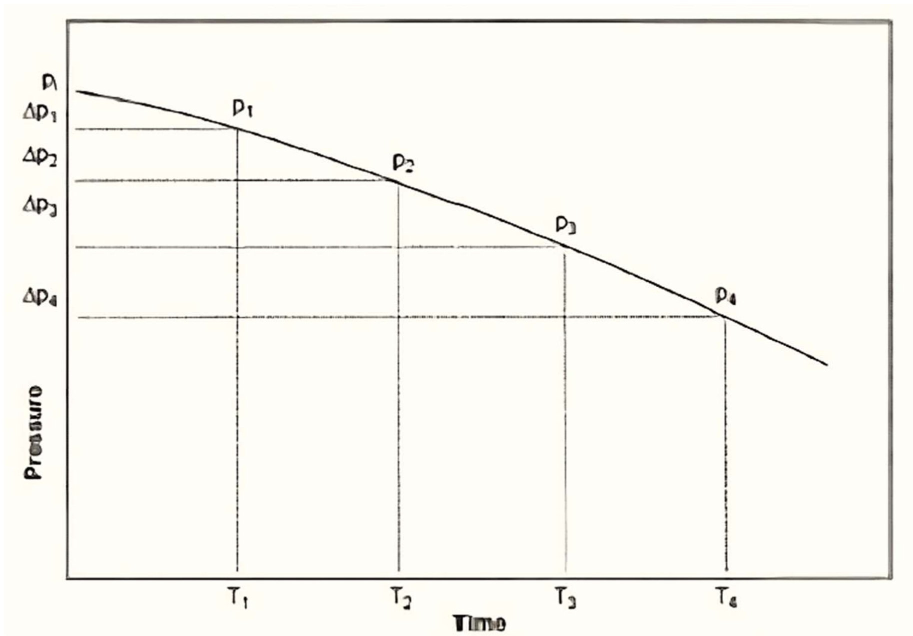
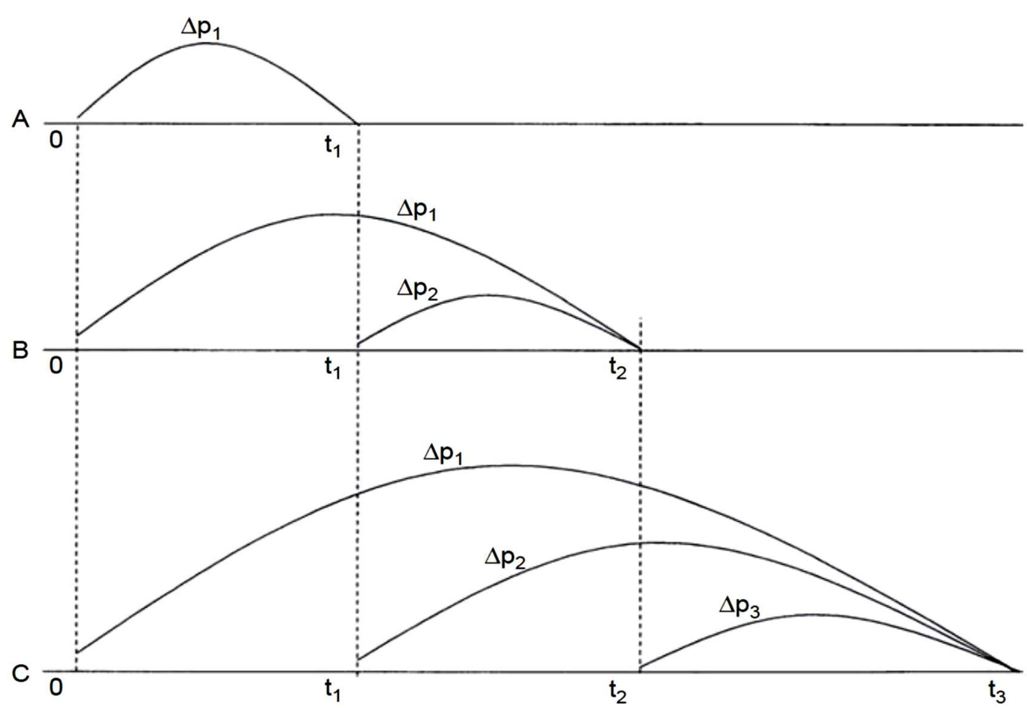
Principle of Superposition

As there will usually be many pressure drops occurring throughout the prediction period, it is necessary to analyze the procedure to be used where these multiple pressure drops are present.



