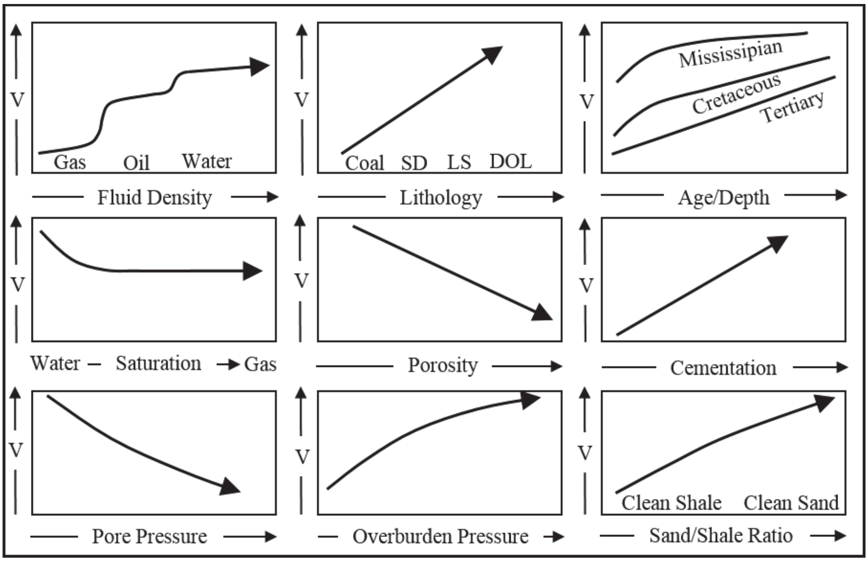

Figure 1.5 Factors affecting Vp (Hiltermann, 2001). Note that those factors working simultaneously.

Numerous equations for calculating rocks seismic velocities with known rock-physics parameters have been published. Two popular equations from Wyllie (1963) and Gassman (1951) are discussed here.Wyllie's equation (popular also as time-average equation) is as follows:

$$\frac{1}{V_p} = \frac{(1-\phi)}{V_m} + \phi\left(\frac{S_w}{V_w} + \frac{1-S_w}{V_{fl}}\right) \qquad (1.6)$$

where V_m= V_P of the matrix, V_{fl}= velocity of pore fluid.

Since this velocity equation based on oversimplified model, it does not work for rocks containing fluid of low velocities ($V_f \leq 1000$ m/s) such as gases and live oils (oils with gas in solution), rock with vugular pores or fractures (e.g. some carbonate rocks), and rocks with loose matrix (e.g. soft and unconsolidated sands).

Gassmann (1951) developed the theory of wave propagation in fluid saturated rocks and come up with the following equations to calculate Vp and Vs (P- and S-wave velocity) of the saturated rocks:

$$V_P = \sqrt{\frac{K_{sat} + \frac{4}{3}\mu_{sat}}{\rho_{sat}}} \qquad (2.7) \qquad\qquad V_s = \sqrt{\frac{\mu_{sat}}{\rho_{sat}}} \qquad (2.8)$$

where K is bulk modulus and μ is shear modulus. The ρ_{sat} is calculated using equation 1.5.

Figure 1.6illustrates the physical meaning of bulk and shear modulus. Bulk modulus is affected more by the pores and its fluid content whereas the shear modulus by the matrix.

Gassmann equation assumes that for a rock with a same matrix and porosity, the shear modulus is independent to pore-fluid or

$$\mu_{sat} = \mu_{dry} \qquad (1.9)$$

where μ_{sat} is shear modulus of saturated rock and μ_{dry} is shear modulus of dry rock.

The bulk modulus can be calculated using equation below:

$$K_{sat} = K_{dry} + \frac{(1 - \frac{K_{dry}}{K_m})^2}{\frac{\phi}{K_{fl}} + \frac{1-\phi}{K_m} - \frac{K_{dry}}{K_m^2}} \qquad (1.10)$$

where sat = saturated rock, dry = dry frame, m = rock matrix, fl = fluid, ϕ = porosity. K_{fl}, K_w, K_{hc} and K_m are bulk modulus of fluid, water, hydrocarbon and matrix.

K_m is usually taken from published data that involved measurements on pure mineral samples (crystals). Mineral values can be averaged using Reuss averaging to estimate K_m for rocks composed of mixed lithologies. Typical K_m values for sandstone and limestone are 40 Gpa and 60 GPa.

The *fluid bulk modulus* K_{fl} can be computed using the following equation:

$$\frac{1}{K_{fl}} = \frac{S_w}{K_w} + \frac{1 - S_w}{K_{hc}} \qquad (1.11)$$

where K_w bulk modulus of water and K_{hc} bulk modulus of hydrocarbon. Equations for estimating the K values of brine, gas, and oil bulk moduli are given by Batzle and Wang (1992). Typical values are K_{gas} = 0.021 GPa, K_{oil} = 0.79 GPa and K_w = 2.38 GPa.

