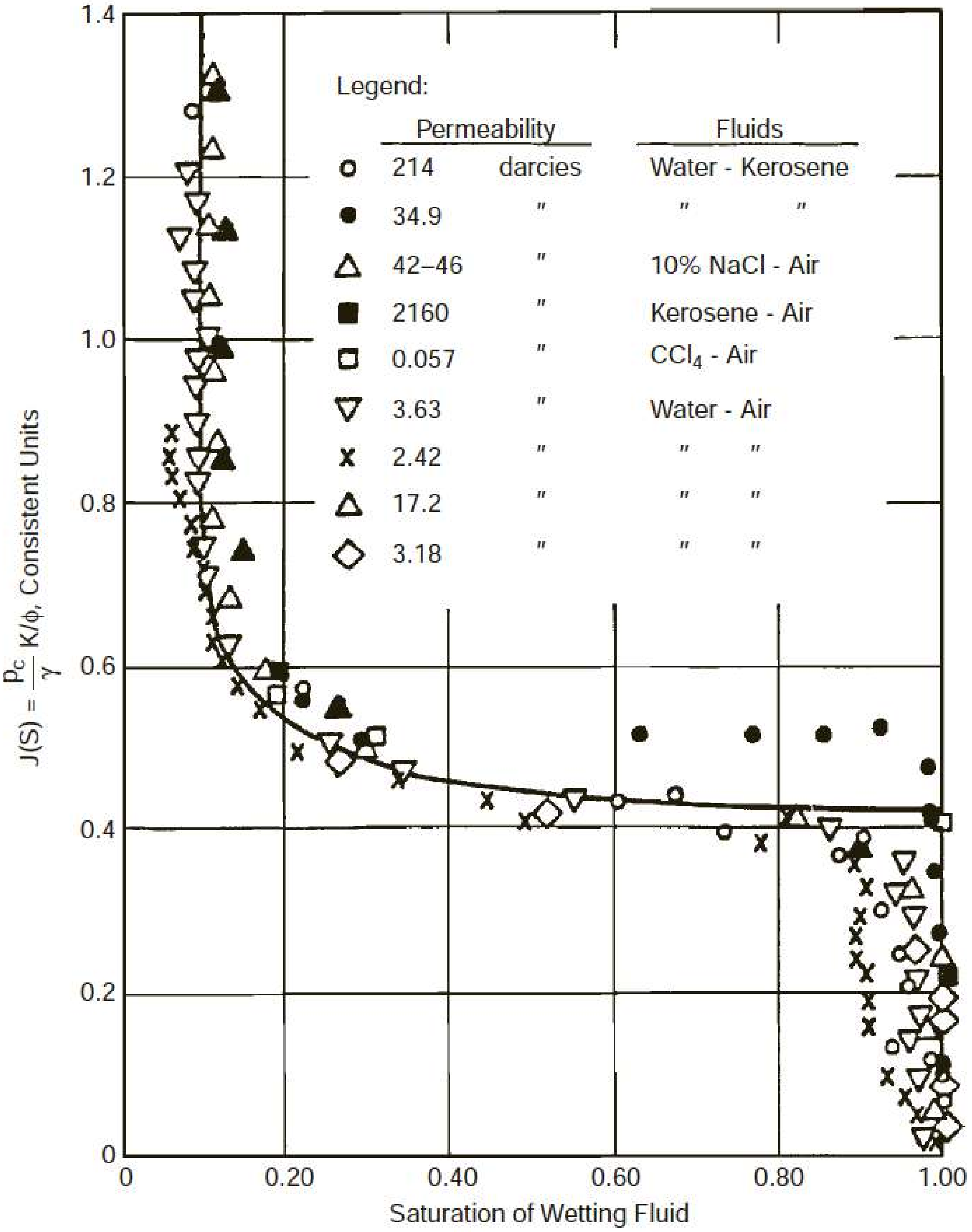


Fig. (5.6): The Leverett J-function for unconsolidated sands. (After Leverett, 1941).

