FLUID MECHANICS

COLLEGE OF PETROLEUM AND MINING ENGINEERING

Dr. Ibrahim Adil Ibrahim Al-Hafidh
Mining Engineering Department
College of Petroleum and Mining Engineering
University of Mosul

Dr.Ibrahim Al-Hafidh
Fluid Mechanics

Email: iibrahim@uomosul.edu.iq

LECTURE 10

c- Flowrate Measurement
The Energy Line and the Hydraulic Grade Line

Dr.Ibrahim Al-Hafidh
Fluid Mechanics

Email: iibrahim@uomosul.edu.iq

c- Flowrate Measurement

Many types of devices using principles involved in the Bernoulli equation have been developed to measure fluid ***velocities*** and ***flowrates***.
Other examples discussed below include devices to measure flowrates in pipes and conduits and devices to measure flowrates in open channels.
In this lecture we will consider "ideal" ***flowmeters***—those devoid of viscous, compressibility, and other "real-world" effects.

Our goal here is to understand the basic operating principles of these simple flowmeters.

Figure (A)

An effective way to measure the flowrate through a pipe is to place some type of restriction within the pipe as shown in Figure (A)and to measure the pressure difference between the low-velocity, high-pressure upstream section (1) and the high-velocity, low-pressure downstream section (2).

Three commonly used types of flowmeters are illustrated:

- The orifice meter.
- The nozzle meter.
- The Venturi meter.

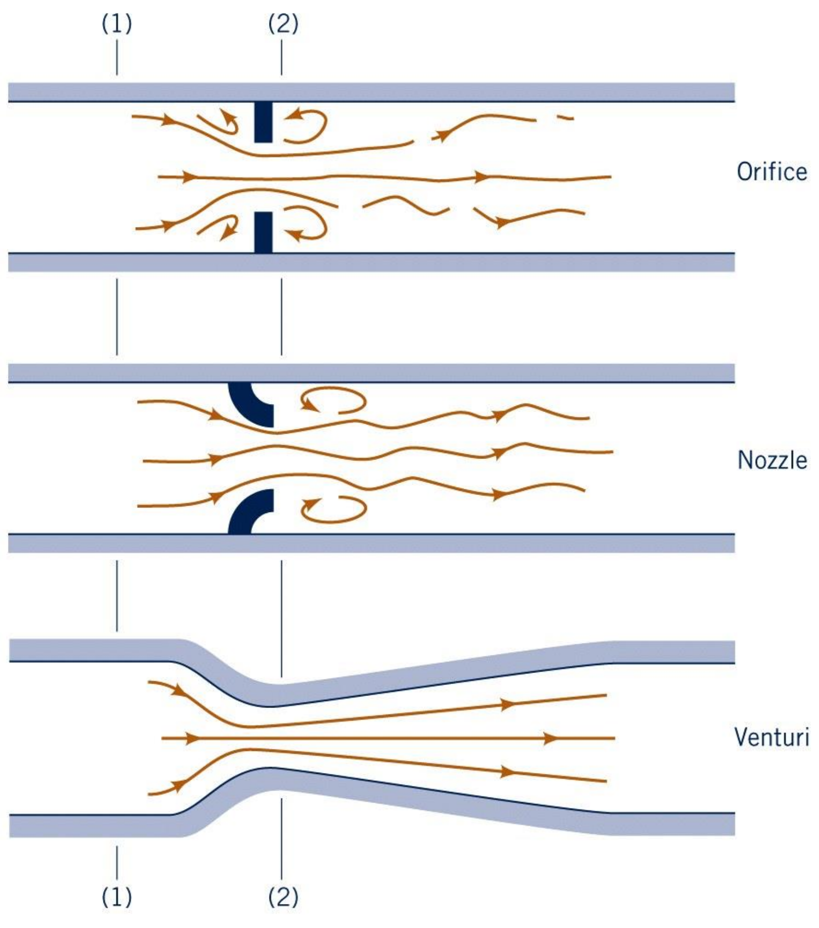


Figure (A)
Typical devices for measuring flowrate in pipes

The operation of each is based on the same physical principles—an increase in velocity causes a decrease in pressure. The difference between them is a matter of cost, accuracy, and how closely their actual operation obeys the idealized flow assumptions.

We assume the flow is horizontal steady, inviscid, and incompressible between points (1) and (2). The Bernoulli equation becomes

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{.........(1)}$$

The effect of nonhorizontal flow can be incorporated easily by including the change in elevation, z_1-z_2, in the Bernoulli equation.

If we assume the velocity profiles are uniform at sections (1) and (2), the continuity equation Eq.(5) lecture 9 can be written as

$$Q = A_1 V_1 = A_2 V_2 \qquad \text{.........(2)}$$

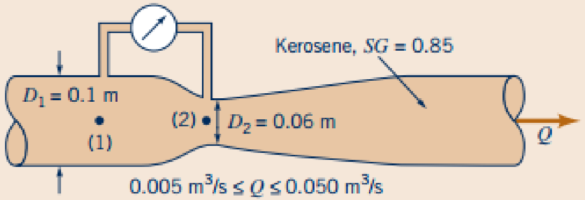
where is the small ($A_2 < A_1$) flow area at section (2). Combination of these two equations results in the following theoretical flowrate

$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}} \qquad \text{.........(3)}$$

Thus, the flowrate can be determined if the pressure difference, $\boldsymbol{p_1 - p_2}$ is measured.

GIVEN Kerosene ($SG = 0.85$) flows through the Venturi meter shown in Fig. E3.11*a* with flowrates between 0.005 and 0.050 m^3/s.

FIND Determine the range in pressure difference, $p_1 - p_2$, needed to measure these flowrates.