K_{dry} represents the incompressibility of the rock frame (including cracks and pores) which often pressure dependent due to cracks closing with increased effective pressure. Accurate values of K_{dry} can be obtained from laboratory measurements of representative core plugs under reservoir pressure. Gassmann theory assumes that fluids are mobile between pores and all stress is carried by K_{dry}. This assumption is violated at "high frequencies" in highly variable and compressible pore systems. Therefore carbonates and other fractured rocks with an abundance of crack-type pores and heterogeneous pore systems are not suitable for standard Gassmann theory.

When V_p and V_s are available, for example from log or core measurement, shear and bulk moduli can be computed back using the following equations:

$$\mu = \rho V_S^2 \qquad (1.12)$$

$$K = \rho\left(V_P^2 - \left(\frac{4}{3}\right)V_S^2\right) \qquad (1.13)$$

If the bulk moduli of the rock are expressed in Giga-Pascal (GPa) and the density in g/cc, then the resulting velocity is expressed in km/s. Table 1.1 gives typical values of Vp, Vs and density of common rocks.

Beside Vp and Vs, another elastic property commonly used in seismic reservoir analysis is the Poisson's ratio σ which is the negative ratio of the transverse strain to the longitudinal strain. In normal practices when laboratory measurement unavailable, Poisson's ratio is calculated as a function of the V_p and V_s as shown in Figure 1.6.

Exercise 1.2 :

A reservoir has porosity ϕ =0.33, ρ_m = 2.65 g/cc, ρ_{water} = 2.65 g/cc, K_m = 40 GPa, K_{water} = 2.38 GPa, K_{dry} = 3.2477 GPa, μ = 3.3056 Gpa. For two different cases: 1) reservoir filled by gas with K_{gas} = 0.021 Gpa, ρ_{gas} = 0.1 g/cc, and 2) reservoir filled by oil with K_{oil} = 1 Gpa, ρ_{oil} = 0.8 g/cc, do the followings (calculate for S_w varies from 0% to 100%):

1. *Calculate Vp for both cases using Gassman and Wyllie equations*
2. *Calculate Vs and Poisson ration for both cases using Gassman equation*
3. *For both cases :*
   a. *Make plots of Vp vs Sw for Gassman and Wyllie and give comments on their differences*
   b. *For Gassman results, make plots of i) S_w vs V_p and V_s, ii) S_w vs Poisson ratio, and iii) Vp vs Sw vs Poisson ratio*
4. *Based on Gassman results find out which elastic property (V_p or V_s or Poisson Ratio) is the best to calculate the gas and oil saturation. Calculate sensitivity as in Exercise 1.1 for each elastic property to justify your answers*

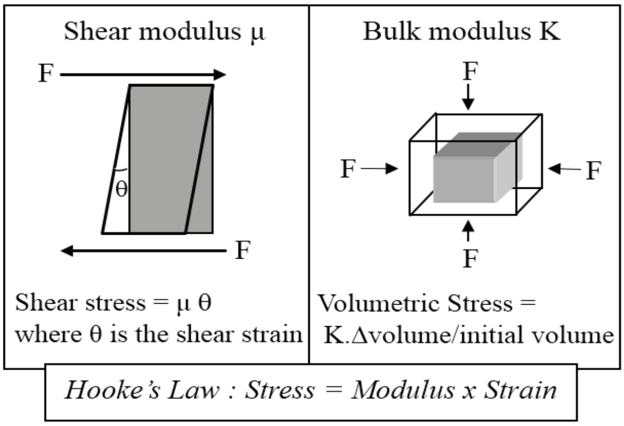

Figure 1.6 Stress-strain relationship illustrating the physical meaning of bulk and shear modulus

Table 1.1Typical rock velocities and densities (Mavko et. al, 1998)

Rock	Vp km/s	Vs km/s	Vp/Vs	Porosity fraction	Density g/cm^3	Impedance 10^6kg/m^{3}m/s
Chalk	2.16	2.03	1.67	0.5	1.85	4.02
Dolomite	5.39	2.97	1.82	0.13	2.59	14
Brine Sandstone	4.09	2.41	1.71	0.16	2.97	9.73
Tight-gas Sandstone	4.67	3.06	1.53	0.05	2.51	11.7
High-porosity Sandstone	3.8	0.15	0.13	0.08	0.13	0.67
Poorly consolidated Sandstone	2.73	1.37	2.02	0.31	2.11	5.77
Limestone	4.63	2.44	1.88	0.15	2.43	14.3

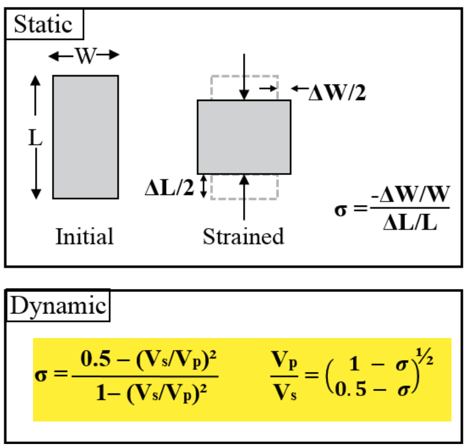

Figure 1.7 Stress-strain relationship illustrating the physical meaning of bulk and shear modulus