Solution:

Step 1. Calculate the J-function using the measured capillary pressure data.

$$J(S_w) = 0.21645 \frac{p_c}{\sigma} \sqrt{\frac{k}{\phi}}$$

$$J(S_w) = 0.21645\left(\frac{p_c}{50}\right)\sqrt{\frac{80}{0.16}} = 0.096799p_c$$

S_w	p_c, psi	$J(S_w) = 0.096799(p_c)$
1.0	0.50	0.048
0.8	0.60	0.058
0.6	0.75	0.073
0.4	1.05	0.102
0.2	1.75	0.169

Step 2. Using the new porosity and permeability values, solve Equation (5.12) for the capillary pressure p_c.

$$p_c = \frac{J(S_w)\sigma}{\left[0.21645\sqrt{\frac{k}{\phi}}\right]}$$

$$p_c = \frac{J(S_w)50}{\left[0.21645\sqrt{\frac{120}{0.19}}\right]}$$

$$p_c = 9.192\,J(S_w)$$

Step 3. Reconstruct the capillary pressure-saturation table.

S_w	$J(S_w)$	$p_c = 9.192\,J(S_w)$
1.0	0.048	0.441
0.8	0.058	0.533
0.6	0.073	0.671
0.4	0.102	0.938
0.2	0.169	1.553

5.1.4 Converting Laboratory Capillary Pressure Data to Reservoir Conditions:

For experimental convenience, it is common in the laboratory determination of capillary pressure to use air-mercury or air-brine systems, rather than the actual water-oil system characteristic of the reservoir. Since the laboratory fluid system does not have the same surface tension as the reservoir system, it becomes necessary to convert laboratory capillary pressure to reservoir capillary pressure. By assuming that the Leverett J-function is a property of rock and does not change from the laboratory to the reservoir, we can calculate reservoir capillary pressure as follows:

$$(p_c)_{res} = (p_c)_{lab}\frac{\sigma_{res}}{\sigma_{lab}} \qquad eq.(5.13)$$

Even after the laboratory capillary pressure has been corrected for surface tension, it may be necessary to make further corrections for permeability and porosity. The reason for this is that the core sample that was used in performing the laboratory capillary pressure test may not be representative of the average reservoir permeability and porosity. If we assume that the J-function will be invariant for a given rock type over a range of porosity and permeability values, then the reservoir capillary pressure can be expressed as:

$$(p_c)_{res} = (p_c)_{lab}\frac{\sigma_{res}}{\sigma_{lab}}\sqrt{\frac{(\phi_{res}k_{core})}{(\phi_{core}k_{res})}} \qquad eq.(5.14)$$

where:

$(p_c)_{res}$ = reservoir capillary pressure.
σ_{res} = reservoir surface or interfacial tension.
k_{res} = reservoir permeability.
ϕ_{res} = reservoir porosity.
$(p_c)_{lab}$ = laboratory-measured capillary pressure.
σ_{lab} = laboratory surface or interfacial tension.
ϕ_{core} = core porosity.
k_{core} = core permeability.

5.1.5 Capillary Pressure and Permeability:

Figure (5.7) shows the air–mercury capillary pressure data as a function of mercury saturation for five different core samples varying in absolute permeability in a range of k_1–k_5 *md*. The data show decreases in permeability have corresponding increases in the capillary pressure at a constant value of mercury saturation. In other words, when iso-saturation data are compared, the sample having a permeability of k_1 *md* has the lowest capillary pressure, whereas the one with permeability of k_5 *md* has the highest capillary pressure. Although the general trend of capillary pressure–saturation curves remains unchanged, the magnitude of the capillary pressures does change when data are compared for samples of different permeabilities. Therefore, the capillary pressure–saturation–permeability relationship is a reflection of the influence of pore sizes and grain sorting. Smaller size pores and poorly sorted grains invariably have lower permeabilities and large capillary pressures.

