



— University of Mosul —
College of Petroleum & Mining Engineering



Partial Derivative

Lecture No.1

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LECTURE CONTENTS:

- Differentiation Rules.
- Partial Derivative.
- Examples.

Differentiation Rules

- Rule 1:

$$F(x)$$

$$F'(x)$$

$$x^n$$

$$n x^{n-1}$$

Example:

$$x^3$$

$$3x^2$$

- Rule 2:

C (constant)

$$0$$

مشتقات الثابت

- Rule 3:

$$C \cdot F(x)$$

$$C \cdot F'(x)$$

اقتران x ثابت

Example:

$$4 \cdot x^3$$

$$4 \cdot 3x^2 = 12x^2$$

- Rule 4:

الاشتقاق الأسّي

$$a^x$$

$$a^x \cdot \ln a \leftarrow \text{اشتقاق الأس}$$

Example:

$$3^x$$

$$3^x \cdot \ln 3$$

(نفذ $\times$ (ln) الأس $\times$ مشتق الأس)

- Rule 5:

$$f \pm g$$

$$f' \pm g'$$

مع وضع افتراضات

Example:

$$x^3 + 3x^2 + 5$$

$$3x^2 + 6x + 0$$

ملاحظات: الاشتقاق يوزع على الجمع والطرح، ولا يوزع على الضرب والقسمة.

- Rule. 6 :

مشتقات الجذر التربيعي $\sqrt{F(x)}$

$$\frac{F'(x)}{2 \cdot \sqrt{F(x)}} \quad \left(\begin{array}{l} \text{مشتقة داخل الجذر} \\ \text{الجذر نفسه} \times 2 \end{array} \right)$$

Example :

$$\sqrt{x^3 + 2x}$$

$$\frac{3x^2 + 2}{2 \cdot \sqrt{x^3 + 2x}}$$

- Rule. 7 :

مشتقة الـ $\ln$ $\ln F(x)$

$$\frac{F'(x)}{F(x)} \quad \left(\begin{array}{l} \text{مشتقة داخل الـ } (\ln) \\ \text{داخل الـ } (\ln) \text{ نفسه} \end{array} \right)$$

Example :

$$\ln(x^2 + 1)$$

$$\frac{2x}{x^2 + 1}$$

$$\ln x$$

$$\frac{1}{x}$$

Rule . 8 : الدقترانات المتلصقة
 $F(x)$

مشتقة الزاوية $\times F'(x)$

$$\sin x$$

$$\cos x \cdot 1$$

$$\cos x$$

$$-\sin x \cdot 1$$

$$\tan x$$

$$\sec^2 x \cdot 1$$

$$\cot x$$

$$-\csc^2 x \cdot 1$$

$$\sec x$$

$$\sec x \cdot \tan x \cdot 1$$

$$\csc x$$

$$-\csc x \cdot \cot x \cdot 1$$

ملاحظة : عند اشتقاق اية اقتران مثلثي يجب ان نضرب الجواب بمشتقة
الزاوية .

Example : $\sin x^2$

$$\cos x^2 \cdot 2x$$

Rule 9: الانفرس $\sin^{-1} x$

$$\frac{1}{\sqrt{1-x^2}} * 1$$

$$\cos^{-1} x$$

$$\frac{-1}{\sqrt{1-x^2}} * 1$$

$$\tan^{-1} x$$

$$\frac{1}{1+x^2} * 1$$

مشتقة الزاوية

Rule 10: مشتقة الضرب $f \cdot g$

$$\begin{array}{c} \text{الأول} \rightarrow f \cdot g' + g \cdot f' \leftarrow \text{مشتقة الأول} \\ \quad \quad \quad \uparrow \quad \quad \quad \uparrow \\ \quad \quad \quad \text{مشتقة الثاني} \quad \text{الثاني} \end{array}$$

Example:

$$\sqrt{x} \cdot \ln x$$

$$\sqrt{x} \cdot \frac{1}{x} + \ln x \cdot \frac{1}{2\sqrt{x}}$$

Rule-11 : $\frac{f}{g}$: $\frac{f}{g}$

$$\frac{f}{g}$$

$$\frac{f \cdot g' - f' \cdot g}{g^2}$$

$$g^2$$

$$(g')^2$$

الاشتقاق الجزئي (Partial Derivative) (14.3)

$$y = f(x) \rightarrow y' = \frac{dy}{dx} = f'(x)$$

$$m = 3u^2 + 4u \rightarrow \frac{dm}{du} = 6u + 4$$

} متغير واحد

$$z = f(x, y) \rightarrow \frac{\partial z}{\partial x} \leftarrow \text{المُتَغَيِّرُ (x) , (y) is constant}$$

$$\frac{\partial z}{\partial y} \leftarrow \text{المُتَغَيِّرُ (y) , (x) is constant}$$

$$\frac{\partial z}{\partial x} = f_x \quad , \quad \frac{\partial z}{\partial y} = f_y$$

Ex: $f(x,y) = 2^{x \cdot y} + x^2 \cdot y + x^y + x^2 \cdot \sin y + \tan^{-1} x + y^3 + x \cdot \cos^{-1} xy$
 find f_x - f_y .

