FLUID MECHANICS

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LECTURE 5

Pipe flow examples

a. Single Pipe.

b. Multiple Pipe Systems.

Pipe Flow Examples

The purpose of this section is to apply these ideas to the solutions of various practical problems. The application of the pertinent equations is straightforward, with rather simple calculations that give answers to problems of engineering importance. The main idea involved is to apply the energy equation between appropriate locations within the flow system, with the head loss written in terms of the friction factor and the minor loss coefficients. We will consider two classes of pipe systems: those containing a single pipe (whose length may be interrupted by various components),

and those containing multiple pipes in parallel, series, or network

A- Single Pipes

The nature of the solution process for pipe flow problems can depend strongly on which of the various parameters are independent parameters (the “given”) and which is the dependent parameter (the “determine”). The three most common types of problems are shown in Table (1) in terms of the parameters involved.

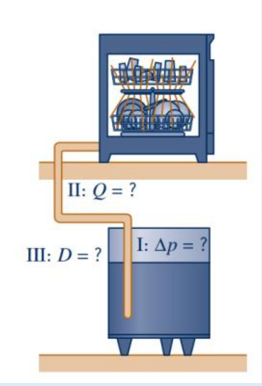
We assume the pipe system is defined in terms of the length of pipe sections used and the number of elbows, bends, and valves needed to convey the fluid between the desired locations. In all instances we assume the fluid properties are given.

Pipe flow problems can be categorized by what parameters are given and what is to be calculated.

In a **Type I** problem we *specify* the **desired flowrate or average velocity** and *determine* the necessary **pressure difference or head loss**. For example, if a flowrate of 2.0 gal/min is required for a dishwasher that is connected to the water heater by a given pipe system as shown by the figure in the margin, what pressure is needed in the water heater?

In a **Type II** problem we *specify* the applied **driving pressure** (or, alternatively, the head loss) and *determine* the **flowrate**. For example, how many gal/min of hot water are supplied to the dishwasher if the pressure within the water heater is 60 psi and the pipe system details (length, diameter, e pipe; number of elbows; etc.) are specified?

In a **Type III** problem, we *specify* the **pressure drop and the flowrate** and *determine* the **diameter of the pipe** needed. For example, what diameter of pipe is needed between the water heater and dishwasher if the pressure in the water heater is 60 psi (determined by the city water system) and the flowrate is to be not less than 2.0 gal/min (determined by the manufacturer)?



Table

Pipe Flow Types

Variable	Type I	Type II	Type III
a. Fluid			
Density	Given	Given	Given
Viscosity	Given	Given	Given
b. Pipe			
Diameter	Given	Given	Determine
Length	Given	Given	Given
Roughness	Given	Given	Given
c. Flow			
Flowrate or Average Velocity	Given	Determine	Given
d. Pressure			
Pressure Drop or Head Loss	Determine	Given	Given

Several examples of these types of problems follow,

EXAMPLE (1) Type I, Determine Pressure Drop

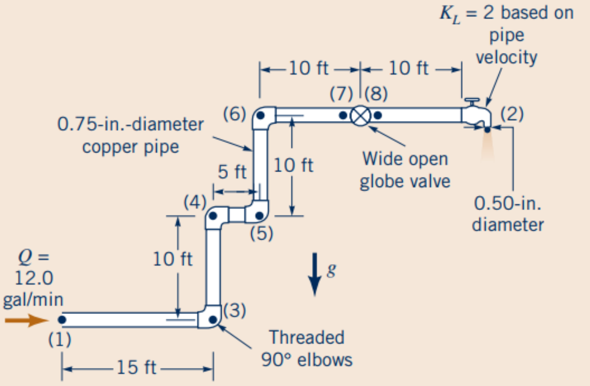
GIVEN Water at 60 °F flows from the basement to the second floor through the 0.75-in. (0.0625-ft)-diameter copper pipe (a drawn tubing) at a rate of $Q = 12.0$ gal/min $= 0.0267$ ft^3/s and exits through a faucet of diameter 0.50 in. as shown in Fig. E8.8*a*.

FIND Determine the pressure at point (1) if

(a) all losses are neglected,

(b) the only losses included are major losses, or

(c) all losses are included.



■ **Figure E8.8*a***

SOLUTION

Since the fluid velocity in the pipe is given by $V_1 = Q/A_1 = Q/(\pi D^2/4) = (0.0267 \text{ ft}^3/\text{s})/[\pi(0.0625 \text{ ft})^2/4] = 8.70$ ft/s, and the fluid properties are $\rho = 1.94$ slugs/ft^3 and $\mu = 2.34 \times 10^{-5}$ lb · s/ft^2 (see Table B.1), it follows that $\text{Re} = \rho V D/\mu = (1.94 \text{ slugs/ft}^3)(8.70 \text{ ft/s})(0.0625 \text{ ft})/(2.34 \times 10^{-5} \text{lb} \cdot \text{s/ft}^2) = 45{,}000$. Thus, the flow is turbulent. The governing equation for either case (a), (b), or (c) is the energy equation given by Eq. 8 ,

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

where $z_1 = 0$, $z_2 = 20$ ft, $p_2 = 0$ (free jet), $\gamma = \rho g = 62.4$ lb/ft^3, and the outlet velocity is $V_2 = Q/A_2 = (0.0267 \text{ ft}^3/\text{s})/[\pi(0.50/12)^2 \text{ft}^2/4] = 19.6$ ft/s. We assume that the kinetic energy coefficients α_1 and α_2 are unity. This is reasonable because turbulent velocity profiles are nearly uniform across the pipe. Thus,

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2^2 - V_1^2) + \gamma h_L \qquad (1)$$

where the head loss is different for each of the three cases.

(a) If all losses are neglected ($h_L = 0$), Eq. 1 gives

$$p_1 = (62.4 \text{ lb/ft}^3)(20 \text{ ft}) + \frac{1.94 \text{ slugs/ft}^3}{2}\left[\left(19.6 \frac{\text{ft}}{\text{s}}\right)^2 - \left(8.70 \frac{\text{ft}}{\text{s}}\right)^2\right]$$

$$= (1248 + 299) \text{ lb/ft}^2 = 1547 \text{ lb/ft}^2$$

or

$$p_1 = 10.7 \text{ psi} \qquad \textbf{(Ans)}$$

COMMENT Note that for this pressure drop, the amount due to elevation change (the hydrostatic effect) is $\gamma(z_2 - z_1) = 8.67$ psi and the amount due to the increase in kinetic energy is $\rho(V_2^2 - V_1^2)/2 = 2.07$ psi.

