

Permeability

Permeability is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids. The rock permeability, k , is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by Henry Darcy in 1856. By analogy with electrical conductors, permeability represents reciprocal of resistance which porous medium offers to fluid flow. Poiseuille's equation for viscous flow **in a cylindrical tube** is a well-known equation

$$v = \frac{d^2 \Delta P}{32 \mu L}$$

Where:

v = fluid velocity, cm/sec

d = tube diameter, cm

Δp = pressure loss over length L ,

μ = fluid viscosity, centipoise

L = length over which pressure loss is measured, cm

A more convenient form of Poiseuille's equation is (multiplied by area=)

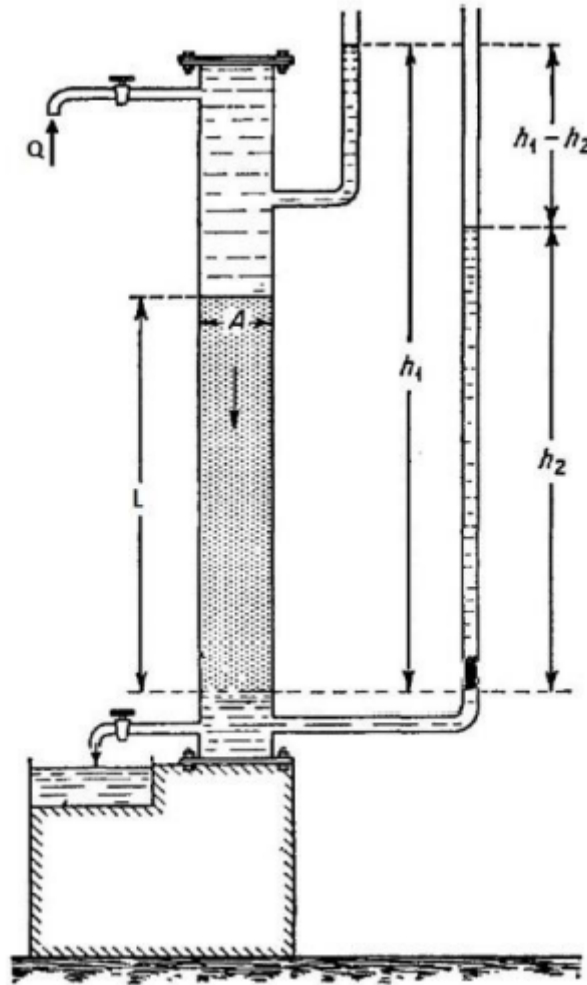
$$Q = \frac{\pi r^4 \Delta P}{8 \mu L}$$

A cast of the flow channel in a rock formation is shown in (Figure below). It is seen that the flow channels are of varying sizes and shapes and are randomly connected. So it is not correct to use the Poiseuille's equation for flow in the porous media.



In 1856, Darcy developed equation to fluid flow (flow of water through sand filter) as schematically in below:

$$Q = KA \frac{h_1 - h_2}{L}$$



Here Q represents the volume rate of flow of water downward through the cylindrical sand pack of cross sectional area A and height h **and K is a proportionality constant.**

Later investigator found that Darcy's law could be extended to other fluid as well as water and that the **constant of proportionality K** could be written as

$$Q = - \frac{KA}{\mu} \frac{dP}{dL}$$

Where:

K = Proportionality constant or permeability, Darcy's

μ = Viscosity of flowing fluid, cp

Δp = Pressure drop per unit length, atm/cm

Q = Volumetric flow rate, This is a linear law, cc/sec

Then the generalized form of Darcy's law

$$Q = \frac{K A \Delta p}{\mu L}$$

S = distance in direction of flow and is always positive, cm

V = volume flux across a unit area of the porous medium in unit time along flow path, s, cm/sec

Z = vertical coordinate, considered positive downward, cm

ρ = density of fluid, gm/cc

g = acceleration of gravity, 980.665 cm/sec²

dp/ds = pressure gradient along s at the point to which V refers, atm/cm

μ = viscosity of fluid, centipoises

k = permeability of the medium, darcys

1.0133×10^6 = dynes/(sq. cm)(atm)

$$\lambda = \frac{k}{\mu} = \text{Mobility}$$

Incompressible fluid and is **horizontal**, then $dz/ds = 0$, $dp/ds = dp/dx$,

Then

$\times (A/A)$ left side

by using of reparative variables,

Integrating between the limits 0 and L in x and P_1 and P_2 where P_1 is the pressure at the inflow face and P_2 the pressure at the outflow face,