In calculating the cumulative water influx into a reservoir at successive intervals, it is necessary to calculate the total water influx from the beginning.



The following procedure will be used to calculate We:

Step 1. Assume that the boundary pressure has declined from its initial value of p_i to p_1 after t_1 days. To determine the cumulative water influx in response to this first pressure drop, $\Delta p_1 = p_i - p_1$ can be simply calculated from Equation

$$W_e = B\, \Delta p_1\, (W_{eD})_{t1}$$

where W_e is the cumulative water influx due to the first pressure drop Δp_1. The dimensionless water influx $(W_{eD})_{t_1}$ is evaluated by calculating the dimensionless time at t_1 days.

Step 2. Let the boundary pressure decline again to p_2 after t_2 days with a pressure drop of $\Delta p_2 = p_1 - p_2$. The cumulative (total) water influx after t_2 days will result from the first pressure drop Δp_1 and the second pressure drop Δp_2, or:

$$W_e = \text{water influx due to } \Delta p_1 + \text{water influx due to } \Delta p_2$$

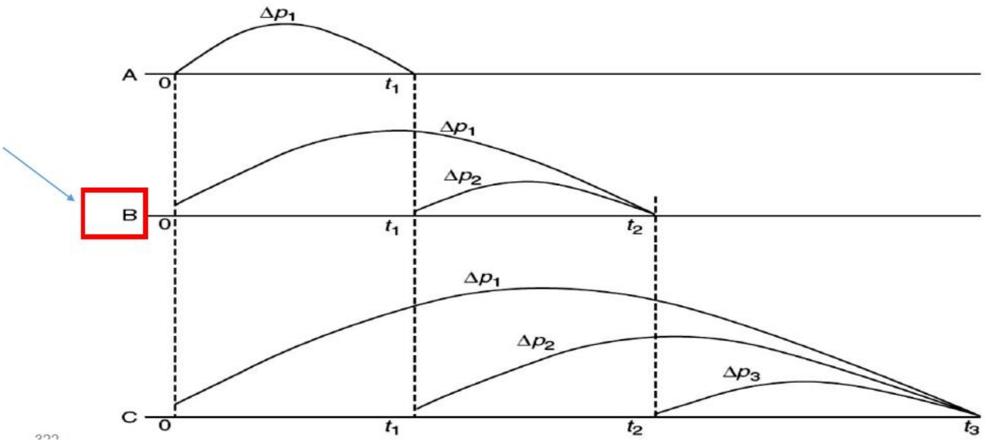
$$W_e = (W_e)_{\Delta p_1} + (W_e)_{\Delta p_2}$$

where

$$(W_e)_{\Delta p_1} = B\, \Delta p_1\, (W_{eD})_{t_2}$$

$$(W_e)_{\Delta p_2} = B\, \Delta p_2\, (W_{eD})_{t_2 - t_1}$$

The above relationships indicate that the effect of the first pressure drop Δp_1 will continue for the entire time t_2, while the effect of the second pressure drop will continue only for $(t_2 - t_1)$ days



Step 3. A third pressure drop of $\Delta p_3 = p_2 - p_3$ would cause an additional water influx as illustrated in section C . The cumulative (total) water influx can then be calculated from:

$$W_e = (W_e)_{\Delta p1} + (W_e)_{\Delta p2} + (W_e)_{\Delta p3}$$

where

$$(W_e)_{\Delta p1} = B\, \Delta p_1\, (W_{eD})_{t3}$$

$$(W_e)_{\Delta p2} = B\, \Delta p_2\, (W_{eD})_{t3-t1}$$

$$(W_e)_{\Delta p3} = B\, \Delta p_3\, (W_{eD})_{t3-t2}$$

- **Van Everdingen and Hurst** also suggested that instead of using the entire pressure drop, a better approximation is to consider that one-half of the pressure drop which is effective (actual) pressure drop during the entire period.