■ Figure E3.11*a*

Solution

If the flow is assumed to be steady, inviscid, and incompressible, the relationship between flowrate and pressure is given by Eq. 3. This can be rearranged to give

$$p_1 - p_2 = \frac{Q^2\rho[1 - (A_2/A_1)^2]}{2\,A_2^2}$$

With the density of the flowing fluid

$$\rho = SG\,\rho_{H_2O} = 0.85(1000\text{ kg/m}^3) = 850\text{ kg/m}^3$$

and the area ratio

$$A_2/A_1 = (D_2/D_1)^2 = (0.06\text{ m}/0.10\text{ m})^2 = 0.36$$

the pressure difference for the smallest flowrate is

$$p_1 - p_2 = (0.005\text{ m}^3\text{/s})^2(850\text{ kg/m}^3)\,\frac{(1 - 0.36^2)}{2\,[(\pi/4)(0.06\text{ m})^2]^2}$$

$$= 1160\text{ N/m}^2 = 1.16\text{ kPa}$$

Likewise, the pressure difference for the largest flowrate is

$$p_1 - p_2 = (0.05)^2(850)\,\frac{(1 - 0.36^2)}{2[(\pi/4)(0.06)^2]^2}$$

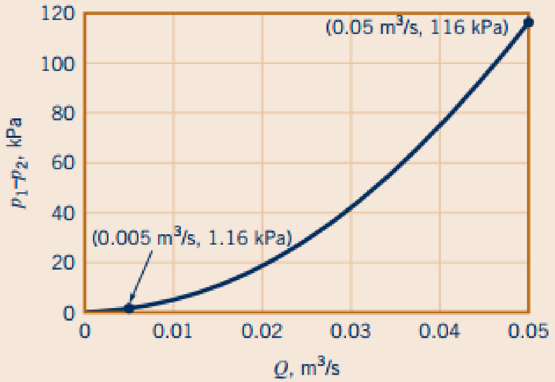
$$= 1.16 \times 10^5\text{ N/m}^2 = 116\text{ kPa}$$

Thus,

$$1.16\text{ kPa} \le p_1 - p_2 \le 116\text{ kPa} \qquad \textbf{(Ans)}$$

COMMENTS These values represent the pressure differences for inviscid, steady, incompressible conditions. The ideal results presented here are independent of the particular flowmeter geometry—an orifice, nozzle, or Venturi meter (see Fig. 3.18).

It is seen from Eq. 3.20 that the flowrate varies as the square root of the pressure difference. Hence, as indicated by the numerical results and shown in Fig. E3.11*b*, a 10-fold increase in flowrate requires a 100-fold increase in pressure difference. This nonlinear relationship can cause difficulties when measuring flowrates over a wide range of values. Such measurements would require pressure transducers with a wide range of operation. An alternative is to use two flowmeters in parallel—one for the larger and one for the smaller flowrate ranges.



■ Figure E3.11*b*

The Energy Line and the Hydraulic Grade Line

As was discussed before, the Bernoulli equation is actually an **energy equation** representing the *partitioning of energy for an inviscid, incompressible, steady flow*. The sum of the various energies of the fluid remains constant as the fluid flows from one section to another. A useful interpretation of the Bernoulli equation can be obtained through use of the concepts of the ***hydraulic grade*** line (**HGL**) and the ***energy line*** (**EL**). These ideas represent a geometrical interpretation of a flow and can often be effectively used to better grasp the fundamental processes involved.

For ***steady, inviscid, incompressible flow*** the total energy remains constant along a streamline. The concept of "head" was introduced by dividing each term in mean equation by the **specific weight** $\gamma = \rho g$, to give the Bernoulli equation in the following form,

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant along streamline}$$

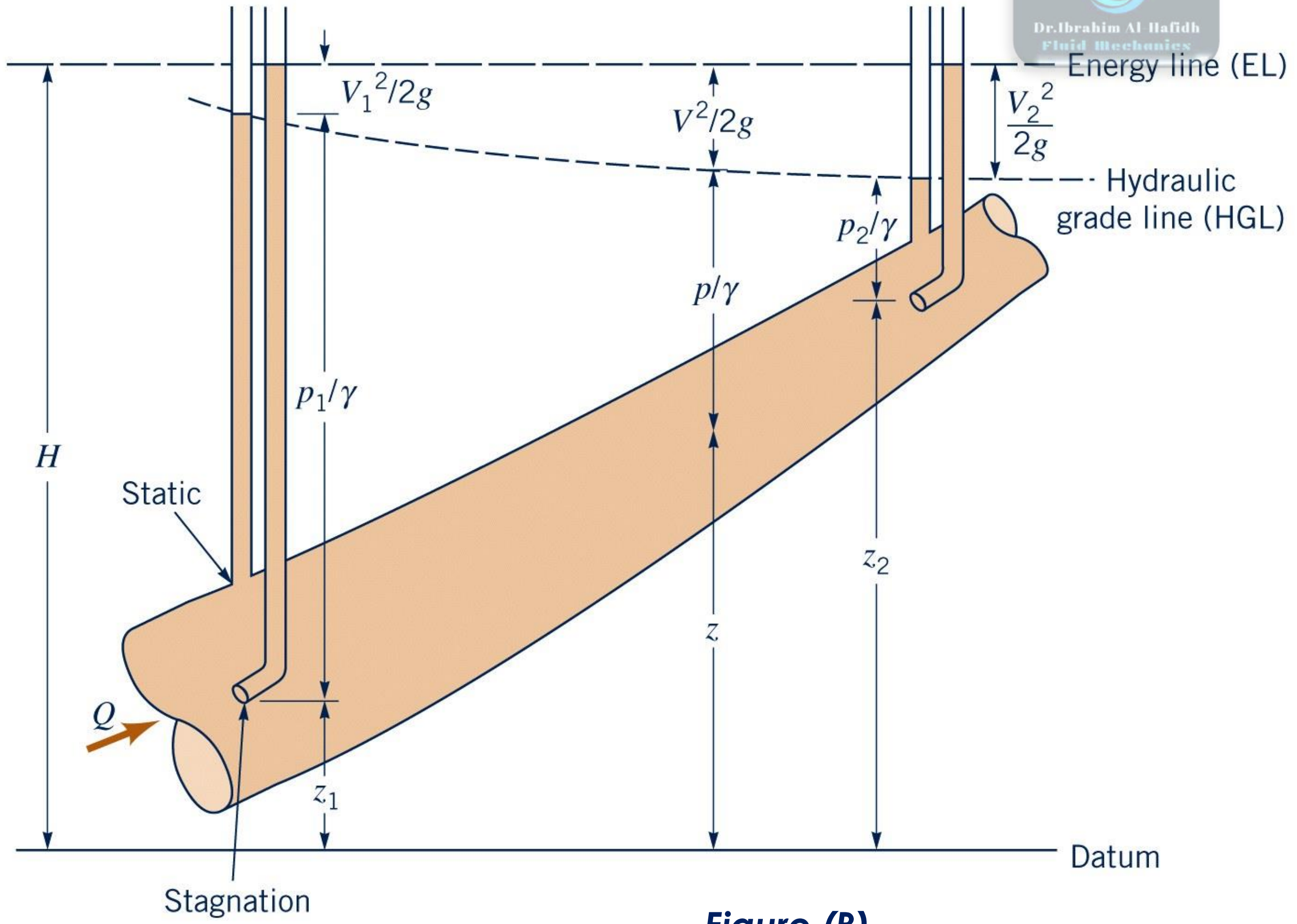


Figure (B)
Representation of the energy line and the hydraulic grade line.

