FLUID MECHANICS

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LECTURE 6

1- Hydrostatic Force on a Plane Surface .

2- Pressure Prism

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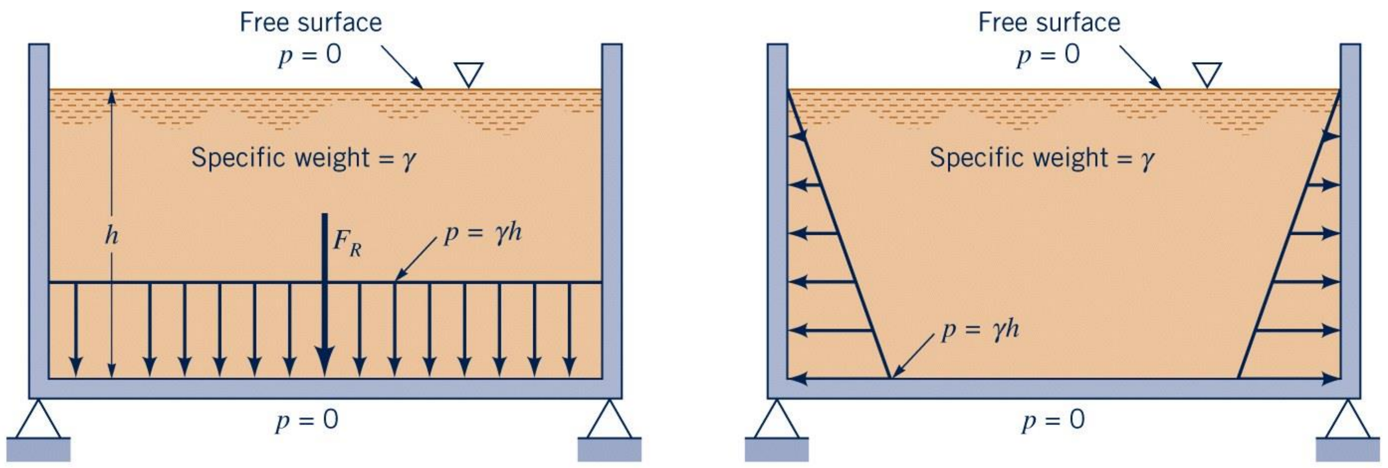
Hydrostatic Force on a Plane Surface.

When a surface is submerged in a fluid, forces develop on the surface due to the fluid. The determination of these forces is important in the design of storage tanks, ships, dams, and other hydraulic structures.
For fluids at rest we know that the force must be *perpendicular* to the surface since there are no shearing stresses present.
We also know that the pressure will vary linearly with depth as shown in Fig. (A) if the fluid is incompressible.

Figure (A)

1) ***Pressure distribution and resultant hydrostatic force on the bottom of an open tank.***
2) ***Pressure distribution on the ends of an open tank.***



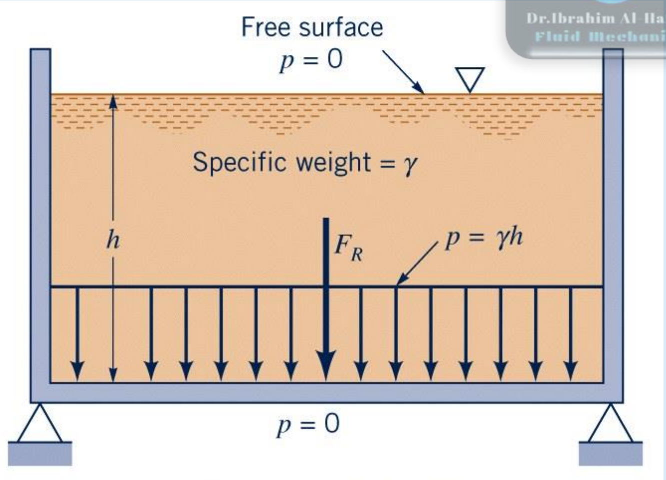
(a) Pressure on tank bottom

(b) Pressure on tank ends

Figure (A.1) ***Figure (A.2)***

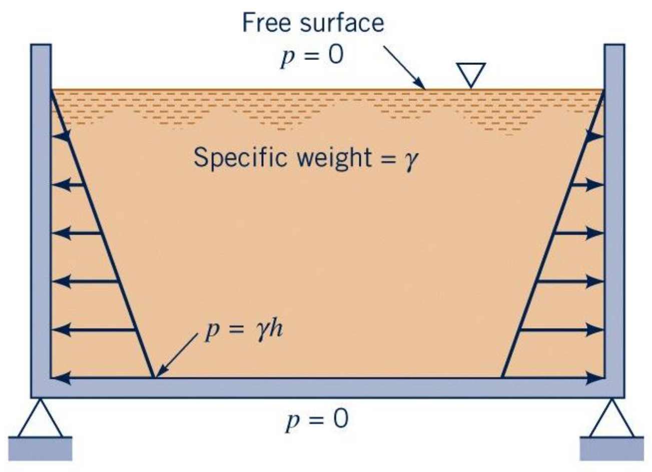
For a horizontal surface, such as **the bottom** of a liquid filled tank (Fig. A.1), the magnitude of the resultant force is simply $F_R = p\,A$, where p is the uniform pressure on the bottom and A is the area of the bottom.

For the open tank shown, $p = \gamma h$.
Note that if atmospheric pressure acts on **both sides of the bottom**, as is illustrated, the ***resultant*** force on the bottom is simply due to the liquid in the tank.



(a) Pressure on tank bottom

Figure (A.1)



(b) Pressure on tank ends

Figure (A.2)

Since the pressure is ***constant*** and ***uniformly distributed*** over the bottom, the resultant ***force*** acts through the **centroid of the area** as shown in Fig.***(A.1)***.

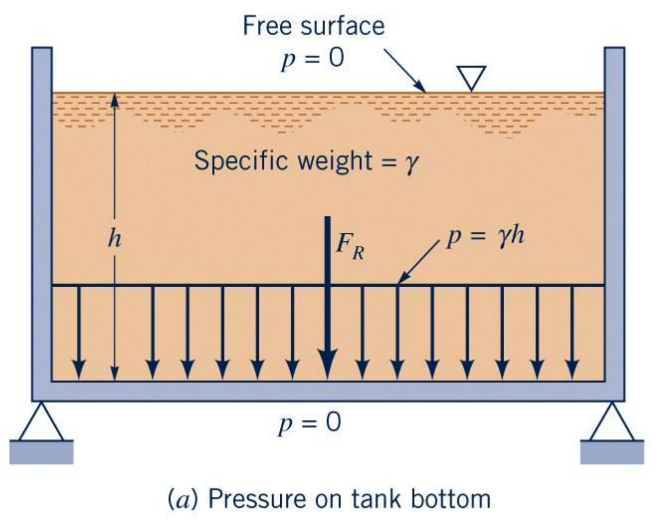


Figure (A.1)

As shown in Fig. **(A.2)**, the pressure on the ends of the tank is ***not uniformly distributed***.