First Period

$$\Delta P_1 = \boxed{\frac{P_i - P_1}{2}}$$

Second Period

$$\Delta P_2 = \frac{P_i - P_1}{2} + \frac{P_1 - P_2}{2} = \boxed{\frac{P_i - P_2}{2}}$$

Third Period

$$\Delta P_3 = \frac{P_1 - P_2}{2} + \frac{P_2 - P_3}{2} = \boxed{\frac{P_1 - P_3}{2}}$$

$$\Delta p_4 = \boxed{\frac{p_2 - p_4}{2}}$$

$$\Delta p_5 = \boxed{\frac{p_3 - p_5}{2}}$$

Example: Calculate the cumulative water influx at the end of (6, 12, 18, and 24 months) into a reservoir with an aquifer of infinite extent.

The predicated boundary pressure at the end of each specified time period is given below:

Time, months	*Boundary pressure, psi*
0	2500
6	2490
12	2472
18	2444
24	2408

The reservoir-aquifer system has the following properties:

	Reservoir	*Aquifer*
radius , ft	2000	∞
h, ft	20	22.78
k, md	50	100
Φ, %	15	20
μ_w, cp	0.5	0.8
c_w, psi^{-1}	$1*10^{-6}$	$0.7*10^{-6}$
c_f, psi^{-1}	$2*10^{-6}$	$0.3*10^{-6}$

Solution

Water influx at the end of 6 months

$C_t = C_w + C_f$

Step 1. Determine water influx constant $B = 1.119 \phi c_t r_e^2 h f$

$$B = 22.4 \text{ bbl/psi}$$

Step 2. Calculate the dimensionless time t_D at 182.5 days.

$$t_D = 6.328 \times 10^{-3} \frac{kt}{\phi \mu_w c_t r_e^2}$$

$$\begin{aligned} t_D &= 0.9888t \\ &= 0.9888\,(182.5) = 180.5 \end{aligned}$$

Step 3. Calculate the first pressure drop Δp_1.

$$\Delta p_1 = \frac{p_i - p_1}{2} \qquad \Delta p_1 = \frac{2500 - 2490}{2} = 5 \text{ psi}$$

Step 4. Determine the dimensionless water influx W_{eD} from Table 10-1 at $t_D = 180.5$ to give:

$$W_{eD} = 69.46$$

Step 5. Calculate the cumulative water influx at the end of 182.5 days

$$W_e = B\, \Delta p_1\, (W_{eD})_{t_1}$$

$$W_e = (20.4)\,(5)\,(69.46) = 7080 \text{ bbl}$$

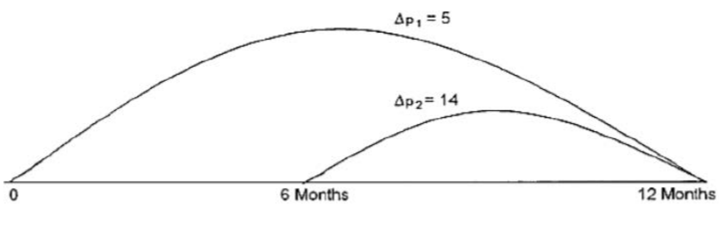
Cumulative water influx after 12 months

Step 1.

$$\Delta p_2 = \frac{p_i - p_2}{2} = \frac{2500 - 2472}{2} = 14 \text{ psi}$$

Step 2. The cumulative (total) water influx at the end of 12 months would result from the first pressure drop Δp_1 and the second pressure drop Δp_2.

$$W_e = (W_e)_{\Delta p_1} + (W_e)_{\Delta p_2}$$



Step 3. Calculate the dimensionless time at 365 days as:

$$t_D = 0.9888t$$
$$= 0.9888\,(365) = 361$$

Step 4. Determine the dimensionless water influx at $t_D = 361$ from Table 10-1 to give:

$$W_{eD} = 123.5$$

Step 5. Calculate the water influx due to the first and second pressure drop, i.e., $(W_e)_{\Delta p1}$ and $(W_e)_{\Delta p2}$, or:

$$(W_e)_{\Delta p_1} = B\ \Delta p_1\ (W_{eD})_{t_2} \qquad (W_e)_{\Delta p_1} = (20.4)(5)(123.5) = 12{,}597 \text{ bbl}$$

$$(W_e)_{\Delta p_2} = B\ \Delta p_2\ (W_{eD})_{t_2 - t_1} \qquad (W_e)_{\Delta p_2} = (20.4)(14)(69.46) = 19{,}838$$

