The MBE as an Equation of a Straight Line

An insight into the general MBE, i.e., Equation below (11-15) may be gained by considering the simplest model that will be discussed is the technique of Havlena and Odeh.

$$N = \frac{N_p \left[B_o + (R_p - R_s) B_g \right] - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_{wi}}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{oi}(1+m) \left[\frac{S_{wi} c_w + c_f}{1 - S_{wi}} \right] \Delta p} \tag{11-15}$$

There are essentially three unknowns in Equation before:

a. The original oil in place N.

b. The cumulative water influx We.

c. The original size of the gas cap as compared to the oil zone size m.

In developing a methodology for determining the above three unknowns, Havlena and Odeh (1963) expressed Equation 11-15 in the following form:

$$N_p[B_o + (R_p - R_s)B_g] + B_w W_p = N[(B_o - B_{oi}) + (R_{si} - R_s)B_g] + \frac{NmB_{oi}}{B_{gi}}(B_g - B_{gi}) + (1+m)NB_{oi}\left[\frac{C_w S_{wi} + C_f}{1 - S_{wi}}\right]\Delta\bar{p} + W_e + W_{inj}B_w + G_{inj}B_{ginj}$$

Apply the technique of Havlena and Odeh in interpreting the material balance as the **equation of a straight line**.

- Np [Bo + (Rp – Rs) Bg] Represents the reservoir volume of cumulative oil and gas produced.
- [We – Wp Bw] Refers to the net water influx that is retained in the reservoir.
- [Ginj Bginj + Winj Bw] This pressure maintenance term represents cumulative fluid injection in the reservoir.

[m Boi (Bg/Bgi – 1)] Represents the net expansion of the gas cap that occurs with the production of Np stock tank barrels of oil (as expressed in bbl/STB of original oil in place).

Havlena and Odeh further expressed equation below in a more condensed form as:

$$F = N\left[E_o + mE_g + E_{w,f}\right] + \left(W_e + W_{inj}B_w + G_{inj}B_{ginj}\right)$$

Assuming, for the purpose of simplicity, that no pressure maintenance by gas or water injection is being considered, the above relationship can be further simplified and written as:

$$\boldsymbol{F = N\left[E_o + mE_g + E_{w,f}\right] + W_e}$$

In which the terms F, Eo, Eg, and Ew,f are defined by the following relationships:

- F represents the underground withdrawal and given by:

$$F = N_p\left[B_o + (R_p - R_s)B_g\right] + B_wW_p$$

$$R_P = \frac{G_P}{N_P}$$

Rp = cumulative produced gas-oil ratio, scf/STB

In terms of the two-phase formation volume factor Bt, the underground withdrawal F can be written as:

$$F = N_p\left[B_t + (R_p - R_{si})B_g\right] + B_wW_p$$

- Eo describes the expansion of oil and its originally dissolved gas and is expressed in terms of the oil formation volume factor as:

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g$$

Or equivalently, in terms of Bt:

$$E_o = B_t - B_{ti}$$

- Eg is the term describing the expansion of the gas-cap gas and is defined by the following expression:

$$E_g = \frac{B_{oi}}{B_{gi}}(B_g - B_{gi})$$

In terms of the two-phase formation volume factor Bt, essentially Bti = Boi or:

$$E_g = \frac{B_{ti}}{B_{gi}}(B_g - B_{gi})$$

- Ef,w represents the expansion of the initial water and the reduction in the pore volume and is given by:

$$E_{w,f} = (1+m)B_{oi}\left[\frac{C_w S_{wi} + C_f}{1 - S_{wi}}\right]\Delta\bar{p}$$

Havlena and Odeh examined several cases of varying reservoir types with Equation below:

$$F=N[E_o+mE_g+E_{f,w}]+W_e$$

and pointed out that the relationship can be rearranged into the form of a straight line.

For example, in the case of a reservoir which has no initial gas cap (i.e., m = 0 for a reservoir with no initial gas cap) or water influx (i.e., We = 0), and negligible formation and water compressibilities (i.e., cf and cw = 0); Equation above reduces to:

$$F=NE_o$$

The above expression suggests that a plot of the parameter F as a function of the oil expansion parameter Eo would yield a straight line with a slope N and intercept equal to zero.

The Straight-Line Solution Method to the MBE

The straight-line solution method requires the plotting of a variable group versus another variable group, with the variable group selection depending on the mechanism of production under which the reservoir is producing. The most important aspect of this method of solution is that it attaché significance the sequence of the plotted points, the direction in which they plot, and to the shape of the resulting plot.

The significance of the straight-line approach is that the sequence of plotting is important and if the plotted data deviates from this straight line, there is some reason for it. This significant observation will provide the engineer with valuable information that can be used in determining the following unknowns:

- Initial oil in place N
- Size of the gas cap m
- Water influx We
- Driving mechanism

The applications of the straight-line form of the MBE in solving reservoir engineering problems are presented next to illustrate the usefulness of this particular form.

Six cases of applications are presented:

1. Case determination of N in volumetric undersaturated reservoirs
2. Case determination of N in volumetric saturated reservoirs
3. Case determination of N and m in gas cap drive reservoirs
4. Case determination of N and We" in water drive reservoirs
5. Case determination of N, m, and We in combination drive reservoirs
6. Case determination of average reservoir pressure, p

Case 1. Volumetric Undersaturated-Oil Reservoirs

Here we assume that there is no gas cap initially present. In this case the production is caused by the expansion of oil and solution gas. Assuming no water or gas injection, the linear form of the MBE, as expressed by Equation below can be written as:

$$F=N[E_o+mE_g+E_{f,w}]+W_e$$

$$F = N_p[B_o + (R_p - R_s)B_g] + B_w W_p$$

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g$$

$$E_g = \frac{B_{oi}}{B_{gi}}(B_g - B_{gi})$$

m= zero

and furthermore, above the bubble point, Rs = Rsi = Rp, since all the gas produced at the surface must have been dissolved in the oil in the reservoir.

$$E_{w,f} = (1+m)B_{oi}\left[\frac{C_w S_{wi} + C_f}{1 - S_{wi}}\right]\Delta\bar{p}$$

Several terms in the above relationship may disappear when imposing the conditions associated with the assumed reservoir driving mechanism. For a volumetric and undersaturated reservoir, the conditions associated with driving mechanism are:

- we = 0, since the reservoir is volumetric
- m = 0, since the reservoir is undersaturate
- Rs = Rsi = Rp, since all produced gas is dissolved in the oil

Applying the above conditions on equation

$$F=N[E_o+mE_g+E_{f,w}]+W_e$$

gives:

$$F = N(E_o + E_{f,w})$$

or

$$N = \frac{F}{E_o + E_{f,w}}$$

Where:

N = initial oil in place, STB

$$F = N_p B_o + W_p B_w$$

$$E_o = B_o - B_{oi}$$

$$E_{f,w} = B_{oi}\left[\frac{c_w S_w + c_f}{1 - S_{wi}}\right]\Delta p$$

$$\Delta p = p_i - \overline{p}_r$$

pi = initial reservoir pressure

pr = volumetric average reservoir pressure