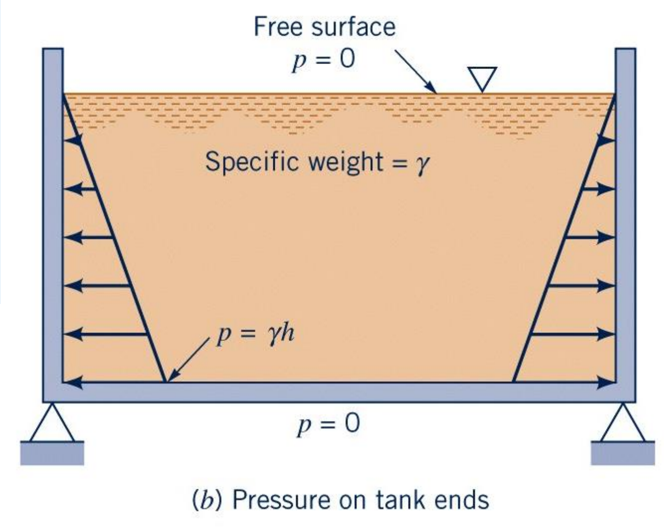


Figure (A.2)

Figure (A)

1) ***Pressure distribution and resultant hydrostatic force on the bottom of an open tank.***
2) ***Pressure distribution on the ends of an open tank.***

For the more general case in which a submerged plane surface is inclined, as is illustrated in Figure (***B***), the determination of the resultant force acting on the surface is more involved.

We will assume that the fluid surface is open to the atmosphere.
Let the plane in which the surface lies intersect the free surface at **0** and make an angle ***ϑ*** with this surface

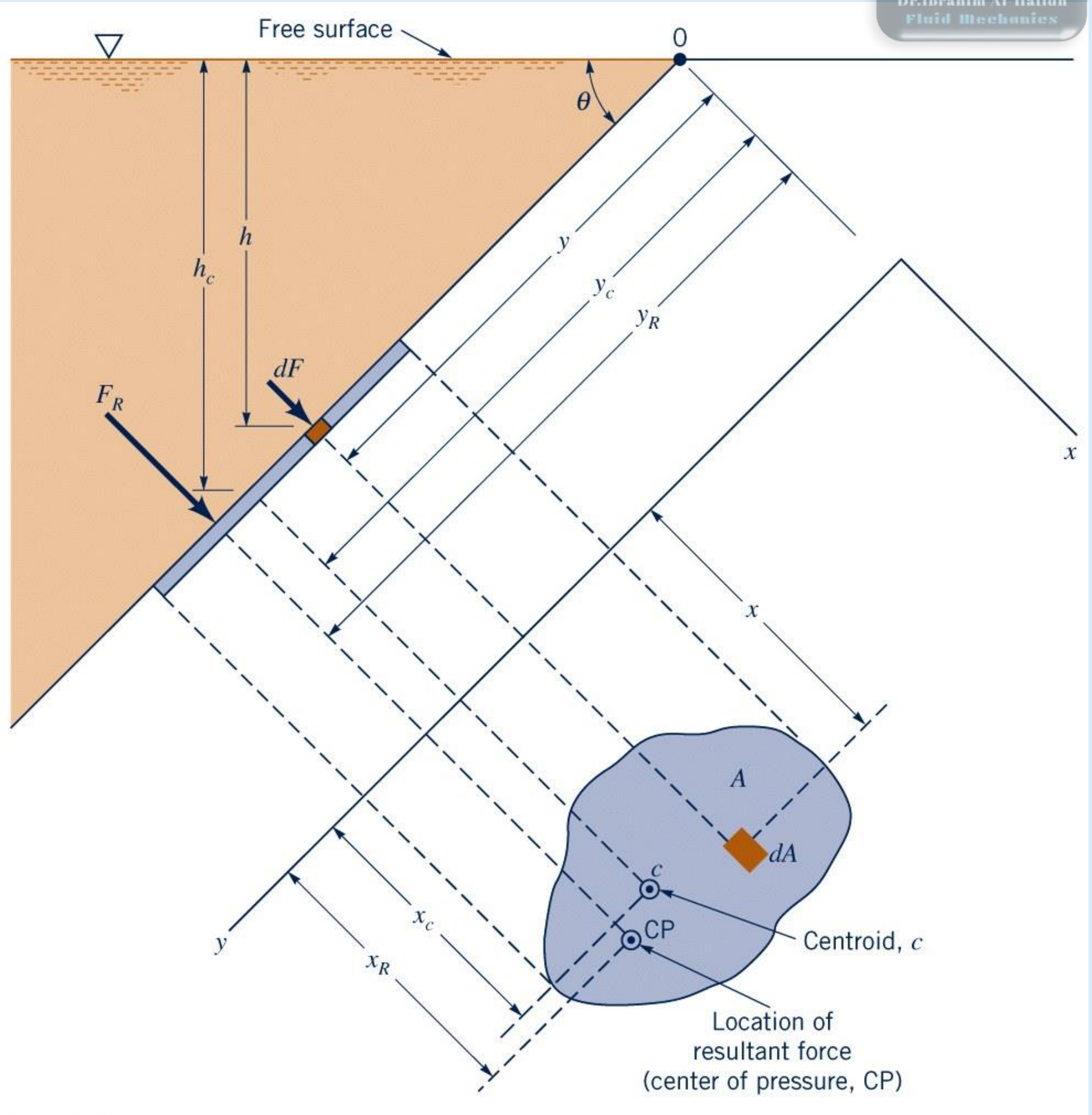


Figure (B): Notation for hydrostatic force on an inclined plane surface of arbitrary shape.

The x–y coordinate system is defined so that 0 is the origin and y = 0 (i.e., the x axis) is directed along the surface as shown. The area can have an arbitrary shape as shown.
We wish to determine the *direction, location, and magnitude of the resultant force* acting on one side of this area due to the liquid in contact with the area.

