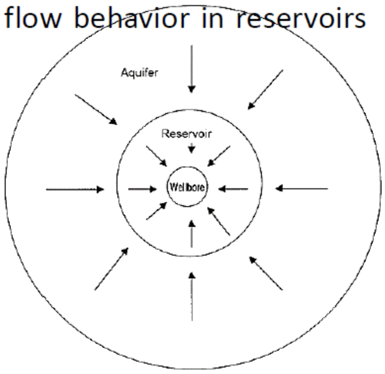
## Unsteady-State Model (Van Everdingen and Hurst Model)

- The mathematical formulations that describe the flow of a crude oil system into a wellbore are identical in form to those equations that describe the flow of water from an aquifer into a cylindrical reservoir.
- The basic of this model is based on the **dimensionless form** of the **diffusivity equation** that is designed to model the transient flow behavior in reservoirs or aquifers.

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}$$




Van Everdingen and Hurst (1949) proposed solutions to the dimensionless diffusivity equation for the following two reservoir-aquifer boundary conditions:

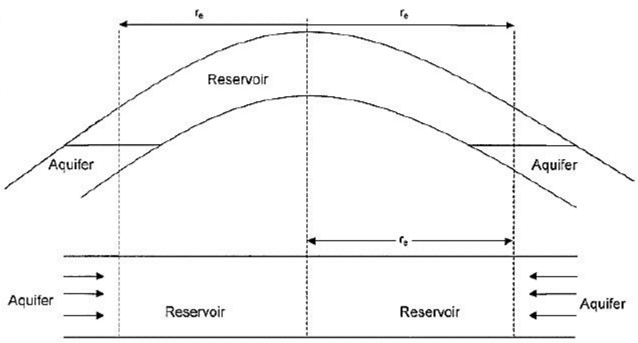
✓ *Constant terminal rate*

✓ *Constant terminal pressure*

- For the constant-terminal-rate boundary condition, the rate of water influx is assumed constant for a given period; and the pressure drop at the reservoir-aquifer boundary is calculated.
- For the constant-terminal-pressure boundary condition, a boundary pressure drop is assumed constant over some finite time period, and the water influx rate is determined.
- Van Everdingen and Hurst solved the diffusivity equation for the aquifer-reservoir system by applying the Laplace transformation to the equation. The authors' solution can be used to determine the water influx in the following systems:
- Edge-water-drive system (radial system)
- Bottom-water-drive system
- Linear-water-drive system

### Edge-Water Drive

- The inner boundary is defined as the interface between the reservoir and the aquifer. The flow across this inner boundary is considered horizontal and encroachment occurs across a cylindrical plane encircling the reservoir. With the interface as the inner boundary, it is possible to impose a constant terminal pressure at the inner boundary and determine the rate of water influx across the interface.



- Van Everdingen and Hurst proposed a solution to the dimensionless diffusivity equation that utilizes the constant terminal pressure condition in addition to the following initial and outer boundary conditions:

*Initial conditions:*

The initial condition : p = pi for all values of radius r

*Outer boundary conditions*

For an infinite aquifer: p = pi at r = ∞

For bounded aquifer: ∂p/∂r=0 at r = ra

- **Van Everdingen and Hurst** assumed that the aquifer is characterized by:
- Uniform Thickness
- Constant Permeability
- Uniform Porosity
- Constant Rock Compressibility
- Constant Water Compressibility

- **Van Everdingen and Hurst** expressed their mathematical relationship for calculating the water influx in a form of a **dimensionless parameter** that is called ***dimensionless water influx*** $W_{eD}$.
- They also expressed the dimensionless water influx as a function of the ***dimensionless time*** $t_D$ and ***dimensionless radius*** $r_D$, thus they made the solution to the diffusivity equation generalized and applicable to any aquifer where the flow of water into the reservoir is essentially radial.

- The solutions were derived for cases of **bounded aquifers** and aquifers of **infinite** extent.
- The authors presented their solution in **tabulated** and **graphical forms.**

- The two dimensionless parameters $t_D$ and $r_D$ are given by:

$$t_D = C\frac{k_a t}{\phi\, \mu_w c_t\, r_e^2} \;\ldots\ldots\ldots\; (18) \qquad r_D = \frac{r_a}{r_e} \ldots\ldots\ldots (19)$$

$$C_t = C_w + C_f \;\ldots\ldots\ldots.(20)$$

where t = time, days
k = permeability of the aquifer, md
ϕ = porosity of the aquifer
$\mu_w$ = viscosity of water in the aquifer, cp
$r_a$ = radius of the aquifer, ft
$r_e$ = radius of the reservoir, ft
$c_w$ = compressibility of the water, $psi^{-1}$
$c_f$ = compressibility of the aquifer formation, $psi^{-1}$
$c_t$ = total compressibility coefficient, $psi^{-1}$

$$t_D = 6.328 \times 10^{-3}\frac{kt}{\phi\mu_w c_t r_e^2} \quad (t_D = day)$$

$$t_D = 0.2637 \times 10^{-3}\frac{kt}{\phi\mu_w c_t r_e^2} \quad (t_D = hours)$$

$$t_D = 2.3097\frac{kt}{\phi\mu_w c_t r_e^2} \quad (t_D = years)$$

The water influx is then given by:

$$W_e = B\Delta p W_{eD} \;\ldots\ldots (21)$$

With;

$$\boldsymbol{B} = 1.119\,\phi c_t r_e^2 h \qquad (22)$$

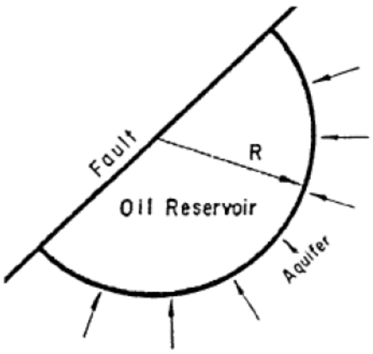
where $W_e$ = cumulative water influx, bbl
$B$ = water influx constant, bbl/psi
$\Delta p$ = pressure drop at the boundary, psi
$W_{eD}$ = dimensionless water influx

