University of Mosul
College of Petroleum & Mining Engineering

Vectors

Lecture No.1

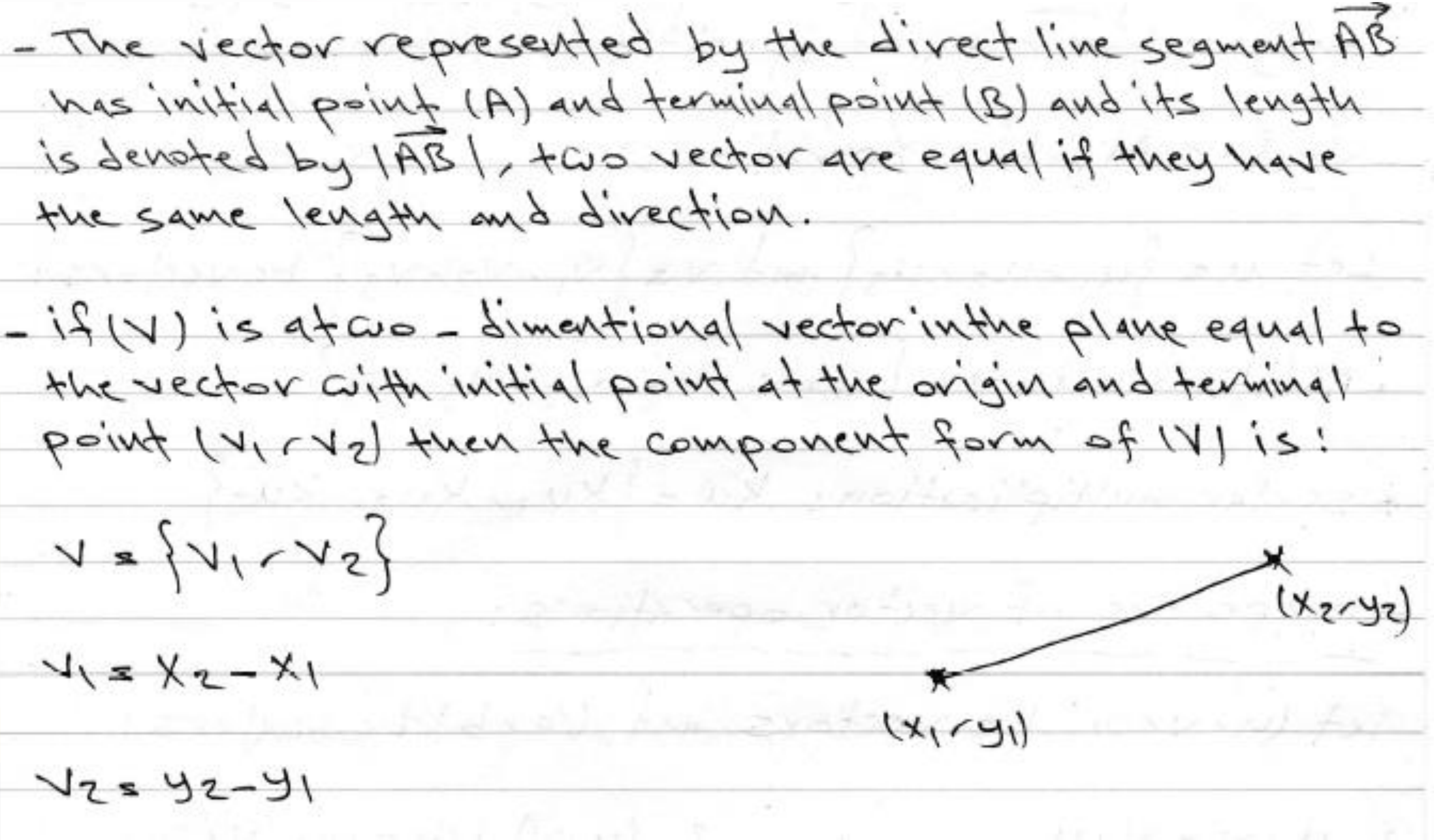
Asst.Lect. Zaid Salahaldeen Thanoon

Petroleum and Refining Engineering Department

Email: zeadsalahaldeen@uomosul.edu.iq

LECTURE CONTENTS:

- Introduction to Vectors.
- Properties of Vector Operations.
- Unit Vectors.
- Examples.