If KA is permitted to equal the flow coefficient C defined with Poiseuille's equation, it is seen that the two expressions are identical such that

$$C = \frac{\pi}{8} \sum_{j=1}^k n_j r_j^4 = kA = \frac{\pi}{8} \sum_{j=1}^k n_j r_j^4$$

“Darcy” is a practical unit of permeability (in honor of Henry Darcy). A porous material has permeability equal to 1 Darcy if a pressure difference of 1 atm will produce a flow rate of $1 \text{ cm}^3/\text{sec}$ of a fluid with 1 cp viscosity through a cube having side 1 cm in length.

Thus

$$1 \text{ darcy} = \frac{1 \left(\frac{\text{cm}^3}{\text{sec}} \right) 1 (\text{cp})}{1 (\text{cm}^2) 1 \left(\frac{\text{atm}}{\text{cm}} \right)} = 0.987 \mu\text{m}^2$$

The dimension of permeability can be established by substituting the units of the other terms into eq.

()

Let

L= length

M= mass

T= time

Then

$$v_s = \frac{L}{T}$$

$$\mu = \frac{M}{LT}$$

$$\rho = \frac{M}{L^3}$$

$$P = \frac{M}{LT^2}$$

$$\frac{dP}{ds} = \frac{M}{L^2 T^2}$$

$$g = \frac{L}{T^2}$$

$$\frac{dz}{ds} = \text{dimensionless}$$

Substituting the dimensions in eq. above

$$\begin{aligned}
\frac{L}{T} &= \frac{k}{M/LT} \left(\frac{M}{L^3 T^2} - \frac{M}{L^3} \frac{L}{T^2} \right) \\
&= \frac{kLT}{M} \left(\frac{M}{L^3 T^2} - \frac{M}{L^3 T^2} \right) \\
&= \frac{k}{LT} \\
k &= L^2
\end{aligned}$$

A rational unit of permeability in the English system of units would be foot squared, cgs system the centimeter squared both were found to be too large a measure to use. Therefore, the petroleum industry adopted as the unit of permeability, the Darcy.

Conversion of Units in Darcy's Law.

Linear Flow: Liquids (or Gases with Volume at Mean Pressure).

Rate in barrels per day:

$$Q = 1.1271 \frac{kA(P_1 - P_2)}{\mu L}$$

Rate in cubic feet per day:

$$Q = 6.3230 \frac{kA(P_1 - P_2)}{\mu L}$$

where

Q is the volume rate of flow,

P_1 and P_2 are in pounds per square inch,

k is in darcys,

μ is in centipoise,

A is in square feet,

L is in feet.

Radial Flow: Liquids (or Gases with Volume at Mean Pressure).

Rate in barrels per day:

$$Q = 7.082 \frac{kh(P_e - P_w)}{\ln(r_e/r_w)}$$

Rate in cubic feet per day:

$$Q = 39.76 \frac{kh(P_e - P_w)}{\ln(r_e/r_w)}$$

Q is the volume rate of flow,

P_e and P_w are in pounds per square inch,

k is in darcys, μ is in centipoises,

h is in feet,

and r_e and r_w are in consistent units.

The above equations describe the flow in the porous medium when the rock is 100 per cent saturated with the flowing fluid.

ملاحظة

إذا فرضنا لدينا حالتان من الجريان

1. Tube

2. Core

نفس الطول ونفس مقطع الجريان و نفس معدل الجريان

في اي النموذجين تكون السرعة اكبر

, Q is same

A pipe > A core

اذن السرعة الظاهرية في الانبوب هي نفس السرعة الحقيقية
وعليه فان السرعة التي في الوسط المسامي هي سرعة ظاهرية

V interstitial =

For steady-state linear flow in oilfield units:

$$Q = \frac{0.001127kA}{\mu B_o} \frac{(p_1 - p_2)}{L}$$

where:

Q = volumetric flowrate, STB/Day.

k = absolute permeability of the rock, millidarcy (md).

A = cross sectional area in the flow direction, ft² .

p₁ = inlet pressure, psig.

p₂ = outlet pressure, psig.

μ = fluid viscosity, cp.

L = length of the rock, ft.

B_o = oil formation volume factor, bbl/STB.