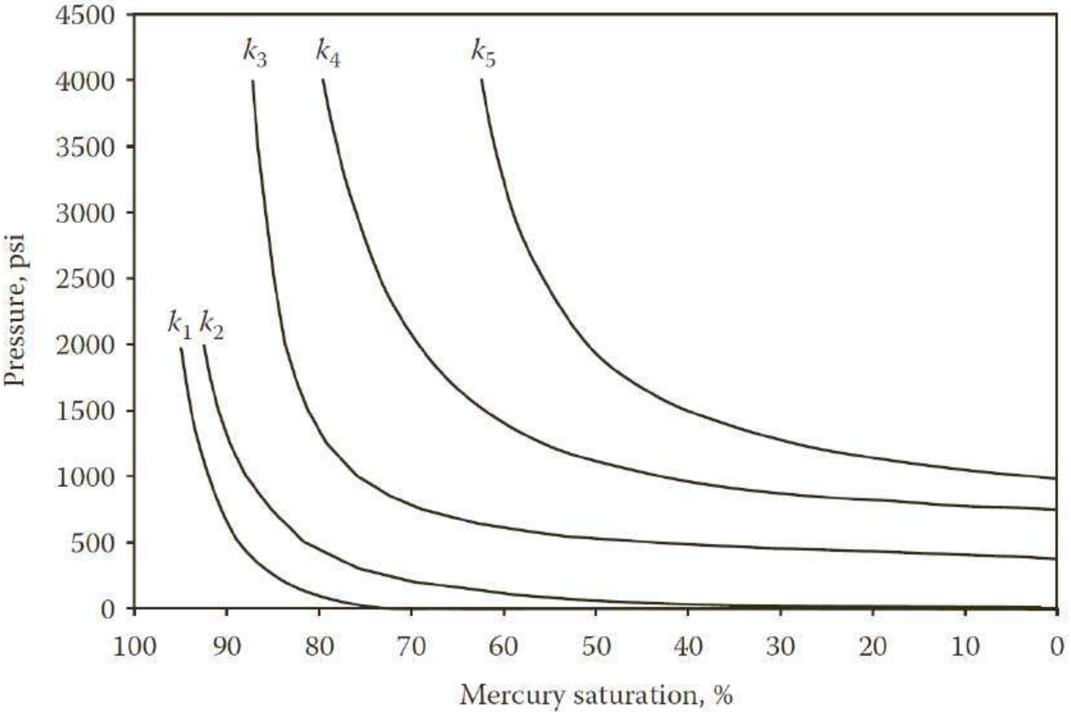


Fig. (5.7): Variation of capillary pressure with permeability ($k_1 > k_2 > k_3 > k_4 > k_5$).

5.1.6 Zonation, Fluid Contacts, and Initial Saturation Distribution in a Reservoir:

The determination of initial fluid contacts in the reservoir, such as the gas–oil contact (GOC) and oil-water contact (OWC), through different methods are available to the engineer’s disposal. These include:

1. A production test which involves the direct determination of GOC and OWC during the drilling of a well prior to setting the casing.
2. The other method includes application of geophysical logs such as electrical and radioactive.
3. In addition to these, SCAL (which includes capillary pressure) represents one of the most reliable methods of determining the fluid contacts in a reservoir.

A great deal of practical information pertaining to zonation, fluid contacts, and initial fluid saturation distribution in a hydrocarbon reservoir prior to its exploitation can be derived from capillary pressure data. However, this requires the conversion of P_c–S_w data to height–saturation data, which can be achieved by considering the generalized form of Equation (5.11):

$$h = \frac{2\sigma \cos\theta}{r\Delta\rho g} \qquad eq. (5.15)$$

By using generalized form of Equation (5.10), Equation (5.15) can be expressed in terms of capillary pressure:

$$p_c = gh\Delta\rho$$

$$h = \frac{p_c}{\Delta\rho g} \qquad eq.(5.16)$$

where:

h = the height above the plane of 0 capillary pressure between the nonwetting and wetting fluids.
P_c = the capillary pressure.
$\Delta\rho$ = the density difference between nonwetting and wetting phase at reservoir conditions.
g = the gravitational constant.

In Equation (5.16), when a value of g is 9.81 m/s^2, P_c is in N/m^2, and the density difference is in kg/m^3,

$$h(m) = \frac{0.102 p_c(N/m^2)}{\Delta\rho(kg/m^3)} \qquad eq.(5.17)$$

Whereas in oil-field units, when g is $32.2\ ft/sec^2$ $(9.81\, m/sec^2 \times 3.28\, ft/m)$, P_c is in lb_{force}/in^2 (multipled by $32.2\ ft/sec^2$ to convert lb_{force} to lb_{mass}), and the density difference is in lb_{mass}/ft^3:

$$h(ft) = \frac{p_c\left(lb_{force}/in.^2\right) \times 32.2(ft/s^2)}{32.2(ft/s^2)\Delta\rho(lb_{mass}/ft^3)} = \frac{144 p_c}{\Delta\rho} \qquad eq.(5.18)$$

Equation (5.18) provides a relationship between capillary pressure and the height above the plane of 0 capillary pressure; that is, capillary pressure data are easily converted to height–saturation data based on which the zonation, fluid contacts, and fluid distribution in a hydrocarbon reservoir are determined. It should be noted, however, that prior to converting P_c–S_w data to h–S_w data, capillary pressures should be converted from laboratory conditions to representative reservoir conditions based on the methods described earlier.

