Bottom-Water Drive

The van Everdingen and Hurst model discussed in the previous lectures is based on the radial diffusivity equation written without a term describing vertical flow from the aquifer.

correctly noted that in many cases reservoirs are situated on top of an aquifer with a continuous horizontal interface between the reservoir fluid and the aquifer water and with a significant aquifer thickness. that in such situations significant bottom-water drive would occur.

The proposed solution technique (van Everdingen and Hurst model), however, is not adequate to describe the vertical water encroachment in bottom-water-drive system. Coats (1962) presented a mathematical model that takes into ***account the vertical flow effects***

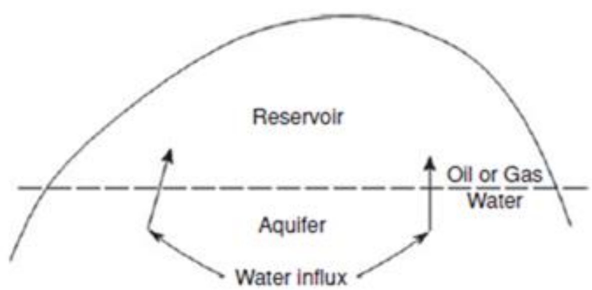

Figure 4 Sketch of bottom-water drive reservoir.

Coats and, later, Allard and Chen added a term to Eq. below to yield the following:

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D}$$

to be as following

$$\frac{\partial^2 p_D}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_D}{\partial_D}+\frac{\partial^2 p_D}{\partial z_D^2}=\frac{\partial p_D}{\partial t_D}$$

They suggested that it is possible to derive a general solution that is applicable to a variety of systems in terms of the dimension-less time tD, dimensionless radius rD, and a newly introduced dimensionless variable Z_D.

$$Z_D=\frac{h}{r_e\sqrt{F_k}}$$

Where:

Z_D = dimensionless vertical distance.

h = aquifer thickness, ft.

where

F_k is the ratio of vertical to horizontal permeability,

or:

$$F_k=\frac{K_v}{K_h}$$

where

k_v = vertical permeability

k_h = horizontal permeability

Using the definitions of dimensionless time, radius, and pressure and introducing a second dimensionless distance, z_D, last Eq. becomes as following:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + F_k \frac{\partial^2 p}{\partial z^2} = \frac{\phi \mu c_t}{0.0002637k}\frac{\partial p}{\partial t}$$

The authors developed a solution to the bottom-water influx that is comparable in form with that of van Everdingen and Hurst.

$$W_e = B\,\Delta p\,W_{eD}$$

$$W_e = B \sum \Delta P W_{eD}$$

They defined the water influx constant B identical to that of Equation in van Everdingen and Hurst ,

$$B = 1.119\,\phi\,c_t\,r_e^2\,h$$

where

We = cumulative water influx, bbl

B = water influx constant, bbl/psi

Δp = pressure drop at the boundary, psi

WeD = dimensionless water influx

Example: Calculate the cumulative water influx as a function of time for the reservoir data and boundary pressure data that follow:

Given:

$r_e = 2000$ ft $\quad$ $r_a = \infty$

$h = 200$ ft $\quad$ $k = 50$ md

$F_k = 0.04$ $\quad$ $\varphi = 0.10$

$\mu = 0.395$ cp $\quad$ $c_t = 8 \times 10^{-6}$ psi^{-1}

Time in days (t)	Average boundary pressure, psi
0	3000
30	2956
60	2917
90	2877
120	2844
150	2811

Solution:

$$r'_D = \frac{r_a}{r_e} = \infty$$

$$z'_D = \frac{h}{r_e F_k^{1/2}} = \frac{200}{2000(0.040)^{1/2}} = 0.5$$

$$B' = 1.119\phi h c_t r_e^2 = 1.119(0.10)(200)(8\times10^{-6})(2000)^2 = 716\, bbl/psi$$

$$t_D = 6.328\times10^{-3}\frac{kt}{\phi\mu c_t r_e^2} = 6.328\times10^{-3}\times\frac{(50t)}{(0.10)(0.395)\left[8(10)^{-6}\right](2000)^2} = 0.2503t$$

- Step Pressures can be calculated as follow:

$\Delta P_0 = 0$

$\Delta P_1 = \frac{Pi - P_1}{2} = \frac{3000 - 2956}{2} = 22\ psi$

$\Delta P_2 = \frac{Pi - P_2}{2} = \frac{3000 - 2917}{2} = 41.5\ psi$

$\Delta P_3 = \frac{P_1 - P_3}{2} = \frac{2956 - 2877}{2} = 39.5\ psi$

$\Delta P_4 = \frac{P_2 - P_4}{2} = \frac{2917 - 2844}{2} = 36.5\ psi$

$\Delta P_5 = \frac{P_3 - P_5}{2} = \frac{2877 - 2811}{2} = 33\ psi$

- Cumulative water influx for each time step can be calculated using the superposition principle:

$$\mathbf{We_{cumulative} = B \sum \Delta P\ We_D}$$

1. At 30 days (t=30):

We_{cum}=716×22×5.038=79,359 bbl

2. At 60 days (t=60)

We_{cum} =716× (22×8.389 +41.5×5.038) = 281,843 bbl

3. At 90 days (t=90):

We_{cum} =716× (22×11.414 +41.5×8.389 +39.5×5.038) = 571,549 bbl

4. At 120 days (t=120):

We_{cum} =716× (22×14.263 +41.5×11.414 +39.5×8.389 +36.5×5.038)

= 932,747 bbl

5. At 150 days (t=150):

$W_{e\,cum}$ = 716× (22×16.994 +41.5×14.263 +39.5×11.414 +36.5×8.389 +33×5.038)
= 1,352,587 bbl

Time in days (t)	Dimensionless time (t_D = 0.2503t)	W_{eD} (from Table)	Average boundary pressure, psi	Step pressure (ΔP)	Water Influx, bbl (W_e)
0	0	0	3000	0	0
30	7.5	5.038	2956	22.0	79,359
60	15.0	8.389	2917	41.5	281,843
90	22.5	11.414	2877	39.5	571,549
120	30.0	14.263	2844	36.5	932,747
150	37.5	16.994	2811	33.0	1,352,587