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline} = H \qquad \text{..........(7)}$$

Each of the terms in this equation has the units of length (feet or meters) and represents a certain type of head. The Bernoulli equation states that the sum of the **pressure head**, the **velocity head**, and the **elevation head** is constant along a streamline. This constant is called the ***total head***, ***H***, and is shown in the figure E.

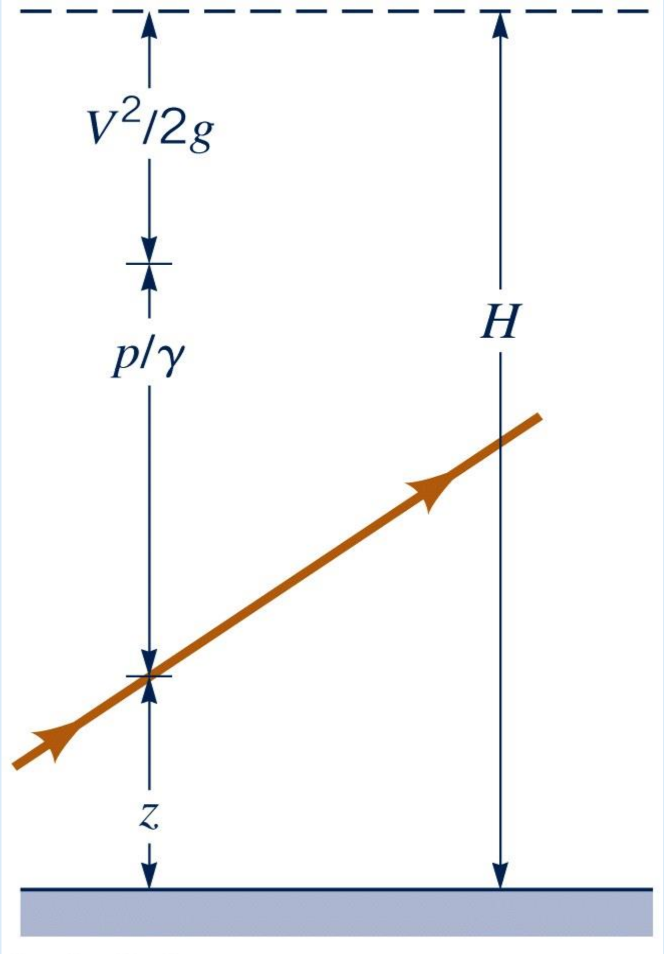
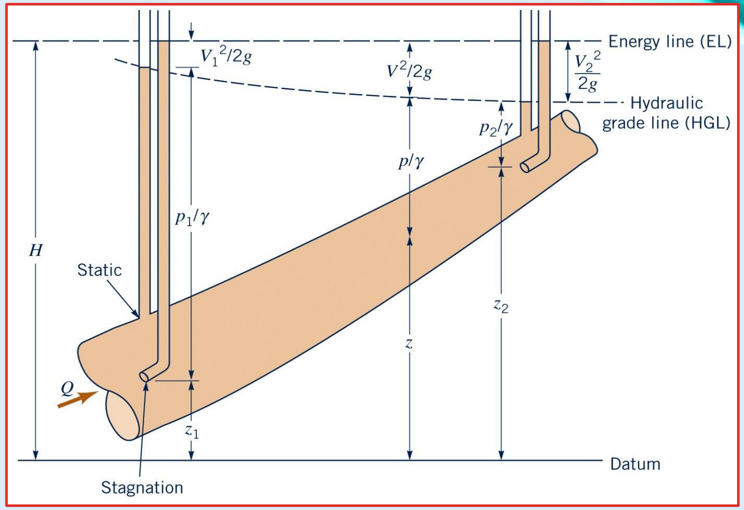


Figure (C)

The energy line is a line that represents the total head available to the fluid. As shown in Fig. B, the elevation of the energy line can be obtained by measuring the stagnation pressure with a Pitot tube. (A Pitot tube is the portion of a Pitot-static tube that measures the stagnation pressure.

The stagnation point at the end of the Pitot tube provides a measurement of the total head (or energy) of the flow.

The **static pressure** tap connected to the piezometer tube shown, on the other hand, measures the sum of the **pressure head** and the **elevation head**, $\frac{p}{\gamma} + z$. This sum is often called the piezometric head. The static pressure tap does not measure the velocity head.



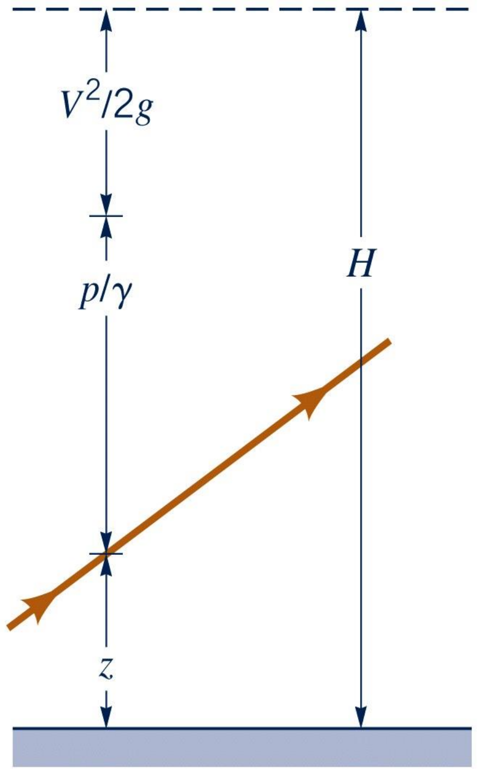


Figure (C)

According to Eq. 7, the total head remains constant along the streamline (provided the assumptions of the Bernoulli equation are valid). Thus, a Pitot tube at any other location in the flow will measure the same total head, as is shown in the figure. The **elevation head**, **velocity head**, and **pressure head** may vary along the streamline, however.