In order to understand the application of height–saturation data to determine the fluid distribution, zonation, and fluid contacts in a reservoir, we consider the drainage capillary pressure curve such as the one in Figure (5.8), the data for which have been converted to reservoir conditions (for an oil–water system) and subsequently to height–saturation, as shown in Figure (5.9). Thus, the figure essentially shows the water saturation distribution in an oil–water system.

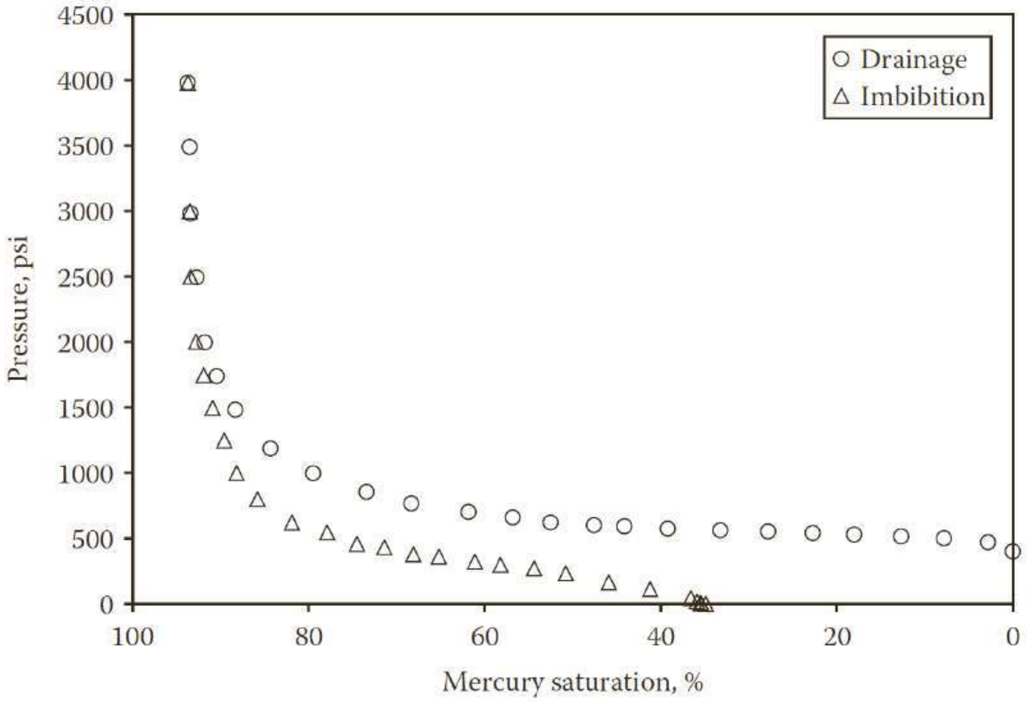


Fig. (5.8): A typical example of drainage and imbibition MICP data.