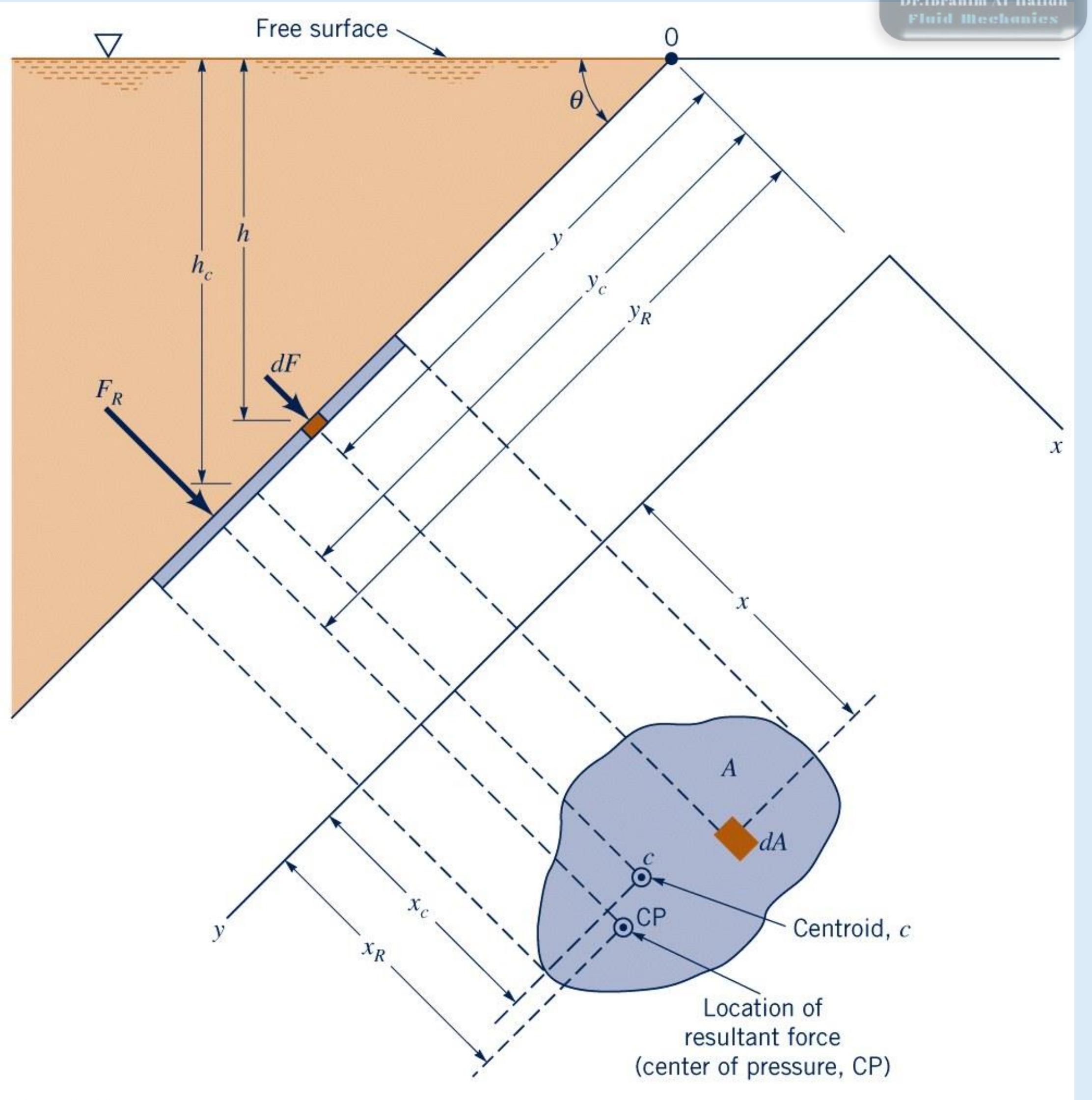


Figure (B): Notation for hydrostatic force on an inclined plane surface of arbitrary shape.

At any given depth, **h**, the force acting on **dA** (the differential area of Fig. **B**) is, $dF = \gamma h \, dA$ and is perpendicular to the surface.

Thus, the magnitude of the resultant force can be found by summing these differential forces over the entire surface. In equation form

$$F_R = \int_A \gamma h \, dA = \int_A \gamma y \sin\theta \, dA$$

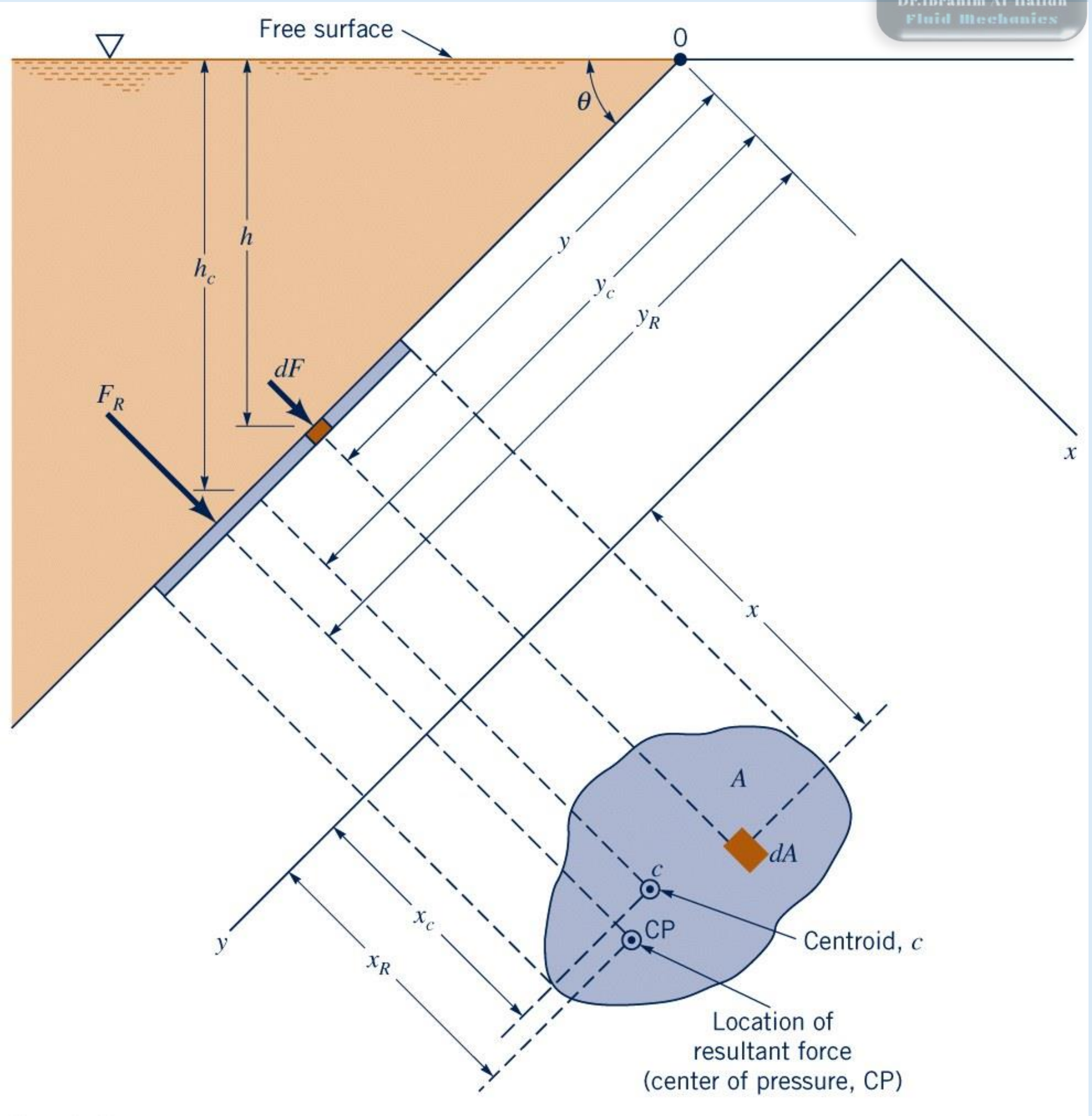


Figure (B): Notation for hydrostatic force on an inclined plane surface of arbitrary shape.

