THE MATERIAL BALANCE EQUATION

When reservoir discovered, it may contain water, gas and oil intermingled or segregated into zones. As it is produced from a hydrocarbon reservoir. Fluid does not leave a void space behind, As the pressure in the reservoir drops during the production of fluids, the remaining fluids and/or reservoir rock expand or nearby water encroaches to fill the space created by any produced fluids. For this reason the sum of the pore volumes occupied by the fluids remaining at pressure p and those added between pressures pi and p must equal the volume at pressure p of the initial pore volume.

The volume of oil produced on the surface aids the reservoir engineer in determining the amount of the expansion or encroachment that occurs in the reservoir. Material balance is a method that can be used to account for the movement of reservoir fluids within the reservoir or to the surface where they are produced. The general material balance equation was first developed by Schilthuis in 1936.

By taking bottom-hole samples of the reservoir fluids under pressure and measuring their relative volumes in the laboratory at reservoir temperature and under various pressures, it is possible to predict how these fluids behave in the reservoir as reservoir pressure declines. Although the connate water and formation compressibilities are quite small, they are, relative to the compressibility of reservoir fluids above their bubble points, significant, and they account for an appreciable fraction of the production above the bubble point.

Range of Compressibilities

| | |
|---|---|
| Formation rock | $3 - 10 \times 10^{-6}$ psi$^{-1}$ |
| Water | $2 - 4 \times 10^{-6}$ psi$^{-1}$ |
| Undersaturated oil | $5 - 100 \times 10^{-6}$ psi$^{-1}$ |
| Gas at 1000 psi | $900 - 1300 \times 10^{-6}$ psi$^{-1}$ |
| Gas at 5000 psi | $50 - 200 \times 10^{-6}$ psi$^{-1}$ |

The general material balance equation is simply a volumetric balance, which states that since the volume of a reservoir (as defined by its initial limits) is a constant, the algebraic sum of the volume changes of the oil, free gas, water, and rock volumes in the reservoir must be zero. For example, if both the oil and gas reservoir volumes decrease, the sum of these two decreases must be balanced by changes of equal magnitude in the water and rock volumes.

In another meaning, the equation is structured to simply keep inventory of all materials entering, leaving, and accumulating in the reservoir.

The MBE, when properly applied, can be used to:

- Estimate initial hydrocarbon volumes in place
- Predict future reservoir performance
- Predict ultimate hydrocarbon recovery under various types of primary driving mechanisms.

Then, in its simplest form, the equation can be written on a volumetric basis as:

Initial volume = volume remaining + volume removed

Since oil, gas, and water are present in petroleum reservoirs, the material balance equation can be expressed for the total fluids or for any one of the fluids presents.

The basic assumptions in the material balance equation (MBE) are as follows:

- Constant temperature. Pressure-volume changes in the reservoir are assumed to occur without any temperature changes. If any temperature changes occur, they are usually sufficiently small to be ignored without significant error.
- Pressure equilibrium. All parts of the reservoir have the same pressure, and fluid properties are therefore constant throughout. Minor variations in the vicinity of the well bores may usually be ignored.

Substantial pressure variation across the reservoir may cause excessive calculation error.

- Constant reservoir volume.
- Reliable production data that must be recorded in order to use the MBE in performing reliable reservoir calculations. These are

1. Oil-production data,
2. Gas-production data,
3. The water-production term, which need represent only the net withdrawals of water.

All production data should be recorded with respect to the same time period. In the development of the general material balance equation, the following terms are used:

N Initial reservoir oil, STB

$Bo_i$ Initial oil formation volume factor, bbl/STB

Np Cumulative produced oil, STB

Bo Oil formation volume factor, bbl/STB

G Initial reservoir gas, SCF

$Bg_i$ Initial gas formation volume factor, bbl/SCF

$G_f$ Amount of free gas in the reservoir, SCF

$R_{soi}$ Initial solution gas-oil ratio, SCF/STB

Rp Cumulative produced gas-oil ratio, SCF/STB

$R_{so}$ Solution gas-oil ratio, SCF/STB

Bg Gas formation volume factor, bbl/SCF

W Initial reservoir water, bbl

Wp Cumulative produced water, STB

Bw Water formation volume factor, bbl/STB

We Water influx into reservoir, bbl

c Total isothermal compressibility, $psi^{-1}$

Δp Change in average reservoir pressure, psia

$Sw_i$ Initial water saturation

$V_f$ Initial pore volume, bbl

$c_f$ Formation isothermal compressibility, $psi^{-1}$

The following expression determines the change in the oil volume:

Initial reservoir oil volume = NBoi

Oil volume at time t and pressure p = (N – Np)Bo

Change in oil volume = NBoi – (N – Np)Bo

initial free gas volume = GBgi

GBgi = NmBoi

When, m expression determines the change in free gas volume:

$$\begin{bmatrix}\text{Ratio of initial free gas}\\ \text{to initial oil volume}\end{bmatrix} = m = \frac{GB_{gi}}{NB_{oi}}$$

$$m = \frac{\text{Initial volume of gas cap}}{\text{Volume of oil initially in place}} = \frac{G\,B_{gi}}{N\,B_{oi}}$$

where m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

Initial volume of the gas cap = G Bgi = m N Boi

The total volume of the hydrocarbon system is then given by:

Initial oil volume + initial gas cap volume = (P.V) (1 – Swi)

$$N\,B_{oi} + m\,N\,B_{oi} = (P.V)(1 - S_{wi})$$

or

$$P.V = \frac{N\,B_{oi}(1+m)}{1 - S_{wi}}$$

Where:

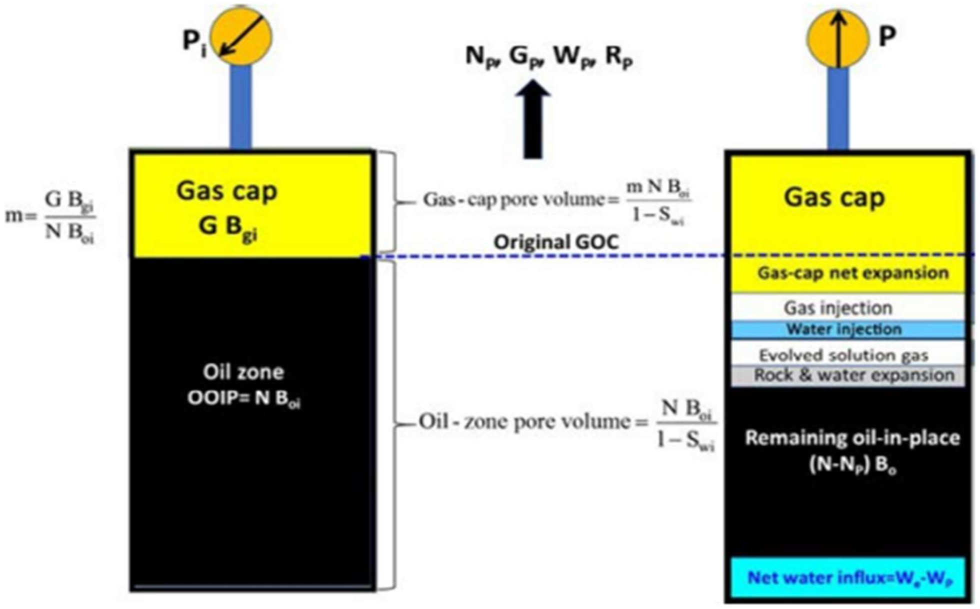
Swi initial water saturation

N initial oil in place, STB

P.V total pore volume, bbl

m ratio of initial gas-cap-gas reservoir volume to initial reservoir oil volume, bbl/bb

Figure below illustrate of treating the reservoir pore as an idealized container, it's resulting of pressure decline. Volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir.



**The concept of the tank model.**

## The MBE can be written in a generalized form as follows:

Pore volume occupied by the oil initially in place at pi+Pore volume occupied by the gas in the gas cap at pi =

Pore volume occupied by the remaining oil at p+

Pore volume occupied by the gas in the gas cap at p+

Pore volume occupied by the evolved solution gas at p+

Pore volume occupied by the net water influx at p+

Change in pore volume due to connate water expansion and pore volume reduction due to rock expansion+

Pore volume occupied by the injected gas at p+

Pore volume occupied by the injected water at p.

The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

## Pore Volume Occupied by the Oil Initially in Place

Volume occupied by initial oil in place = N Boi

Where:

N oil initially in place, STB

Boi oil formation volume factor at initial reservoir pressure pi, bbl/STB

## Pore Volume Occupied by the Gas in the Gas Cap

Volume of gas cap = mNBoi

where

m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

## Pore Volume Occupied by the Remaining Oil

$$\text{Volume of the remaining oil} = \left(N - N_p\right) B_o$$

Where:

Np cumulative oil production, STB

Bo oil formation volume factor at reservoir pressure p, bbl/STB

## Pore Volume Occupied by the Gas Cap at Reservoir Pressure p

As the reservoir pressure drops to a new level p, the gas in the gas cap expands and occupies a larger volume. Assuming no gas is produced from the gas cap during the pressure decline, the new volume of the gas cap can be determined as:

$$\text{Volume of the gas cap at } p = \left[\frac{m\ N\ B_{oi}}{B_{gi}}\right] B_g$$

Where:

Bgi gas formation volume factor at initial reservoir pressure, bbl/scf

Bg current gas formation volume factor, bbl/scf

## Pore Volume Occupied by the Evolved Solution Gas

This volumetric term can be determined by applying the following material balance on the solution gas:

$$\begin{bmatrix}\text{volume of the evolved}\\ \text{solution gas}\end{bmatrix} =$$

$$\begin{bmatrix}\text{volume of gas initially}\\ \text{in solution}\end{bmatrix} - \begin{bmatrix}\text{volume of gas}\\ \text{produced}\end{bmatrix} - \begin{bmatrix}\text{volume of gas}\\ \text{remaining in solution}\end{bmatrix}$$