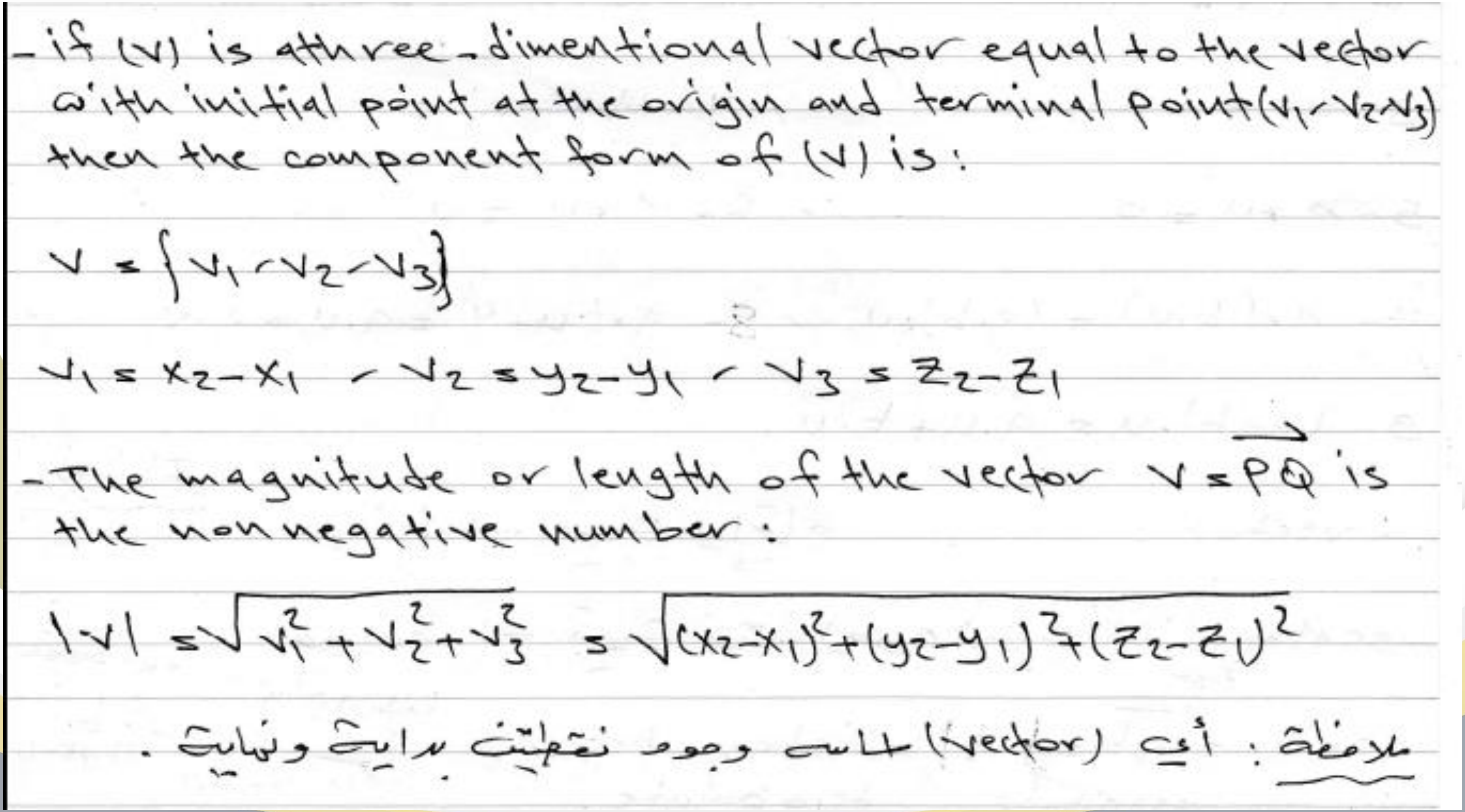
- The vector represented by the direct line segment $\overrightarrow{AB}$ has initial point (A) and terminal point (B) and its length is denoted by $|\overrightarrow{AB}|$, two vector are equal if they have the same length and direction.

- if (V) is a two - dimentional vector in the plane equal to the vector with initial point at the origin and terminal point (v_1, v_2) then the component form of (V) is:

$V = \{v_1, v_2\}$

$v_1 = x_2 - x_1$

$v_2 = y_2 - y_1$



- if (v) is athree-dimentional vector equal to the vector with initial point at the origin and terminal point (v_1, v_2, v_3) then the component form of (v) is:

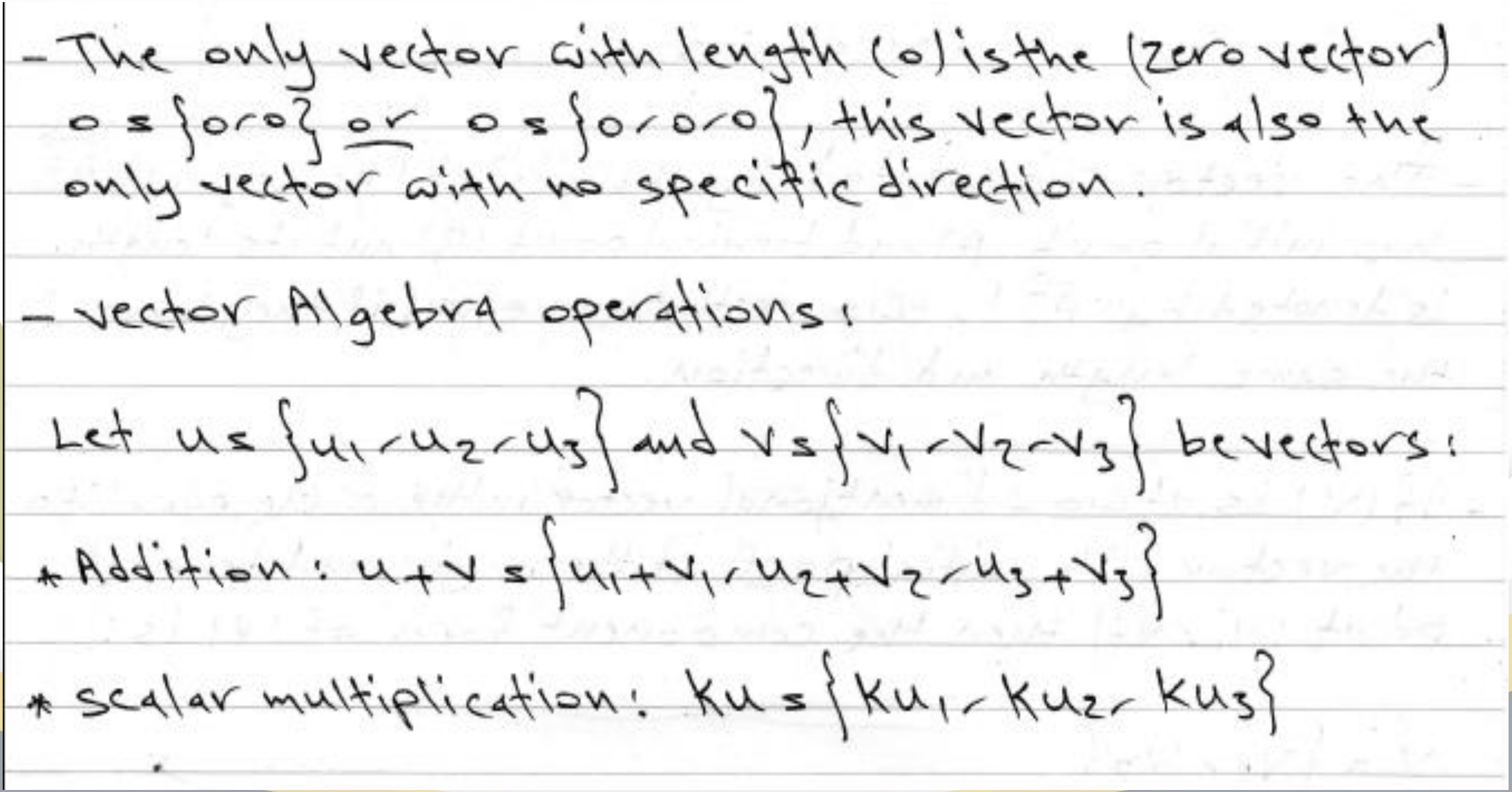
$$v = \{v_1, v_2, v_3\}$$

$$v_1 = x_2 - x_1 \quad , \quad v_2 = y_2 - y_1 \quad , \quad v_3 = z_2 - z_1$$

- The magnitude or length of the vector $v = \overrightarrow{PQ}$ is the nonnegative number:

$$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

ملاحظة: أي (vector) لابد وجود نقطتين بداية ونهاية.