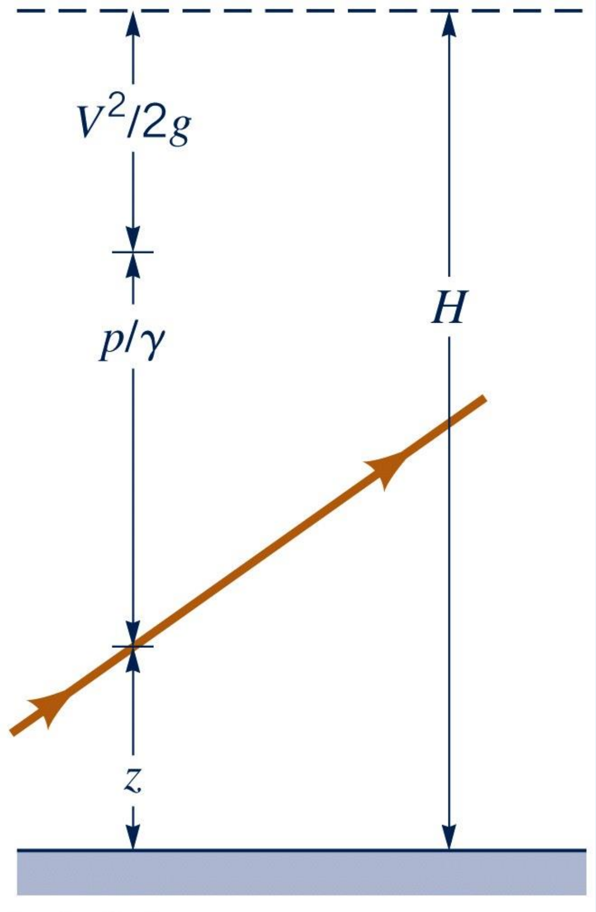


Figure (C)

The energy line and hydraulic grade line for flow from a large tank are shown in Fig. D. **If the flow is steady, incompressible, and inviscid, the energy line is horizontal and at the elevation of the liquid in the tank** (since the fluid velocity in the tank and the pressure on the surface are zero).

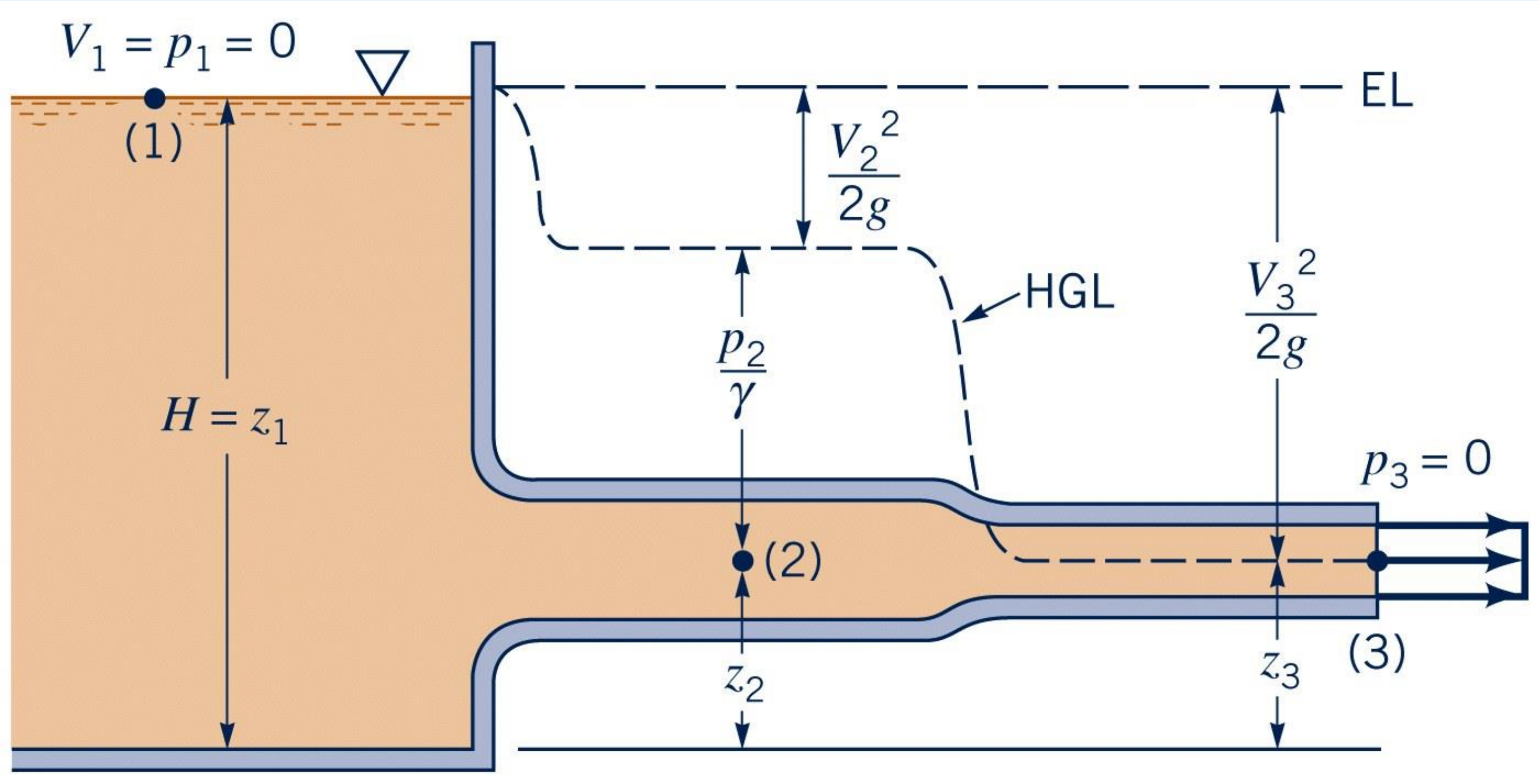
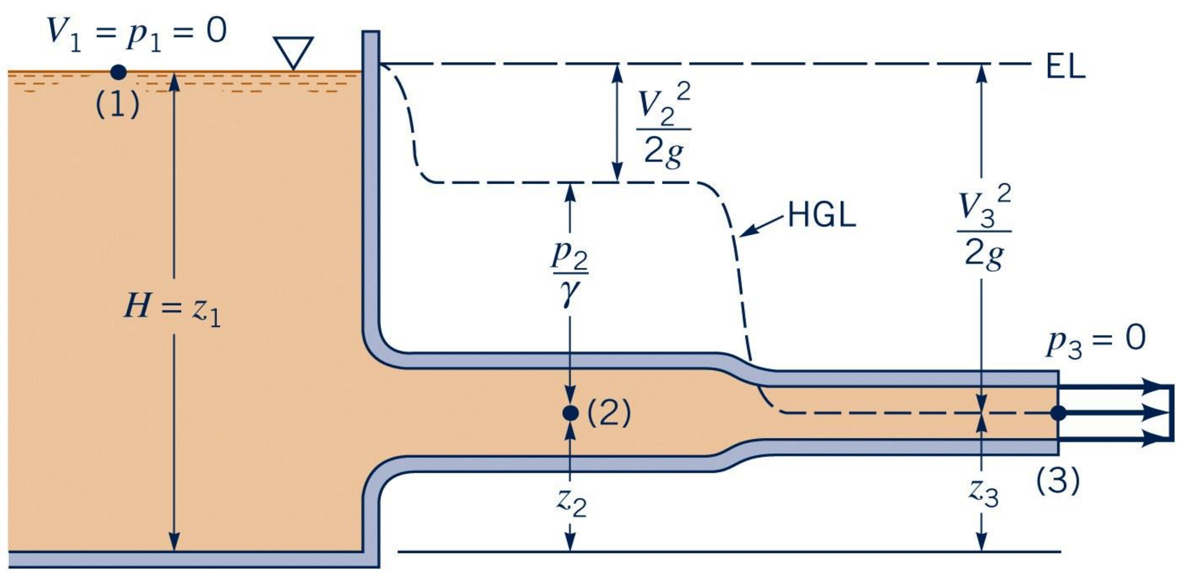


