NUMERICAL ANALYSIS

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Lecture 3 & 4

FINITE DIFFERENCE

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Lagrange Interpolation Formula for Unequal Interval

The corresponding values for x_1 , x_2 , x_3 , … … … are $f(x_1)$, $f(x_2)$, $f(x_3)$

The formula given below is used to find the value of $f(x)$ for any value of x

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2)$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3) + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$$

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Example

Using Lagranges interpolation formula, find the value of y corresponding to $x = 10$ from the following table:

x	5	6	9	11
y	12	13	14	16

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x	5	6	9	11
y	12	13	14	16

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2)$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3) + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$$

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13$$

$$= \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= 14.667$$

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Example

Using Lagrange's interpolation formula, find the value of y corresponding to $x = 5$ from the following table:

x	1	2	3	4	7
y	2	4	8	16	128

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$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} f(x_1)$$

$$+\frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} f(x_2)$$

$$+\frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} f(x_3)$$

$$+\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} f(x_4)$$

$$+\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} f(x_5)$$

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$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+\frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+\frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

$$+\frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+\frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128 = 32.933$$

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Example

Sate Lagrange's interpolation formula, using unequal interval. From an experiment, we get the following values of a function $f(x)$

x	1	2	-4
$f(x)$	3	-5	4

Represent the function $f(x)$ approximately by polynomial of degree 2.

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$$f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} f(x_1) + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$f(x) = \frac{(x-2)(x+4)}{(1-2)(1+4)} \times 3 + \frac{(x-1)(x+4)}{(2-1)(2+4)} \times (-5) + \frac{(x-1)(x-2)}{(-4-1)(-4-2)} \times 4$$

$$= \frac{-3}{5}\left(x^2+2x-8\right) - \frac{5}{6}\left(x^2+3x-4\right) + \frac{4}{30}\left(x^2-3x+2\right)$$

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$$= \left(\frac{-3}{5} - \frac{5}{6} + \frac{4}{30}\right)x^2 + \left(\frac{-6}{5} - \frac{15}{6} - \frac{12}{30}\right)x + \left(\frac{24}{5} + \frac{20}{6} + \frac{8}{30}\right)$$

$$= \frac{-13}{10}x^2 - \frac{41}{10}x + \frac{42}{5}$$

$$= \frac{-1}{10}\left(13x^2 + 41x - 84\right)$$

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Newton's Divided Difference Interpolation

تستخدم هذه الطريقة عندما تكون الزيادة في قيم x غير منتظمة

$$f(x) = f(a) + \Delta f(a)(x - x_0) + \Delta^2 f(a)(x - x_0)(x - x_1)$$

$$+ \Delta^3 f(a)(x - x_0)(x - x_1)(x - x_2) + \dots\dots\dots\dots\dots$$

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Example

Find $f(9.2)$ from the given values, using Newton's divided difference interpolation

x	8	9	9.5	11
$f(x)$	2.079442	2.197225	2.251292	2.397895

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x	$f(x)$	Δ	Δ^2	Δ^3
8	2.079442			
		0.117783		
9	2.197225		-0.006433	
		0.108134		0.000411
9.5	2.251292		-0.0052	
		0.097735		
11	2.397895			

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$$\Delta = \frac{\text{اللاحق} - \text{السابق}}{\text{الفترة بينهم}} = \frac{2.197225 - 2.079442}{9 - 8} = 0.117783$$

$$\Delta^2 = \frac{0.108134 - 0.117783}{9.5 - 8} = -0.006433$$

$$\Delta^3 = \frac{-0.0052 - (-0.006433)}{11 - 8} = 0.000411$$

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$$f(x) = f(a) + \Delta f(a)(x - x_0) + \Delta^2 f(a)(x - x_0)(x - x_1)$$

$$+ \Delta^3 f(a)(x - x_0)(x - x_1)(x - x_2)$$

$$f(9.2) = 2.079443 + 0.117783(9.2 - 8) - 0.006433(9.2 - 8)(9.2 - 9)$$

$$+ 0.000411(9.2 - 8)(9.2 - 9)(9.2 - 9.5)$$

$$= 2.219209$$

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Example

Find $f(1.2)$ from the given values, using Newton's divided difference interpolation

x	1	1.5	2.5	3	4
$f(x)$	0	8.625	43.875	72	153

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x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
1	0				
		17.25			
1.5	8.625		12		
		35.25		1	
2.5	43.875		14		0
		56.25		1	
3	72		16.5		
		81			
4	153				

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$$\Delta = \frac{\text{اللاحق} - \text{السابق}}{\text{الفترة بينهم}} = \frac{8.625 - 0}{1.5 - 1} = 17.25$$

$$\Delta^2 = \frac{35.25 - 17.25}{2.5 - 1} = 12$$

$$\Delta^3 = \frac{14 - 12}{3 - 1} = 1$$

$$\Delta^4 = \frac{1 - 1}{4 - 1} = 0$$

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$$f(x) = f(a) + \Delta f(a)(x - x_0) + \Delta^2 f(a)(x - x_0)(x - x_0)$$

$$+ \Delta^3 f(a)(x - x_0)(x - x_1)(x - x_2)$$

$$f(1.2) = 0 + 17.25\,(1.2 - 1) + 12\,(1.2 - 1)(1.2 - 1.5)$$

$$+ 1\,(1.2 - 1)(1.2 - 1.5)(1.2 - 2.5)$$

$$= 2.808$$

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Example

Find the values of $f(2), f(8)$ and $f(15)$, using Newton's divided difference interpolation, for given the following table:

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

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x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
4	48				
		52			
5	100		15		
		97		1	
7	294		21		0
		202		1	
10	900		27		0
		310		1	
11	1210		33		
		409			
13	2028				

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$$\Delta = \frac{\text{اللاحق} - \text{السابق}}{\text{الفترة بينهم}} = \frac{100 - 48}{5 - 4} = 52$$

$$\Delta^2 = \frac{97 - 52}{7 - 4} = 15$$

$$\Delta^3 = \frac{21 - 15}{10 - 4} = 1$$

$$\Delta^4 = \frac{1 - 1}{11 - 4} = 0$$

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$$f(x) = f(a) + \Delta f(a)(x - x_0) + \Delta^2 f(a)(x - x_0)(x - x_0)$$

$$+ \Delta^3 f(a)(x - x_0)(x - x_1)(x - x_2)$$

$$f(2) = 48 + 52\,(2-4) + 15\,(2-4)(2-5) \quad + 1\,(2-4)(2-5)(2-7) = 4$$

$$f(8) = 48 + 52\,(8-4) + 15\,(8-4)(8-5) + 1\,(8-4)(8-5)(8-7) = 448$$

$$f(15) = 48 + 52\,(15-4) + 15\,(15-4)(15-5) + 1\,(15-4)(15-5)(15-7)$$

$$= 3150$$

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