FLUID MECHANICS

COLLEGE OF PETROLEUM AND MINING ENGINEERING

Dr. Ibrahim Adil Ibrahim Al-Hafidh
Mining Engineering Department
College of Petroleum and Mining Engineering
University of Mosul

Email: iibrahim@uomosul.edu.iq

LECTURE 7

Hydrostatic Force on a Curved Surface.

Dr.Ibrahim Al-Hafidh
Fluid Mechanics

Email: iibrahim@uomosul.edu.iq

Hydrostatic Force on a Curve Surface.

Many surfaces such as those associated with dams, pipes, and tanks are nonplanar. Therefor, to study an analysis the force on the curve surface is extremely important.

For example, consider a curved portion of the swimming pool shown in Fig. A. We wish to find the resultant fluid force acting on section BC (which has a unit length perpendicular to the plane of the paper) shown in Fig. B.

Figure (A)

We first isolate a volume of fluid that is bounded by the surface of interest, in this instance section BC, the horizontal plane surface ***AB***, and the vertical plane surface ***AC***.

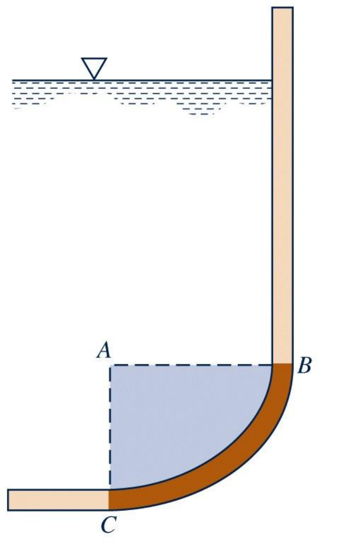


Figure (B)

The free-body diagram for this volume is shown in Fig.c. The magnitude and location of forces ***F1*** and ***F2*** can be determined from the relationships for planar surfaces. The weight, ***W***,$(\gamma * Vol)$ is simply the specific weight of the fluid times the enclosed volume and acts through the center of gravity ***(CG)*** of the mass of fluid contained within the volu

The forces $\boldsymbol{F_H}$ and $\boldsymbol{F_V}$ represent the components of the force that the tank ***exerts on the fluid.***

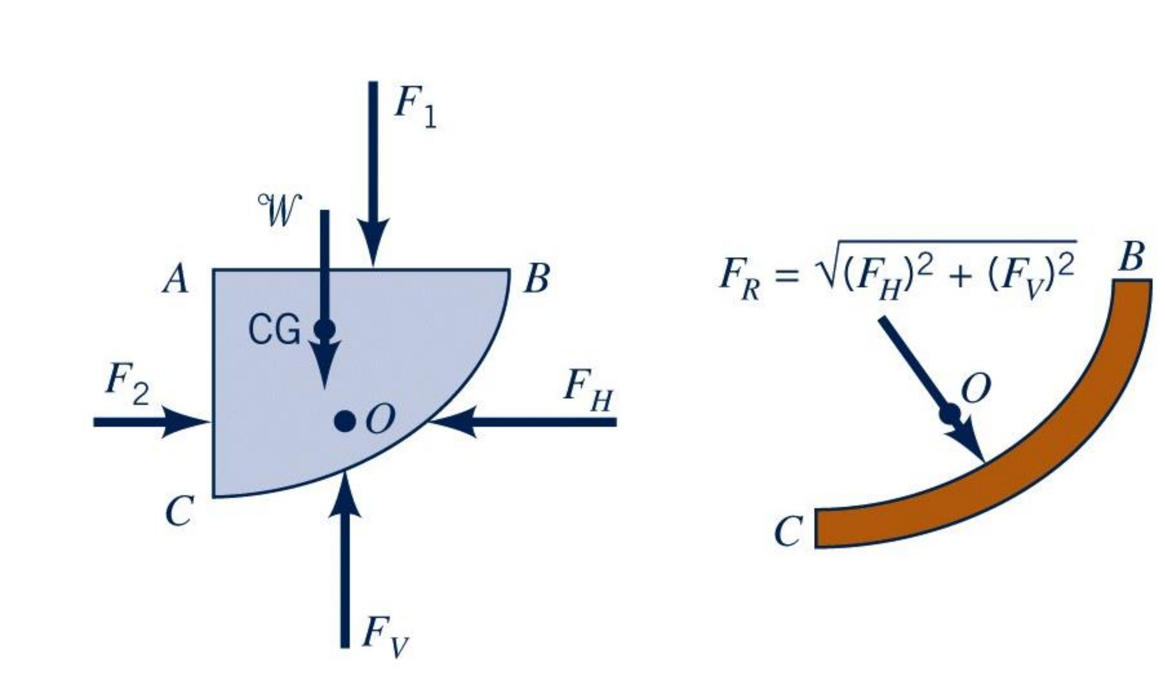


Figure (C) ***Figure (D)***

In order for this force system to be in equilibrium, the horizontal component $\boldsymbol{F_H}$ must be equal in magnitude and collinear with $\boldsymbol{F_2}$, and the vertical component $\boldsymbol{F_V}$ equal in magnitude and collinear with the resultant of the vertical forces $\boldsymbol{F_1}$ and $\boldsymbol{W}$.