Where, h= y sin θ.

$$F_R = \gamma \sin\theta \int_A y\, dA$$

The integral appearing in the above equation is the *first moment of the area* with respect to the x axis, so we can write

$$\int_A y\, dA = y_c A$$

Where, y_c is the *y* coordinate of the centroid of area *A* measured from the x axis which passes through *0*.

$$F_R = \gamma A y_c \sin\theta$$

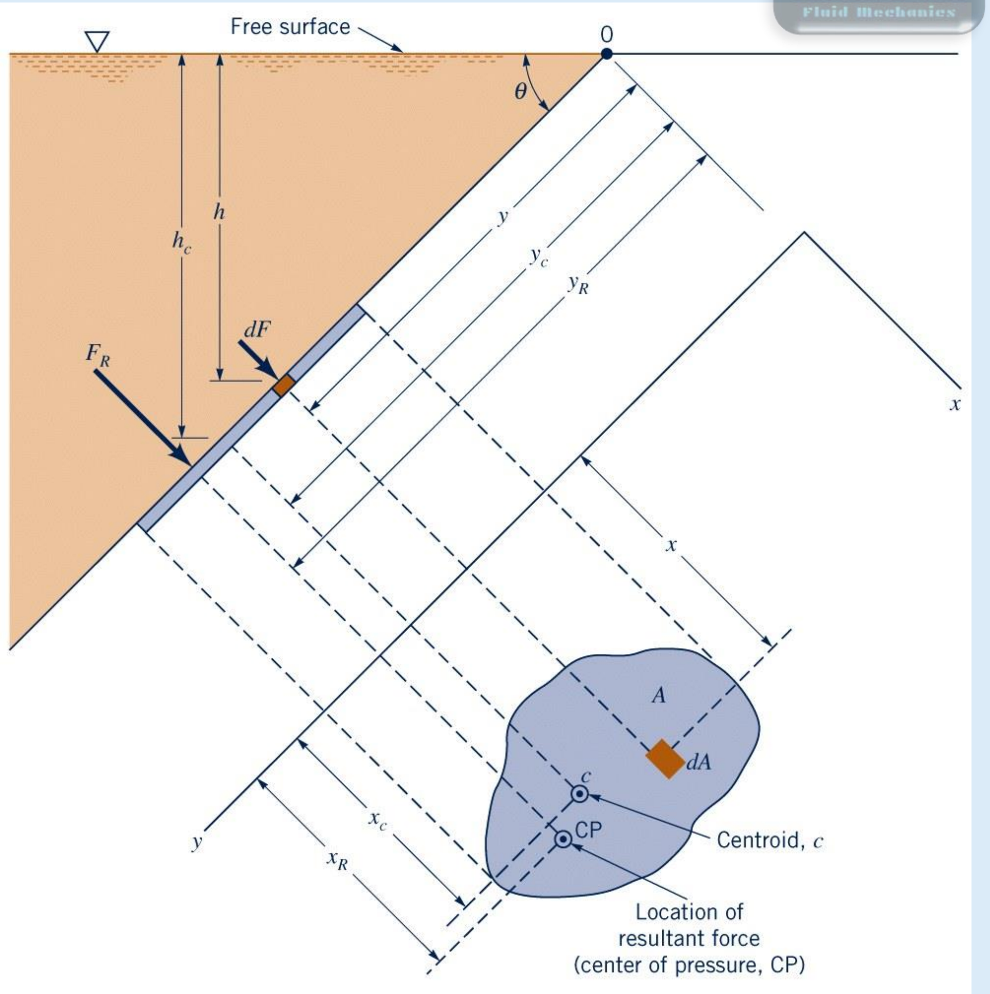


Figure (B): Notation for hydrostatic force on an inclined plane surface of arbitrary shape.

$$F_R = \gamma h_c A$$ ...Eq. A

The magnitude of the resultant fluid force is equal to the pressure acting at the centroid of the area multiplied by the total area.

Where, h_c is the vertical distance from the fluid surface to the centroid of the area.

Note that the magnitude of the force is independent of the angle . As indicated by the figure (C), it depends only on the specific weight of the fluid, the total area, and the depth of the centroid of the area below the surface. F_R must be perpendicular to the surface.

Our intuition might suggest that the resultant force should pass through the (c) *centroid of the area*, this is not actually the case.

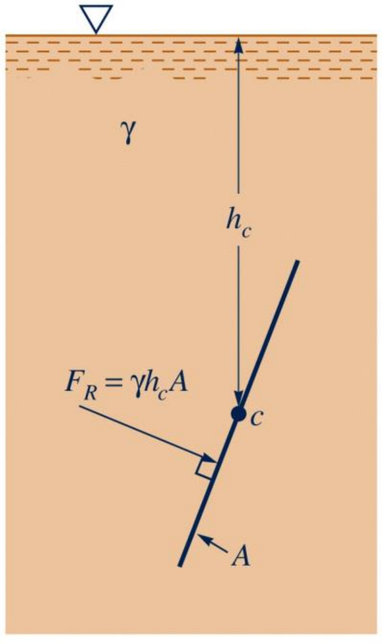


Figure (C)

$$F_R = \gamma h_c A$$

The *y* coordinate, y_R of the resultant force can be determined by summation of **moments around the x axis**. That is, the moment of the resultant force must equal the moment of the distributed pressure force.

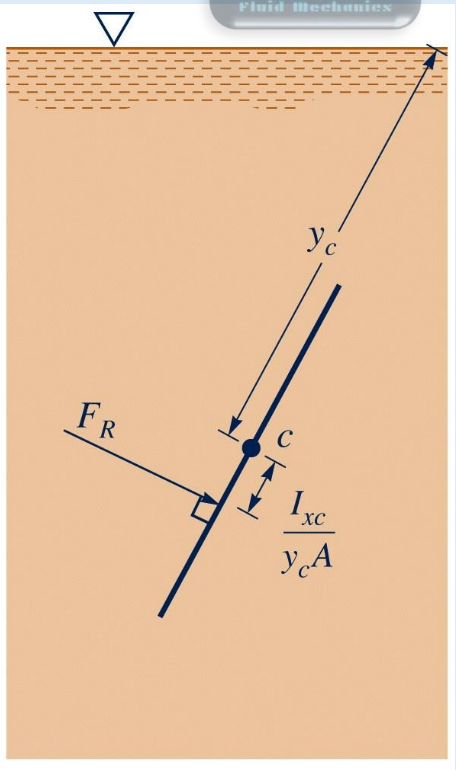


Figure (D)

The resultant fluid force does not pass through the centroid of the area.