Figure (D)
The energy line and hydraulic grade line for flow from a tank.

The hydraulic grade line (HGL) lies a distance of one velocity head V2/2g, below the energy line. Thus, a change in fluid velocity due to a change in the pipe diameter results in a change in the elevation of the hydraulic grade line.

At the pipe outlet the pressure head is **zero** (gage), so the pipe elevation and the hydraulic grade line coincide.



The distance from the pipe to the hydraulic grade line indicates the pressure within the pipe, as is shown in Fig. E. ***If*** the pipe lies **below** the hydraulic grade line, the pressure within the pipe is positive (**above atmospheric**). ***If*** the pipe lies **above** the hydraulic grade line, the pressure is negative (**below atmospheric**). Thus, a scale drawing of a pipeline and the hydraulic grade line can be used to readily indicate regions of positive or negative pressure within a pipe.

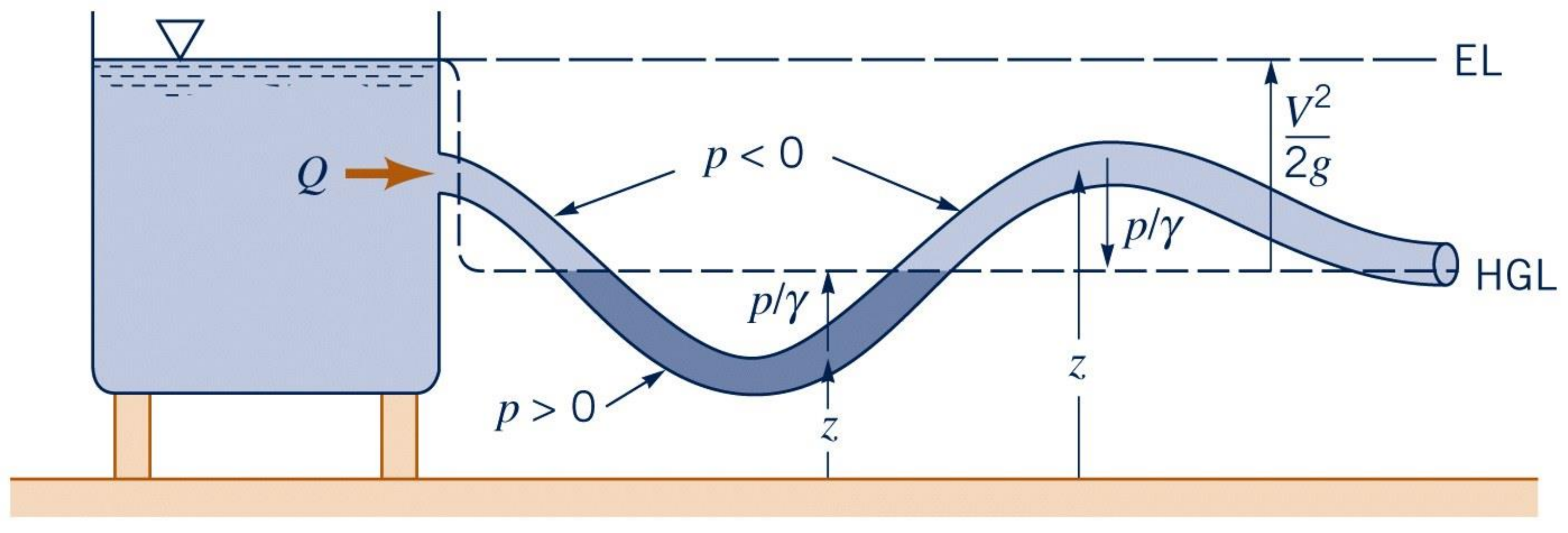


Figure (E)
Use of the energy line and the hydraulic grade line.