- Equation 21 assumes that the water is encroaching in a radial form.
- Quite often, water does not encroach on all sides of the reservoir, or the reservoir is not circular in nature. In these cases, some modifications must be made in Equation 21 to properly describe the flow mechanism.
- One of the simplest modifications is to introduce the **encroachment angle** to the water influx constant B as:

$$f = \frac{\theta}{360} \qquad (23)$$

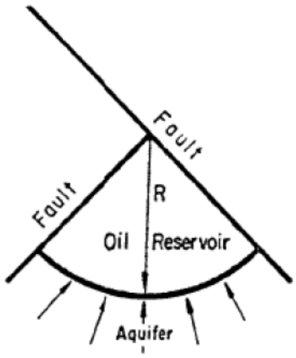
$$\boldsymbol{B} = 1.119\,\phi\, c_t\, r_e^2\, h\, f \qquad (24)$$

- θ is the angle subtended by the reservoir circumference, i.e., for a full circle θ = 360° and for semicircle reservoir against a fault θ =180°.




f = 0.5

θ= 180°




f =0.25

θ= 90°

**Example 8.1.** Calculate the water influx after 100 days, 200 days, 400 days, and 800 days into a reservoir the boundary pressure of which is suddenly lowered and held at 2724 psia ($p_i = 2734$ psia).

*Given:*

| | |
|---|---|
| $\phi = 0.20$ | $k = 83$ md |
| $c_t = 8(10)^{-6}$ psi$^{-1}$ | $r_R = 3000$ ft |
| $r_e = 30{,}000$ ft | $\mu = 0.62$ cp |
| $\theta = 360°$ | $h = 40$ ft |

SOLUTION: From Eq. (8.6):

$$t_D = \frac{0.0002637(83)t}{0.20(0.62)[8(10)^{-6}]3000^2} = 0.00245t$$

From Eq. (8.9):

$$B' = 1.119(0.20)[8(10)^{-6}](3000^2)(40)\left(\frac{360}{360}\right) = 644.5$$

At 100 days $t_D = 0.00245(100)(24) = 5.88$ dimensionless time units. From the $r_e/r_R = 10$ curve of Fig. 8.8 find corresponding to $t_D = 5.88$, $W_{eD} = 5.07$ dimensionless influx units. This same value may also be found by interpolation of Table 8.1, since below $t_D = 15$ the aquifer behaves essentially as if it were infinite, and no values are given in Table 8.2. Since $\Delta p = 2734 - 2724 = 10$ psi, and water influx at 100 days from Eq. (8.8) is

$$W_e = B'\Delta p W_{eD} = 644.5(10)(5.07) = 32{,}680 \text{ bbl}$$

Similarly at

| | | | |
|---|---|---|---|
| $t$ = 100 days | 200 days | 400 days | 800 days |
| $t_D$ = 5.88 | 11.76 | 23.52 | 47.04 |
| $W_{eD}$ = 5.07 | 8.43 | 13.90 | 22.75 |
| $W_e$ = 32,680 | 54,330 | 89,590 | 146,600 |

To estimate the **cumulative water influx**, use the following equation:

$$We = B' \sum \Delta p \, W_{eD}$$

**Example 8.2.** Suppose in Ex. 8.1 at the end of 100 days the reservoir boundary pressure suddenly drops to $p_2 = 2704$ psia (i.e., $\Delta p_2 = p_1 - p_2 = 20$ psi, *not* $p_i - p_2 = 30$ psi). Calculate the water influx at 400 days total time.

The water influx due to the first pressure drop $\Delta p_1 = 10$ psi at 400 days was calculated in Example 8.1 to be 89,590 bbl. This will be the same even though a second pressure drop occurs at 100 days and continues to 400 days. This second drop will have acted for 300 days, or a dimensionless time of $t_D = 0.0588 \times 300 = 17.6$. From Fig. 8.8 or Table 8.2 for $r_e/r_R = 10$ $W_{eD} = 11.14$ for $t_D = 17.6$ and the water influx is

$$\Delta W_{e2} = B' \times \Delta p_2 \times W_{eD2} = 644.5 \times 20 \times 11.14 = 143{,}600 \text{ bbl}$$

$$W_{e2} = \Delta W_{e1} + \Delta W_{e2} = B' \times \Delta p_1 \times W_{eD1} + B' \times \Delta p_2 \times W_{eD2} = B' \Sigma \Delta p W_{eD}$$

$$= 644.5\,(10 \times 13.90 + 20 \times 11.14)$$

$$= 89{,}590 + 143{,}600 = 233{,}190 \text{ bbl}$$