(b) If the only losses included are the major losses, the head loss is

$$h_L = f\frac{\ell}{D}\frac{V_1^2}{2g}$$

From Table 1 the roughness for a 0.75-in.-diameter copper pipe (drawn tubing) is $\varepsilon = 0.000005$ ft so that $\varepsilon/D = 8 \times 10^{-5}$. With this ε/D and the calculated Reynolds number (Re = 45,000), the value of f is obtained from the Moody chart as $f = 0.0215$. Note that the Colebrook equation (Eq. 8.35a) would give the same value of f. Hence, with the total length of the pipe as $\ell = (15 + 5 + 10 + 10 + 20)$ ft $= 60$ ft and the elevation and kinetic energy portions the same as for part (a), Eq. 1 gives

$$\begin{aligned} p_1 &= \gamma z_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho f\frac{\ell}{D}\frac{V_1^2}{2} \\ &= (1248 + 299)\ \text{lb/ft}^2 \\ &\quad + (1.94\ \text{slugs/ft}^3)(0.0215)\left(\frac{60\ \text{ft}}{0.0625\ \text{ft}}\right)\frac{(8.70\ \text{ft/s})^2}{2} \\ &= (1248 + 299 + 1515)\ \text{lb/ft}^2 = 3062\ \text{lb/ft}^2 \end{aligned}$$

or

$$p_1 = 21.3\ \text{psi} \qquad \textbf{(Ans)}$$

COMMENT Of this pressure drop, the amount due to pipe friction is approximately $(21.3 - 10.7)$ psi $= 10.6$ psi.

(c) If major and minor losses are included, Eq. 1 becomes

$$p_1 = \gamma z_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + f\gamma\frac{\ell}{D}\frac{V_1^2}{2g} + \sum \rho K_L\frac{V^2}{2}$$

or

$$p_1 = 21.3\ \text{psi} + \sum \rho K_L\frac{V^2}{2} \qquad \textbf{(2)}$$

where the 21.3 psi contribution is due to elevation change, kinetic energy change, and major losses [part (b)], and the last term represents the sum of all of the minor losses. The loss coefficients of the components ($K_L = 1.5$ for each elbow and $K_L = 10$ for the wide-open globe valve) are given in Table 4 (except for the loss coefficient of the faucet, which is given in Fig. E8.8a as $K_L = 2$). Thus,

$$\begin{aligned} \sum \rho K_L\frac{V^2}{2} &= (1.94\ \text{slugs/ft}^3)\frac{(8.70\ \text{ft})^2}{2}[10 + 4(1.5) + 2] \\ &= 1321\ \text{lb/ft}^2 \end{aligned}$$

or

$$\sum \rho K_L\frac{V^2}{2} = 9.17\ \text{psi} \qquad \textbf{(3)}$$

Note that we did not include an entrance or exit loss because points (1) and (2) are located within the fluid streams, not within an attaching reservoir where the kinetic energy is zero. Thus, by combining Eqs. 2 and 3 we obtain the entire pressure drop as

$$p_1 = (21.3 + 9.17)\ \text{psi} = 30.5\ \text{psi} \qquad \textbf{(Ans)}$$

This pressure drop calculated by including all losses should be the most realistic answer of the three cases considered.

COMMENTS More detailed calculations will show that the pressure distribution along the pipe is as illustrated in Fig. E8.8b for cases (a) and (c)—neglecting all losses or including all losses. Note that not all of the pressure drop, $p_1 - p_2$, is a "pressure loss." The pressure change due to the elevation and velocity changes is completely reversible. The portion due to the major and minor losses is irreversible.

This flow can be illustrated in terms of the energy line and hydraulic grade line concepts introduced in Section 3.7. As is shown in Fig. E8.8c, for case (a) there are no losses and the energy line (EL) is horizontal, one velocity head ($V^2/2g$) above the hydraulic grade line (HGL), which is one pressure head (γz) above the pipe itself. For cases (b) or (c) the energy line is not horizontal. Each bit of friction in the pipe or loss in a component reduces the available

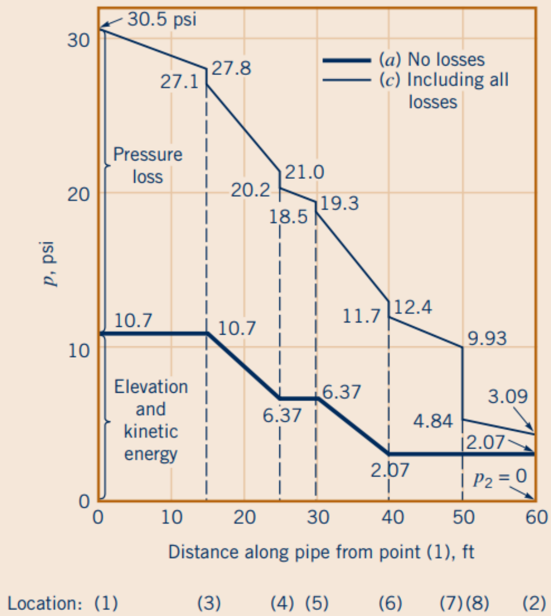


Figure E8.8b

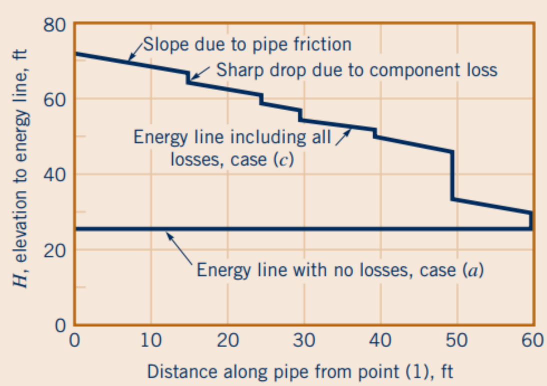


Figure E8.8c

energy, thereby lowering the energy line. Thus, for case (a) the total head remains constant throughout the flow with a value of

$$H = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{(1547\ \text{lb/ft}^2)}{(62.4\ \text{lb/ft}^3)} + \frac{(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0$$
$$= 26.0\ \text{ft.}$$
$$= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 = \cdots$$

For case (c) the energy line starts at

$$H_1 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$
$$= \frac{(30.5 \times 144)\text{lb/ft}^2}{(62.4\ \text{lb/ft}^3)} + \frac{(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 = 71.6\ \text{ft}$$

and falls to a final value of

$$H_2 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = 0 + \frac{(19.6\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 20\ \text{ft}$$
$$= 26.0\ \text{ft}$$

The elevation of the energy line can be calculated at any point along the pipe. For example, at point (7), 50 ft from point (1),

$$H_7 = \frac{p_7}{\gamma} + \frac{V_7^2}{2g} + z_7$$
$$= \frac{(9.93 \times 144)\ \text{lb/ft}^2}{(62.4\ \text{lb/ft}^3)} + \frac{(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 20\ \text{ft}$$
$$= 44.1\ \text{ft}$$

The head loss per foot of pipe is the same all along the pipe. That is,

$$\frac{h_L}{\ell} = f\frac{V^2}{2gD} = \frac{0.0215(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)(0.0625\ \text{ft})} = 0.404\ \text{ft/ft}$$

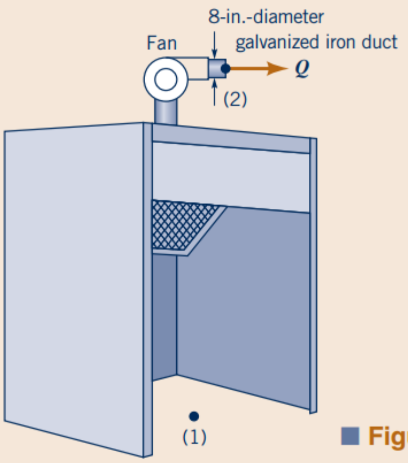
Thus, the energy line is a set of straight-line segments of the same slope separated by steps whose height equals the head loss of the minor component at that location. As is seen from Fig. E8.8*c*, the globe valve produces the largest of all the minor losses.