Total Energy of Flow

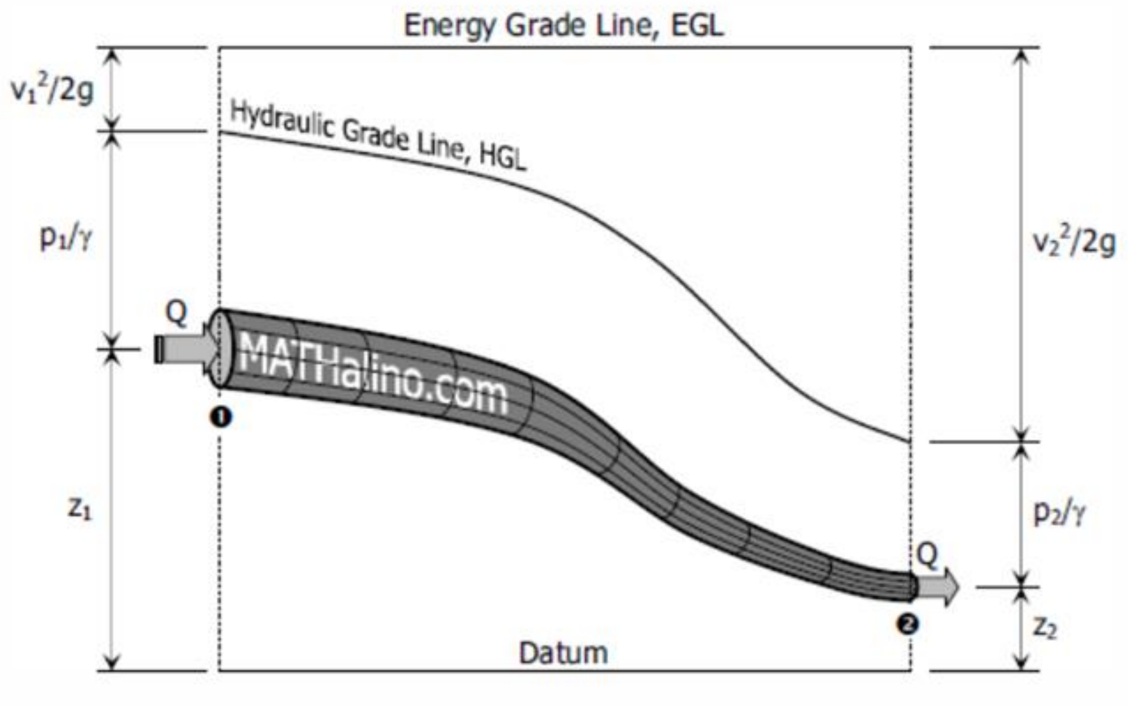
The total energy or head in a fluid is the sum of kinetic and potential energies. Recall that potential energies are pressure energy and elevation energy.

Total energy = Kinetic energy + Pressure energy + Elevation energy
Total head = Velocity head + Pressure head + Elevation head

In symbol, the total head energy is

$$E = \frac{v^2}{2g} + \frac{P}{\gamma} + z$$

Where:
v = mean velocity of flow (m/sec in SI and ft/sec in BG)
p = fluid pressure (N/m² or Pa in SI and lb/ft² or psf in BG)
z = position of fluid above or below the datum plane (m in SI and ft in BG)
g = gravitational acceleration (9.81 m/sec² in SI and 32.2 ft/sec² in BG)
γ = Specific weight of fluid (N/m³ in SI and lb/ft³ in BG)