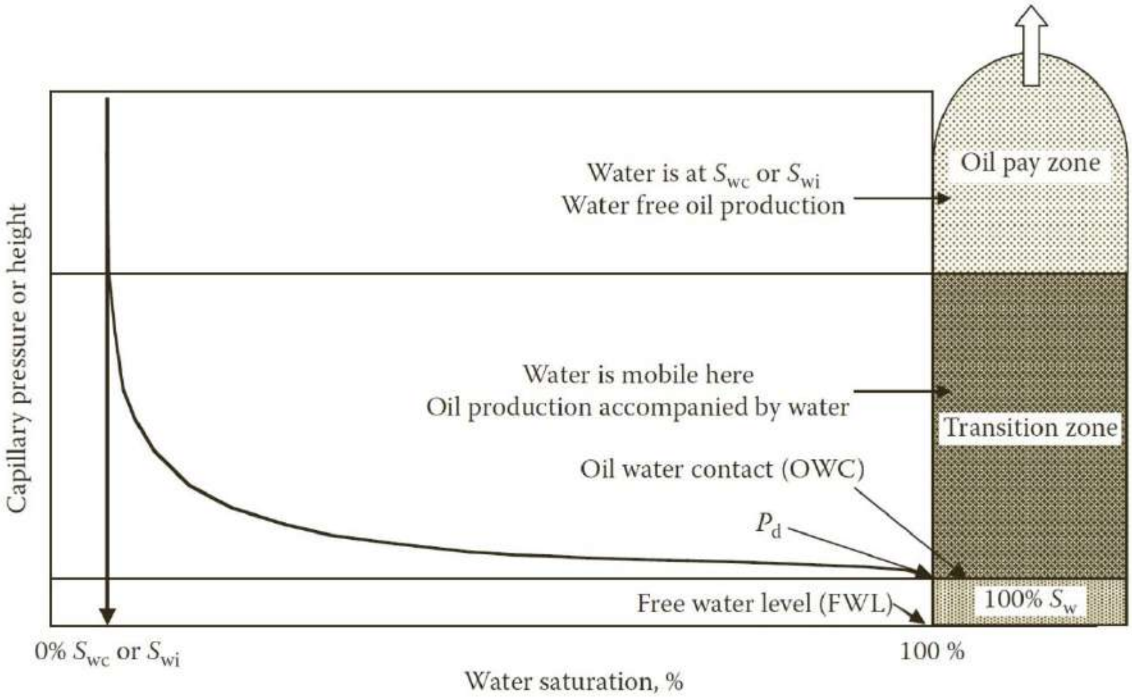


Fig. (5.9): Profile of fluid distribution, zonation, and fluid contacts based on the capillary pressure or height versus water saturation data.

5.1.6.1 Free Water Level, FWL:

From the capillary pressure standpoint, the FWL occurs at zero capillary pressure at which water saturation is 100%. Consequently, the FWL is represented by the base of the height–saturation curve, as schematically illustrated in Figure (5.9), below which a water zone or aquifer may exist. However, moving upward vertically from the base of the height–saturation curve in Figure (5.9), the water saturation is still 100% up to a certain finite value of "h" that extends from h = 0, which is an outcome of the capillary entry pressure or displacement pressure (P_d) or the threshold pressure. In terms of depth, there is a difference between the FWL and the minimum depth at which 100% water saturation still exists, which is a manifestation of the largest pore size in the reservoir. On the other hand, if this pore size is so large that there is no capillary rise in this size pore, then the FWL and the 100% water saturation level will be the same, basically implying the absence or nonexistence of the displacement pressure. This is, however, not the case with most reservoirs because even the largest pore is small enough to result in some capillary rise. (Slider, 1976) states that in the case of very tight reservoirs, P_d values can be large; similarly, low-permeability chalk reservoirs are also characterized by high capillary entry pressures, thus resulting in a difference of many feet between the two heights as outlined earlier. FWL can be expressed mathematically by the following relationship:

$$FWL = OWC + \frac{144p_d}{\Delta\rho} \qquad eq.(5.19)$$

where:

p_d = displacement pressure, psi (from capillary pressure curve).
$\Delta\rho$ = density difference, *lb*$_{mass}$*/ft*^{3}.
FWL = free-water level, *ft.*
OWC = Oil-Water contact, *ft.*

5.1.6.2 Oil-Water Contact, OWC:

As shown in Figure (5.9), the OWC and 100% water saturation point on the height–saturation curve is represented by the coordinates p_c = p_d; S_w = 100%. In Equation (5.18), P_c can be replaced by p_d to express the OWC in terms of height above the FWL:

$$OWC = \frac{144p_d}{\Delta p} \qquad eq.(5.20)$$

where:

OWC = Oil-Water contact, *ft.*
p_d = displacement pressure, psi (from capillary pressure curve).
$\Delta\rho$ = density difference, *lb*$_{mass}$*/ft*^{3}.