Pipe flow problems in which it is desired to determine the flowrate for a given set of conditions (Type II problems) often require trial-and-error or numerical root-finding techniques. This is because it is necessary to know the value of the friction factor to carry out the calculations, but the friction factor is a function of the unknown velocity (flowrate) in terms of the Reynolds number. The solution procedure is indicated in Example 2.

EXAMPLE (2) Type II, Determine Flowrate

GIVEN The fan shown in Fig. E8.10*a* is to provide airflow through the spray booth and fume hood so that workers are protected from harmful vapors and aerosols while mixing chemicals within the hood. For proper operation of the hood, the flowrate is to be between 6 ft^3/s and 12 ft^3/s. With the initial setup the flowrate is 9 ft^3/s, the loss coefficient for the system is 5, and the duct is short enough so that major losses are negligible. It is proposed that when the factory is remodeled the 8-in.-diameter galvanized iron duct will be 100 ft long and the total loss coefficient will be 10.

FIND Determine if the flowrate will be within the required 6 ft^3/s to 12 ft^3/s after the proposed remodeling. Assume that the head added to the air by the fan remains constant.



■ **Figure E8.10*a***

SOLUTION

We can determine the head that the fan adds to the air by considering the initial situation (i.e., before remodeling). To do this we write the energy equation between section (1) in the room and section (2) at the exit of the duct as shown in Fig. E8.10*a*.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \qquad (1)$$

Since we are dealing with air, we can assume any change in elevation is negligible. We can also assume the pressure inside the room and at the exit of the duct is equal to atmospheric pressure and the air in the room has zero velocity. Therefore, Eq. 1 reduces to

$$h_p = \frac{V_2^2}{2g} + h_L \qquad (2)$$

The diameter of the duct is given as 8 in., so the velocity at the exit can be calculated from the flowrate, where $V = Q/A = (9 \text{ ft}^3/\text{s})/[\pi(8/12)^2/4] = 25.8 \text{ ft/s}$. For the original configuration the duct is short enough to neglect major losses and only minor losses contribute to the total head loss. This head loss can be found from $h_{L,\text{minor}} = \Sigma K_L V^2/(2g) = 5(25.8 \text{ ft/s})^2/[2(32.2 \text{ ft/s}^2)] = 51.6 \text{ ft}$. With this information the simplified energy equation, Eq. 2, can now be solved to find the head added to the air by the fan.

$$h_p = \frac{(25.8 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 51.6 \text{ ft} = 61.9 \text{ ft}$$

The energy equation now must be solved with the new configuration after remodeling. Using the same assumptions as before gives the same reduced energy equation as shown in Eq. 2. With the increase in duct length to 100 ft the duct is no longer short enough to neglect major losses. Thus,

$$h_p = \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$

where $V_2 = V$ and $\sum K_L = 10$. We can now rearrange and solve for the velocity in ft/s.

$$V = \sqrt{\frac{2gh_p}{1 + f\frac{\ell}{D} + \sum K_L}} = \sqrt{\frac{2(32.2 \text{ ft/s}^2)(61.9 \text{ ft})}{1 + f\left(\frac{100 \text{ ft}}{8/12 \text{ ft}}\right) + 10}}$$

$$= \sqrt{\frac{3990}{11 + 150f}} \qquad (3)$$

The value of f is dependent on Re, which is dependent on V, an unknown.

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(2.38 \times 10^{-3} \text{ slugs/ft}^3)(V)(\frac{8}{12} \text{ ft})}{3.74 \times 10^{-7} \text{ lb}\cdot\text{s/ft}^2}$$

or

$$\text{Re} = 4240V \qquad (4)$$

where again V is in feet per second.

Also, since $\varepsilon/D = (0.0005 \text{ ft})/(8/12 \text{ ft}) = 0.00075$ (see Table 1 for the value of ε), we know which particular curve of the Moody chart is pertinent to this flow. Thus, we have three relationships (Eqs. 3, 4, and $\varepsilon/D = 0.00075$ curve of Fig. 3) from which we can solve for the three unknowns, f, Re, and V. This is done easily by an iterative scheme as follows.

It is usually simplest to assume a value of f, calculate V from Eq. 3, calculate Re from Eq. 4, and look up the new value of f in the Moody chart for this value of Re. If the assumed f and the new f do not agree, the assumed answer is not correct—we do not have the solution to the three equations. Although values of f, V, or Re could be assumed as starting values, it is usually simplest to assume a value of f because the correct value often lies on the relatively flat portion of the Moody chart for which f is quite insensitive to Re.

Thus, we assume $f = 0.019$, approximately the large Re limit for the given relative roughness. From Eq. 3 we obtain

$$V = \sqrt{\frac{3990}{11 + 150(0.019)}} = 17.0 \text{ ft/s}$$

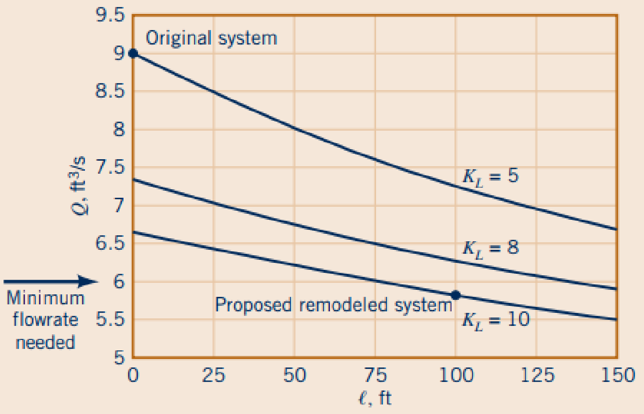
and from Eq. 4

$$\text{Re} = 4240(17.0) = 72{,}100$$

With this Re and ε/D, Fig. 3 gives $f = 0.022$, which is not equal to the assumed solution of $f = 0.019$ (although it is close!). We try again, this time with the newly obtained value of $f = 0.022$, which gives $V = 16.7$ ft/s and Re = 70,800. With these values, Fig. 3 gives $f = 0.022$, which agrees with the assumed value. Thus, the solution is $V = 16.7$ ft/s, or

$$Q = VA = (16.7 \text{ ft/s})\left(\frac{\pi}{4}\right)\left(\frac{8}{12} \text{ ft}\right)^2 = 5.83 \text{ ft}^3/\text{s} \qquad \textbf{(Ans)}$$

COMMENT It is seen that operation of the system after the proposed remodeling will not provide enough airflow to protect workers from inhalation hazards while mixing chemicals within the hood. By repeating the calculations for different duct lengths and different total minor loss coefficients, the results shown in Fig. E8.10*b* are obtained, which give flowrate as a function of duct length for various values of the minor loss coefficient. It will be necessary to redesign the remodeled system (e.g., larger fan, shorter ducting, larger-diameter duct) so that the flowrate will be within the acceptable range. In many companies, teams of occupational safety and health experts and engineers work together during the design phase of remodeling (or when a new operation is being planned) to consider and prevent potential negative impacts on workers' safety and health in an effort called "Prevention through Design." They also may be required to ensure that exhaust from such a system exits the building away from human activity and into an area where it will not be drawn back inside the facility.