Or

$$\begin{bmatrix}\text{volume of the evolved}\\ \text{solution gas}\end{bmatrix} = [N R_{si} - N_p R_p - (N - N_p) R_s] B_g$$

Where:

Np cumulative oil produced, STB

Rp net cumulative produced gas-oil ratio, scf/STB

Rs current gas solubility factor, scf/STB

Bg current gas formation volume factor, bbl/scf

Rsi gas solubility at initial reservoir pressure, scf/STB

## Change in Pore Volume Due to Initial Water and Rock Expansion

The component describing the reduction in the hydrocarbon pore volume due to the expansion of initial (connate) water and the reservoir rock cannot be neglected for an undersaturated oil reservoir. The water compressibility $c_w$ and rock compressibility cf are generally of the same order of magnitude as the compressibility of the oil. The effect of these two components, however, can be generally neglected for gas-cap-drive reservoir or when the reservoir pressure drops below the bubble-point pressure.

The compressibility coefficient c, which describes the changes in the volume (expansion) of the fluid or material with changing pressure, is given by:

$$c = \frac{-1}{V}\frac{\partial V}{\partial p}$$

or

$$\Delta V = V c \, \Delta p$$

where ΔV represents the net changes or expansion of the material as a result of changes in the pressure. Therefore, the reduction in the pore volume due to the expansion of the connate-water in the oil zone and the gas cap is given by:

$$Connate - water\ expansion = [(pore\ volume)S_{wi}]c_w \Delta p$$

Substituting for the pore volume (P.V) with Equation below

$$N\ B_{oi} + m\ N\ B_{oi} = (P.V)(1 - S_{wi})$$

or

$$P.V = \frac{N\,B_{oi}(1+m)}{1 - S_{wi}}$$

gives:

$$\text{Expansion of connate water} = \frac{N\ B_{oi}(1+m)}{1 - S_{wi}} S_{wi}\ c_w\ \Delta p$$

Combining the expansions of the connate water and formation as represented by two above equations gives:

Total changes in the pore volume

$$= N\ B_{oi}(1+m)\left(\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right)\ \Delta p$$

## Pore Volume Occupied by the Injection Gas and Water

Assuming that $G_{inj}$ volumes of gas and $W_{inj}$ volumes of water have been injected for pressure maintenance, the total pore volume occupied by the two injected fluids is given by:

$$\text{Total volume} = G_{inj}\,B_{ginj} + W_{inj}\,B_w$$

Where:

| | |
|---|---|
| $G_{inj}$ | cumulative gas injected, scf |
| $Bg_{inj}$ | injected gas formation volume factor, bbl/scf |
| $W_{inj}$ | cumulative water injected, STB |
| $Bw$ | water formation volume factor, bbl/STB |

Combining Equations

$$\text{Volume occupied by initial oil in place} = N\,B_{oi}$$

Through

$$\text{Total volume} = G_{inj}\,B_{ginj} + W_{inj}\,B_w$$

with generalized form of MBE equation and rearranging gives:

$$N = \frac{N_pB_o + (G_p - N_pR_s)B_g - (W_e - W_p\,B_w) - G_{inj}B_{ginj} - W_{inj}B_w}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

where:

| | |
|---|---|
| $N$ | initial oil-in-place, STB. |
| $Gp$ | cumulative gas produced, scf. |
| $Np$ | cumulative oil produced, STB. |
| $Rsi$ | gas solubility at initial pressure, scf/STB. |
| m | ratio of gas-cap gas volume to oil volume, bbl/bbl. |
| $Bgi$ | gas formation volume factor at $pi$ , bbl/scf. |
| $Bg_{inj}$ | gas formation volume factor of the injected gas, bbl/scf. |

The cumulative gas produced Gp can be expressed in terms of the cumulative gas-oil ratio Rp and cumulative oil produced Np by

$$G_p = R_p N_p$$

### Combining Above Equation with Following Equation

$$N = \frac{N_pB_o + (G_p - N_pR_s)B_g - (W_e - W_p B_w) - G_{inj}B_{ginj} - W_{inj}B_w}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

gives:

$$N = \frac{N_p\left[B_o + (R_p - R_s)B_g\right] - (W_e - W_p B_w) - G_{inj}B_{ginj} - W_{inj}B_{wi}}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + m\,B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

The above relationship is referred to as the material balance equation (MBE). A more convenient form of the MBE can be determined by introducing the concept of the total (two-phase) formation volume factor $Bt$ into the equation. This oil PVT property is defined as:

$$B_t = B_o + (R_{si} - R_s)B_g$$

Introducing $Bt$ into Equation

$$N = \frac{N_p\left[B_o + (R_p - R_s)B_g\right] - (W_e - W_p B_w) - G_{inj}B_{ginj} - W_{inj}B_{wi}}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + m\,B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

and assuming, for the sake of simplicity, no water or gas injection gives:

$$N = \frac{N_p\left[B_t + (R_p - R_{si})B_g\right] - (W_e - W_pB_w)}{(B_t - B_{ti}) + mB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$