

PROPERTIES OF INTEGRAL :-

$$\int_a^b dx = b - a \Rightarrow \int_a^b c \, dx = c(b - a)$$

$$\int_a^b c \cdot f(x) \, dx = c \cdot \int_a^b f(x) \, dx$$

$$\int_a^a f(x) \, dx = 0$$

$$\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$$

$$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx, c \in [a, b]$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$$

$$\int_a^b [f(x) \pm g(x)] \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx$$

$$\text{if } f(x) \geq 0 \Rightarrow \int_a^b f(x) \, dx \geq 0$$

$$\text{if } f(x) \leq g(x) \Rightarrow \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx$$

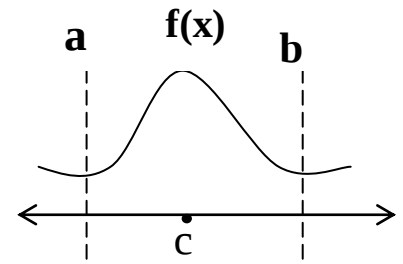
$$\int [f(x)]^n f'(x) \, dx = \frac{[f(x)]^{n+1}}{n+1} + C, n \neq -1$$

EXAM :-

$$\int_6^{10} dx = 10 - 6 = 4$$

$$\int_6^{10} 5 \, dx = 5(10 - 6) = 5 \cdot 4 = 20$$

$$\int_1^3 x \, dx = \left. \frac{x^2}{2} \right|_1^3 = \frac{1}{2}(9 - 1) = 4$$



$$\int_2^5 (x+5)dx = \frac{x^2}{2} + 5x \Big|_2^5 = 25.5$$

$$\int_0^1 (x^2 + 3x)dx = \frac{x^3}{3} + \frac{3}{2}x^2 \Big|_0^1 = \frac{11}{6}$$

$$\int_1^3 (2x^2 + 5x)^2 (4x + 5)dx = \frac{(2x^2 + 5x)^3}{3} \Big|_1^3 = ?$$

$$\int_1^3 (2x^2 + 6x)^2 (2x + 3)dx = \frac{1}{2} \int_1^3 (2x^2 + 6x)^2 (4x + 6)dx =$$

$$= \frac{(2x^2 + 6x)^3}{3} \Big|_1^3 = ?$$

$$\int_0^1 (x^3 + 1)^2 dx = \int_0^1 (x^6 + 2x^3 + 1)dx = \frac{x^7}{7} + \frac{x^4}{2} + x \Big|_0^1 = ?$$

EQUATION OF STRAIGHT LINE :

معادلة الخط المستقيم

The general form of straight line equation is :-

$$ax + by + c = 0$$

OR

$$y = mx + b$$

EXAM : - Find the equation of the curve whose slope at any point p(x,y) is $2x+1$ and passing through the point (1,3) .

$$m = \frac{dy}{dx} = 2x + 1$$

$$y = \int \frac{dy}{dx} dx = \int (2x + 1)dx \Rightarrow \boxed{y = x^2 + x + C} \Leftarrow \text{general curves}$$

$$(1,3) \in \text{curves} \Rightarrow 3 = 1^2 + 1 + C \Rightarrow C = 1$$

$$\boxed{y = x^2 + x + 1} \Leftarrow \text{special curve}$$

EXAM : - Find the equation of the curve whose slope at any point p(x,y) is

$$4x^3 + 18x^2 + 8x + 3$$

and passing through the point (1,11) .

$$m = \frac{dy}{dx} = 4x^3 + 18x^2 + 8x + 3$$

$$y = \int (4x^3 + 18x^2 + 8x + 3) dx$$

$$y = x^4 + 6x^3 + 4x^2 + 3x + C$$

$$11 = 1 + 6 + 4 + 3 + C \Rightarrow C = -3$$

$$y = x^4 + 6x^3 + 4x^2 + 3x - 3$$

DOUBLE INTEGRATION :