■ **Figure E8.10*b***

In pipe flow problems for which the diameter is the unknown (Type III), an iterative or numerical root-finding technique is required. This is, again, because the friction factor is a function of the diameter—through both the Reynolds number and the relative roughness. Thus, neither $Re = \frac{\rho V D}{\mu} = 4\rho Q/\pi\mu D$ nor ε/D are known unless D is known. Examples 3 illustrate this.

EXAMPLE (3) Type III without Minor Losses, Determine Diameter

GIVEN Air at standard temperature and pressure flows through a horizontal, galvanized iron pipe ($\varepsilon = 0.0005$ ft) at a rate of 2.0 ft^3/s. The pressure drop is to be no more than 0.50 psi per 100 ft of pipe.

FIND Determine the minimum pipe diameter.

SOLUTION

We assume the flow to be incompressible with $\rho = 0.00238\ \text{slugs/ft}^3$ and $\mu = 3.74 \times 10^{-7}\ \text{lb} \cdot \text{s/ft}^2$. Note that if the pipe were too long, the pressure drop from one end to the other, $p_1 - p_2$, would not be small relative to the pressure at the beginning, and compressible flow considerations would be required. For example, a pipe length of 200 ft gives $(p_1 - p_2)/p_1 = [(0.50\ \text{psi})/(100\ \text{ft})](200\ \text{ft})/14.7\ \text{psia} = 0.068 = 6.8\%$, which is probably small enough to justify the incompressible assumption.

With $z_1 = z_2$ and $V_1 = V_2$ the energy equation (Eq. 8.) becomes

$$p_1 = p_2 + f \frac{\ell}{D} \frac{\rho V^2}{2} \tag{1}$$

where $V = Q/A = 4Q/(\pi D^2) = 4(2.0\ \text{ft}^3/\text{s})/\pi D^2$, or

$$V = \frac{2.55}{D^2}$$

where D is in feet. Thus, with $p_1 - p_2 = (0.5\ \text{lb/in.}^2)(144\ \text{in.}^2/\text{ft}^2)$ and $\ell = 100$ ft, Eq. 1 becomes

$$p_1 - p_2 = (0.5)(144)\ \text{lb/ft}^2$$
$$= f \frac{(100\ \text{ft})}{D} (0.00238\ \text{slugs/ft}^3) \frac{1}{2} \left(\frac{2.55}{D^2} \frac{\text{ft}}{\text{s}} \right)^2$$

or

$$D = 0.404 f^{1/5} \tag{2}$$

where D is in feet. Also $\text{Re} = \rho V D/\mu = (0.00238\ \text{slugs/ft}^3)[(2.55/D^2)\ \text{ft/s}]D/(3.74 \times 10^{-7}\ \text{lb} \cdot \text{s/ft}^2)$, or

$$\text{Re} = \frac{1.62 \times 10^4}{D} \tag{3}$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005}{D} \tag{4}$$

Thus, we have four equations [Eqs. 2, 3, 4, and either the Moody chart, the Colebrook equation (8.35*a*) or the Haaland equation (8.35*b*)] and four unknowns (f, D, ε/D, and Re) from which the solution can be obtained by trial-and-error methods.

If we use the Moody chart, it is probably easiest to assume a value of f, use Eqs. 2, 3, and 4 to calculate D, Re, and ε/D, and then compare the assumed f with that from the Moody chart. If they do not agree, try again. Thus, we assume $f = 0.02$, a typical value, and obtain $D = 0.404(0.02)^{1/5} = 0.185$ ft, which gives $\varepsilon/D = 0.0005/0.185 = 0.0027$ and $\text{Re} = 1.62 \times 10^4/0.185 = 8.76 \times 10^4$. From the Moody chart we obtain $f = 0.027$ for these values of ε/D and Re. Since this is not the same as our assumed value of f, we try again. With $f = 0.027$, we obtain $D = 0.196$ ft, $\varepsilon/D = 0.0026$, and $\text{Re} = 8.27 \times 10^4$, which in turn give $f = 0.027$, in agreement with the assumed value. Thus, the diameter of the pipe should be

$$D = 0.196\ \text{ft} \qquad \textbf{(Ans)}$$

COMMENT If we use the Colebrook equation (Eq. 8.35*a*) with $\varepsilon/D = 0.0005/0.404 f^{1/5} = 0.00124/f^{1/5}$ and $\text{Re} = 1.62 \times 10^4/0.404 f^{1/5} = 4.01 \times 10^4/f^{1/5}$, we obtain

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

or

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{3.35 \times 10^{-4}}{f^{1/5}} + \frac{6.26 \times 10^{-5}}{f^{3/10}} \right)$$

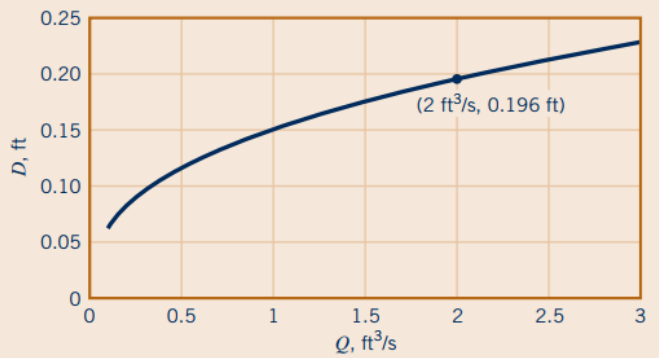
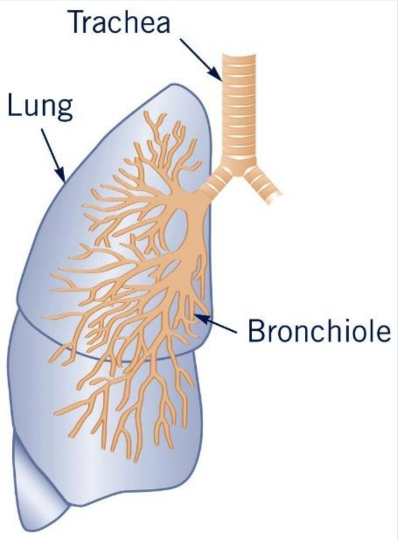


Figure E8.12

By using a root-finding technique on a computer or calculator, the solution to this equation is determined to be $f = 0.027$, and hence $D = 0.196$ ft, in agreement with the Moody chart method.

