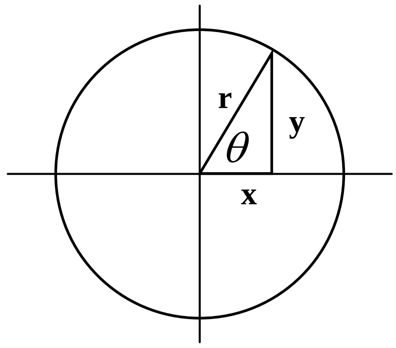
الدوال المثلثية



$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{Adjance}{\text{Hypotenuse}} = \frac{x}{r}$$

By phythaghore theorem we get :

$$\sin\theta = \frac{y}{r} \Rightarrow \sin^2\theta = \frac{y^2}{r^2} \quad , \quad \cos\theta = \frac{x}{r} \Rightarrow \cos^2\theta = \frac{x^2}{r^2}$$

$$\sin^2\theta + \cos^2\theta = \frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{x^2 + y^2}{r^2} = \frac{r^2}{r^2} = 1$$

$$\therefore \quad \boxed{\sin^2\theta + \cos^2\theta = 1}$$

1) $\theta = 0$

$$\cos\theta = \frac{x}{r} = \frac{r}{r} = 1 \Rightarrow \cos\theta = 1$$

$$\sin\theta = \frac{y}{r} = \frac{0}{r} = 0 \Rightarrow \sin\theta = 0$$

2) $\theta = 90^\circ$

$$\cos\theta = \frac{x}{r} = \frac{0}{r} = 0 \Rightarrow \cos\theta = 0$$

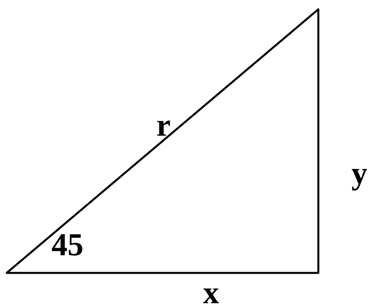
$$\sin\theta = \frac{y}{r} = \frac{r}{r} = 1 \Rightarrow \sin\theta = 1$$

3) $\theta = 45^\circ$

$$x^2 + y^2 = r^2 \quad , \quad y = x$$

$$x^2 + x^2 = r^2 \Rightarrow 2x^2 = r^2 \Rightarrow r = x\sqrt{2}$$

$$y^2 + y^2 = r^2 \Rightarrow 2y^2 = r^2 \Rightarrow r = y\sqrt{2}$$



$$\cos\theta = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta = \frac{y}{y\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

4) $\theta = 30^\circ$

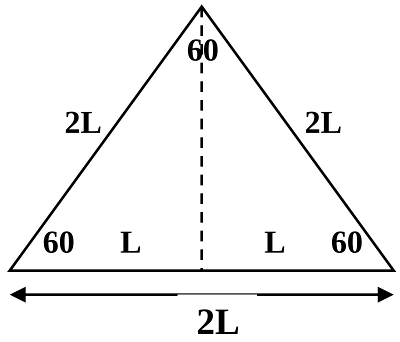
$x^2 + y^2 = r^2$

$L^2 + y^2 = 4L^2 \Rightarrow y^2 = 3L^2 \Rightarrow y = \sqrt{3}L$

by same we get : $x = \sqrt{3}L$

$$\cos\theta = \frac{x}{r} = \frac{\sqrt{3}L}{2L} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

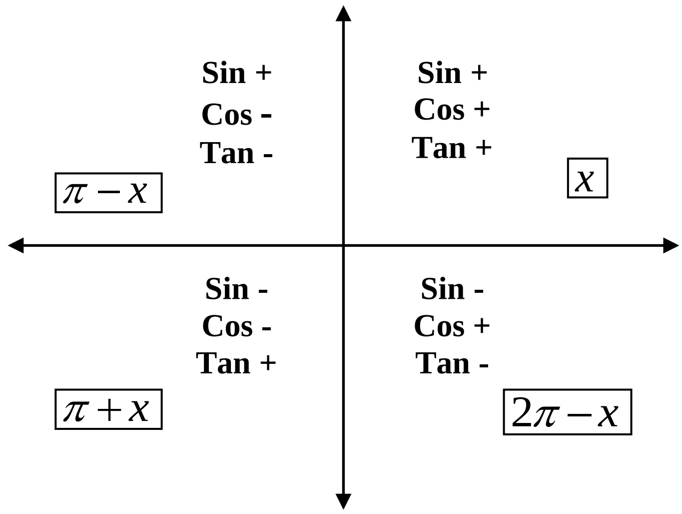
$$\sin\theta = \frac{y}{r} = \frac{L}{2L} \Rightarrow \sin\theta = \frac{1}{2}$$