For radial flow of fluids into a wellbore, darcy's law may be expressed in radial coordinates in field units as:

$$Q = \frac{0.001127.2\pi.kh(p_e - p_w)}{\mu B_o \ln \left(\frac{r_e}{r_w} \right)}$$

$$Q = \frac{0.00708kh(p_e - p_w)}{\mu B_o \ln \left(\frac{r_e}{r_w} \right)}$$

where:

Q = volumetric flow rate, STB/Day.

k = absolute permeability of the rock, millidarcy (md).

h = pay thickness, ft.

pe= pressure at external radius, psig.

pw = pressure at wellbore, psig.

μ = fluid viscosity, cp.

L = length of the rock, ft.

Bo = oil formation volume factor, bbl/STB.

re = external drainage area, ft.

rw = wellbore radius, ft.

ln = natural logarithm.

مطلوب اثبات ذلك

$$q = \frac{2\pi kh(p_e - p_w)}{\mu \left[\ln \frac{r_e}{r_w} \right]}$$

The Klinkenberg Effect

Darcy's modified law for gases is not applicable at low gas pressures (gas densities). Klinkenberg (1941) reported a variation in the permeability test results with the pressure when gas is used as testing fluid. Klinkenberg found that for a given porous medium as the mean pressure increased the calculated permeability decreased. Mean pressure is defined as follow:

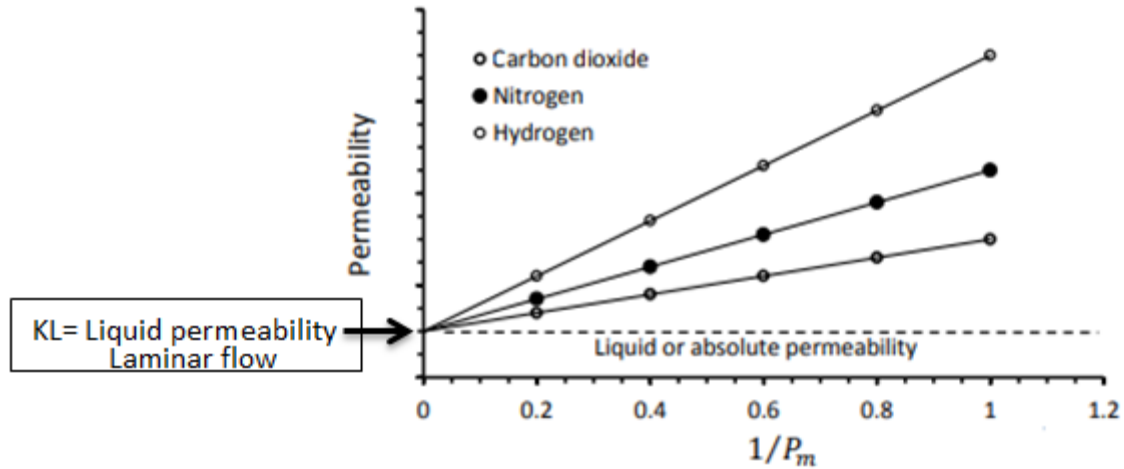
$$P_m = \frac{(P_1 + P_2)}{2}$$

This variation caused by "gas slippage" phenomenon. The phenomenon of gas slippage occurs when the diameter of the capillary opening approaches the mean free path of the gas. When, flowing of the gas through the porous media the velocity at the solid wall cannot, in general, be considered zero, but a so called "slip" velocity at the wall must be taken into account. This effect becomes significant when the mean free path of the gas molecules is of comparable magnitude as the pore size. When the mean free path is such smaller than the pore size, the slip velocity becomes negligibly small. As in liquids the mean free path of molecules is of the order of the molecular diameter, so the no-slip condition always applied in liquid flow. The mean free path of a gas is a function of molecular size and the kinetic energy of the gas. Therefore the "Klinkenberg Effect" is a function of the gas that is used as testing fluid and the conditions of the test like as pressure and temperature. Figure below is a plot of the permeability of the porous medium as determined at various mean pressures using hydrogen, nitrogen and carbon dioxide as the testing fluids.