By repeating the calculations for various values of the flowrate, Q, the results shown in Fig. E8.12 are obtained. Although an increase in flowrate requires a larger diameter pipe (for the given pressure drop), the increase in diameter is minimal. For example, if the flowrate is doubled from 1 ft^3/s to 2 ft^3/s, the diameter increases from 0.151 ft to 0.196 ft.

B- Multiple Pipe Systems



In many pipe systems there is more than one pipe involved. The complex system of tubes in our lungs (beginning as shown by the figure here, with the relatively large-diameter trachea and ending in tens of thousands of minute bronchioles after numerous branchings)

and the maze of pipes in a city's water distribution system are typical of such systems. The governing mechanisms for the flow in ***multiple pipe systems*** are the same as for the single pipe systems discussed in this lecture. However, because of the numerous unknowns involved, additional complexities may arise in solving for the flow in multiple pipe systems. Some of these complexities are discussed in this section.

The simplest multiple pipe systems can be classified into ***series*** or ***parallel*** flows, as are shown in Fig. 12. below.

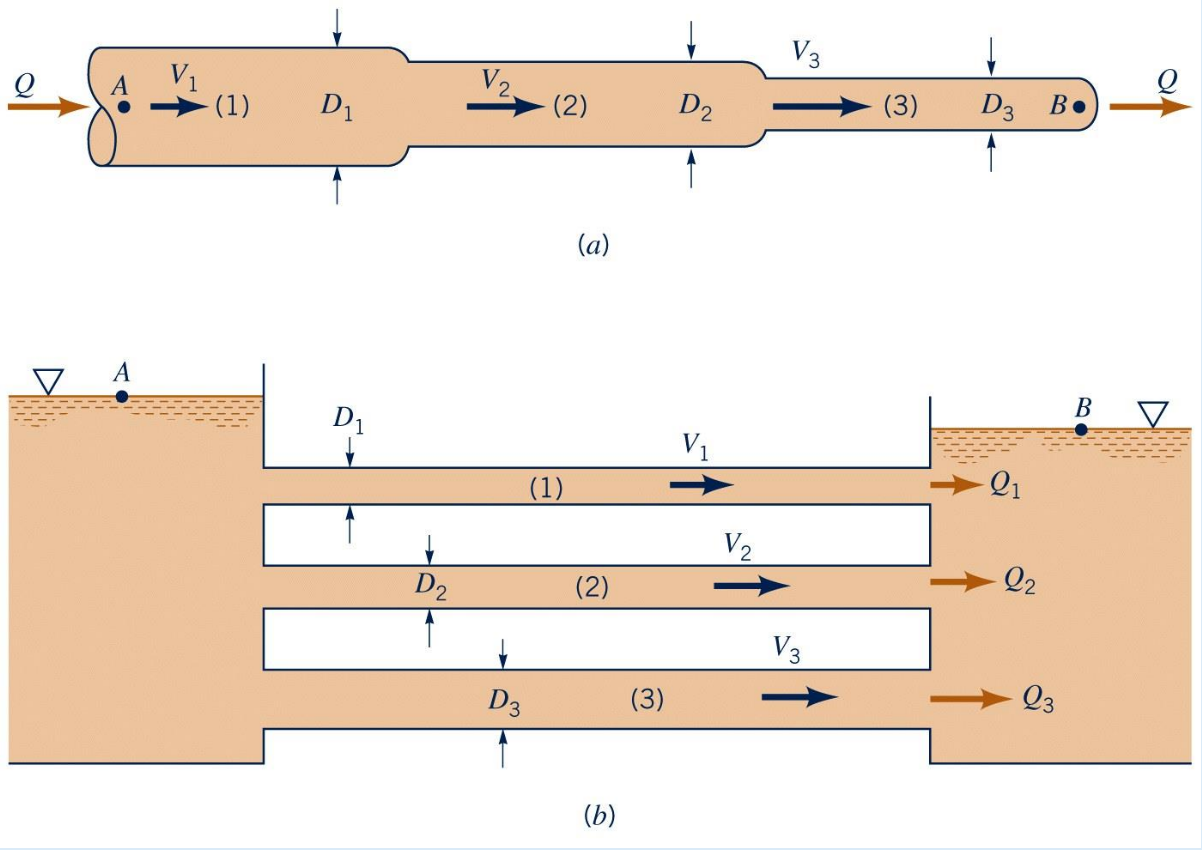


Figure 12 (*a*) Series and (*b*) parallel pipe systems.

In a fluid circuit there is a balance between the pressure drop (Δp) the flowrate or velocity (***Q*** or ***V***), and the flow resistance as given in terms of the friction factor and minor loss coefficients (***f*** and K_L). For a simple flow [$\Delta p = f(l/D)(\rho V^2/2)$], it follows that $\Delta p = Q^2 \widetilde{R}$, where $\widetilde{R}$, a measure of the resistance to the flow, is proportional to ***f***.

The fluid equations are generally nonlinear (doubling the pressure drop does not double the flowrate unless the flow is laminar).

One of the simplest multiple pipe systems is that containing pipes in *series*, as is shown in Fig. 12*a*. Every fluid particle that passes through the system passes through each of the pipes. Thus, the flowrate (but not the velocity) is the same in each pipe, and the head loss from point *A* to point *B* is the sum of the head losses in each of the pipes. The governing equations can be written as follows:

$$Q_1 = Q_2 = Q_3$$

And

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

where the subscripts refer to each of the pipes. In general, the friction factors will be different for each pipe because the Reynolds numbers ($R_{e_i} = \rho V_i D_i/\mu$) and the relative roughnesses (ε_i/D_i) will be different.

If the flowrate is given, it is a straightforward calculation to determine the head loss or pressure drop (Type I problem). If the pressure drop is given and the flowrate is to be calculated (Type II problem), an iteration scheme is needed. In this situation none of the friction factors $\boldsymbol{f_i}$, are known, so the calculations may involve more trial-and-error attempts than for corresponding single pipe systems. The same is true for problems in which the pipe diameter (or diameters) is to be determined (Type III problems).

Another common multiple pipe system contains pipes in ***parallel***, as is shown in Fig. 12 *b*. In this system a fluid particle traveling from *A* to *B* may take any of the paths available, with the total flowrate equal to the sum of the flowrates in each pipe. However, by writing the energy equation between points *A* and *B* it is found that the head loss experienced by any fluid particle traveling between these locations is the same, independent of the path taken. Thus, the governing equations for parallel pipes are,

$$Q = Q_1 + Q_2 + Q_3$$

And

$$h_{L_1} = h_{L_2} = h_{L_3}$$

Again, the method of solution of these equations depends on what information is given and what is to be calculated.