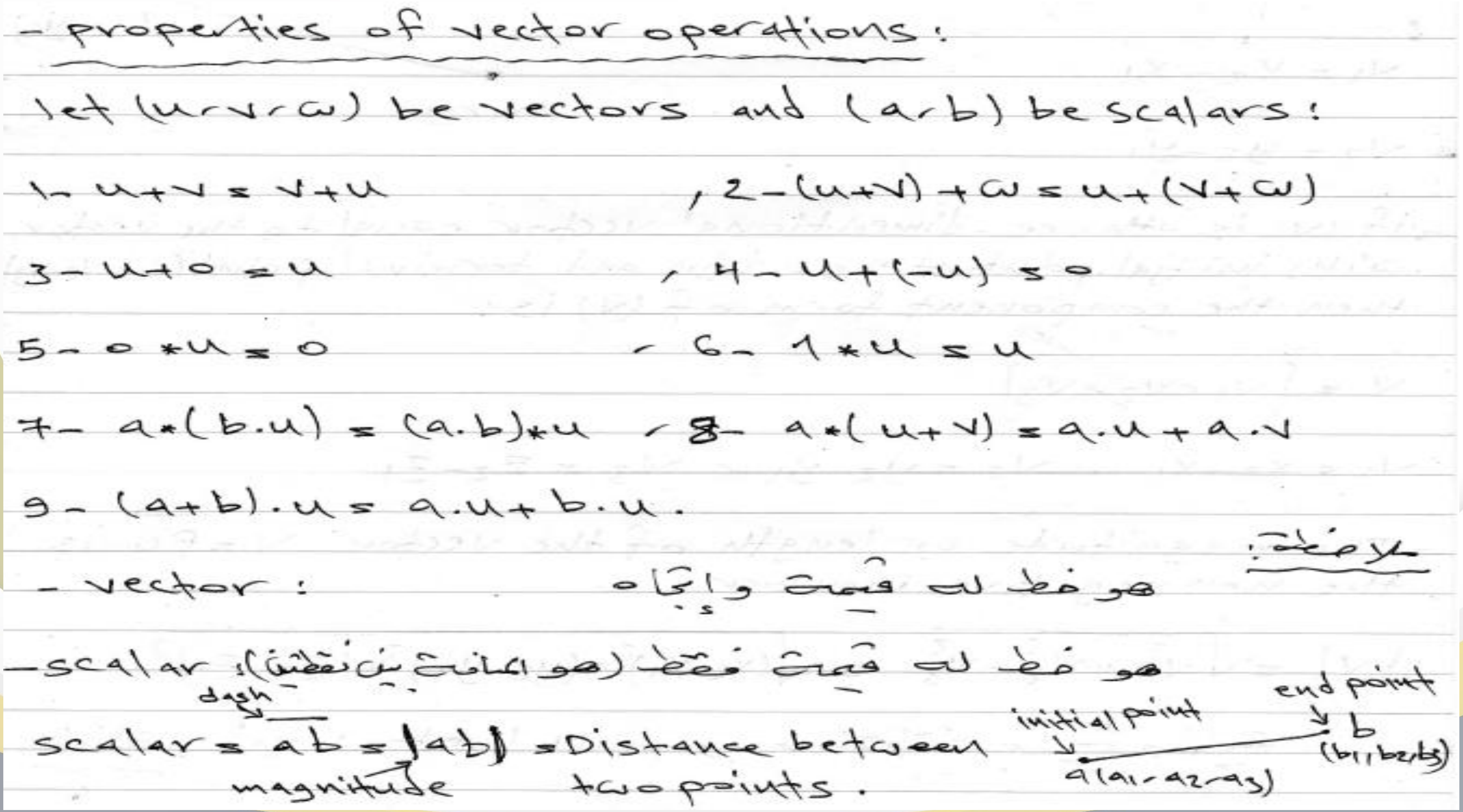
- The only vector with length (0) is the (zero vector) $0 = \{0,0\}$ or $0 = \{0,0,0\}$, this vector is also the only vector with no specific direction.

- vector Algebra operations:

Let $u = \{u_1, u_2, u_3\}$ and $v = \{v_1, v_2, v_3\}$ be vectors:

* Addition: $u + v = \{u_1 + v_1, u_2 + v_2, u_3 + v_3\}$

* scalar multiplication: $ku = \{ku_1, ku_2, ku_3\}$



- properties of vector operations:

let (u,v,w) be vectors and (a,b) be scalars:

1- $u+v = v+u$, 2- $(u+v)+w = u+(v+w)$

3- $u+0 = u$, 4- $u+(-u) = 0$

5- $0 * u = 0$, 6- $1 * u = u$

7- $a*(b.u) = (a.b)*u$, 8- $a*(u+v) = a.u + a.v$

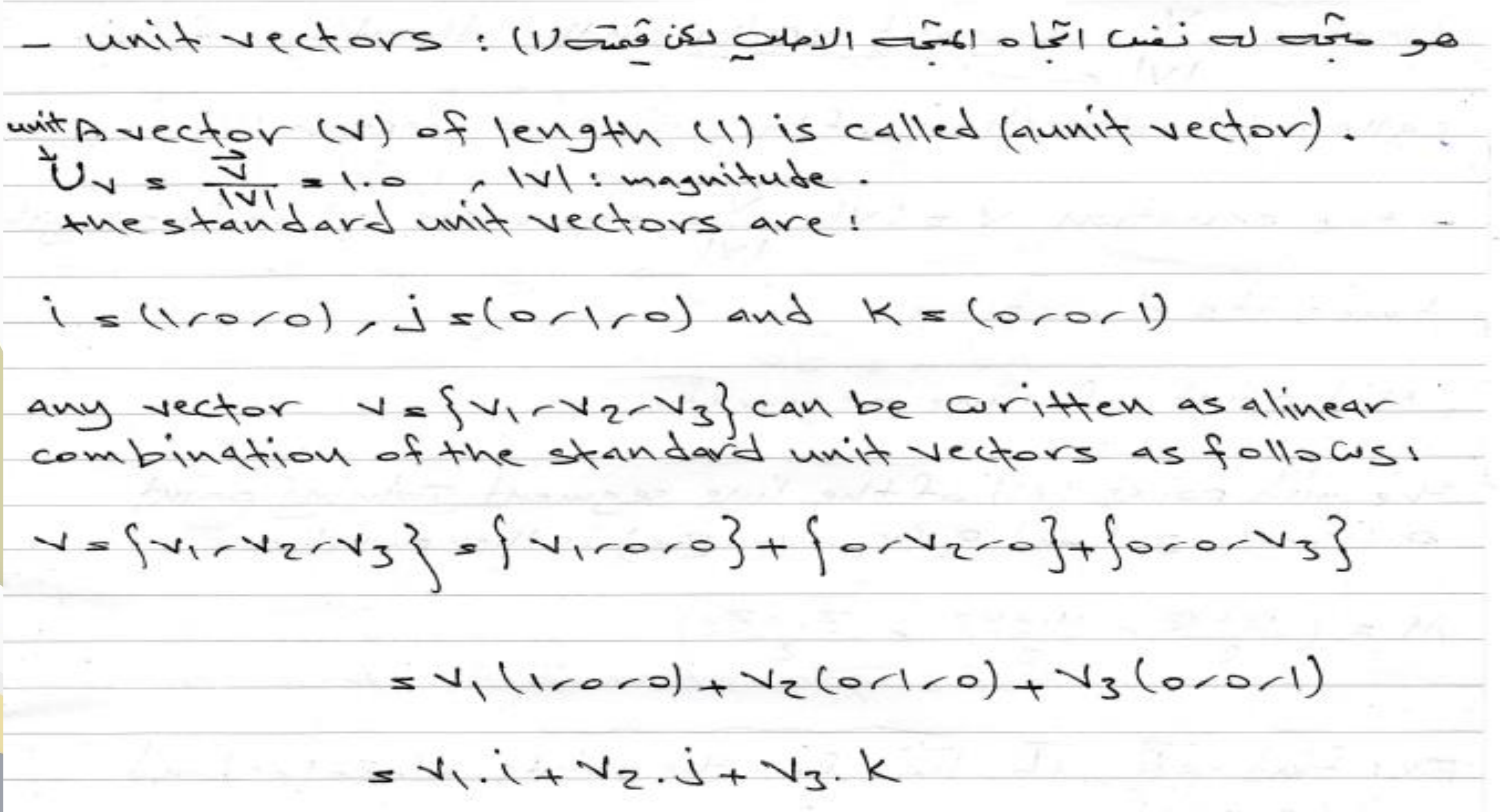
9- $(a+b).u = a.u + b.u$.

ملاحظة:

- vector : هو خط له قيمة واتجاه

-scalar : هو خط له قيمة فقط (هو المسافة بين نقطتين)

scalar $= \overset{\text{dash}}{\overline{ab}} = \underset{\text{magnitude}}{|ab|}$ = Distance between two points.



- unit vectors : هو متجه له نفس اتجاه المتجه الاصلي لكن قيمته (1)

unit A vector (V) of length (1) is called (aunit vector).

$\vec{U}_v = \frac{\vec{v}}{|v|} = 1.0$, $|v|$: magnitude .

the standard unit vectors are :

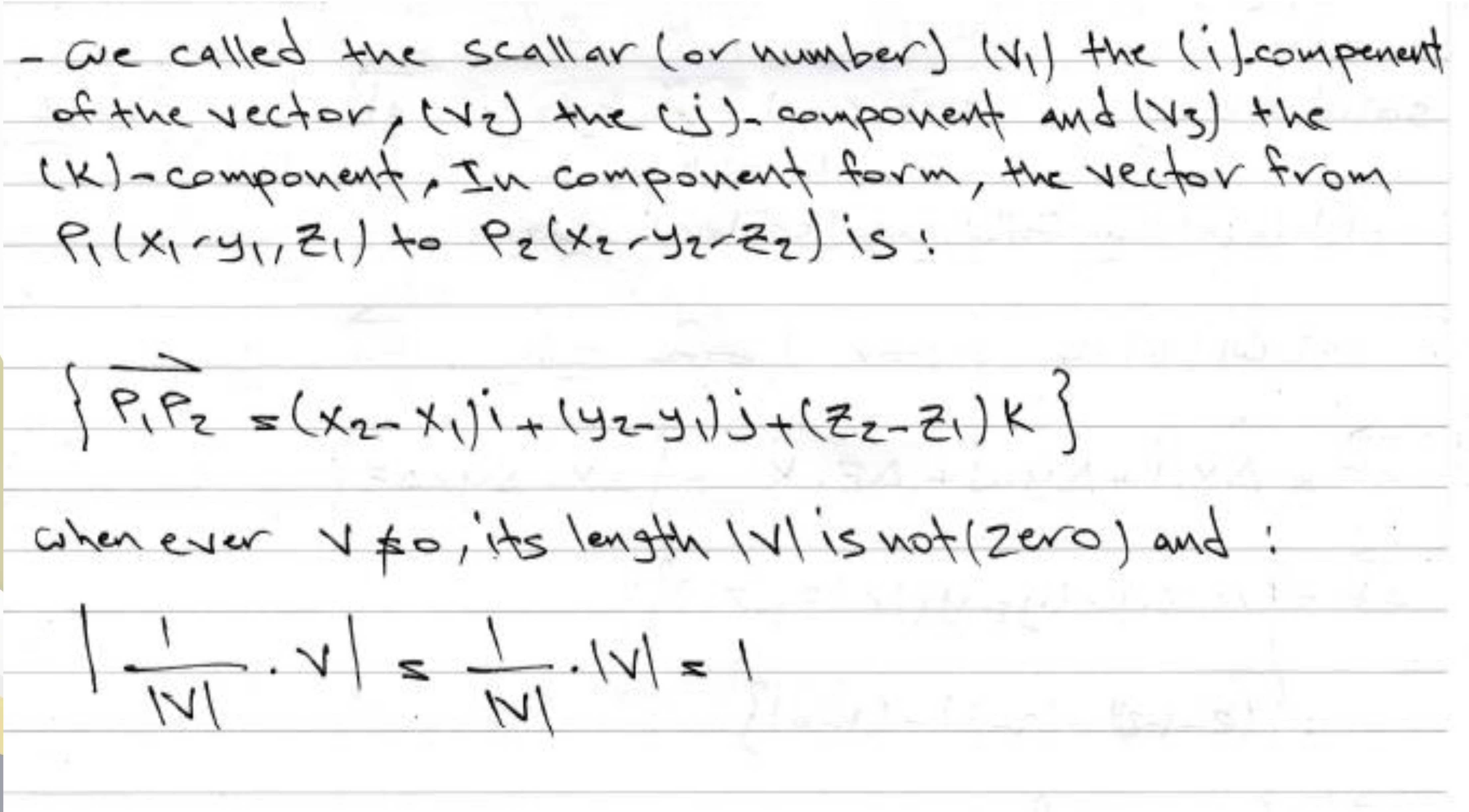
$i = (1,0,0)$, $j = (0,1,0)$ and $K = (0,0,1)$

