University of Mosul
College of Petroleum & Mining Engineering

Mathematics II

Lecture (1)

Hyperbolic Function

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LECTURE CONTENTS

Derivatives

Integration

Hyperbolic Functions

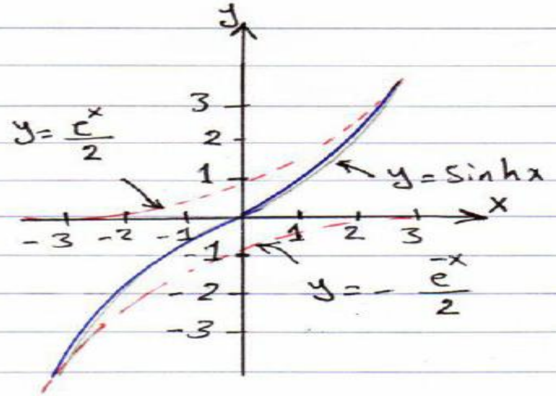
The hyperbolic functions are formed by taking combinations of the two expressions and occur frequently in mathematical and engineering applications. In this section we give a brief introduction to these functions, their graphs, their derivatives, their integrals, and their inverse functions.

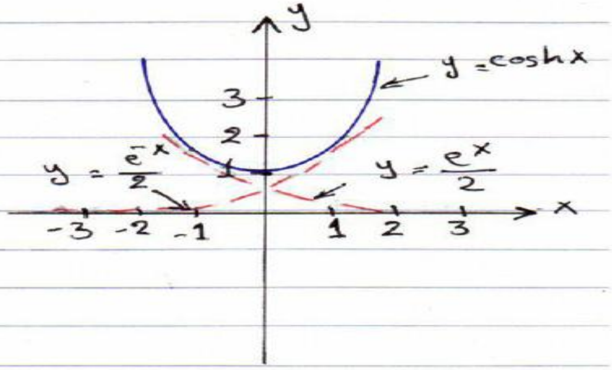
Definitions and Identites

The hyperbolic sine and hyperbolic cosine functions are defined by the equations

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$



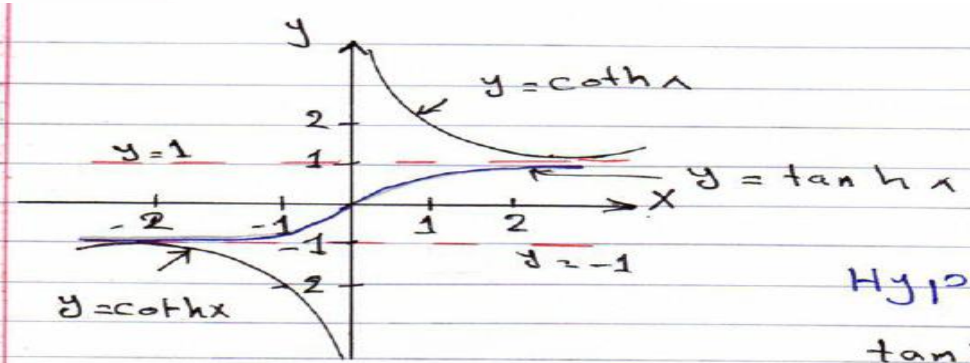


Hyperbolic sine

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

Hyperbolic cosine

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

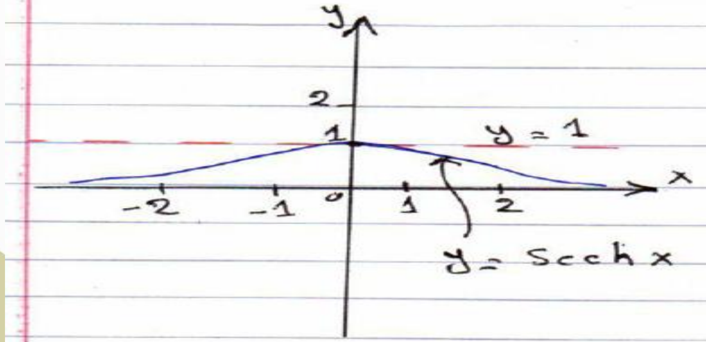


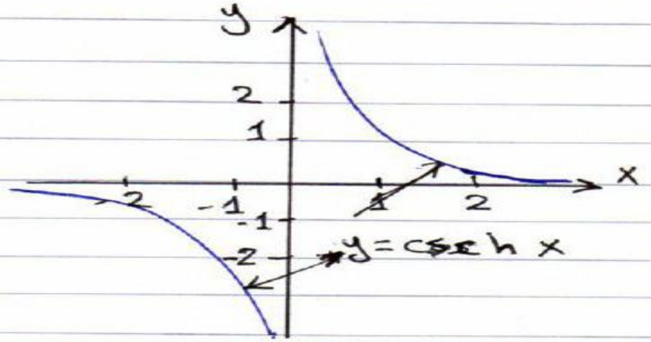
Hyperbolic tangent

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

Hyperbolic cotangent

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$





Hyperbolic secant

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^{x} + e^{-x}}$$

Hyperbolic cosecant

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}}$$

Fig. (1): The six basic hyperbolic functions (2)

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Identities for hyperbolic functions

$$\cosh^2 x - \sin^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\tanh^2 x = 1 - \operatorname{sech}^2 x$$

$$\coth^2 x = 1 + \operatorname{csch}^2 x$$

Derivatives of hyperbolic functions

1) $\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$

2) $\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$

3) $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$

4) $\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$

5) $\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$

6) $\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \frac{du}{dx}$