Another type of multiple pipe system called a ***loop*** is shown in Fig. 13. In this case the flowrate through pipe (1) equals the sum of the flowrates through pipes (2) and (3), or $Q_1 = Q_2 + Q_3$.

As can be seen by writing the energy equation between the surfaces of each reservoir, the head loss for pipe (2) must equal that for pipe (3), even though the pipe sizes and flowrates may be different for each. That is,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2}$$

for a fluid particle traveling through pipes (1) and (2), While, for fluid that travels through pipes (1) and (3).

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3}$$

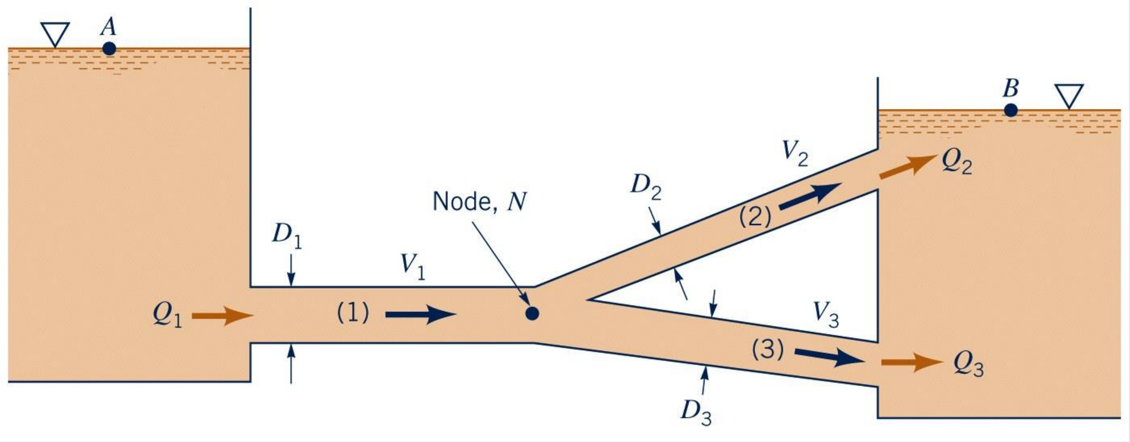


Figure 13 Multiple-pipe loop system

These can be combined to give $h_{L_2} = h_{L_3}$. This is a statement of the fact that fluid particles that travel through pipe (2) and particles that travel through pipe (3) all originate from common conditions at the junction (or node, *N*) of the pipes and all end up at the same final conditions.

The flow in a relatively simple looking multiple pipe system may be more complex than it appears initially. The branching system termed the ***three-reservoir problem*** shown in Fig. 14 is such a system.

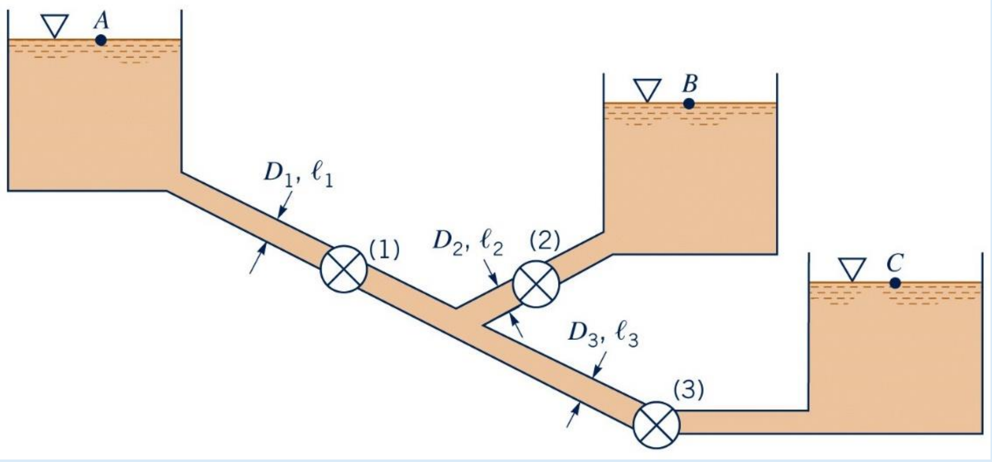


Figure 14 A three reservoir system.

Three reservoirs at known elevations are connected together with three pipes of known properties (length, diameter, and roughness). The problem is to determine the flowrates into or out of the reservoirs. If valve (1) were closed, the fluid would flow from reservoir B to C, and the flowrate could be easily calculated.
Similar calculations could be carried out if valves (2) or (3) were closed with the others open.

With all valves open, however, it is not necessarily obvious which direction the fluid flows. For the conditions indicated in Fig. 14, it is clear that fluid flows from reservoir ***A*** because the other two reservoir levels are lower. Whether the fluid flows into or out of reservoir ***B*** depends on the elevation of reservoirs ***B*** and ***C*** and the properties (length, diameter, roughness) of the three pipes. In general, the flow direction is not obvious, and the solution process must include the determination of this direction. This is illustrated in Example

EXAMPLE (4) Three-Reservoir, Multiple Pipe System

GIVEN Three reservoirs are connected by three pipes as are shown in Fig. E8.14. For simplicity we assume that the diameter of each pipe is 1 ft, the friction factor for each is 0.02, and because of the large length-to-diameter ratio, minor losses are negligible.

FIND Determine the flowrate into or out of each reservoir.

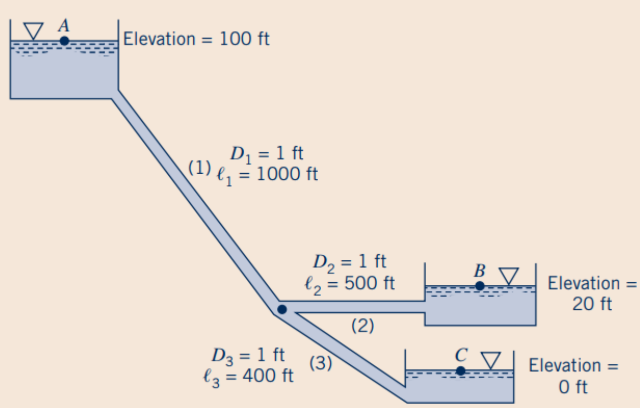


Figure E8.14

SOLUTION

It is not obvious which direction the fluid flows in pipe (2). However, we assume that it flows out of reservoir B, write the governing equations for this case, and check our assumption. The continuity equation requires that $Q_1 + Q_2 = Q_3$, which, since the diameters are the same for each pipe, becomes simply

$$V_1 + V_2 = V_3 \tag{1}$$

The energy equation for the fluid that flows from A to C in pipes (1) and (3) can be written as