any vector $v = \{v_1, v_2, v_3\}$ can be written as alinear combination of the standard unit vectors as follows:

$$v = \{v_1, v_2, v_3\} = \{v_1, 0, 0\} + \{0, v_2, 0\} + \{0, 0, v_3\}$$

$$= v_1(1,0,0) + v_2(0,1,0) + v_3(0,0,1)$$

$$= v_1 . i + v_2 . j + v_3 . K$$

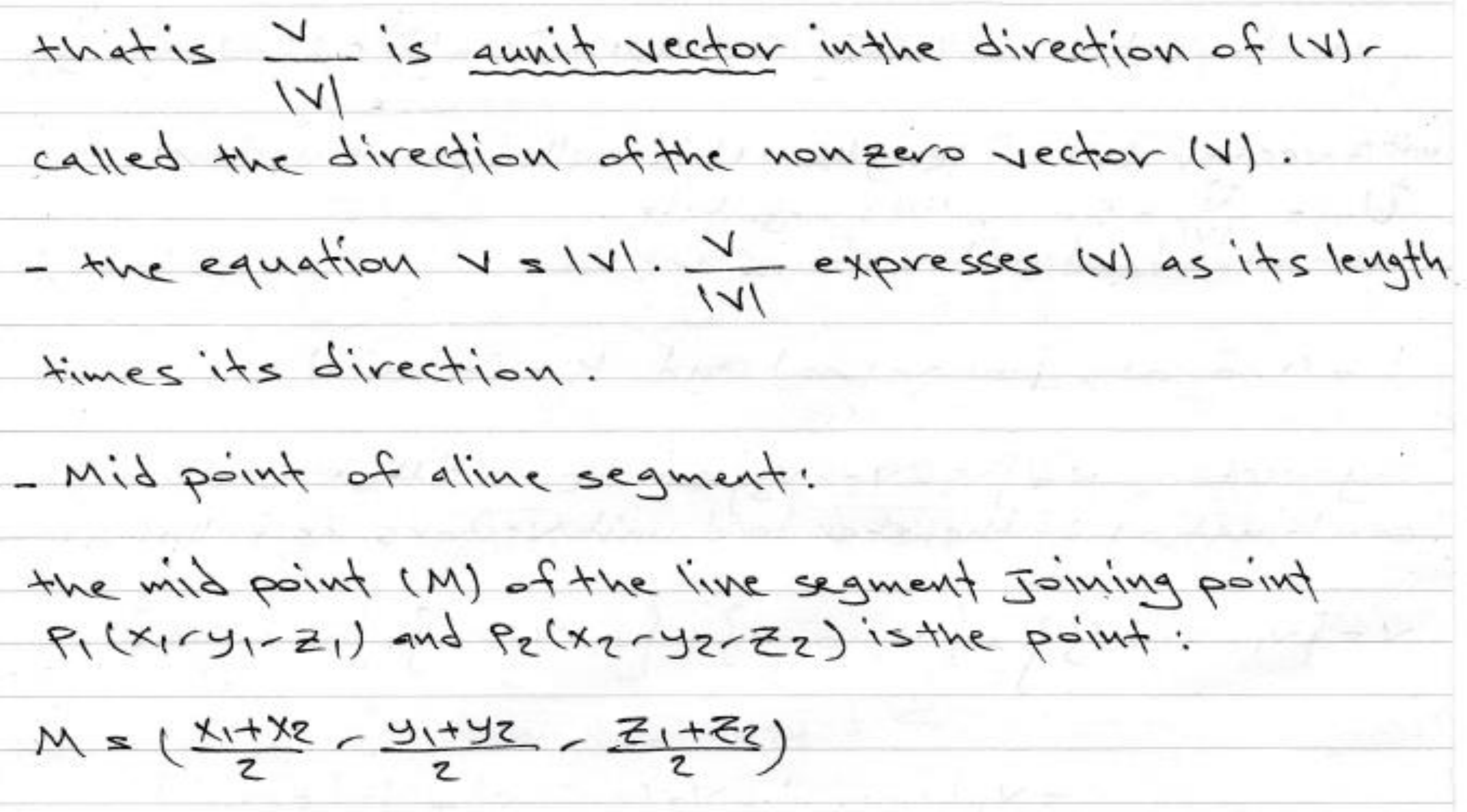


- we called the scallar (or number) (v_1) the (i)-compenent of the vector, (v_2) the (j)-component and (v_3) the (k)-component, In component form, the vector from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is:

$$\left\{ \overrightarrow{P_1P_2} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k \right\}$$

when ever $v \neq 0$, its length $|v|$ is not (zero) and:

$$\left| \frac{1}{|v|} \cdot v \right| = \frac{1}{|v|} \cdot |v| = 1$$



that is $\frac{V}{|V|}$ is aunit vector inthe direction of (V),

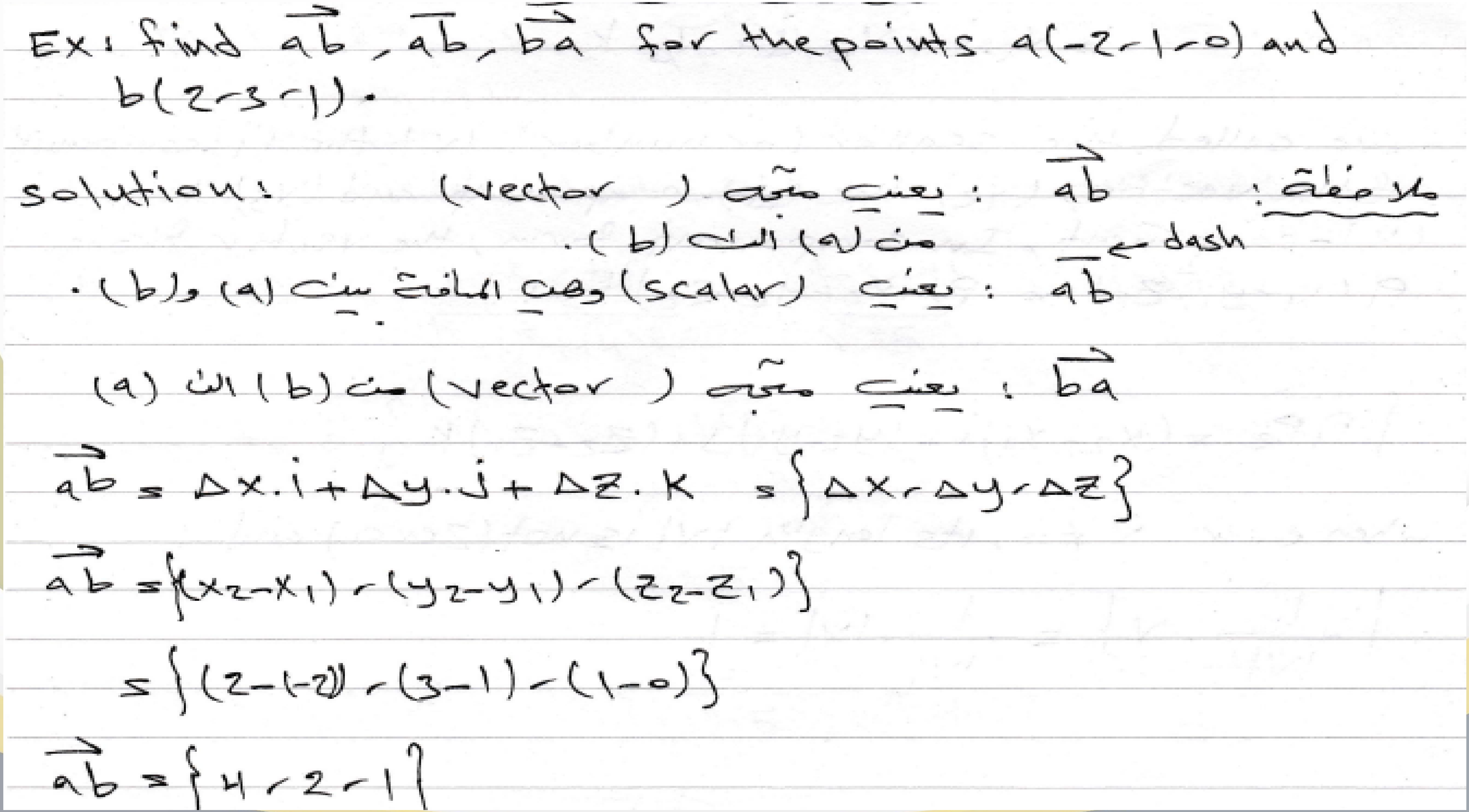
called the direction of the nonzero vector (V).

- the equation $V = |V| . \frac{V}{|V|}$ expresses (V) as its length times its direction.

- Mid point of aline segment:

the mid point (M) of the line segment Joining point $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$



Ex: find $\overrightarrow{ab}$, $\overline{ab}$, $\overrightarrow{ba}$ for the points a(-2,1,0) and b(2,3,1).

solution:

ملاحظة: $\overrightarrow{ab}$: يعني متجه (vector) من (a) الى (b).

dash ←

$\overline{ab}$: يعني (scalar) وهي المسافة بين (a) و(b).