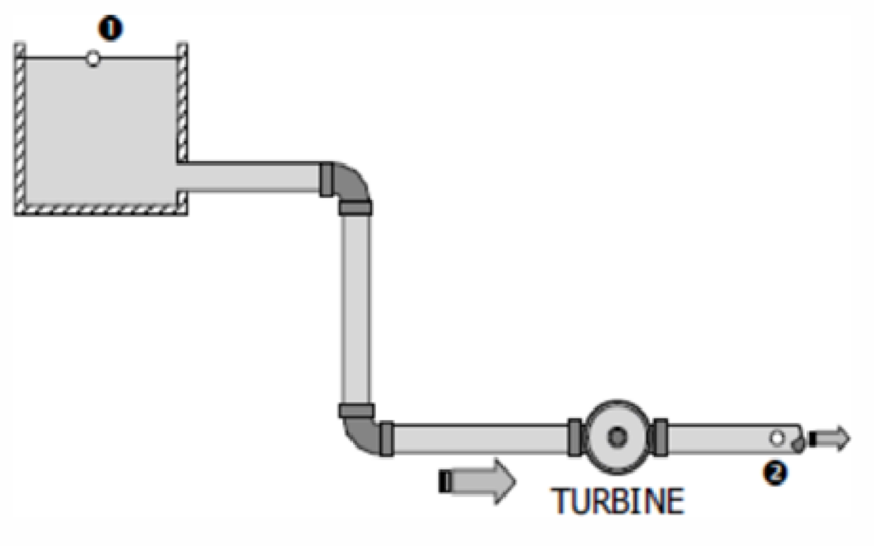


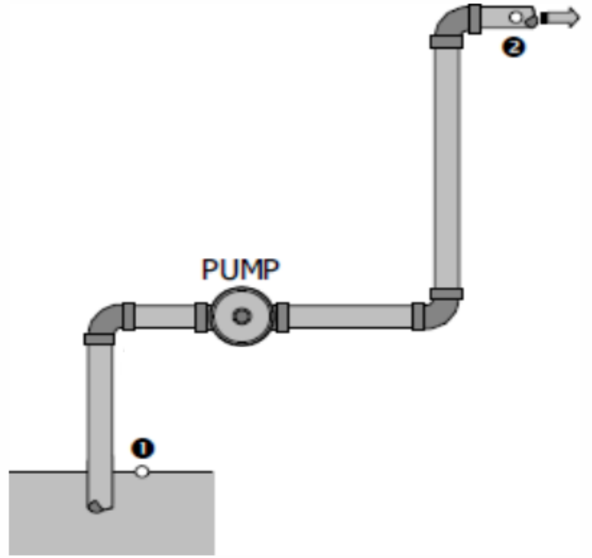
Energy Equation with Pump

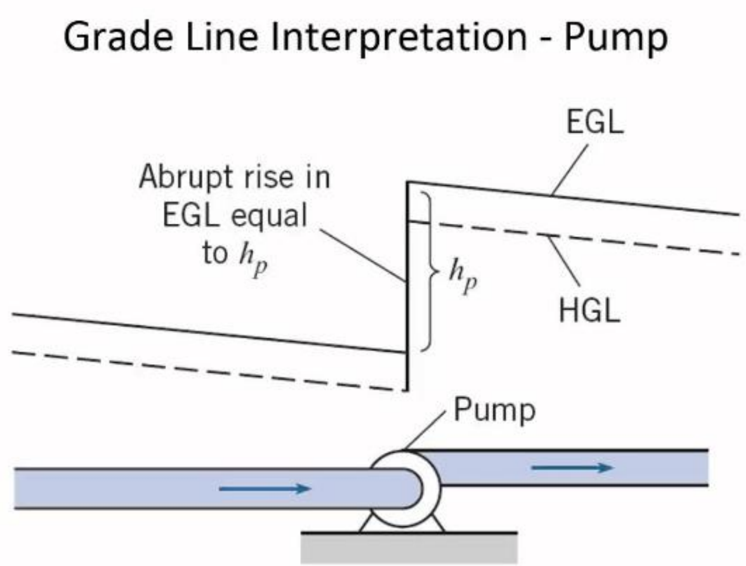
In most cases, pump is used to raise water from lower elevation to higher elevation. In a more technical term, the use of pump is basically to increase the energy of flow. The pump consumes electrical energy (Pinput) and delivers flow energy (Poutput).

Energy Equation with Turbine

Turbines extract flow energy and converted it into mechanical energy which in turn converted into electrical energy.







Hydraulic and Energy Grade Lines

Hydraulic Grade Line (HGL)

Hydraulic grade line, also called hydraulic gradient and pressure gradient, is the graphical representation of the potential head (pressure head + elevation head). It is the line to which liquid rises in successive piezometer tubes. The line is always at a distance ($p/\gamma + z$) above the datum plane.

Characteristics of HGL

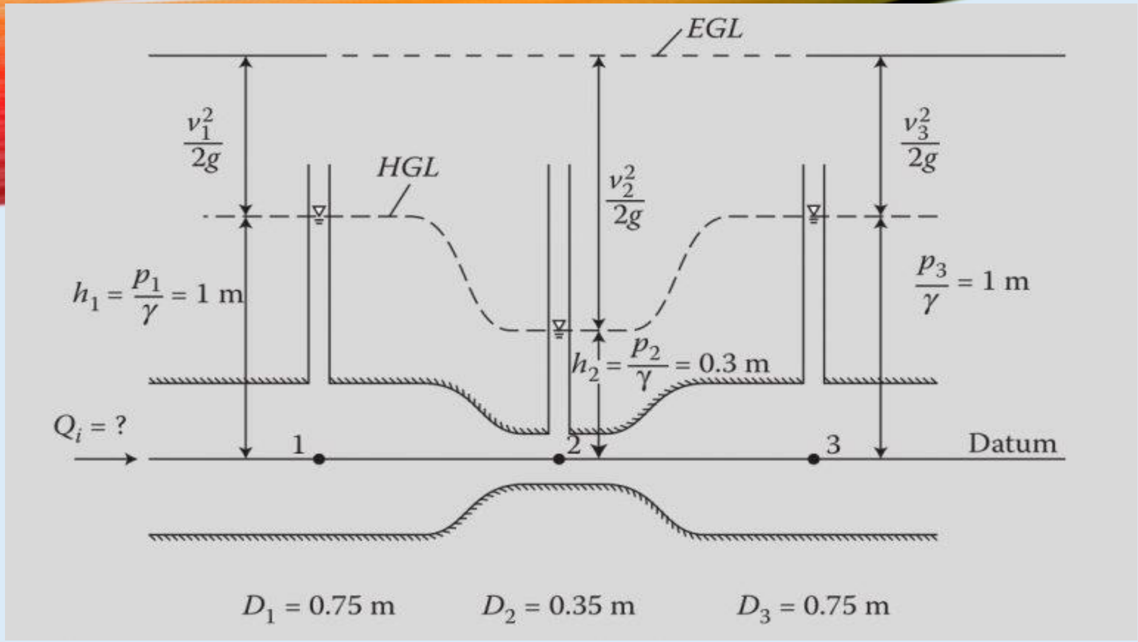
- HGL slopes downward in the direction of flow but it may rise or fall due to change in pressure.
- HGL is parallel to EGL for uniform pipe cross section.
- For horizontal pipes with constant cross section, the drop in pressure gradient between two points is equivalent to the head lost between these points.

Energy Grade Line (*EGL*)

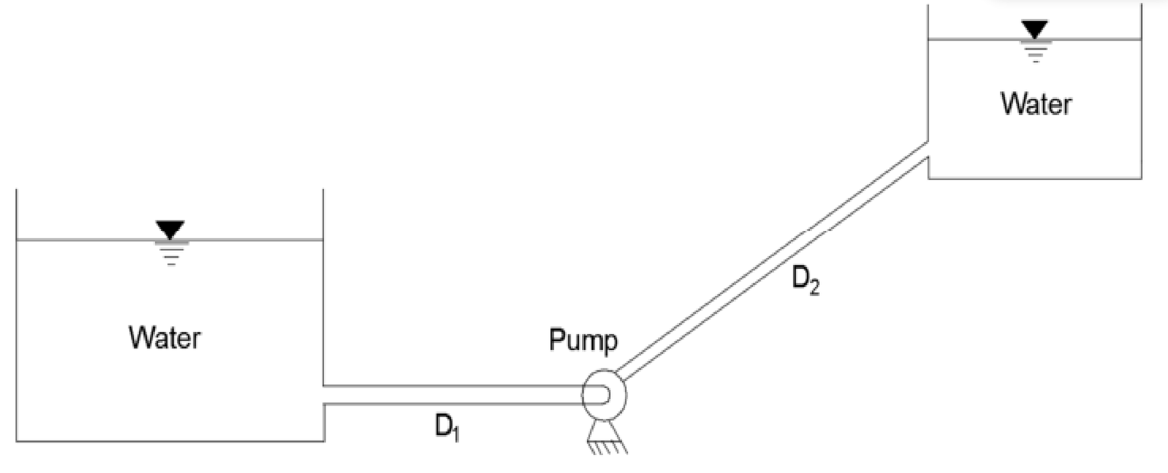
Energy grade line is always above the hydraulic grade line by an amount equal to the velocity head. Thus, the distance of energy gradient above the datum plane is always ($v^2/2g + p/\gamma + z$). Energy grade line therefore is the graphical representation of the total energy of flow.

