

Properties of exp. fun.

$$\ln e = 1, \quad e^0 = 1, \quad e = 2.718...$$

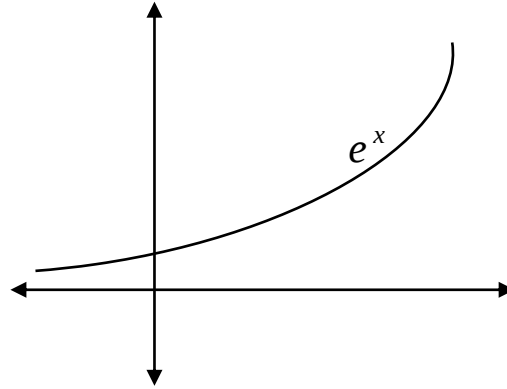
$$e^{a+b} = e^a e^b$$

$$e^{ax} = (e^a)^x = (e^x)^a$$

its domain is all real no.

its range are $y > 0$

the graph of $y = e^x$ is



أشتقاق الدوال الاسية :

$$\boxed{\frac{d}{dx} e^x = e^x}$$

Proof:

$$\text{let } y = e^x$$

$$\ln y = \ln e^x = x \ln e = x$$

$$\frac{1}{y} \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = y = e^x$$

and in general form

$$\boxed{\frac{d}{dx} e^u = e^u \frac{du}{dx}}$$

EXAM: Find $\frac{dy}{dx}$ to :

$$y = e^{3x} \Rightarrow \frac{dy}{dx} = e^{3x} (3) = 3e^{3x}$$

$$y = e^{-x} \Rightarrow \frac{dy}{dx} = e^{-x} (-1) = -e^{-x}$$

$$y = xe^x \Rightarrow \frac{dy}{dx} = xe^x + e^x (1) = xe^x + e^x$$

$$y = e^{x^3} + \ln \ln x + x^3 + e \Rightarrow \frac{dy}{dx} = e^{x^3} (3x^2) + \frac{1}{x \ln x} + 3x^2 + 0$$

EXAM : Simplify

$$y = e^{2 \ln x} \Rightarrow e^{\ln x^2} = x^2$$

$$y = e^{2 \ln \frac{x}{y}} \Rightarrow e^{\ln \left(\frac{x}{y}\right)^2} = \left(\frac{x}{y}\right)^2$$

$$\int e^x dx = e^x + C$$

and in general form :

$$\int e^u du = e^u + C$$

EXAM : Find

$$\int e^{4x} dx = \frac{1}{4} \int 4e^{4x} dx = \frac{1}{4} e^{4x} + C$$

$$\int e^{-2x} dx = \frac{-1}{2} \int -2e^{-2x} dx = \frac{-1}{2} e^{-2x} + C$$

$$\int e^{2\ln x} dx = \int e^{\ln x^2} dx = \int x^2 dx = \frac{x^3}{3} + C$$

$$\int x^2 e^{-2x^3} dx = \frac{-1}{6} \int -6x^2 e^{-2x^3} dx = \frac{-1}{6} e^{-2x^3} + C$$

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \ln |e^x - e^{-x}| + C$$

Exponential fun. of type a^x , a^u

$$a^x = e^{x \ln a} = e^{\ln a^x} = a^x$$

اشتقاق الدالة الاسية من نوع a^x

$$\frac{d}{dx} a^x = a^x \ln a$$

and in general form :

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$$

EXAM: Find $\frac{dy}{dx}$ to :

$$y = 3^{x^4} \Rightarrow \frac{dy}{dx} = 3^{x^4} \cdot \ln 3 \cdot (4x^3)$$

$$y = 7^{x^2} \Rightarrow \frac{dy}{dx} = 7^{x^2} \cdot \ln 7 \cdot (2x)$$

$$y = 10^{\ln x} \Rightarrow \frac{dy}{dx} = 10^{\ln x} \cdot \ln 10 \cdot \left(\frac{1}{x}\right)$$

تكامل الدالة الاسية من نوع a^x

$$\int a^u du = \frac{a^u}{\ln a} + C, \quad a > 0, \quad a \neq 1$$

EXAM : Find

$$\int 3^x dx = \frac{3^x}{\ln 3} + C$$

$$\int 7^{x^2} x dx = \frac{1}{2} \int 7^{x^2} 2x dx = \frac{1}{2} \frac{7^{x^2}}{\ln 7} + C$$

EXAM : a) Find the area bounded by the curve $y = e^{-4x}$ and the x-axis from $x=-1, 1$

b) The area in (a) rotated about the x-axis, find the volume generated ?

$$(a) \quad A = \int_a^b y dx = \int_{-1}^1 e^{-4x} dx = \left. \frac{-1}{4} e^{-4x} \right|_{-1}^1$$

$$= \frac{-1}{4} (e^{-4} - e^4) \text{ unit}^2$$

$$(b) \quad V_x = \pi \int_a^b y^2 dx = \pi \int_{-1}^1 (e^{-4x})^2 dx = \pi \int_{-1}^1 e^{-8x} dx$$

$$= \left. \frac{-\pi}{8} e^{-8x} \right|_{-1}^1 = ? \text{ unit}^3$$

HOMEWORK

1) Show that

(a) $y = xe^{-x}$ satisfies the eq. $xy' = (1-x)y$

(b) $y = xe^{-x^2/2}$ satisfies the eq. $xy' = (1-x^2)y$

2) Find $\frac{dy}{dx}$ to :

$$y = \ln(1 - xe^{-x}) \quad , \quad y = \frac{e^x}{\ln x} \quad , \quad y = x^3 e^{-x}$$

3) Find

$$\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}} dx \quad , \quad \int \sqrt{e^{x^2}} dx$$