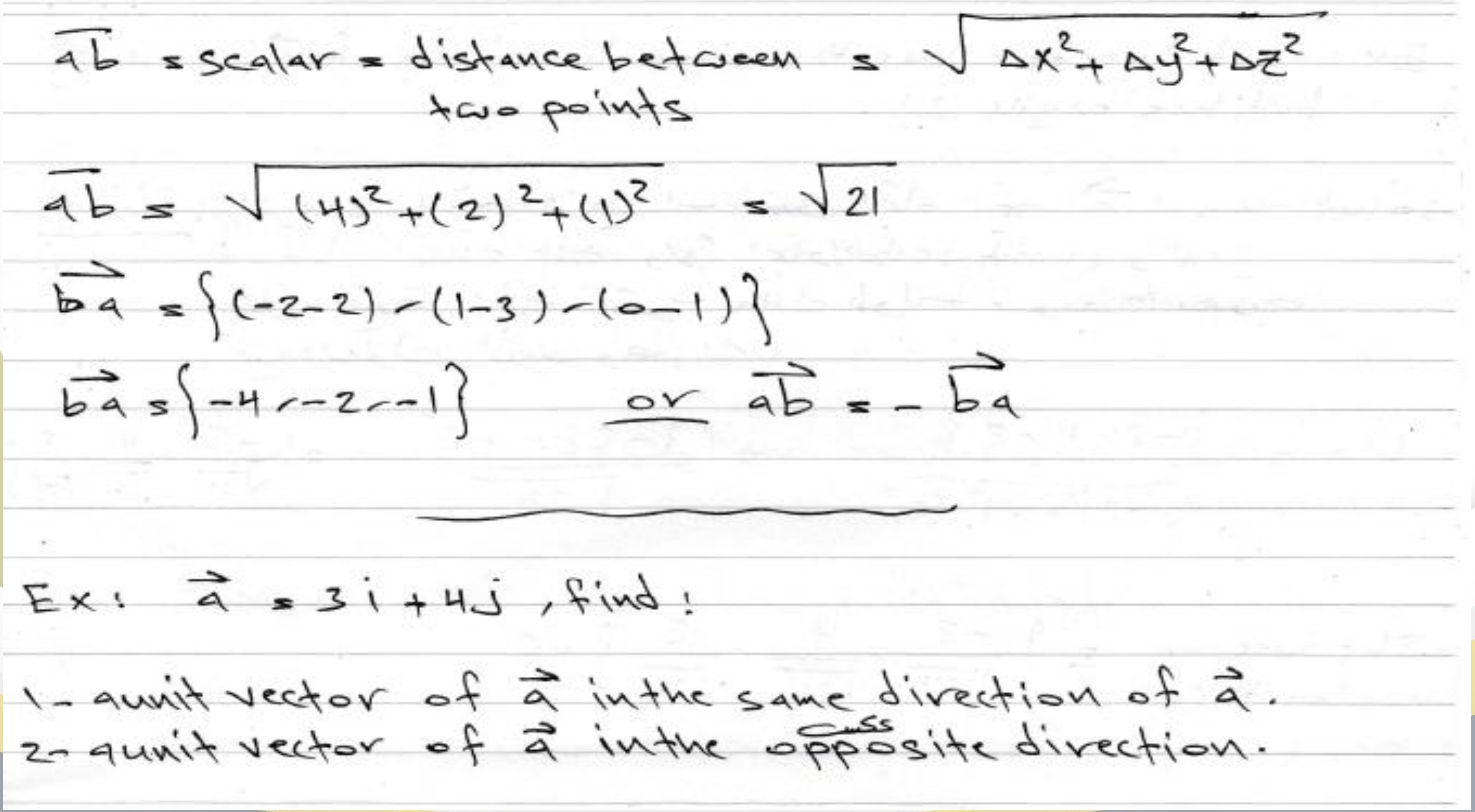
$\overrightarrow{ba}$: يعني متجه (vector) من (b) الى (a)

$$\overrightarrow{ab} = \Delta x.i + \Delta y.j + \Delta z.K = \{\Delta x, \Delta y, \Delta z\}$$

$$\overrightarrow{ab} = \{(x_2-x_1),(y_2-y_1),(z_2-z_1)\}$$

$$= \{(2-(-2)),(3-1),(1-0)\}$$

$$\overrightarrow{ab} = \{4,2,1\}$$



$\overline{ab}$ = scalar = distance between two points $= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

$\overline{ab} = \sqrt{(4)^2 + (2)^2 + (1)^2} = \sqrt{21}$

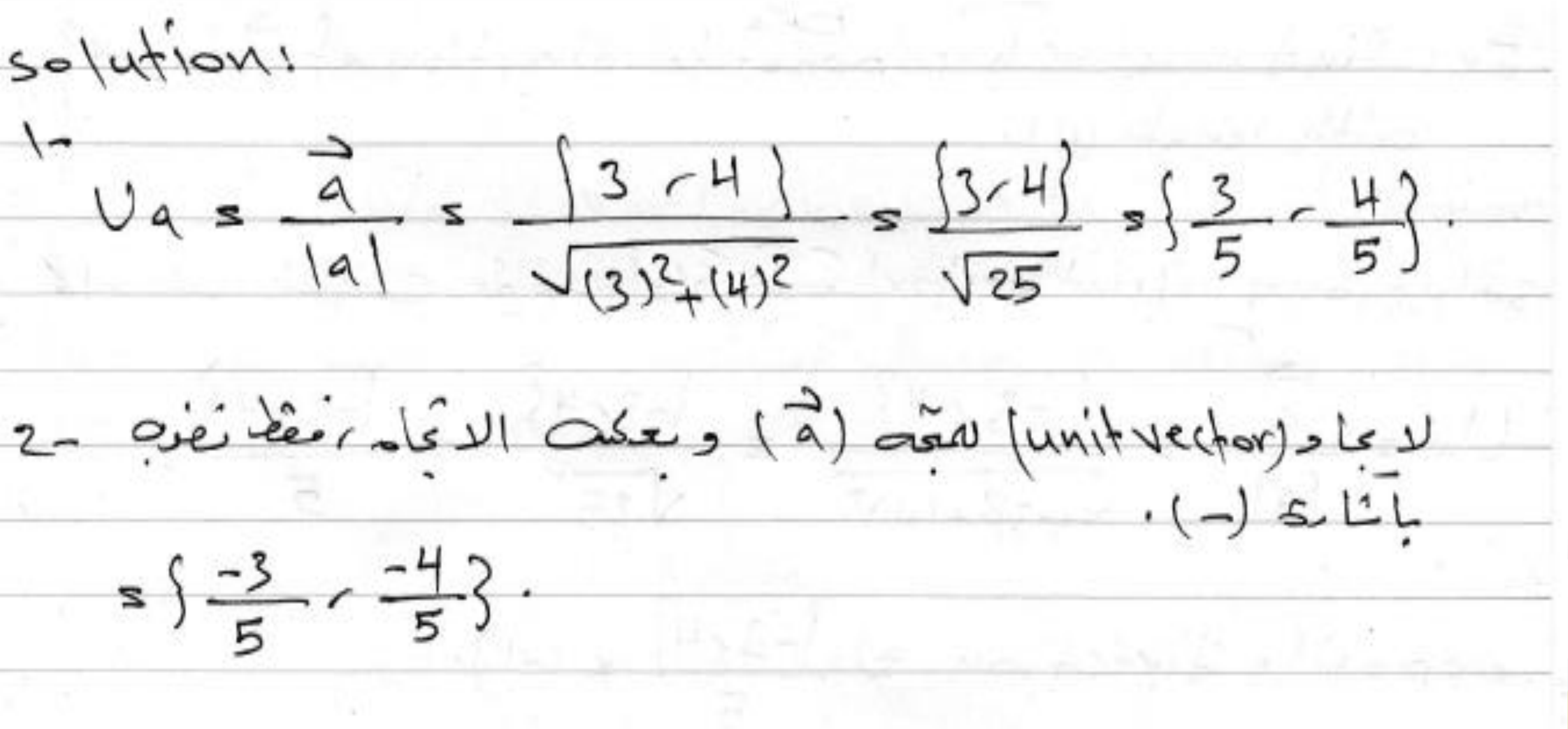
$\overrightarrow{ba} = \{(-2-2), (1-3), (0-1)\}$

$\overrightarrow{ba} = \{-4, -2, -1\}$ or $\overrightarrow{ab} = -\overrightarrow{ba}$

Ex: $\vec{a} = 3i + 4j$, find:

1- aunit vector of $\vec{a}$ inthe same direction of $\vec{a}$.