Step 6. Calculate total (cumulative) water influx after one year.

$$W_e = 12{,}597 + 19{,}938 = 32{,}435 \text{ bbl}$$

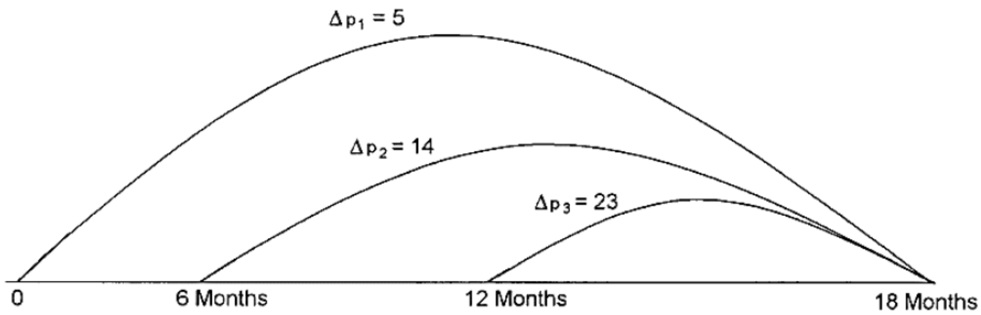
Water influx after 18 months

Step 1.

$$\Delta p_3 = \frac{p_1 - p_3}{2} \qquad \Delta p_3 = \frac{2490 - 2444}{2} = 23\,\text{psi}$$

Step 2. Calculate the dimensionless time after 18 months.

$$t_D = 0.9888\ t$$
$$= 0.9888\,(547.5) = 541.5$$



Step 3. Determine the dimensionless water influx at:

$$t_D = 541.5 \text{ from Table 10-1} \qquad W_{eD} = 173.7$$

Step 4. The first pressure drop will have been effective the entire 18 months, the second pressure drop will have been effective for 12 months, and the last pressure drop will have been effective only 6 months, as shown in Figure above.

Time, days	t_D	Δp	W_{eD}	$B\Delta p\ W_{eD}$
547.5	541.5	5	173.7	17,714
365	361	14	123.5	35,272
182.5	180.5	23	69.40	32,291

$W_e = 85{,}277$ bbl

$$(W_e)_{\Delta p_1} = B\ \Delta p_1\ (W_{eD})_{t_3}$$

$$(W_e)_{\Delta p_2} = B\ \Delta p_2\ (W_{eD})_{t_3 - t_1}$$

$$(W_e)_{\Delta p_3} = B\ \Delta p_3\ (W_{eD})_{t_3 - t_2}$$

Water influx after two years

The first pressure drop has now been effective for the entire two years, the second pressure drop has been effective for 18 months, the third pressure drop has been effective for 12 months, and the fourth pressure drop has been effective only 6 months. Summary of the calculations is given below:

Time, days	t_D	Δp	W_{eD}	$B\Delta p\ W_{eD}$
730	722	5	221.8	22,624
547.5	541.5	14	173.7	49,609
365	631	23	123.5	57,946
182.5	180.5	32	69.40	45,343

$W_e = 175{,}522$ bbl

Practice:

A reservoir-aquifer system subtend an angle of 140 at the centre and the aquifer radius is estimated to be 112,000 ft. using the reservoir pressure and dimensionless water influx values in the table below, find the water influx at each time step, using Van Everdingen- Hurst.

Time (yrs)	Pressure (psia)	W_{eD}
0	4014	0
1	3941	6.8712
2	3870	11.2763
3	3800	14.8854
4	3732	17.8373
5	3664	20.2521

Additional data for the **aquifer** are:

h, ft	100
k, md	200
Φ, %	19
μ_w, cp	0.55
c_t, psi^{-1}	$2.5 * 10^{-6}$
r_D	8

Solution:

Time (yrs)	Pressure (psia)	ΔP (psia)	WeD	We (bbl)
0	4014		0	
1	3941	36.5	6.8712	1016116.35
2	3870	72	11.2763	3671938.56
3	3800	70.5	14.8854	7453298.95
4	3732	69	17.8373	12122474.28
5	3664	68	20.2521	17495331.4