To measure the location of **resultant force** (F_R) from the surface of water,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{...Eq. B}$$

I_{xc}, is the second moment of the area with respect to an axis passing through its centroid and parallel to the x-axis.

The resultant force does not pass through the centroid but for nonhorizontal surfaces is always below it, since $I_{xc}/y_c A > 0$

The x coordinate, x_R for the resultant force can be determined in a similar manner by summing moments of the y axis.

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$ ...Eq. C

I_{xyc}, is the product of inertia with respect to an orthogonal coordinate system passing through the centroid of the area and formed by a translation of the x–y coordinate system.

*If the submerged area is symmetrical with respect to an axis passing through the centroid and parallel to either the **x** or **y** axis, the resultant force must lie along the line **x = x_c**, since I_{xyc} is identically **zero** in this case.*

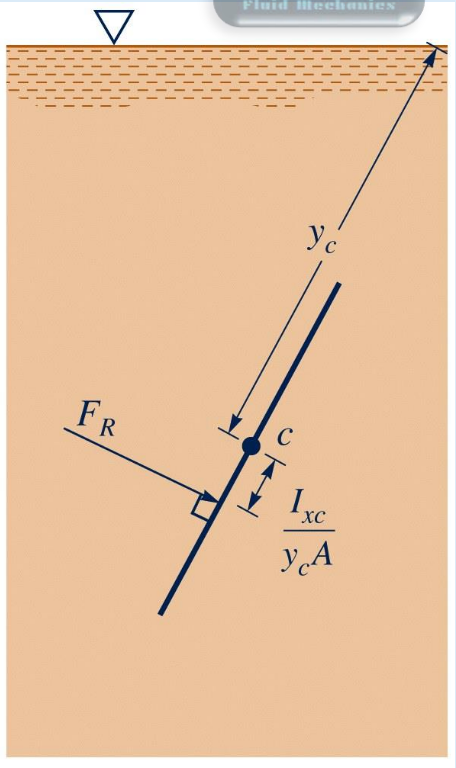


Figure (D)

The resultant fluid force does not pass through the centroid of the area.

The point through which the resultant force acts is called the *center of pressure*.

It is to be noted from Equations **B**, **C** that y_c increases the center of pressure moves closer to the centroid of the area.

Since $y_c = h_c/\sin\vartheta$, the distance y_c will increase if the depth of submergence, h_c, increases.
Thus, the hydrostatic force on the right-hand side of the gate shown in figure (E) acts closer to the centroid of the gate than the force on the left-hand side.

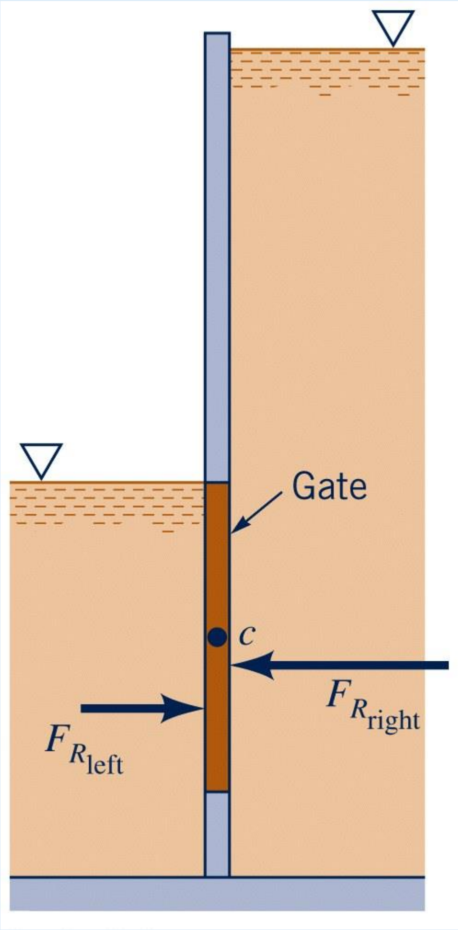
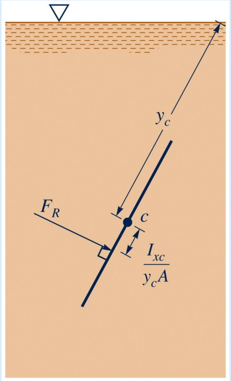
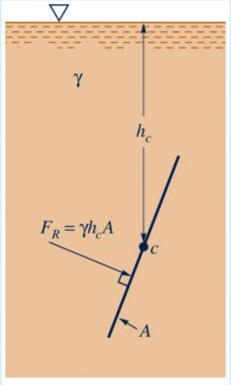


Figure (E)

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$





Centroidal coordinates and moments of inertia for some common areas are given in Figure (F)