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The six hyperbolic functions, being rational combinations of the differentiable functions e^{x} and e^{-x}, have derivatives at every point at which they are defined (1–6). Again, there are similarities with trigonometric functions.

The derivative formulas are derived from the derivative of e^{u}

$$\frac{d}{dx}(\sinh u) = \frac{d}{dx}\left(\frac{e^{u} - e^{-u}}{2}\right) \quad \text{Definition of } \sinh u$$

$$= \frac{e^{u}\,du/dx + e^{-u}\,du/dx}{2} \quad \text{Derivative of } e^{u}$$

$$= \cosh u \frac{du}{dx} \quad \text{Definition of } \cosh u$$

Integral formulas for hyperbolic functions

1) $\int \sinh u\,du = \cosh u + C$

2) $\int \cosh u\,du = \sinh u + C$

3) $\int \operatorname{sech}^2 u\,du = \tanh u + C$

4) $\int \operatorname{csch}^2 u\,du = -\coth u + C$

5) $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

6) $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

7) $\int \tanh u \cdot du = \ln(\cosh u) + C$

8) $\int \coth u \cdot du = \ln(\sinh u) + C$

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Examples

a) $\frac{d}{dt}\left(\tanh\sqrt{1+t^2}\right) =$

Sol.

$$= \operatorname{sech}^2\sqrt{1+t^2} \cdot \frac{d}{dt}\sqrt{1+t^2}$$

$$= \frac{t^2}{\sqrt{1+t^2}} \operatorname{sech}^2\sqrt{1+t^2}$$

b) $\int \coth 5x \, dx$

Sol.

$$= \int \frac{\cosh 5x}{\sinh 5x} \, dx$$

assume $u = \sinh 5x$, $du = 5\cosh 5x \, dx$

$$= \frac{1}{5}\int \frac{du}{u}$$

$$= \frac{1}{5}\ln|u| + c$$

$$= \frac{1}{5}\ln|\sinh 5x| + C$$

c) $\int_0^1 \sinh^2 x \, dx =$

Sol.

$$= \int_0^1 \frac{\cos 2x - 1}{2} \, dx$$

$$= \frac{1}{2} \int_0^1 (\cosh 2x - 1) \, dx$$

$$= \frac{1}{2} \left[\frac{\sinh 2x}{2} - x \right]_0^1$$

$$= \frac{\sinh 2}{4} - \frac{1}{2} \simeq 0.406$$

d) $\int_0^{\ln 2} 4 e^x \sinh x \, dx$

Sol.

$$= \int_0^{\ln 2} 4 e^x \, \frac{e^x - e^{-x}}{2} \, dx$$

$$= \int_0^{\ln 2} (2 e^{2x} - 2) \, dx$$

$$= \left[e^{2x} - 2x \right]_0^{\ln 2} =$$

$$= (e^{2\ln 2} - 2\ln 2) - (1 - 0)$$

$$= 4 - 2\ln 2 - 1 = 1.61$$