متوسط المسار الحر للغاز هو دالة لعاملين

1. للحجم الجزيئي
2. الطاقة الحركية للغاز

وبالتالي "تأثير Klinkenberg" هو دالة للغاز المستخدم كسائل اختبار وظروف الاختبار مثل الضغط ودرجة الحرارة. الشكل ادناه عبارة عن مخطط لنفاذية الوسط المسامي تم تحديده عند ضغوط متوسطة مختلفة باستخدام الهيدروجين والنيتروجين وثاني أكسيد الكربون كاختبار سوائل.



Permeability of Core Sample to Three Different Gases and Different Mean Pressure

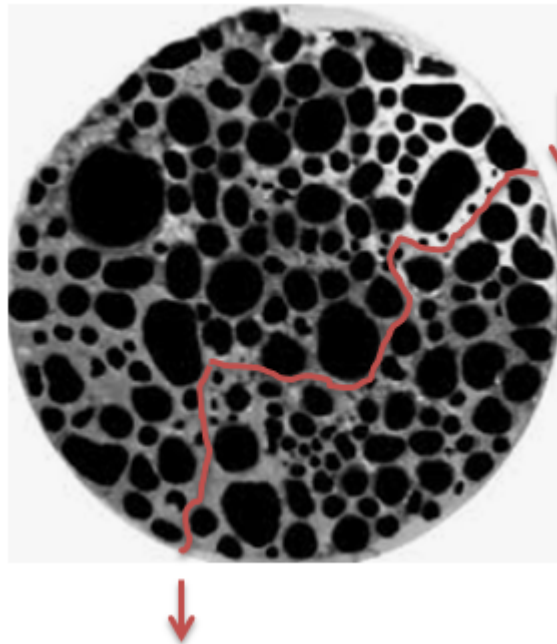
لوحظ مختبريا ان ظاهرة الانزلاق تختفي عندما يكون متوسط ضغط الفحص اكبر من 150 psi

كما يجب الاشارة الى ان حجم فتحات المسام تكون اكبر عند الضغط الاقل

$Q_{in} = Q_{out}$ liquid state

$Q_{in} > Q_{out}$ gas state

ولهذا فأننا نقوم مختبريا بحساب Q_m



$K_g =$

اذن افضل معادلة تمثل هذه العلاقة هي

$y = a + bx$ (straight line)

