Steady-State Model (Schilthuis Model)

- Schilthuis (1936) proposed that for an aquifer that is flowing under the steady-state flow regime, the flow behavior could be described by Darcy's equation.

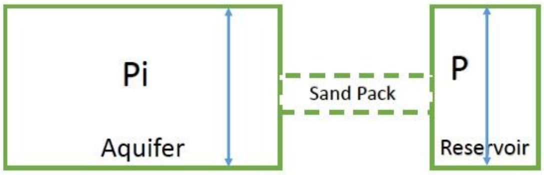
$$ew = \frac{dWe}{dt} = \frac{0.00708 k_a h_a}{\mu_w \ln(\frac{r_a}{r_e})} (p_i - p) \qquad (8) \quad or \qquad ew = C(p_i - p) \qquad (9)$$

where e_w = rate of water influx, bbl/day
k = permeability of the aquifer, md
h = thickness of the aquifer, ft
r_a = radius of the aquifer, ft
r_e = radius of the reservoir
t = time, days
Pi = initial aquifer pressure, psi
P = avg. reservoir pressure, psi

The parameter **C** is called "**The Water Influx Constant**", and expressed in bbl/D/psia.

- The parameter C may be estimated by combining Equation 1 with 9.

$$e_w = Q_o B_o + Q_g B_g + Q_w B_w$$



Example: Calculate water influx constant : p_i = 3500 psia, p = 3000 psia,
Q_o = 32000 STB/D, B_0= 1.4 bbl/STB, GOR =900 SCF/STB, Rs = 700 SCF/STB, B_g= 0.00082 bbl/ SCF, Q_w= 0, B_w= 1.0 bbl/STB.

Solution:

$$ew = Q_o B_0 + Q_g B_g + Q_w B_w$$

$$e_w = \frac{dW_e}{dt} = B_o \frac{dN_p}{dt} + (GOR - R_s)\frac{dN_p}{dt} B_g + \frac{dW_p}{dt} B_w$$

$$e_w = (1.4)(32{,}000) + (900 - 700)(32{,}000)(0.00082) + 0$$
$$= 50{,}048 \text{ bbl/day}$$

Step 2. Solve for the water influx constant from Equation 9.

$$\frac{dWe}{dt} = ew = C(p_i - p)$$

$$C = \frac{50{,}048}{(3500 - 3000)} = 100 \text{ bbl/day/psi}$$

Note that the pressure drops contributing to influx are the cumulative pressure drops from the initial pressure.

In terms of the cumulative water influx We, Equation 9 is integrated to give the Schilthuis expression for water influx as:

$$\int_o^{We} dW_e = \int_o^t C(p_i - p)\,dt$$

$$or \quad We = C\int_0^t (p_i - p)\,dt \qquad (10)$$

where W_e = cumulative water influx, bbl

C = water influx constant, bbl/day/psi

t = time, days

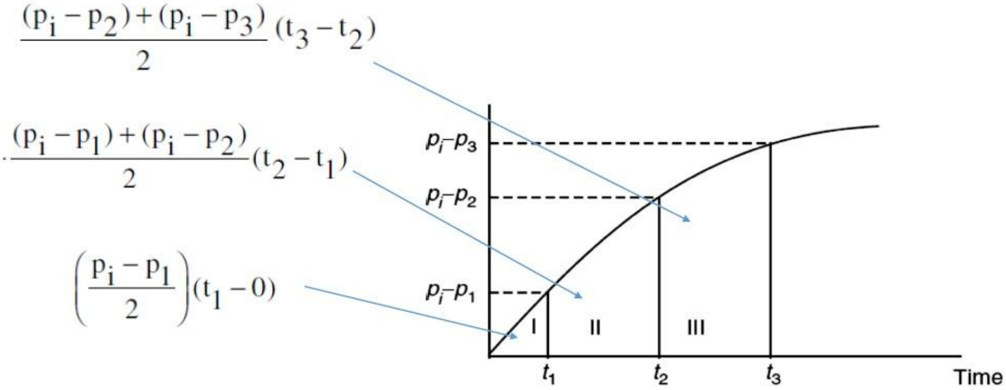
p_i = initial reservoir pressure, psi

p = pressure at the oil-water contact at time t, psi

When the pressure drop (pi – p) is plotted versus the time t, **the area under the curve** represents the integral $\int_0^t (p_i - p)\, dt$, and can be calculated by trapezoidal rule.

$$\int_0^t (p_i - p) \quad dt = \text{area}_{I} + \text{area}_{II} + \text{area}_{III} + \text{etc.} = \left(\frac{p_i - p_1}{2}\right)(t_1 - 0)$$
$$+ \frac{(p_i - p_1) + (p_i - p_2)}{2}(t_2 - t_1) + \frac{(p_i - p_2) + (p_i - p_3)}{2}(t_3 - t_2)$$
$$+ \text{etc.}$$

$$We = C \sum_0^t (\Delta p)\, \Delta t \qquad (11)$$



Example: The pressure history of a water drive oil reservoir is given below:

The aquifer is under a steady-state flowing condition with an estimated water influx constant of 130 bbl/day/psi. Calculate the cumulative water influx after 100, 200, 300, and 400 days using the steady-state model.

t ,days	P, psia
0	3500
100	3450
200	3410
300	3380
400	3340

Solution:

1. Calculate the total pressure drop at each time, t.

t ,days	P, psia	$(P_i\text{-}P)$
0	3500	0
100	3450	50
200	3410	90
300	3380	120
400	3340	160

2. Calculate the cumulative water influx **We** after 100 days.

$$We = C\left[\frac{(p_i - p)}{2}(t_1 - 0)\right] = 130 \times \left(\frac{50}{2}\right)(100 - 0) = 325000\ bbl$$

3. Calculate the cumulative water influx **We** after 200 days.

$$We = C\left[\frac{(p_i - p)}{2}(t_1 - 0) + \frac{(p_i - p_1) + (p_i - p_2)}{2}(t_2 - t_1)\right]$$

$$W_e = 130\left[\left(\frac{50}{2}\right)(100 - 0) + \left(\frac{50 + 90}{2}\right)(200 - 100)\right] = 1,235,000\ \text{bbl}$$

4. Calculate the cumulative water influx **We** after 300 days.

$$W_e = 130\left[\left(\frac{50}{2}\right)(100) + \left(\frac{50 + 90}{2}\right)(200 - 100) + \left(\frac{120 + 90}{2}\right)(300 - 200)\right] = 2,600,000\ \text{bbl}$$

5. Calculate the cumulative water influx **We** after 400 days.

$$W_e = 130\left[2500 + 7000 + 10,500 + \left(\frac{160 + 120}{2}\right)(400 - 300)\right]$$

$$= 4,420,000\ \text{bbl}$$

Modified Steady-State Model (Hurst Model)

- The dimensionless radius r_a/r_e may be replaced with time-dependent function as given below;

$$r_a/r_e = a\,t \qquad (12)$$

Substituting Equation 12 into Equation 8 gives:

$$ew = \frac{dWe}{dt} = \frac{0.00708 k_a h_a}{\mu_w \ln(\frac{r_a}{r_e})}(p_i - p)$$

$$ew = \frac{dWe}{dt} = \frac{0.00708 k_a h_a}{\mu_w \ln(at)}(p_i - p) \qquad (13)$$

- The Hurst modified steady-state equation can be written in a more simplified form as:

$$ew = \frac{dWe}{dt} = \frac{C}{\ln(at)}\,(p_i - p) \qquad (14)$$

and in terms of the cumulative water influx;

$$We = C\int_0^t \frac{(p_i - p)}{\ln(at)}dt \qquad (15)$$

$$We = C\sum_0^t \frac{\Delta p}{\ln(at)}\,\Delta t \qquad (16)$$

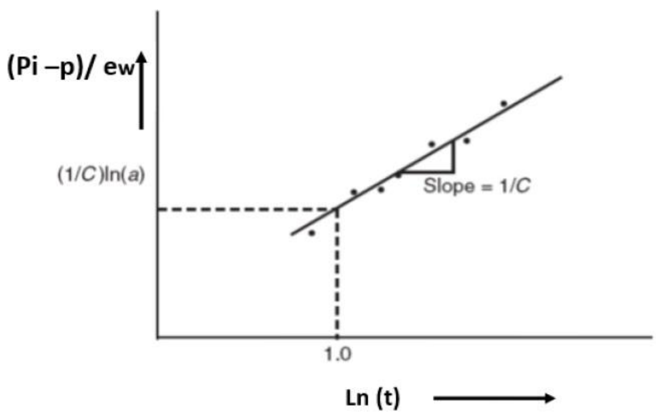
- The Hurst modified steady-state equation contains **two unknown constants**, a and C, that must be determined from the reservoir-aquifer pressure and water influx historical data.
- The procedure of determining the constants a and C is based on Equation 14 as a linear relationship.

$$\left(\frac{p_i - p}{e_w}\right) = \frac{1}{C}\ln(at)$$

or

$$\frac{p_i - p}{ew} = \frac{1}{C}\ln(a) + \frac{1}{C}\ln(t) \qquad (17)$$

- Equation 17 indicates that a plot of (pi − p)/ew versus ln(t) will be a straight line with a slope of 1/C and intercept of (1/C)ln(a).



Problem (Schiltuis model)

The pressure history of a water-drive oil reservoir is given below:

t, days	p, psi
0	4000
120	3950
220	3910
320	3880
420	3840

The aquifer is under a steady-state flowing condition with an estimated water influx constant of 80 bbl/day/psi. Using the steady-state model, calculate and plot the cumulative water influx as a function of time.