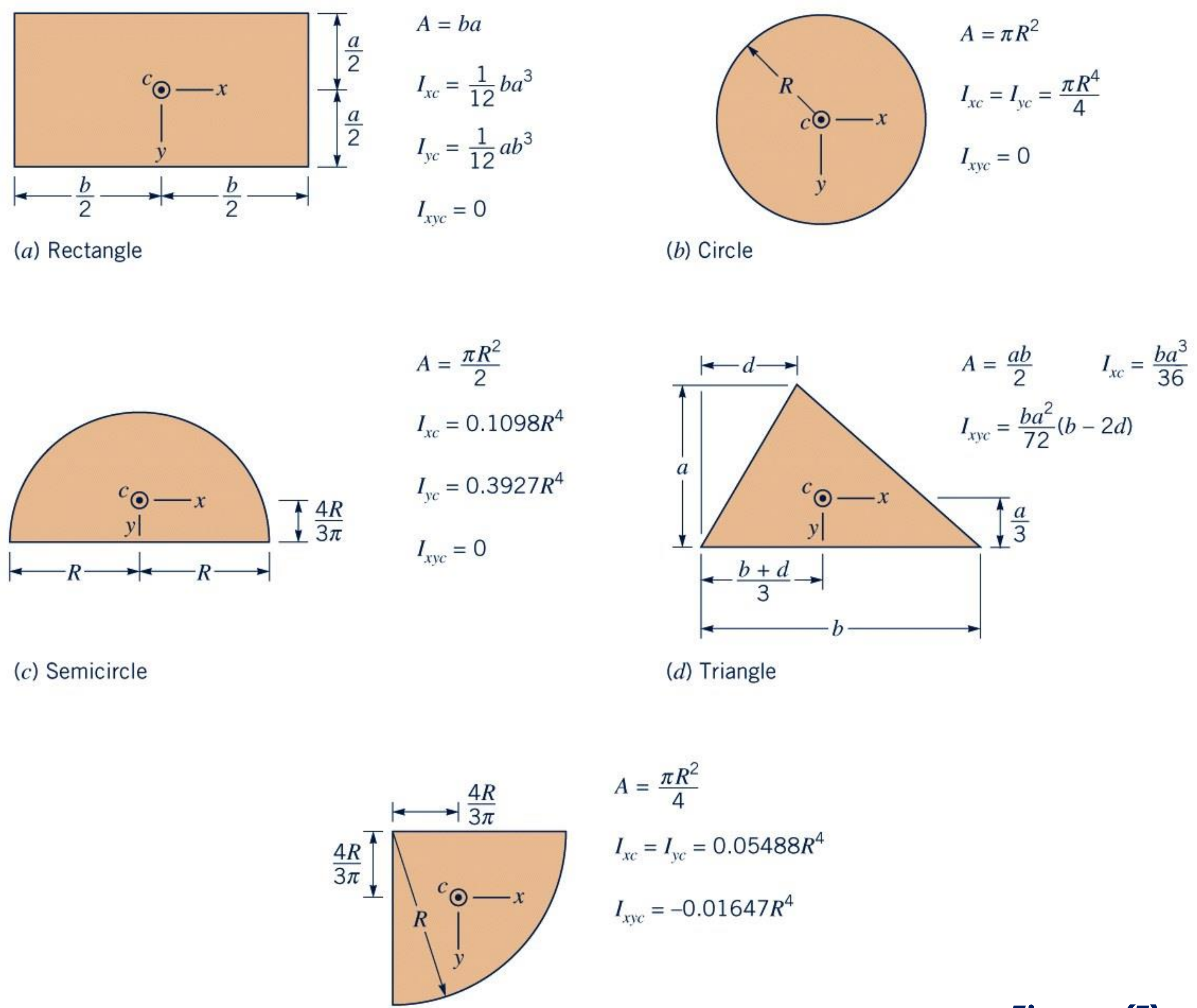


Figure (F)

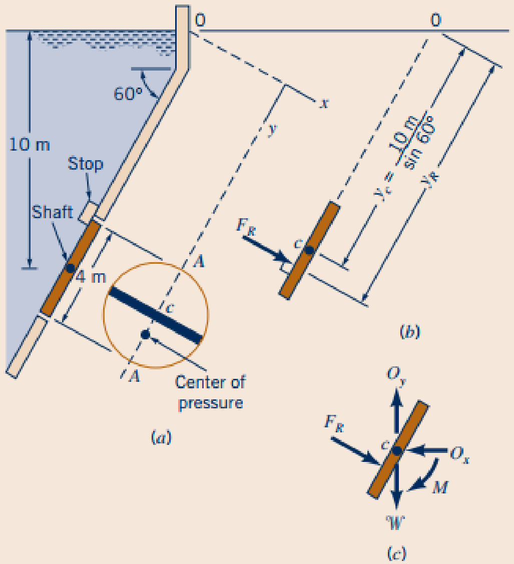
EXAMPLE 2.6 Hydrostatic Force on a Plane Circular Surface

GIVEN The 4-m-diameter circular gate of Fig. E2.6*a* is located in the inclined wall of a large reservoir containing water ($\gamma = 9.80 \text{ kN/m}^3$). The gate is mounted on a shaft along its horizontal diameter, and the water depth is 10 m above the shaft.

FIND Determine

(a) the magnitude and location of the resultant force exerted on the gate by the water and

(b) the moment that would have to be applied to the shaft to open the gate.



■ Figure E2.6*a–c*

SOLUTION

(a) To find the magnitude of the force of the water we can apply Eq. 2.18,

$$F_R = \gamma h_c A$$

and since the vertical distance from the fluid surface to the centroid of the area is 10 m, it follows that

$$F_R = (9.80 \times 10^3 \text{ N/m}^3)(10 \text{ m})(4\pi \text{ m}^2)$$
$$= 1230 \times 10^3 \text{ N} = 1.23 \text{ MN} \qquad \textbf{(Ans)}$$

To locate the point (center of pressure) through which F_R acts, we use Eqs. B & C

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad y_R = \frac{I_{xc}}{y_c A} + y_c$$

For the coordinate system shown, $x_R = 0$ since the area is symmetrical, and the center of pressure must lie along the diameter *A-A*. To obtain y_R, we have from Fig. F

$$I_{xc} = \frac{\pi R^4}{4}$$

and y_c is shown in Fig. E2.6*b*. Thus,

$$y_R = \frac{(\pi/4)(2 \text{ m})^4}{(10 \text{ m}/\sin 60°)(4\pi \text{ m}^2)} + \frac{10 \text{ m}}{\sin 60°}$$
$$= 0.0866 \text{ m} + 11.55 \text{ m} = 11.6 \text{ m}$$

and the distance (along the gate) below the shaft to the center of pressure is

$$y_R - y_c = 0.0866 \text{ m} \qquad \textbf{(Ans)}$$

We can conclude from this analysis that the force on the gate due to the water has a magnitude of 1.23 MN and acts through a point along its diameter *A-A* at a distance of 0.0866 m (along the gate) below the shaft. The force is perpendicular to the gate surface as shown in Fig. E2.6*b*.

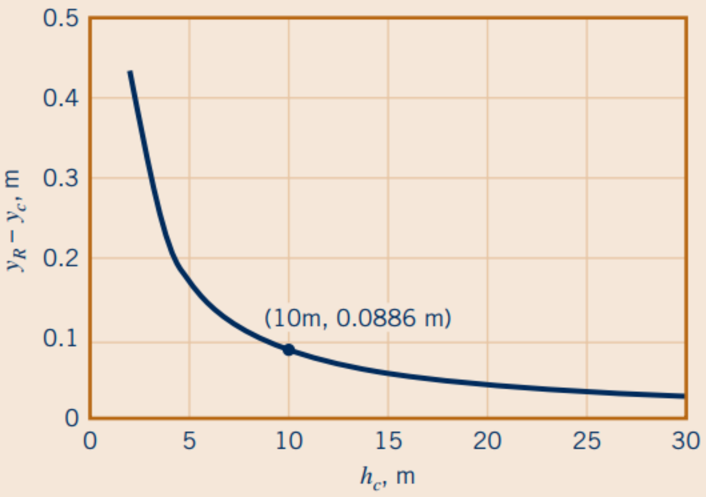
COMMENT By repeating the calculations for various values of the depth to the centroid, h_c, the results shown in Fig. E2.6*d* are obtained. Note that as the depth increases, the distance between the center of pressure and the centroid decreases.