In terms of depth, the OWC is defined as the uppermost depth in the reservoir where a 100% water saturation exists, which can be mathematically expressed as:

$$OWC = FWL - \frac{144p_d}{\Delta p} \qquad eq.(5.21)$$

Example 5.2:

The reservoir capillary pressure-saturation data of the Big Butte Oil Reservoir is shown graphically in Figure (5.10). Geophysical log interpretations and core analysis establish the WOC at (5023 *ft*). The following additional data are available:

- Oil density = 43.5 *lb/ft*^{3}.
- Water density = 64.1 *lb/ft*^{3}.
- Interfacial tension = 50 dynes/cm.

Calculate:

- Connate-water saturation (S_{wc}).
- Depth to FWL.
- Thickness of the transition zone.
- Depth to reach 50% water saturation.

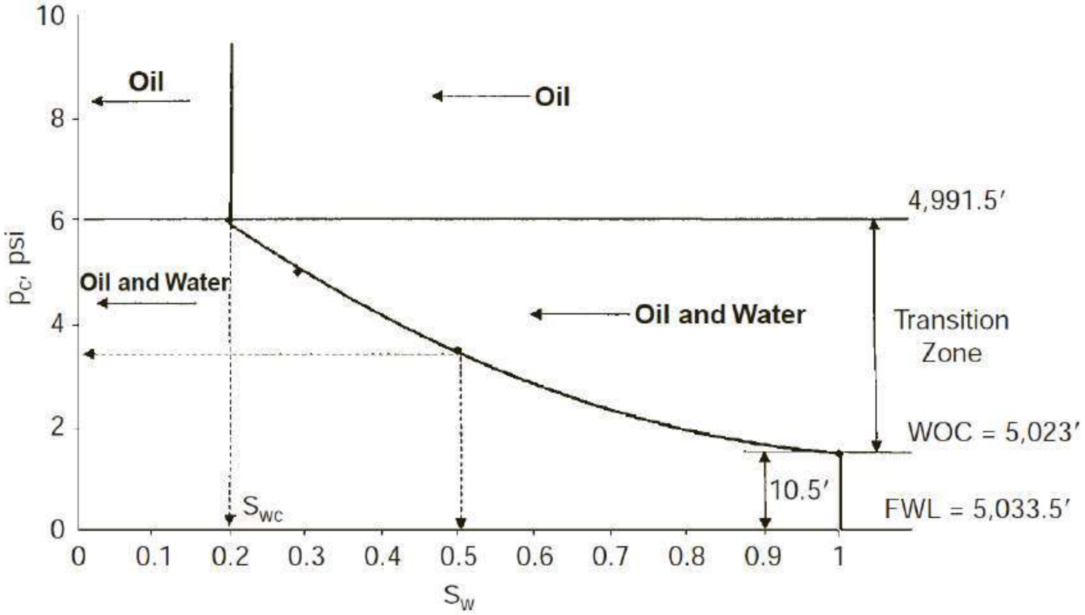


Fig (5.10): Capillary pressure-saturation data.

Solution:

a. From Figure (5.10), connate-water saturation is 20%.

b. Applying Equation (5.19) with a displacement pressure of 1.5 psi gives:

$$FWL = 5023 + \frac{(144)(1.5)}{(64.1 - 43.5)} = 5033.5\, ft$$

c. Thickness of transition zone:

$$TZ\ thickness = \frac{144(6.0 - 1.5)}{(64.1 - 43.5)} = 31.5 ft$$

d. P_c at 50% water saturation = 3.5 psia.

5.1.6.3 Transition Zone:

The presence of transition zones in petroleum reservoirs is a manifestation of the capillary forces, the absence of which would essentially result in complete segregation of the fluid phases. The relationship as expressed in Equation (5.15), when applied for $h_{@swi} - h_{owc}$ suggests that the transition zone thickness is basically influenced by (1) the radius of the pore, r, and (2) the difference in density, Δρ. Therefore, a reservoir rock system having small pore sizes has a large transition zone (typical characteristics of low-permeability chalk reservoirs) than a system comprised of large pore sizes. Additionally, the more uniform the pore sizes (well-sorted grains), the flatter the middle portion of the capillary pressure curve, thus resulting in thinner transition zones. Such uniform pore size systems will also have high permeabilities; thus, the thickness of the transition zone can also be indirectly related to permeability; a high-permeability reservoir rock system has shorter transition zones than low-permeability reservoirs. As far as the influence of Δρ is concerned, from a practical standpoint, in a gas reservoir having a gas–water contact, the thickness of the transition zone will be small since the density difference is significantly large (gravity dominating over capillarity). Similarly, in the case of a light oil (smaller density), the density difference is large, resulting in a lower thickness of the transition zone. In the case of heavy oils (larger density), the density difference is small, thus increasing the thickness of the transition zone. However, a combined effect of r and Δρ would normally dictate the thickness of the transition zone. (Ahmed, 2019) points out that the thickness of the transition zone may range from a few feet to several hundred feet in some reservoirs. Figure (5.11) shows the variations of transition zone with fluid gravity and permeability.