5) $\theta = 60^\circ$

$$\cos\theta = \frac{x}{r} = \frac{L}{2L} \Rightarrow \cos\theta = \frac{1}{2}$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}L}{2L} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$



	0	45	30	60	90	π	$\frac{3\pi}{2}$	2π
Sin	**0**	$1/\sqrt{2}$	$1/2$	$\sqrt{3}/2$	**1**	**0**	**-1**	**0**
Cos	**1**	$1/\sqrt{2}$	$\sqrt{3}/2$	$1/2$	**0**	**-1**	**0**	**1**
Tan	**0**	**1**	$1/\sqrt{3}$	$\sqrt{3}$	∞	**0**	∞	**0**

Some Rules of Trig. Fun. :-

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\csc^2\theta = 1 + \cot^2\theta$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\sin^2 A = \frac{1}{2}(1-\cos 2A)$$

$$\cos^2 A = \frac{1}{2}(1+\cos 2A)$$

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

$$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$$

EXAM :

$$\sin(120) = \sin(\pi - 60) = +\sin 60 = \frac{\sqrt{3}}{2}$$

$$\sin(240) = \sin(\pi + 60) = -\sin 60 = -\frac{\sqrt{3}}{2}$$

$$\sin(300) = \sin(2\pi - 60) = -\sin 60 = -\frac{\sqrt{3}}{2}$$

$$\sin(330) = \sin(2\pi - 30) = -\sin 30 = -\frac{1}{2}$$

$$\sin(\frac{\pi}{2} - \theta) = \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta = \cos\theta$$

$$\cos(\frac{\pi}{2} - \theta) = \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta = \sin\theta$$

Derivative of Trig. Fun. مشتقات الدوال المثلثية

$\boxed{1}$ $\frac{d}{dx}\sin x = \cos x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

$\boxed{2}$ $\frac{d}{dx}\cos x = -\sin x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

$\boxed{3}$ $\frac{d}{dx}\tan x = \sec^2 x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$

$\boxed{4}$ $\frac{d}{dx}\cot x = -\csc^2 x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$

$\boxed{5}$ $\frac{d}{dx}\sec x = \sec x \tan x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$

$\boxed{6}$ $\frac{d}{dx}\csc x = -\csc x \cot x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$

Proof :

$\boxed{1}$ $\frac{d}{dx}\sin x = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x)-\sin x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin x \cos\Delta x + \sin\Delta x \cos x - \sin x}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{-\sin x[1-\cos\Delta x]+\sin\Delta x \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\sin x[1-\cos\Delta x]}{\Delta x} + \lim_{\Delta x \to 0} \frac{\sin\Delta x \cos x}{\Delta x}$$

$$= -\sin x \lim_{\Delta x \to 0} \frac{[1-\cos\Delta x]}{\Delta x} + \cos x \lim_{\Delta x \to 0} \frac{\sin\Delta x}{\Delta x}$$

$$= -\sin x\,(0) + \cos x\,(1) = \cos x$$

$\boxed{2}$ $\frac{d}{dx}\cos x = \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x)-\cos x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cos x \cos\Delta x - \sin x \sin\Delta x - \cos x}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{-\cos x[1-\cos\Delta x]-\sin x \sin\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cos x[1-\cos\Delta x]}{\Delta x} - \lim_{\Delta x \to 0} \frac{\sin\Delta x \sin x}{\Delta x}$$

$$= -\cos x \lim_{\Delta x \to 0} \frac{[1-\cos\Delta x]}{\Delta x} - \sin x \lim_{\Delta x \to 0} \frac{\sin\Delta x}{\Delta x}$$

$$= -\cos x\,(0) + -\sin x\,(1) = -\sin x$$

EXAM :

$$y = \cos 4x \Rightarrow \frac{dy}{dx} = -\sin 4x\,(4) = -4\sin 4x$$

$$y = e^{\tan x} \Rightarrow \frac{dy}{dx} = e^{\tan x}(\sec^2 x)$$

$$y = Ln\sin 5x \Rightarrow \frac{dy}{dx} = \frac{5\cos 5x}{\sin 5x} = 5\cot 5x$$

$$y = (x^2+1)^{\sin x}$$

$$Lny = Ln(x^2+1)^{\sin x} = \sin x\ Ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x\ \frac{2x}{x^2+1} + Ln(x^2+1).\cos x$$

$$\frac{dy}{dx} = y\left[\sin x\ \frac{2x}{x^2+1} + Ln(x^2+1).\cos x\right]$$

$$\frac{dy}{dx} = (x^2+1)^{\sin x}\left[\sin x\ \frac{2x}{x^2+1} + Ln(x^2+1).\cos x\right]$$

$$y = \frac{\sin x}{1+\cos x} \Rightarrow \frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