(b) The moment required to open the gate can be obtained with the aid of the free-body diagram of Fig. E2.6*c*. In this diagram $\mathcal{W}$ is the weight of the gate and O_x and O_y are the horizontal and vertical reactions of the shaft on the gate. We can now sum moments about the shaft

$$\sum M_c = 0$$

and, therefore,

$$M = F_R\,(y_R - y_c)$$
$$= (1230 \times 10^3 \text{ N})(0.0866 \text{ m})$$
$$= 1.07 \times 10^5 \text{ N} \cdot \text{m} \qquad \textbf{(Ans)}$$



■ Figure E2.6*d*

Pressure Prism

An informative and useful graphical interpretation can be made for the force developed by a fluid acting on a plane rectangular area. Consider the pressure distribution along a vertical wall of a tank of constant width **b**, which contains a liquid having a specific weight $\boldsymbol{\gamma}$.

Since the pressure must vary linearly with depth, we can represent the variation as is shown in Figure (G.a), where the pressure is equal to zero at the upper surface and equal to $\boldsymbol{\gamma h}$ at the bottom.
It is apparent from this diagram that the average pressure occurs at the depth ***h/2*** and, therefore, the resultant force acting on the rectangular area ***A=bh*** is

$$F_R = p_{\text{av}}\, A = \gamma \left(\frac{h}{2}\right) A \qquad \text{...Eq. D}$$

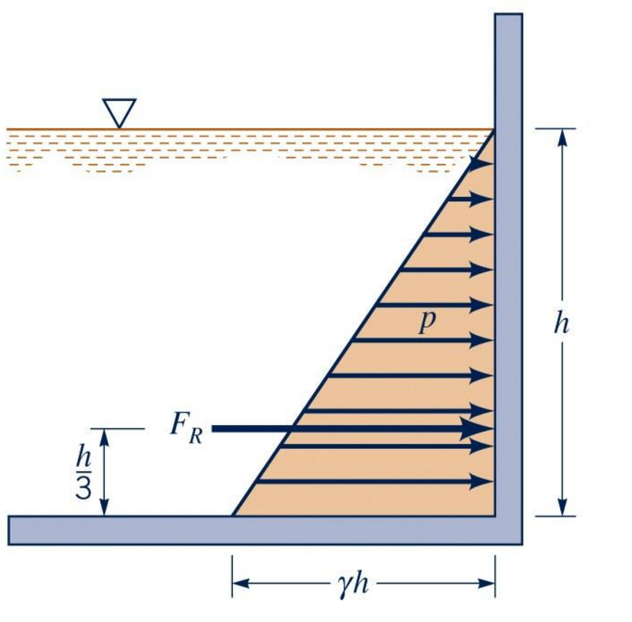


Figure (G)

which is the same result as obtained from Eq. (A).

We can draw the three-dimensional representation of the pressure distribution as shown in Figure (G.b)

The base of this "volume" in pressure-area space is the ***plane surface*** of interest, and its altitude at each point is the ***pressure***.
This volume is called the ***pressure prism***, and it is clear that the magnitude of the resultant force acting on the rectangular surface is equal to the volume of the pressure prism.
Thus, for the prism of Figure (G.B) the fluid force is:

$$F_R = \text{volume} = \frac{1}{2}(\gamma h)(bh) = \gamma\left(\frac{h}{2}\right)A \quad \text{...Eq. E}$$

where ***bh*** is the area of the rectangular surface, A .

The resultant force must pass through the centroid of the pressure prism. For the volume under consideration the centroid is located along the vertical axis of symmetry of the surface and at a distance of (***h/3***) above the base (*since the centroid of a triangle is located at h/3 above its base*).

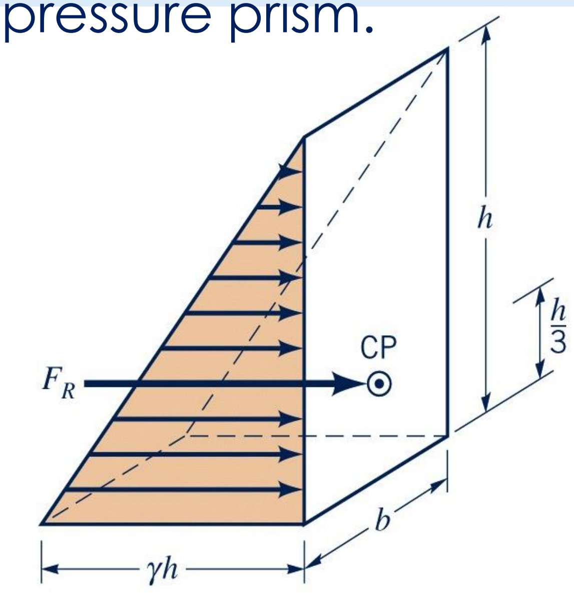


Figure (G) (*b*)

This same graphical approach can be used for plane rectangular surfaces that do not extend up to the fluid surface, as illustrated in Figure **H**. In this instance, the cross section of the pressure prism is **trapezoidal**. However, the resultant force is still equal in magnitude to the volume of the pressure prism, and it passes through the centroid of the volume. Specific values can be obtained by decomposing the pressure prism into two parts, ABDE and BCD, as shown in Fig. **H.b**.

Thus,

$$F_R = F_1 + F_2$$

The location can be determined by summing moments about some convenient axis, such as one passing through A. In this instance

$$F_R y_A = F_1 y_1 + F_2 y_2$$

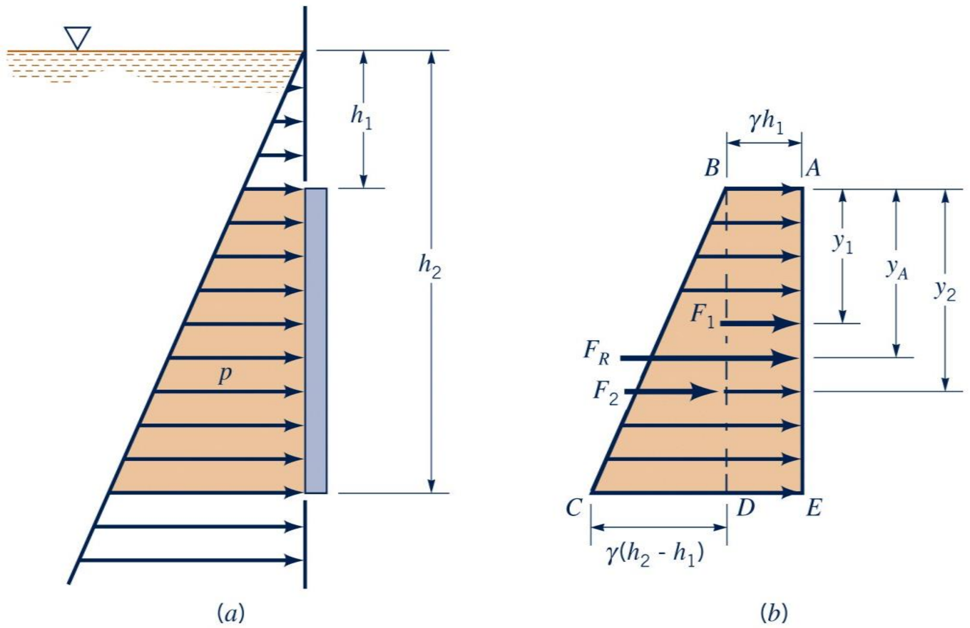


Figure (H)

For inclined plane rectangular surfaces the pressure prism can still be developed, and the cross section of the prism will generally be trapezoidal, as is shown in Figure **I**.