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

By using the fact that $p_A = p_C = V_A = V_C = z_C = 0$, this becomes

$$z_A = f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

For the given conditions of this problem we obtain

$$100 \text{ ft} = \frac{0.02}{2(32.2 \text{ ft/s}^2)} \frac{1}{(1 \text{ ft})} [(1000 \text{ ft})V_1^2 + (400 \text{ ft})V_3^2]$$

or

$$322 = V_1^2 + 0.4V_3^2 \quad (2)$$

where V_1 and V_3 are in ft/s. Similarly the energy equation for fluid flowing from B and C is

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

or

$$z_B = f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

For the given conditions this can be written as

$$64.4 = 0.5V_2^2 + 0.4V_3^2 \quad (3)$$

Equations 1, 2, and 3 (in terms of the three unknowns V_1, V_2, and V_3) are the governing equations for this flow, provided the fluid flows from reservoir B. It turns out, however, that there is no solution for these equations with positive, real values of the velocities. Although these equations do not appear to be complicated, there is no simple way to solve them directly. Thus, a trial-and-error solution is suggested. This can be accomplished as follows. Assume a value of $V_1 > 0$, calculate V_3 from Eq. 2, and then V_2 from Eq. 3. It is found that the resulting V_1, V_2, V_3 trio does not satisfy Eq. 1 for any value of V_1 assumed. There is no solution to Eqs. 1, 2, and 3 with real, positive values of V_1, V_2, and V_3. Thus, our original assumption of flow out of reservoir B must be incorrect.

To obtain the solution, assume the fluid flows into reservoirs B and C and out of A. For this case the continuity equation becomes

$$Q_1 = Q_2 + Q_3$$

or

$$V_1 = V_2 + V_3 \quad (4)$$

Application of the energy equation between points A and B and A and C gives

$$z_A = z_B + f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}$$

and

$$z_A = z_C + f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

which, with the given data, become

$$258 = V_1^2 + 0.5\,V_2^2 \quad (5)$$

and

$$322 = V_1^2 + 0.4\,V_3^2 \quad (6)$$

Equations 4, 5, and 6 can be solved as follows. By subtracting Eq. 5 from 6 we obtain

$$V_3 = \sqrt{160 + 1.25V_2^2}$$

Thus, Eq. 5 can be written as

$$258 = (V_2 + V_3)^2 + 0.5V_2^2$$
$$= (V_2 + \sqrt{160 + 1.25V_2^2})^2 + 0.5V_2^2$$

or

$$2V_2\sqrt{160 + 1.25V_2^2} = 98 - 2.75V_2^2 \quad (7)$$

which, upon squaring both sides, can be written as

$$V_2^4 - 460\,V_2^2 + 3748 = 0$$

By using the quadratic formula, we can solve for V_2^2 to obtain either $V_2^2 = 452$ or $V_2^2 = 8.30$. Thus, either $V_2 = 21.3$ ft/s or $V_2 = 2.88$ ft/s. The value $V_2 = 21.3$ ft/s is not a root of the original equations. It is an extra root introduced by squaring Eq. 7, which with $V_2 = 21.3$ becomes "1140 = −1140." Thus, $V_2 = 2.88$ ft/s and from Eq. 5, $V_1 = 15.9$ ft/s. The corresponding flowrates are

$$Q_1 = A_1V_1 = \frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}(1\text{ ft})^2\,(15.9\text{ ft/s})$$
$$= 12.5\text{ ft}^3\text{/s from } A \quad \textbf{(Ans)}$$

$$Q_2 = A_2V_2 = \frac{\pi}{4}D_2^2V_2 = \frac{\pi}{4}(1\text{ ft})^2\,(2.88\text{ ft/s})$$
$$= 2.26\text{ ft}^3\text{/s into } B \quad \textbf{(Ans)}$$

and

$$Q_3 = Q_1 - Q_2 = (12.5 - 2.26)\text{ ft}^3\text{/s}$$
$$= 10.2\text{ ft}^3\text{/s into } C \quad \textbf{(Ans)}$$

Note the slight differences in the governing equations depending on the direction of the flow in pipe (2)—compare Eqs. 1, 2, and 3 with Eqs. 4, 5, and 6.

COMMENT If the friction factors were not given, a trial-and-error procedure similar to that needed for Type II problems (see Section 8.5.1) would be required.

The ultimate in multiple pipe systems is a ***network*** of pipes such as that shown in Fig. 15. Networks like these often occur in city water distribution systems and other systems that may have multiple "**inlets**" and "**outlets**." The direction of flow in the various pipes is by no means obvious—in fact, it may vary in time, depending on how the system is used from time to time.

The solution for pipe network problems is often carried out by use of node and loop equations similar in many ways to that done in electrical circuits. For example, the continuity equation requires that for each *node* (the junction of two or more pipes) the net flowrate is zero. What flows into a node must flow out at the same rate. In addition, the net pressure difference completely around a ***loop*** (starting at one location in a pipe and returning to that location) must be zero. By combining these ideas with the usual head loss and pipe flow equations, the flow throughout the entire network can be obtained. Of course, trial-and-error solutions are usually required because the direction of flow and the friction factors may not be known. Such a solution procedure using matrix techniques is ideally suited for computer use.

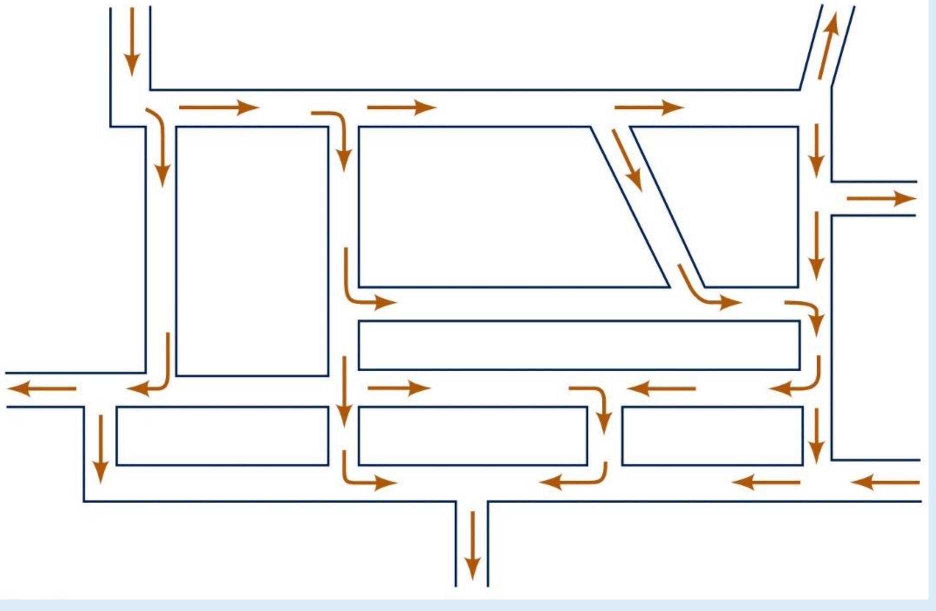
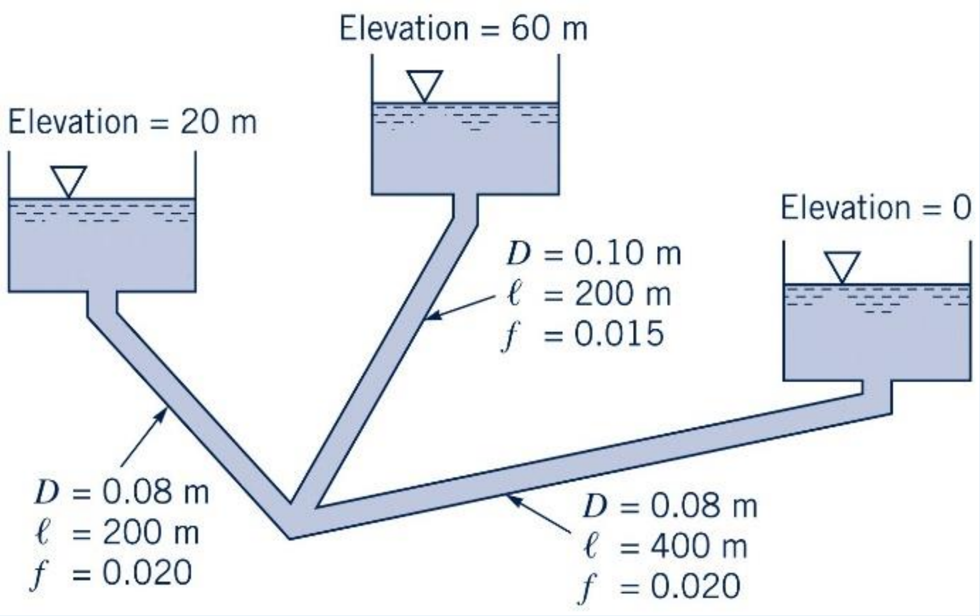