منقطة الزاوية

solution:

$$f_x = \frac{\partial f}{\partial x} = 2^{x \cdot y} \cdot \ln 2 \cdot y + (2x) \cdot y + y \cdot x^{y-1} + (2x) \cdot \sin y + \frac{1}{1+x^2} * 1$$

$$+ 0 + x \cdot \frac{-1}{\sqrt{1-(xy)^2}} * y + \cos^{-1} xy \cdot (1)$$

$$f_y = \frac{\partial f}{\partial y} = 2^{x \cdot y} \cdot \ln 2 \cdot x + x^2 \cdot (1) + x^y \cdot \ln x \cdot (1) + x^2 \cdot \cos y + 0 + 3y^2$$

$$+ x \cdot \frac{-1}{\sqrt{1-(xy)^2}} \cdot (x)$$

Ex: find $\frac{\partial \omega}{\partial x}$ and $\frac{\partial \omega}{\partial y}$ if $\omega = x^2y - e^{xy} + \sin 5x^3y$

solution:

$$\frac{\partial \omega}{\partial x} = 2xy - e^{xy} \cdot \ln e \cdot y + \cos 5x^3y \cdot (15x^2y)$$

• (1) قاعدة $\ln(e)$: أبسط

$$\frac{\partial \omega}{\partial x} = 2xy - e^{xy} \cdot 1 \cdot y + \cos 5x^3y \cdot (15x^2y)$$

$$\frac{\partial \omega}{\partial y} = x^2 - e^{xy} \cdot \ln e \cdot x + \cos 5x^3y \cdot (5x^3)$$

Ex: if $w = \ln \sqrt{x^2 + y^2}$ find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

solution:

$$\frac{\partial w}{\partial x} = \frac{1}{\sqrt{x^2 + y^2}} * \frac{2x}{2\sqrt{x^2 + y^2}} = \frac{x}{x^2 + y^2}$$

$$\frac{\partial w}{\partial y} = \frac{1}{\sqrt{x^2 + y^2}} * \frac{2y}{2\sqrt{x^2 + y^2}} = \frac{y}{x^2 + y^2}$$

Ex: find f_x and f_y if $f(x, y) = \frac{2y}{y + \cos x}$

$$f_x = \frac{\partial f}{\partial x} = \frac{(y + \cos x) * 0 - 2y * (-\sin x)}{(y + \cos x)^2} = \frac{2y \cdot \sin x}{(y + \cos x)^2}$$

$$\begin{aligned} f_y = \frac{\partial f}{\partial y} &= \frac{(y + \cos x) * 2 - 2y * (1)}{(y + \cos x)^2} = \frac{2y + 2\cos x - 2y}{(y + \cos x)^2} \\ &= \frac{2\cos x}{(y + \cos x)^2} \end{aligned}$$

Ex: $f(x, y, z, w) = x^2 \cdot e^{2y+3z} \cdot \cos 4w$ - find $\frac{\partial f}{\partial x}$,

$$\frac{\partial f}{\partial y} , \frac{\partial f}{\partial z} , \frac{\partial f}{\partial w} .$$

Solution:

$$\frac{\partial f}{\partial x} = 2x \cdot e^{2y+3z} \cdot \cos 4w .$$

$$\frac{\partial f}{\partial y} = x^2 \cdot e^{2y+3z} \cdot \underbrace{\ln e}_{(1)} \cdot (2) \cdot \cos 4w = x^2 \cdot 2e^{2y+3z} \cdot \cos 4w .$$

$$\frac{\partial f}{\partial z} = x^2 \cdot e^{2y+3z} \cdot \underbrace{\ln e}_{(1)} \cdot (3) \cdot \cos 4w = 3x^2 \cdot e^{2y+3z} \cdot \cos 4w .$$

$$\frac{\partial f}{\partial w} = x^2 \cdot e^{2y+3z} \cdot (-\sin 4w) \cdot (4) = -4x^2 e^{2y+3z} \cdot \sin 4w .$$

Ex: find f_x and f_y for the function:

$$f(x,y) = \sin^2(x-3y)$$

solution: $f(x,y) = \sin(x-3y) \cdot \sin(x-3y)$

$$\begin{aligned} f_x &= \sin(x-3y) * \cos(x-3y) * 1 + \sin(x-3y) * \cos(x-3y) * 1 \\ &= 2 \sin(x-3y) \cdot \cos(x-3y) \end{aligned}$$

$$\begin{aligned} f_y &= \sin(x-3y) * \cos(x-3y) * -3 + \sin(x-3y) * \cos(x-3y) * -3 \\ &= -6 \sin(x-3y) \cdot \cos(x-3y) \end{aligned}$$