Although it is usually convenient to measure distances along the inclined surface, the pressures developed depend on the vertical distances as illustrated

The use of pressure prisms for determining the force on submerged plane areas is convenient if the area is rectangular so the volume and centroid can be easily determined.

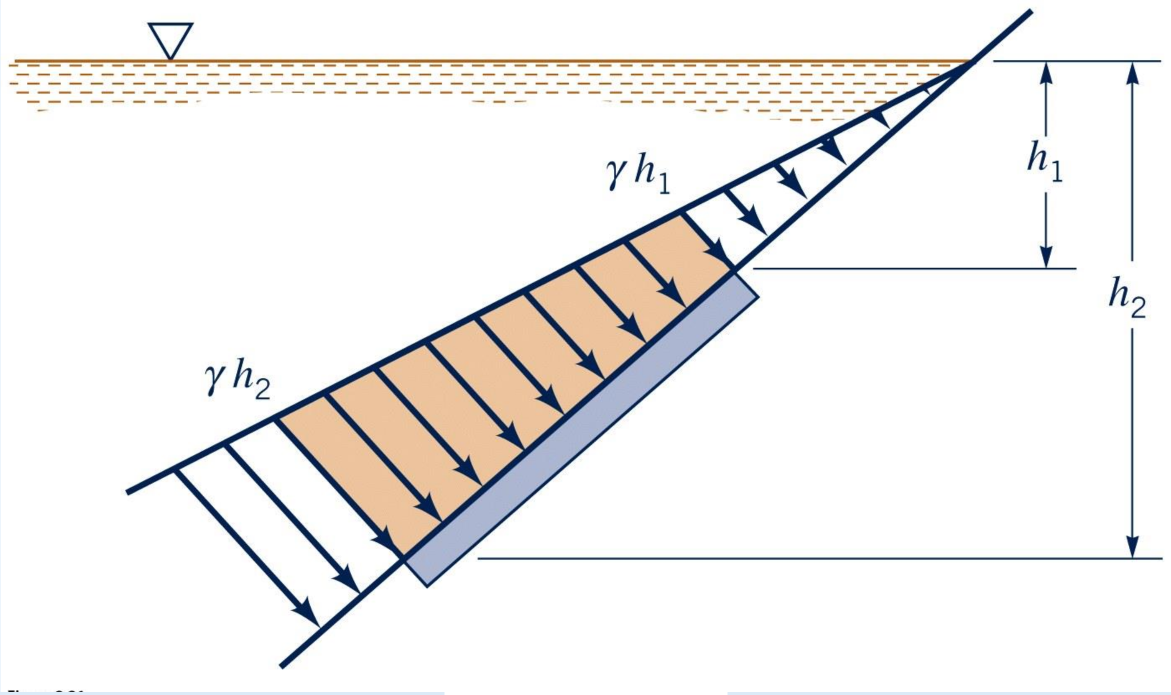
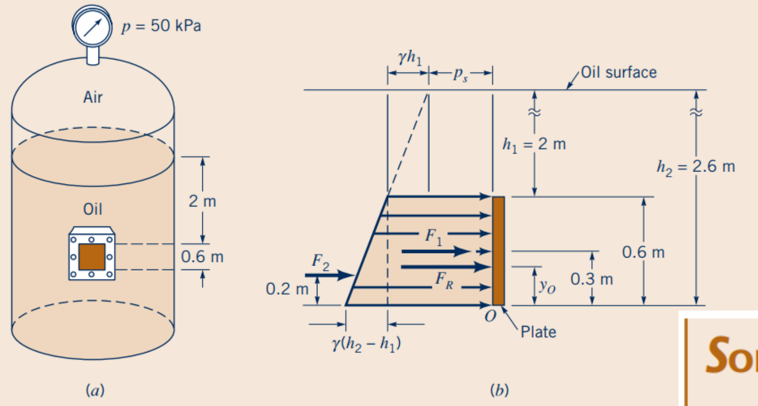


Figure (I)

EXAMPLE 2.8 Use of the Pressure Prism Concept

GIVEN A pressurized tank contains oil ($SG = 0.90$) and has a square, 0.6-m by 0.6-m plate bolted to its side, as is illustrated in Fig. E2.8*a*. The pressure gage on the top of the tank reads 50 kPa, and the outside of the tank is at atmospheric pressure.

FIND What is the magnitude and location of the resultant force on the attached plate?



■ Figure E2.8

SOLUTION

The pressure distribution acting on the inside surface of the plate is shown in Fig. E2.8*b*. The pressure at a given point on the plate is due to the air pressure, p_s, at the oil surface and the pressure due to the oil, which varies linearly with depth as is shown in the figure. The resultant force on the plate (having an area A) is due to the components, F_1 and F_2, where F_1 and F_2 are due to the rectangular and triangular portions of the pressure distribution, respectively. Thus,

$$\begin{aligned} F_1 &= (p_s + \gamma h_1)\,A \\ &= [50 \times 10^3 \text{ N/m}^2 \\ &\quad + (0.90)(9.81 \times 10^3 \text{ N/m}^3)(2 \text{ m})](0.36 \text{ m}^2) \\ &= 24.4 \times 10^3 \text{ N} \end{aligned}$$

and

$$\begin{aligned} F_2 &= \gamma\left(\frac{h_2 - h_1}{2}\right)A \\ &= (0.90)(9.81 \times 10^3 \text{ N/m}^3)\left(\frac{0.6 \text{ m}}{2}\right)(0.36 \text{ m}^2) \\ &= 0.954 \times 10^3 \text{ N} \end{aligned}$$

The magnitude of the resultant force, F_R, is therefore

$$F_R = F_1 + F_2 = 25.4 \times 10^3 \text{ N} = 25.4 \text{ kN} \qquad \textbf{(Ans)}$$

The vertical location of F_R can be obtained by summing moments around an axis through point O so that

$$F_R y_O = F_1(0.3 \text{ m}) + F_2(0.2 \text{ m})$$

or

$$\begin{aligned} y_O &= \frac{(24.4 \times 10^3 \text{ N})(0.3 \text{ m}) + (0.954 \times 10^3 \text{ N})(0.2 \text{ m})}{25.4 \times 10^3 \text{ N}} \\ &= 0.296 \text{ m} \end{aligned} \qquad \textbf{(Ans)}$$

Thus, the force acts at a distance of 0.296 m above the bottom of the plate along the vertical axis of symmetry.

COMMENT Note that the air pressure used in the calculation of the force was gage pressure. Atmospheric pressure does not affect the resultant force (magnitude or location), since it acts on both sides of the plate, thereby canceling its effect.