Figure 15 A general pipe network.

Example 5 : The three water-filled tanks shown in figure are connected by pipes as indicated. If minor losses are neglected, determine the flow rate in each pipe.



Solution:

Assume the fluid flows from A to B and A to C

Thus $Q_1 = Q_2 + Q_3$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$\frac{\pi}{4}(0.1)^2 V_1 = \frac{\pi}{4}(0.08)^2 V_2 + \frac{\pi}{4}(0.08)^2 V_3$$

$$V_1 = 0.64 V_2 + 0.64 V_3 \quad \text{--- (1)}$$

For the fluid flowing from A to B, $P_A = P_B = 0$ and $V_A = V_B = 0$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g}$$

$$Z_A = Z_B + f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g}$$

$$60 - 20 = (0.015)\left(\frac{200}{0.1}\right)\left(\frac{V_1^2}{19.62}\right) + (0.02)\left(\frac{200}{0.08}\right)\left(\frac{V_2^2}{19.62}\right)$$

$$40 = 1.529 V_1^2 + 2.55 V_2^2 \quad \text{--- (2)}$$

Similary for fluid flowing from A to C with

$P_A = P_C = 0$ and $V_A = V_C = 0$

$$Z_A = Z_C + f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{l_3}{D_3}\frac{V_3^2}{2g}$$

$$60 = (0.015)\left(\frac{200}{0.1}\right)\left(\frac{V_1^2}{19.62}\right) + (0.02)\left(\frac{400}{0.08}\right)\left(\frac{V_3^2}{19.62}\right)$$

Hence,

$$60 = 1.529 V_1^2 + 5.1 V_3^2 \quad \text{--- (3)}$$

Solve Eqs(1), (2), and(3) for V_1, V_2 and V_3

From eqs (1) and (3)

$$60 = 1.529\,(0.64)^2\,(V_2 + V_3)^2 + 5.1\,V_3^2$$

$$95.8 = (V_2 + V_3)^2 + 8.14\,V_3^2 \quad \text{—— (4)}$$

subtract Eq (2) from Eq. (3)

$$60 - 40 = 5.1\,V_3^2 - 2.55\,V_2^2 \qquad \div\ 2.55$$

$$7.84 = 2V_3^2 - V_2^2 \rightarrow V_2^2 = 2V_3^2 - 7.84$$

$$\therefore V_2 = \sqrt{2V_3^2 - 7.84} \quad \text{—— (5)}$$

From eqs (4) and (5)

$$8.14\,V_3^2 + \left(\sqrt{2V_3^2 - 7.84} + V_3\right)^2 - 95.8 = 0$$

$$8.14\,V_3^2 + 2V_3^2 - 7.84 + 2V_3\sqrt{2V_3^2 - 7.84} + V_3^2 - 95.8 = 0$$

$$11.14\,V_3^2 + 2V_3\sqrt{2V_3^2 - 7.84} - 103.64 = 0$$

$$2V_3\sqrt{2V_3^2 - 7.84} = 103.64 - 11.14\,V_3^2 \quad \text{—— (6)}$$

square both sides and rearrange the equation

$$4V_3^2\,(2V_3^2 - 7.84) = 10743.3 - 2309.1\,V_3^2 + 124.1\,V_3^4 \quad (103.64 - 11.14V_3^2)^2$$

$$8V_3^4 - 31.36\,V_3^2 = 124.1\,V_3^4 - 2309.1\,V_3^2 + 10743.3$$

$$116.1\,V_3^4 - 2277.74\,V_3^2 + 10743.3 = 0 \quad \text{(divided by 116.1)}$$

$$V_3^4 - 19.63\,V_3^2 + 92.5 = 0$$

By using quadratic formula

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$V_3^2 = \frac{19.62 \mp \sqrt{(19.62)^2 + (4 * 92.5)}}{2} \Rightarrow V_3^2 = 11.77 \text{ or } 7.86$$

Thus $V_3 = 3.43\ m/s$ or $V_3 = 2.8\ m/s$

Note: The value $V_3 = 3.43\ m/s$ is not a solution of the original equations, Eq. (1), (2) and (3). with this value the right hand side of eq. (6) is negative (i.e $103.64 - 11.14 V_3^2 = 103.6 - 11.14 (3.43)^2 = -24.5$

Thus $V_3 = 2.8\ m/s$ ←

From equation (3) → $60 = 1.529 V_1^2 + 5.1 (2.8)^2$

$\therefore V_1 = 3.62\ m/s$ ←

And from equation (1) → $V_1 = 0.64 V_2 + 0.64 V_3$

$3.62 = 0.64 V_2 + 0.64 * (2.8) \Rightarrow$

$V_2 = 2.86\ m/s$ ←

$\therefore Q_1 = A_1 V_1 = \frac{\pi}{4} (0.1)^2 * 3.62 = 0.028\ \frac{m^3}{s}$

$Q_2 = A_2 V_2 = \frac{\pi}{4} (0.08)^2 * 2.86 = 0.0143\ \frac{m^3}{s}$

$Q_3 = A_3 V_3 = \frac{\pi}{4} (0.08)^2 * 2.8 = 0.0141\ \frac{m^3}{s}$