EXAM :

if $y = \sin^2 3x$, *prove that* $\dfrac{dy}{dx} = 3\sin 6x$

$$\frac{dy}{dx} = 2\sin 3x\ \cos 3x\,(3)$$

$$\because\quad 2\sin 3x\ \cos 3x = \sin 6x$$

$$\therefore \frac{dy}{dx} = \sin 6x \cdot (3) = 3\sin 6x$$

EXAM : Show that y=cosx & y=sinx are solution of the eq. $y'' + y = 0$

1) $y = \cos x \Rightarrow y' = -\sin x \Rightarrow y'' = -\cos x$

$y'' + y = -\cos x + \cos x = 0 = L.H.S$

2) $y = \sin x \Rightarrow y' = \cos x \Rightarrow y'' = -\sin x$

$y'' + y = -\sin x + \sin x = 0 = L.H.S$

EXAM : Show that y =A sinx+ B cosx is solution of the eq. $y'' + y = 0$

$$y = A\sin x + B\cos x$$
$$y' = A\cos x - B\sin x$$
$$y'' = -A\sin x - B\cos x$$
$$y'' + y = -A\sin x - B\cos x + A\sin x + B\cos x = 0 = L.H.S$$

Trig. Fun. Integration تكاملات الدوال المثلثية

$\boxed{1}$ $\int \cos u \, du = \sin u + C$

$\boxed{2}$ $\int \sin u \, du = -\cos u + C$

$\boxed{3}$ $\int \sec^2 u \, du = \tan u + C$

$\boxed{4}$ $\int \csc^2 u \, du = -\cot u + C$

$\boxed{5}$ $\int \sec u \tan u \, du = \sec u + C$

$\boxed{6}$ $\int \csc u \cot u \, du = -\csc u + C$

EXAM :

$$\int \cos 5x \, dx = \frac{1}{5}\sin 5x + C$$

$$\int \sec^2 3x \, dx = \frac{1}{3}\tan 3x + C$$

$$\int e^{\cos x}\sin x \, dx = -e^{\cos x} + C$$

$$\int \frac{\sec^2 2x}{1+\tan 2x}dx = \frac{1}{2}\int \frac{2\sec^2 2x}{1+\tan 2x}dx = \frac{1}{2}Ln\left|1+\tan 2x\right| + C$$

$$\int \tan u \, du = -\int \frac{-\sin u}{\cos u}du = -Ln\left|\cos u\right| = Ln\left|\sec u\right| + C$$

$$\int \tan^2 3x \, dx = \int (\sec^2 3x - 1)\,dx = \int \sec^2 3x dx - \int 1\,dx = \frac{1}{3}\tan 3x - x + C$$

$$\int \sin^4 x \cos x \, dx = \frac{1}{5}\sin^5 x + C$$

$$\int \cos^4 x \sin x \, dx = -\frac{1}{5}\cos^5 x + C$$

$$\int \frac{\cos x}{\sin^2 x}dx = \int \frac{1}{\sin x}\frac{\cos x}{\sin x}dx = \int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{\cos x}{\sin^2 x} dx = \int \cos x \sin^{-2} x \, dx = -\sin^{(-1)} x = -\csc x + C$$

$$\int \frac{1}{1+\sin x} dx = \int \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} dx = \int \frac{1-\sin x}{1-\sin^2 x} dx$$

$$= \int \frac{1-\sin x}{\cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx = \int (\sec^2 x - \sin x \cos^{-2} x) dx$$

$$= \tan x + \frac{\cos^{(-1)} x}{-1} = \tan x - \sec x + C$$

HOME WORK

(1) *if* $y = 2\cos^2 x - 2\cos^4 x - \frac{1}{8}$, *show that* $\frac{dy}{dx} = \sin 4x$

(2) *show that* $y = x \sin x$ *is solution to*

$$y'' + y = 2\cos x \quad \& \quad y^{(4)} + y'' = -2\cos x$$

(3) *Find* $\frac{dy}{dx}$ *to*

$$y = \frac{\csc x}{\tan x} \ , \ y = \frac{1}{\cot x} \ , \ y = \frac{\sin x \sec x}{1 + x \tan x} \ , \ y = \frac{\cot x}{1+\csc x}$$

(4) *Find*

$$\int \sec x (\sec x + \tan x) dx$$

$$\int \sec x (\tan x + \cos x) dx$$

$$\int \frac{\sec x}{\cos x} dx$$

$$\int \frac{\sin 2x}{\cos x} dx$$

$$\int \frac{1}{1+\cos 2x} dx$$

$$\int (1+\sin^2 x \csc x) \, dx$$