2- aunit vector of $\vec{a}$ inthe عكس opposite direction.

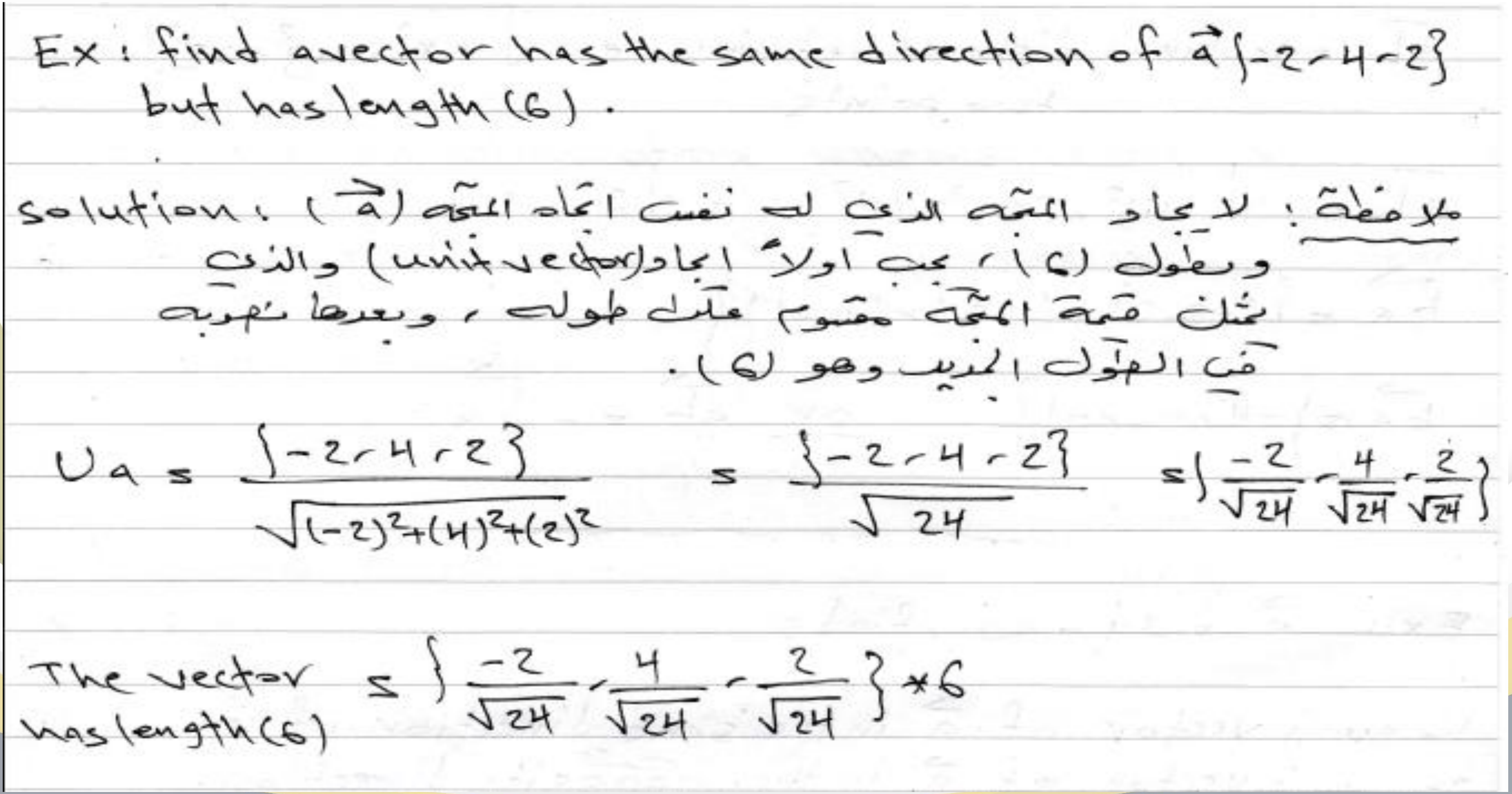


solution:

1- $U_a = \frac{\vec{a}}{|a|} = \frac{\{3 , 4\}}{\sqrt{(3)^2+(4)^2}} = \frac{\{3 , 4\}}{\sqrt{25}} = \{\frac{3}{5} , \frac{4}{5}\}.$

2- لايجاد (unit vector) لمتجه ($\vec{a}$) وبعكس الاتجاه، فقط نضرب
باشارة (−).

$= \{\frac{-3}{5} , \frac{-4}{5}\}.$



Ex: find a vector has the same direction of $\vec{a}\{-2, 4, 2\}$ but has length (6).

solution: ملاحظة: لايجاد المتجه الذي له نفس اتجاه المتجه ($\vec{a}$) وبطول (6)، يجب اولاً ايجاد (unit vector) والذي يمثل قيمة المتجه مقسوم على طوله، وبعدها نضربه في الطول الجديد وهو (6).

$$U_a = \frac{\{-2, 4, 2\}}{\sqrt{(-2)^2+(4)^2+(2)^2}} = \frac{\{-2, 4, 2\}}{\sqrt{24}} = \left\{\frac{-2}{\sqrt{24}}, \frac{4}{\sqrt{24}}, \frac{2}{\sqrt{24}}\right\}$$

The vector has length (6) $= \left\{\frac{-2}{\sqrt{24}}, \frac{4}{\sqrt{24}}, \frac{2}{\sqrt{24}}\right\} * 6$