University of Mosul

College of Petroleum & Mining Engineering

Mathematics I

Lecture (2)

Derivatives

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LECTURE CONTENTS

Derivatives

Derivatives

The problem of finding the tangent line a curve and the problem of finding the velocity of an object both involve finding the same type of limit, as we saw in limit object. This special type of limit is called a derivative and we will see that it can be interpreted as a rate of change in any of the sciences or engineering.

1. **Tangents and the derivative at a point**
   To find a tangent to an arbitrary curve $y = f(x)$ at a point p $(x_o, f(x_o))$, we calculated the slope of the secant through p and near by point Q $(x_o + h\ f(x_o+h)$. We then investigate the limit of the slope as $h \to 0$ figure 1. If the limit exists , we call it the slope of the curve at p and define the tangent at p to be the line through p having this slope. As shown in figure 1.

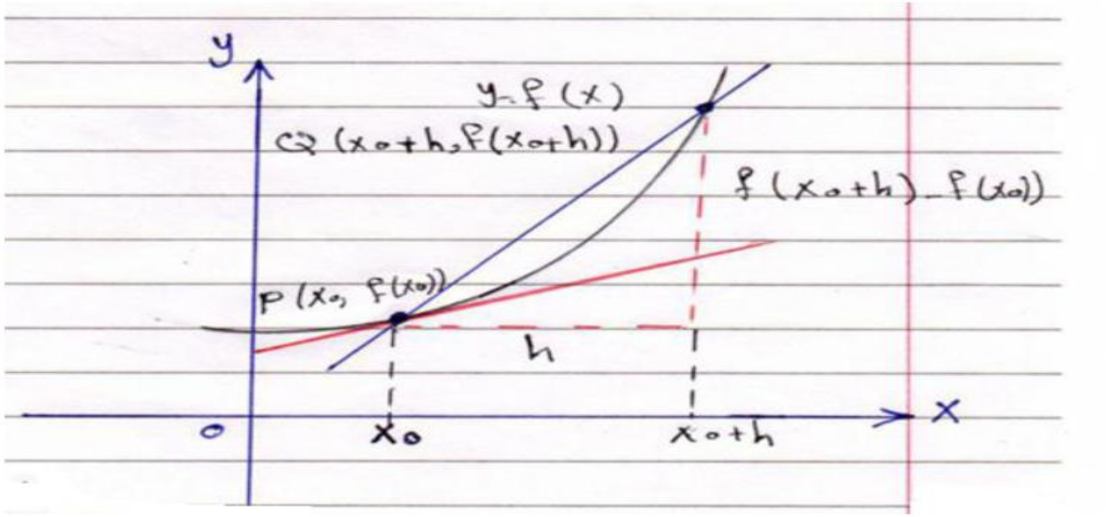

Figure (1)

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The slope of the curve $y = f(x)$ at the point p (x_o + h f (x_o +h) is the number

$$m = \lim_{h\to 0} \frac{f(x_o+h)-f(x)}{h} \quad \textbf{(provided the limit exists)}$$

The tangent line to the curve at p is the line through p with this slope.

Example (1)

a) Find the slope of the curve $y = \frac{1}{x}$ at any point $x = a \neq 0$. what is the slope at the point $x = -1$?

b) Where does the slope equal $\frac{-1}{4}$?

c) What happens to the tangent to the curve at the point (a, 1/a) as a changes?

Solution

a) Here $f(x) = 1/x$. The slope at (a, 1/a) is

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{a+h}-\frac{1}{a}}{h}$$
$$= \lim_{h\to 0} \frac{1}{h}\frac{a-(a+h)}{a(a+h)} = \lim_{h\to 0} \frac{-h}{h\,a\,(a+h)}$$

$$= \lim_{h\to 0} \frac{-1}{a\,(a+h)} = \frac{-1}{a^2}$$

Notice how we had to keep writing $\lim_{h\to 0}$ before each fraction until the stage where we could evaluate the limit by substituting h = 0 .

The number a may be positive or negative but not 0. When a =-1 , the slope is $\frac{-1}{(-1)^2} = -1$, as shown in figure 2.

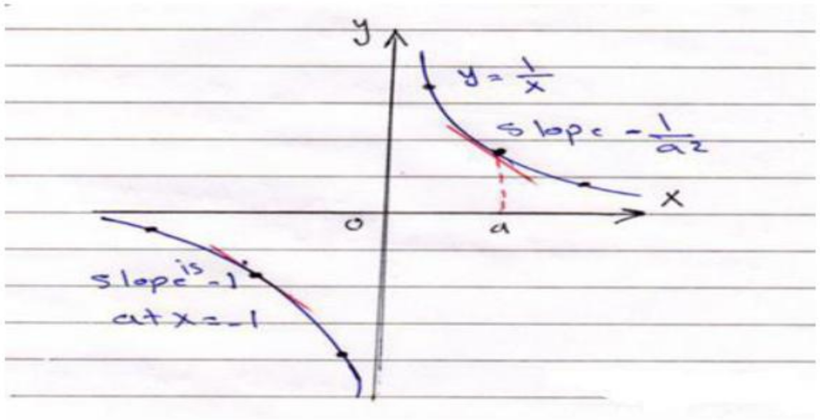

Figure (2)

b) The slope of $y = \frac{1}{x}$ at the point where x =a is $-1/a^2$. It will be provided that -1/4