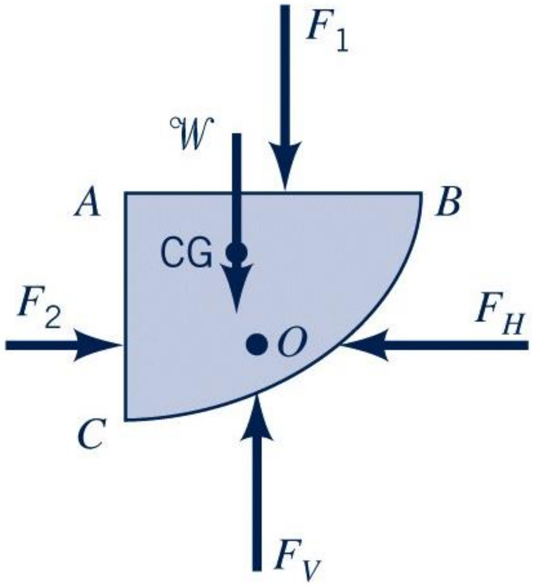


Figure (C)

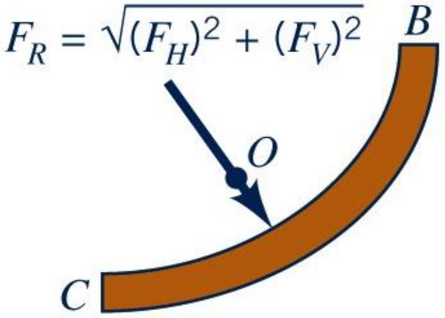


Figure (D)

This follows since the three forces acting on the fluid mass (***F_2***, the resultant of ***F_1*** and ***W***, and the resultant force that the tank exerts on the mass) must form a ***concurrent*** force system. That is, from the principles of statics, it is known that when a body is held in equilibrium by three nonparallel forces, they must be concurrent (their lines of action intersect at a common point) and coplanar. Thus,

$$F_H = F_2$$
$$F_V = F_1 + \mathcal{W}$$

and the magnitude of the resultant is obtained from the equation

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

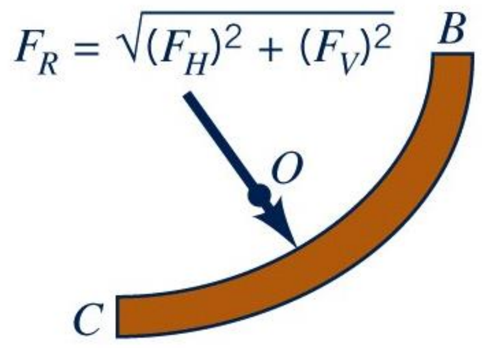


Figure (D)

The resultant passes through the point ***O***, which can be located by summing moments about an appropriate axis. The **resultant force** of the fluid acting on the ***curved surface BC*** is equal and opposite in direction to that obtained from the free-body diagram of Fig. C. The desired fluid force is shown in Fig. D.

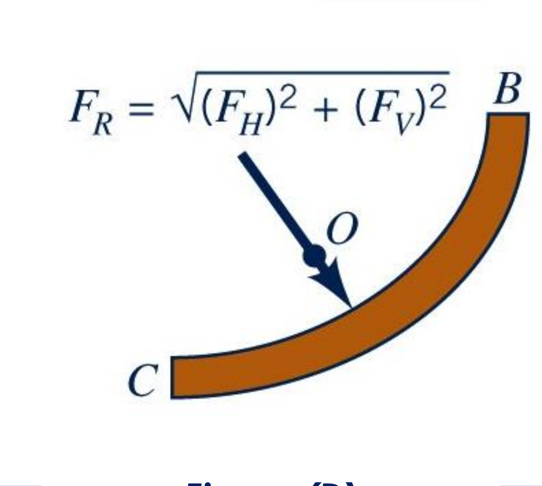


Figure (D)

This same general approach can also be used for determining the force on curved surfaces of pressurized, closed tanks. If these tanks contain a gas, the weight of the gas is usually **negligible** in comparison with the forces developed by the pressure.