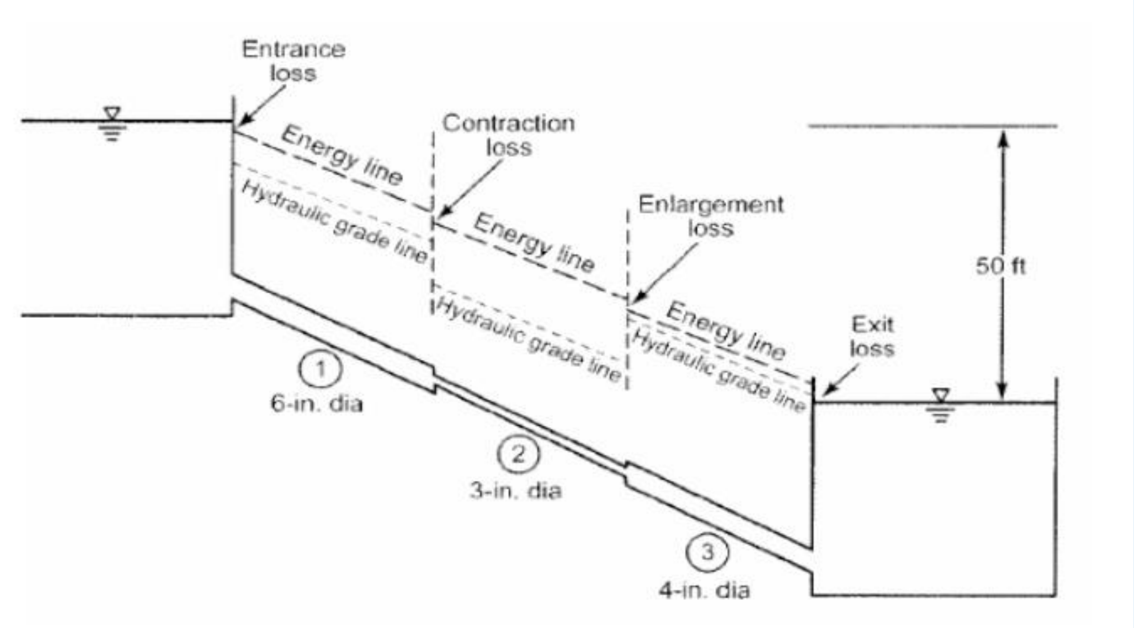
Characteristics of *EGL*

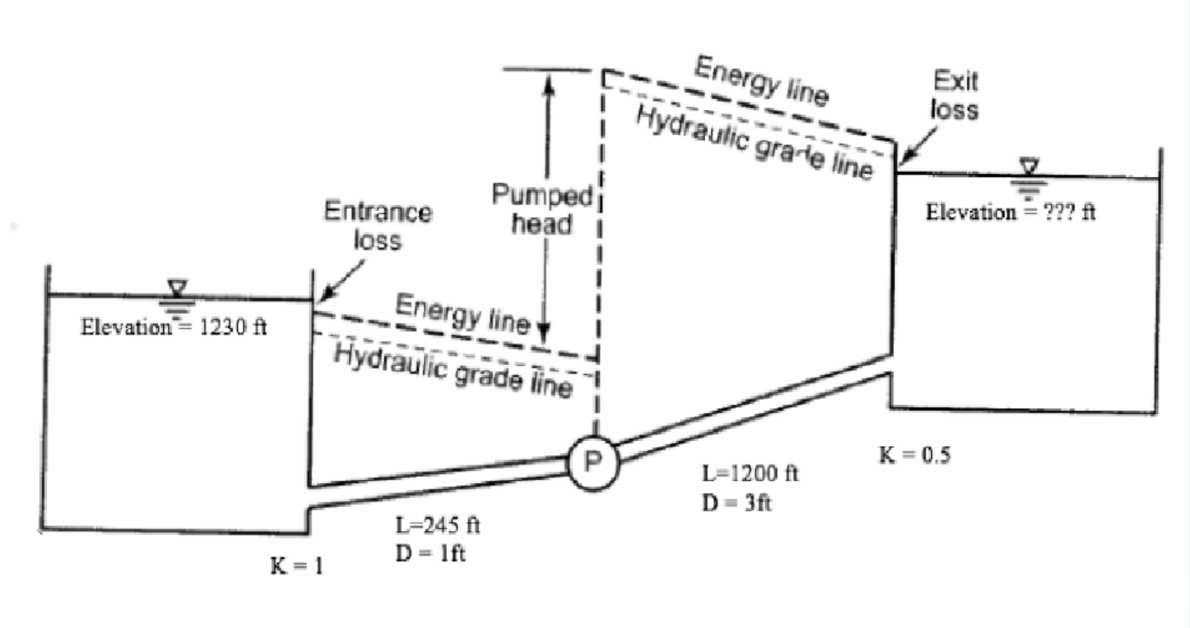
- *EGL* slopes downward in the direction of flow and will only rise with the presence of pump.
- The vertical drop of *EGL* between two points is the head lost between those points.
- *EGL* is parallel to *HGL* for uniform pipe cross section.
- *EGL* is always above the *HGL* by $v^2/2g$.
- Neglecting head loss, *EGL* is horizontal.



For the shown figure below, if $D_1 > D_2$. Draw the HGL and EGL.







For the shown figure below, if $D_3 > D_1 > D_2$. Draw the HGL and EGL.