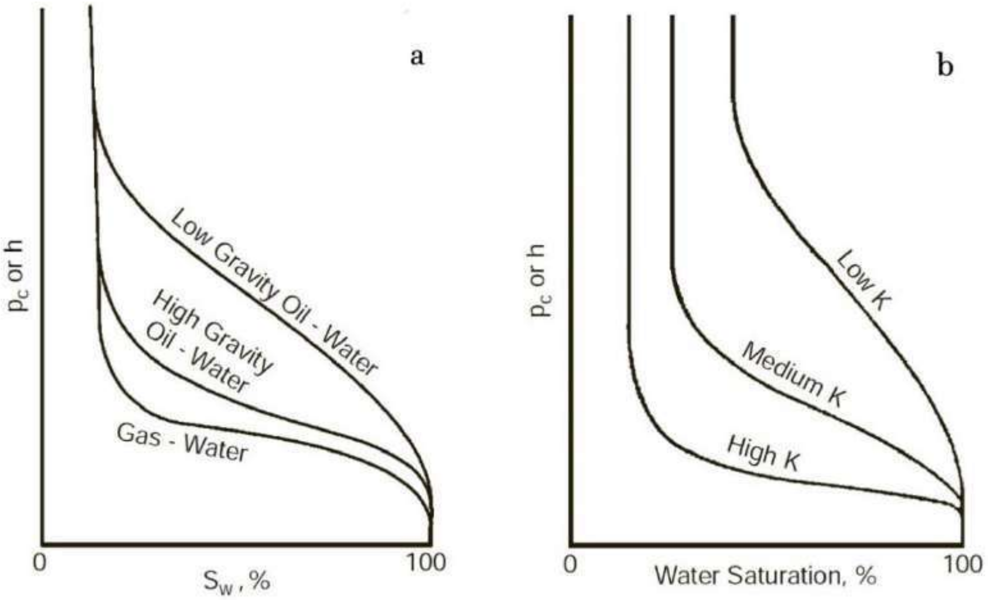


Fig. (5.11): Transition zone variation with (a) fluid gravity, (b) permeability.

Example 5.3:

A four-layer oil reservoir is characterized by a set of reservoir capillary pressure-saturation curves as shown in Figure (5.12). The following data are also available:

Layer	Depth, ft	Permeability, md
1	4000-4010	80
2	4010-4020	100
3	4020-4035	70
4	4035-4060	90