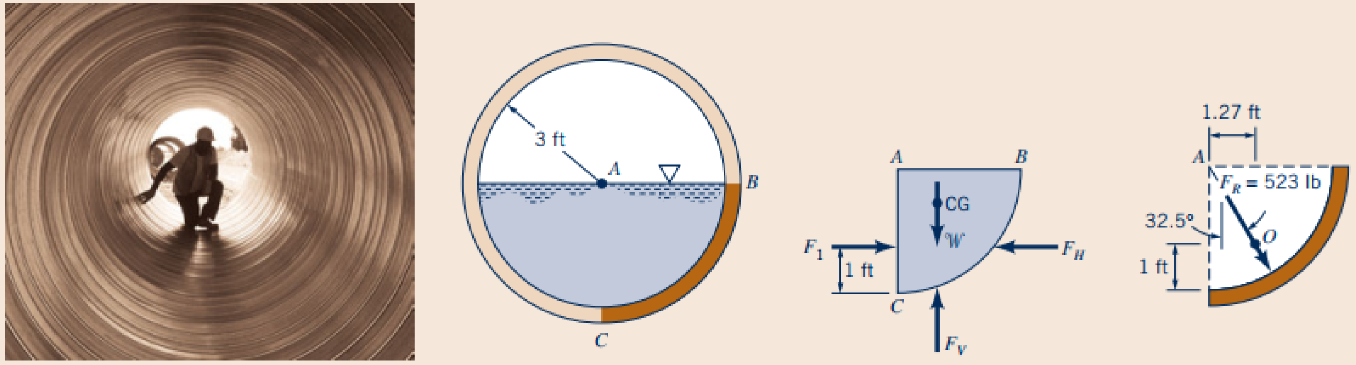
Thus, the forces (such as in Fig. C) on horizontal and vertical projections of the curved surface of interest can simply be expressed as the internal pressure times the appropriate projected area.

$$F_R = p_{av}\, A = \gamma\left(\frac{h}{2}\right)A$$

EXAMPLE 2.9 Hydrostatic Pressure Force on a Curved Surface

GIVEN A 6-ft-diameter drainage conduit of the type shown in Fig. E2.9*a* is half full of water at rest, as shown in Fig. E2.9*b*.

FIND Determine the magnitude and line of action of the resultant force that the water exerts on a 1-ft length of the curved section *BC* of the conduit wall.



(*a*) (*b*) (*c*) (*d*)

Figure E2.9 (Photograph courtesy of CONTECH Construction Products, Inc.)

SOLUTION

We first isolate a volume of fluid bounded by the curved section *BC*, the horizontal surface *AB*, and the vertical surface *AC*, as shown in Fig. E2.9*c*. The volume has a length of 1 ft. The forces acting on the volume are the horizontal force, F_1, which acts on the vertical surface *AC*, the weight, $\mathcal{W}$, of the fluid contained within the volume, and the horizontal and vertical components of the force of the conduit wall on the fluid, F_H and F_V, respectively.

The magnitude of F_1 is found from the equation

$$F_1 = \gamma h_c A = (62.4 \text{ lb/ft}^3)(\tfrac{3}{2} \text{ ft})(3 \text{ ft}^2) = 281 \text{ lb}$$

and this force acts 1 ft above *C* as shown. The weight $\mathcal{W} = \gamma \forall$, where $\forall$ is the fluid volume, is

$$\mathcal{W} = \gamma \forall = (62.4 \text{ lb/ft}^3)(9\pi/4 \text{ ft}^2)(1 \text{ ft}) = 441 \text{ lb}$$

and acts through the center of gravity of the mass of fluid, which according to Fig. 2.18 is located 1.27 ft to the right of *AC* as shown. Therefore, to satisfy equilibrium

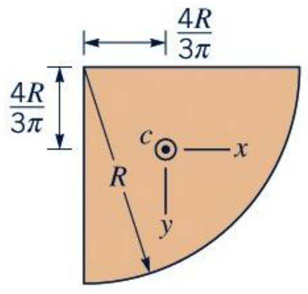
$$F_H = F_1 = 281 \text{ lb} \qquad F_V = \mathcal{W} = 441 \text{ lb}$$

and the magnitude of the resultant force is

$$F_R = \sqrt{(F_H)^2 + (F_V)^2} = \sqrt{(281 \text{ lb})^2 + (441 \text{ lb})^2} = 523 \text{ lb} \qquad \textbf{(Ans)}$$

The force the water exerts *on* the conduit wall is equal, but *opposite in direction*, to the forces F_H and F_V shown in Fig. E2.9*c*. Thus, the resultant force *on the conduit wall* is shown in Fig. E2.9*d*. This force acts through the point *O* at the angle shown.

COMMENT An inspection of this result will show that the line of action of the resultant force passes through the center of the conduit. In retrospect, this is not a surprising result since at each point on the curved surface of the conduit the elemental force due to the pressure is normal to the surface, and each line of action must pass through the center of the conduit. It therefore follows that the resultant of this concurrent force system must also pass through the center of concurrence of the elemental forces that make up the system.



$A = \dfrac{\pi R^2}{4}$

$I_{xc} = I_{yc} = 0.05488R^4$

$I_{xyc} = -0.01647R^4$

(*e*) Quarter circle