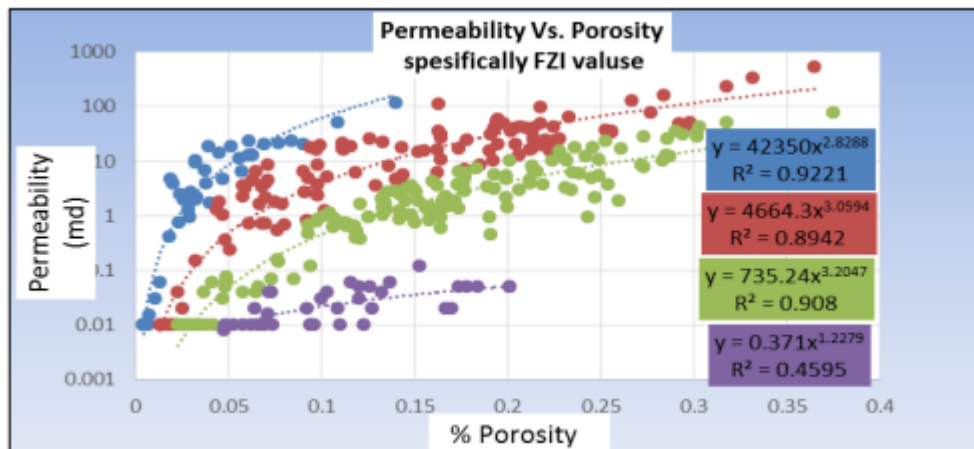
$K_g = K_L(1 +$

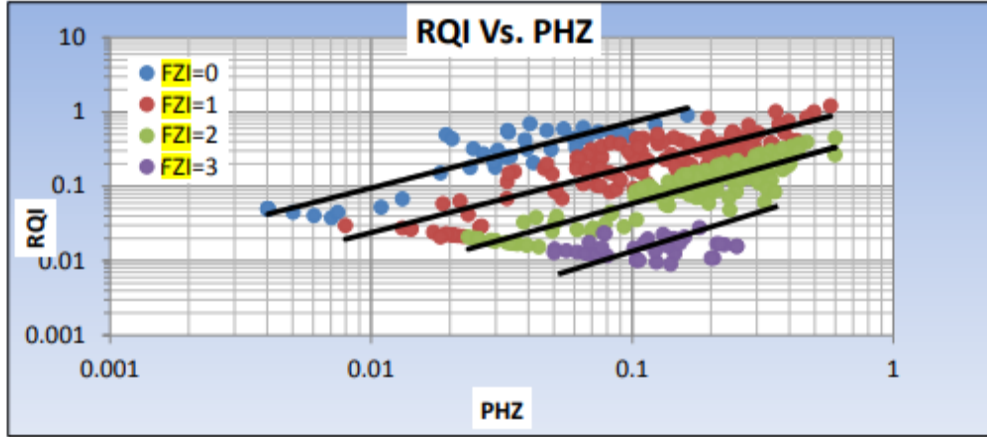
$K_g = K_L +$

$K_g = K_L +$

porosity and permeability relationship

نتيجة الفحوصات المختبرية وجد ان هناك علاقة بين المسامية والنفذية





hydraulic flow units (HFU) is utilized within the field of oil industry so as to enhance the prediction of permeability measurement in uncored interval. The abovementioned notion (HFU) is closely connected to the flow zone indicator (FZI) considered as a function of the reservoir quality index (RQI). (FZI) and (RQI) depend on porosity and permeability cores. These up to date techniques are considered as basic for hybrid soft computing methods and are utilized to predict permeability

$$RQI = 0.0314 \sqrt{\frac{K}{\Phi}}$$

$$\left(\frac{\Phi}{1-\Phi}\right) = \Phi_z$$

$$FZI = \frac{RQI}{\Phi_z}$$

since

$$\Phi_z = \frac{\Phi_e}{1-\Phi_e}$$

By applying the logarithm of two sides of the equation stated above gives:

$$\log RQI = \log FZI + \log \phi_z$$

normalized porosity : pore volume-to-grain volume ratio effective porosity (fraction)

Hydraulic Flow Units: (HFUs) are known as correlatable and mappable zones inside a reservoir ,so control fluid movement. It is very closely connected to FZI.

Flow Zone Indicator (FZI): Flow Zone Indicator is an effective and practical value to evaluate the flow properties of a reservoir and it creates a relationship between petrophysical characteristics at small-scale, for example (core sample), and large-scale, as (well sections). Besides, the FZI offers the representation of the flow zones that depend on the surface area and path of flow. Incorporated well log data is the basis of the FZI. Permeability, which is the indicator of rock's flow properties, is based on porosity properties (distribution and connectivity).

Pressure Potential & Pressure Gradient in Static Fluid Columns

التطبيقات العملية لجهد الجاذبية وجهد الضغط في الحالات الساكنة والتي من الممكن ملاحظتها في جوف البئر well bore و منظومة المكن reservoir system.

في حالة وجود مقاييس للضغط ممكن انزالها الى قعر البئر سيكون من الممكن ان نسجل قيمة الضغط عند اعماق مختلفة, وبالتالي يمكن الحصول على قياسات مختلفة لكثافات السوائل المكمية.

وعليه فان الضغوط المكمية يمكن ان تقسم الى ثلاث انواع:

1- Lithostatic (overburden) pressure

Press. Gradient =1 psi/ft

وهو يمثل التدرج الناتج عن وزن الصخور من السطح الى المكن

2- Pore pressure

Reservoir gradient = 0.433 psi/ft for fresh water

= 0.465 psi/ft for salt water

وهذا الضغط له علاقة بنوعية المائع المكملي وقد اصطلح على حساب تدرج ال pore pressure بدلالة الماء وتتغير قيمة التدرج بتغير ملوحة الماء salinity اي هو دلالة لكثافة الموائع.

3- Grain pressure = overburden pressure-pore pressure

Then, if the case of reservoir calculations,

Reservoir fluid gradient = Reservoir fluid specific gravity × Hydraulic gradient
fresh water

Reservoir fluid gradient = × 0.433 = psi/ft

P= Patm.+ (gradient) × Depth

P= Patm.+ Overburden gradient at sea bed OBG

Then

P= Patm.+ [0.433 × psea-water(specific gravity of sea water) × h]

specific gravity of sea water 1.03 gm/cc

h= ft

ρ= gm/cc

Hydraulic gradients in reservoirs vary from a maximum near 0.500 psi/ft for brines to 0.433 psi/ft for fresh water at 60°F, depending on the pressure, temperature, and salinity of the water.

