Chain Rule

Lecture No.3

Asst.Lect. Zaid Salahaldeen Thanoon

Petroleum and Refining Engineering Department

Email: zeadsalahaldeen@uomosul.edu.iq

LECTURE CONTENTS:

- The Chain Rule.
- Examples.

- if $\omega = f(x, y)$ is differentiable and $x = x(t), y = y(t)$ are differentiable function of (t), then:

$$\boxed{\frac{\partial \omega}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}}$$

- if $\omega = f(x, y, z)$ is differentiable and (x, y, z) are differentiable function of (t) then:

$$\boxed{\frac{\partial \omega}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial t}}$$

- if $\omega = f(x, y, z)$, $x = g(r, s)$, $y = h(r, s)$ and $z = k(r, s)$
if all four functions are differentiable, then:

$$\boxed{\begin{aligned} \frac{\partial \omega}{\partial r} &= \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial r} \\ \frac{\partial \omega}{\partial s} &= \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial s} \end{aligned}}$$

Ex : $y = \sin x$, $x = U^2$ find $\frac{\partial y}{\partial U}$.

"مابّا

solution :

$$\frac{\partial y}{\partial U} = \frac{\partial y}{\partial x} \cdot \frac{\partial x}{\partial U} , \frac{\partial y}{\partial x} = \cos x * 1 , \frac{\partial x}{\partial U} = 2U$$

$$\Rightarrow \frac{\partial y}{\partial U} = 2U * \cos x .$$

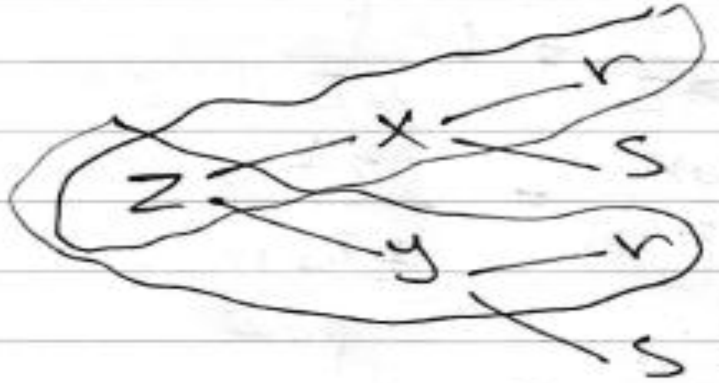
$z = f(x, y)$ find $\frac{\partial z}{\partial U}$.

$$\frac{\partial z}{\partial U} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial U} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial U}$$

Ex: $z = x^2 \cdot e^y + y^3 \cdot \sin x$, $x = r^2 \cdot s$, $y = r^3 \cdot s^2$

find $\frac{\partial z}{\partial r}$.

solution:



$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial z}{\partial x} = e^y * 2x + y^3 \cdot \cos x * 1$$

$$\frac{\partial x}{\partial r} = 2rs$$

$$\frac{\partial z}{\partial y} = x^2 \cdot e^y \cdot \ln e * 1 + 3y^2 \cdot \sin x$$

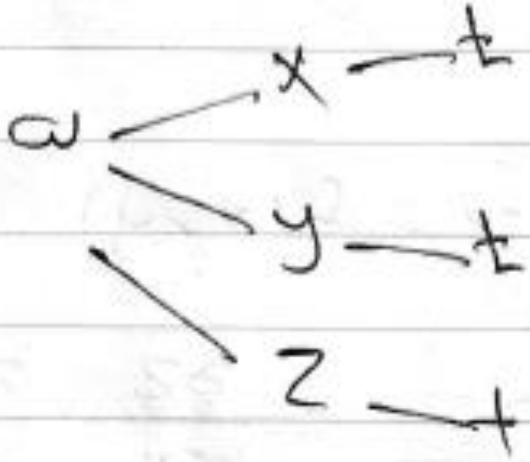
$$\frac{\partial y}{\partial r} = 3r^2 \cdot s^2$$

$$\Rightarrow \frac{\partial z}{\partial r} = (2e^y x + y^3 \cdot \cos x) * 2sr + (x^2 \cdot e^y + 3y^2 \sin x) * 3r^2 s^2$$

Ex: By using chain rule express $\frac{\partial \omega}{\partial t}$ as a function of (t) for the following below, then find $\frac{\partial \omega}{\partial t}$ at the given value of (t) !

1- $\omega = 2y \cdot e^{x} - \ln z, \quad x = \ln(t^2+1), \; y = \tan^{-1} t, \; z = e^{t}, \; t = 1$

solution!



$$\frac{\partial \omega}{\partial t} = \frac{\partial \omega}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial \omega}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial \omega}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$$\frac{\partial \omega}{\partial x} = 2y \cdot e^{x} \cdot \ln e \times 1 - 0 = 2ye^{x}$$

$$\frac{\partial w}{\partial y} = 2e^x$$

$$\frac{\partial w}{\partial z} = -\frac{1}{z}$$

$$\frac{\partial x}{\partial t} = \frac{2t}{t^2+1}$$

$$\frac{\partial y}{\partial t} = \frac{1}{t^2+1} \quad , \quad \frac{\partial z}{\partial t} = e^t \cdot \ln e + 1 = e^t$$

$$\Rightarrow \frac{\partial w}{\partial t} = 2ye^x * \frac{2t}{t^2+1} + 2e^x * \frac{1}{t^2+1} + \left(\frac{-1}{z}\right) * e^t$$

$$\frac{\partial w}{\partial t} = 4ye^x\left(\frac{t}{t^2+1}\right) + \frac{2e^x}{t^2+1} - \frac{e^t}{z}$$

to find $\frac{\partial \omega}{\partial t}$ at $t = 1$

$$\frac{\partial \omega}{\partial t} = 4 * (\tan^{-1} t) * \cancel{e^{(\tan^{-1}(t^2+1))}} * \frac{t}{\cancel{t^2+1}} + \frac{2\cancel{e^{(\tan^{-1}(t^2+1))}}}{\cancel{t^2+1}} - \frac{\cancel{e^{t}}}{\cancel{e^{t}}}$$

$$\frac{\partial o}{\partial t} = 4t \cdot \tan^{-1} t + 1 \qquad \text{at } t = 1$$

$$\frac{\partial \omega}{\partial t} = 4 * 1 \cdot \tan^{-1}(1) + 1$$