— University of Mosul —
College of Petroleum & Mining Engineering

Mathematics II

Lecture (2)

Transcedental Function

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LECTURE CONTENTS

Inverse Function

Derivatives

Integration

Transcendental Functions

A function that is not algebraic (cannot expressed in terms of algebra) is called transcendental function.
The trancendental functions are:

1- Trigonometric functions
2- Inverse trigonometric functions
3- Logarithmic functions
4- Exponential functions

* Inverse Functions

A function that undoes, or inverts, the effect of a function f is called the inverse of f

One-to-One Functions

A function is a rule that assigns a value from its range to each element in its domain (i.e for each value of x, there is only one value of y)

For example:
$y = x^3$ one-to-one function
$y = 4x-2$ one-to-one function
$y = x^2$ not one-to-one function

Some functions assign the same range value to more than one element in the domain

The symbol for the inverse function is f^{-1}

$$x \longrightarrow f \longrightarrow y \longrightarrow f^{-1} \longrightarrow x = f^{-1}(f(x))$$

Only one-to-one function have inverse

To find the inverse of a function $f(x)$.

1- Express x in terms of y $(x = f(y))$

2- Interchange x and y in the formula of step 1, we get the function $g(x)$ which is the inverse of $f(x)$.

3- Checking the inverse function by finding $f(g(x))$ and $g(f(x))$, if $f(g(x)) = g(f(x)) = x$, the $f(x)$ and $g(x)$ are inverses of one another.

Example (1)

Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x

Sol.

1- solve for x in terms of y

$$y = \frac{1}{2}x + 1$$
$$2y = x + 2$$
$$x = 2y - 2$$

2- Interchange x and y

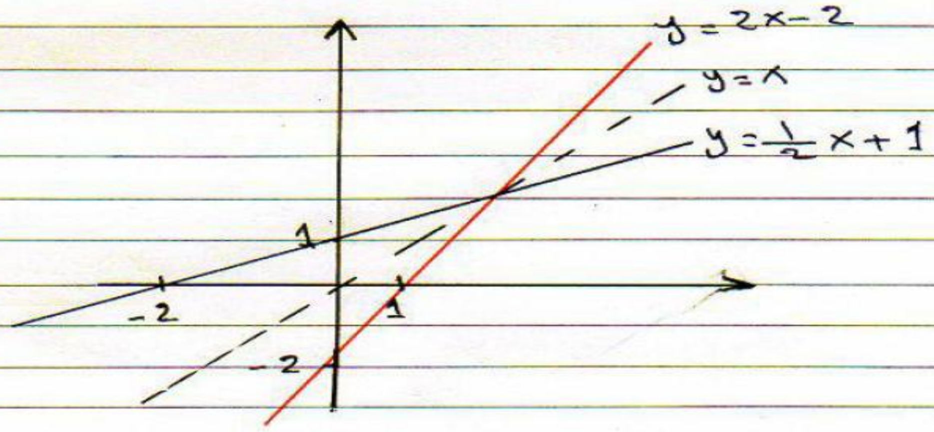
$$y = 2x - 2$$

The inverse of the function $y = \frac{1}{2}x + 1$ is the function $f^{-1}(x) = 2x - 2$ (Figure 1)

3- To check, we verify that both composites give the identity function

$$f^{-1}(f(x)) = 2\left(\frac{1}{2}x + 1\right) - 2 = x + 2 - 2 = x$$

$$f(f^{-1}(x)) = \frac{1}{2}(2x - 2) + 1 = x - 1 + 1 = x$$



Figure(1)

Example (2)

Find the inverse of $y = x/4 + 3$

Sol.

$y = \dfrac{x+12}{4}$, y is one-to-one function

1- Find $x = f(y)$

$4y = x + 12$, $x = 4y - 12 = f(y)$

2- $y = 4x - 12$

3- check $f(x) = \dfrac{x+12}{4}$, $g(x) = 4x - 12$

$f(g(x)) = \dfrac{(4x-12)+12}{4} = x$

$g(f(x)) = 4\left(\dfrac{x+12}{4}\right) - 12 = x = f(g(x))$

$g(x)$ is the inverse of $f(x)$

Example (3)

Derivative the inverse of $y = \frac{1}{2}x + 1$

Sol.

$$y = \frac{1}{2}x + 1$$

$$2y = x + 2$$

$$x = 2y - 2$$

Interchange x and y

$\therefore \; y = 2x - 2$

The inverse of the function $y = \frac{1}{2}x + 1$ is the function $f^{-1}(x) = 2x - 2$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{1}{2}x + 1\right) = \frac{1}{2}$$

$$\frac{d}{dx} f^{-1}(x) = \frac{d}{dx}(2x - 2) = 2$$