EXAM :

$$\int_0^1 \int_0^x (3 - x - y) dy dx$$

$$= \int_0^1 \left(3y - xy - \frac{1}{2}y^2 \right) \Big|_0^x dx = \int_0^1 \left(3x - x^2 - \frac{1}{2}x^2 \right) dx = \int_0^1 \left(3x - \frac{3}{2}x^2 \right) dx$$

$$= \left(\frac{3}{2}x^2 - \frac{1}{2}x^3 \right) \Big|_0^1 = 1$$

EXAM :

$$\int_1^2 \int_y^{y^2} dx dy$$

$$= \int_1^2 x \Big|_y^{y^2} dy = \int_1^2 (y^2 - y) dy = \left(\frac{1}{3}y^3 - \frac{1}{2}y^2 \right) \Big|_1^2 = \frac{5}{6}$$

EXAM :

$$\int_0^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y dx dy$$

$$= \int_0^{\sqrt{2}} yx \Big|_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} dx = \int_0^{\sqrt{2}} y \sqrt{4-2y^2} + y \sqrt{4-2y^2} dx = 2 \int_0^{\sqrt{2}} y \sqrt{4-2y^2} dx$$

$$\left. \frac{2}{-4} \frac{(4-2y^2)^{\frac{3}{2}}}{\frac{3}{2}} \right|_0^{\sqrt{2}} = \frac{8}{3}$$

TRIPLE INTEGRATION :**EXAM :**

$$\begin{aligned}
& \int_0^1 \int_0^x \int_{-y^2}^{x^2} (x+1) dz dy dx \\
&= \int_0^1 \int_0^x (x+1) z \Big|_{-y^2}^{x^2} dy dx = \int_0^1 \int_0^x (x+1) x^2 + (x+1) y^2 dy dx \\
&= \int_0^1 \int_0^x x^3 + x^2 + xy^2 + y^2 dy dx = \int_0^1 x^3 y + x^2 y + \frac{1}{3} xy^3 + \frac{1}{3} y^3 \Big|_0^x dx \\
&= \int_0^1 x^4 + x^3 + \frac{1}{3} x^4 + \frac{1}{3} x^3 dx = \int_0^1 \frac{4}{3} x^4 + \frac{4}{3} x^3 dx = \frac{4}{15} x^5 + \frac{1}{3} x^4 \Big|_0^1 = \frac{9}{15}
\end{aligned}$$

EXAM :

$$\begin{aligned}
& \int_0^1 \int_0^x \int_{-y^2}^{x^2} dz dy dx \\
&= \int_0^1 \int_0^x z \Big|_{-y^2}^{x^2} dy dx = \int_0^1 \int_0^x x^2 + y^2 dy dx \\
&= \int_0^1 x^2 y + \frac{1}{3} y^3 \Big|_0^x dx = \int_0^1 x^3 + \frac{1}{3} x^3 dx \\
&= \int_0^1 \frac{4}{3} x^3 dx = \frac{1}{3} x^4 \Big|_0^1 = \frac{1}{3} \\
&= \int_0^1 x^4 + x^3 + \frac{1}{3} x^4 + \frac{1}{3} x^3 dx = \int_0^1 \frac{4}{3} x^4 + \frac{4}{3} x^3 dx = \frac{4}{15} x^5 + \frac{1}{3} x^4 \Big|_0^1 = \frac{9}{15}
\end{aligned}$$

HOME WORK

1) Find the equation of the curve whose slope at any point $p(x,y)$ is
 $m = (x^4 + 16x + 4)^2 (x^3 + 4)$
 and passing through the point $(2,1)$.

2) Find the equation of the curve whose slope at any point $p(x,y)$ is
 $m = x(x + 5)^2$
 and passing through the point $(2,1)$.

3) Find :

$$\int_0^1 \int_0^3 x \sqrt{x^2 + y} \, dy \, dx$$

$$\int_0^1 \int_{-1}^{\sqrt{y}} y \, dx \, dy$$

$$\int_0^1 \int_x^{x^2} \int_{x-y}^{x+y} (x + 2y + 4z) \, dz \, dy \, dx$$

$$\int_0^1 \int_0^2 \int_0^3 (z^3 y^2 x) \, dx \, dy \, dz$$