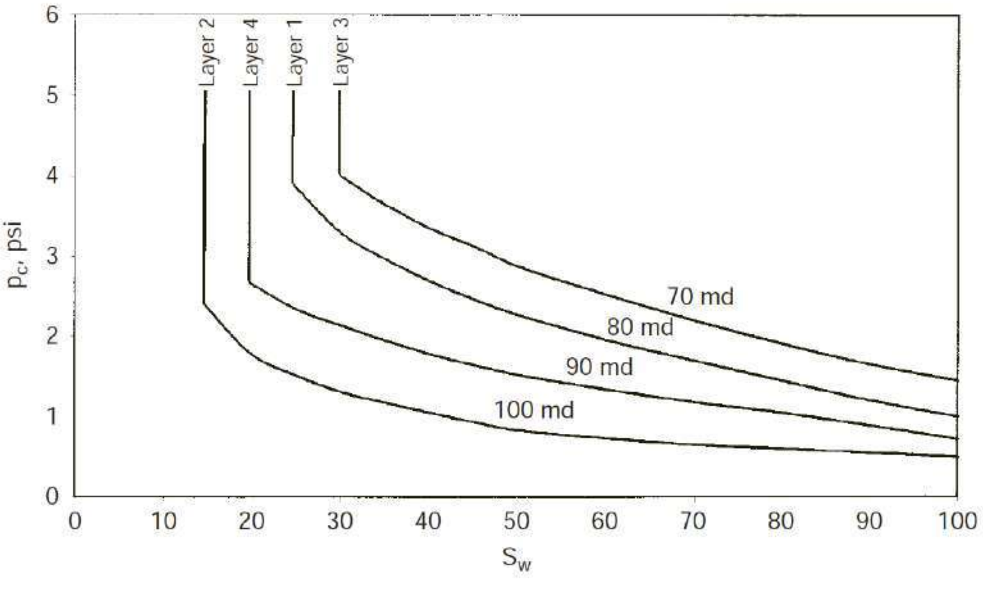


Fig. (5.12): Variation of p_c with k.

WOC = 4060 *ft.*
Water density = 65.2 *lb/ft*^{3}.
Oil density = 55.2 *lb/ft*^{3}.

Calculate and plot water saturation versus depth for this reservoir.

Solution:

Step 1. Establish the FWL by determining the displacement pressure p_d for the bottom layer, i.e., Layer 4, and apply Equation (5.19):

- P_d = 0.75 psi

$$FWL = 4060 + \frac{(144)(0.75)}{(65.2 - 55.2)} = 4070.8\,ft$$

Step 2. The top of the bottom layer is located at a depth of 4,035 ft, which is 35.8 ft above the FWL. Using that height h of 35.8 ft, calculate the capillary pressure at the top of the bottom layer:

$$p_c = \left(\frac{h}{144}\right)\Delta\rho = \left(\frac{35.8}{144}\right)(65.2 - 55.2) = 2.486\,psi$$

- From the capillary pressure-saturation curve designated for layer 4, read the water saturation that corresponds to a p_c of 2.486 to give S_w = 0.23.

- Assume different values of water saturations and convert the corresponding capillary pressures into height above the FWL by applying Equation (5.18).

$$h = \frac{144 p_c}{\rho_w - \rho_o}$$

S_w	p_c, psi	h, ft	Depth = FWL – h
0.23	2.486	35.8	4035
0.25	2.350	33.84	4037
0.30	2.150	30.96	4040
0.40	1.800	25.92	4045
0.50	1.530	22.03	4049
0.60	1.340	19.30	4052
0.70	1.200	17.28	4054
0.80	1.050	15.12	4056
0.90	0.900	12.96	4058

Step 3. The top of layer 3 is located at a distance of 50.8 ft from the FWL (i.e., h = 4,070.8 – 4,020 = 50.8 ft). Calculate the capillary pressure at the top of the third layer:

- $p_c = \left(\frac{50.8}{144}\right)(65.2 - 55.2) = 3.53\, psi$
- The corresponding water saturation as read from the curve designated for layer 3 is 0.370.
- Construct the following table for layer 3.

S_w	p_c, psi	h, ft	Depth = FWL - h
0.37	3.53	50.8	4020
0.40	3.35	48.2	4023
0.50	2.75	39.6	4031
0.60	2.50	36.0	4035

Step 4. Distance from the FWL to the top of layer 2 is:

- $h = 4070.8 - 4010 = 60.8\, ft$
- $p_c = \left(\frac{60.8}{144}\right)(65.2 - 55.2) = 4.22\, psi$
- $S_w\ at\ p_c\ of\ 4.22\, psi\ is\ 0.15$
- The distance from the FWL to the bottom of the layer is 50.8 ft, which corresponds to a p_c of 3.53 psi and an S_w of 0.15. This indicates that the second layer has a uniform water saturation of 15%.

Step 5. For layer 1, the distance from the FWL to the top of the layer is

- $h = 4070 - 4000 = 70.8\, ft$
- $p_c = \left(\frac{70.8}{144}\right)(10) = 4.92\, psi$
- S_w at the top layer 1 = 0.25.
- The capillary pressure at the bottom of the layer is 3.53 psi with a corresponding water saturation of 0.27.

Step 6. Figure (5.13) documents the calculated results graphically. The figure indicates that layer 2 will produce 100% oil while all remaining layers produce oil and water simultaneously.