This equation is equivalent to $a^2 = 4$ so a =2 or a =-2 . The curve has slope -1/4 at the two points $(2, \frac{1}{2})$ and $(-2, -\frac{1}{2})$ as shown in figure 3

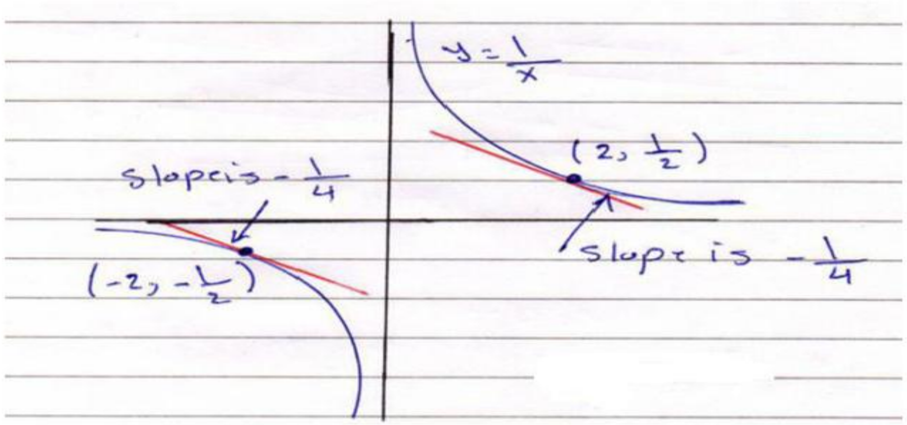

Figure (3)

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c) The slope $\frac{-1}{a^2}$ is always negetive if $a \neq 0$. As $a \to 0^+$, the slope approaches $-\infty$ and the tangent becomes increasingly steep figure 3. We see this situation again as $a \to 0^-$. As a moves away from the origin in either direction, the slope approaches 0 and the tangent levels off to become horizontal.

Differentiation Rules

1. Derivative of a constant function

If has the constant value $f(x) = c$

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

Example (1)

$\frac{d}{dx}5 = 0$

2. Power Rule (General version)

If n is any real number, then

$$\frac{d}{dx}x^n = nx^{n-1}$$

For all x where the powers x^n and x^{n-1}

Example (2)

a) x^3 b) $x^{2/3}$ c) $x^{\sqrt{2}}$ d) $\frac{1}{x^4}$ e) $x^{-4/3}$ f) $\sqrt{x^{2+\pi}}$

Solution

a) $x^3 = 3x^2$

b) $x^{2/3} = \frac{d}{dx}\left(x^{\frac{2}{3}}\right) = \frac{2}{3}x^{-1/3}$

c) $x^{\sqrt{2}} = \frac{d}{dx}\left(x^{\sqrt{2}}\right) = \sqrt{2}x^{\sqrt{2}-1}$

d) $\frac{1}{x^4} = \frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{d}{dx}(x^{-4}) = -4x^{-5} = \frac{-4}{x^5}$

e) $\frac{d}{dx}x^{-4/3} = -\frac{4}{3}x^{-\left(\frac{4}{3}\right)-1} = -\frac{4}{3}x^{-7/3}$

f) $\sqrt{x^{2+\pi}} = \frac{d}{dx}\left(\sqrt{x^{2+\pi}}\right) = \frac{d}{dx}\left(x^{1+\left(\frac{\pi}{2}\right)}\right)$

$$= \left(1+\frac{\pi}{2}\right)x^{1+\left(\frac{\pi}{2}\right)-1} = \frac{1}{2}(2+\pi)\sqrt{x^{\pi}}$$

3. Derivative Sum Rule

If u and v are differentiable functions of x, then their sum u+v is differentiable at every point where u and v are both differentiable. At such points,

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

Example (3)

Find the derivative of the polynomial

$$y = x^3 + \left(\frac{4}{3}\right)x^2 - 5x + 1$$

Solution

$\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}\left(\frac{4}{3}x^2\right) - \frac{d}{dx}5x + \frac{d}{dx}(1)$

$= 3x^2 + \left(\frac{4}{3}\right) * 2x - 5 + 0$

$= 3x^2 + \left(\frac{8}{3}\right)x - 5$

4. Derivative Product Rule

If u and v are differentiable functions of x, then so is their product uv , and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

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