DEF.: The inverse sine fun. Denoted by $\sin^{-1}x$ is defined to be the inverse of restricted sine fun. $\sin x \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

DEF.: The inverse cosine fun. Denoted by $\cos^{-1}x$ is defined to be the inverse of restricted cosine fun. $\cos x \quad 0 \le x \le \pi$

DEF.: The inverse tangent fun. Denoted by $\tan^{-1}x$ is defined to be the inverse of restricted tangent fun. $\tan x \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

DEF.: The inverse secant fun. Denoted by $\sec^{-1}x$ is defined to be the inverse of restricted secant fun. $\sec x \quad 0 \le x \le \pi \quad , x \ne \frac{\pi}{2}$

<u>**NOTE**</u> **:** $\sin^{-1}x \ne \frac{1}{\sin x}$

<u>**EXAM**</u> **:**

(1) $if \quad x = \sin^{-1}\frac{1}{2} \;,\; find\ value\ \ of \quad x$

$$x = \sin^{-1}\frac{1}{2} \Rightarrow \sin x = \cancel{\sin}\ \cancel{\sin^{-1}}\frac{1}{2} \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$$

(2) $simplify \quad \sin(\sin^{-1}\frac{1}{2})$

$$\because \sin^{-1}\frac{1}{2} = \frac{\pi}{6} \Rightarrow \sin(\sin^{-1}\frac{1}{2}) = \sin(\frac{\pi}{6}) = \frac{1}{2}$$

(3) $show\ \ that \quad \cos(\sin^{-1}\frac{1}{2}) = \frac{\sqrt{3}}{2}$

$$\because \cos x = \sqrt{1-\sin^2 x}$$

$$\therefore \cos(\sin^{-1}x) = \sqrt{1-\sin^2\sin^{-1}\frac{1}{2}} = \sqrt{1-\sin^2\frac{\pi}{6}} = \sqrt{1-(\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$$

Inverse Trigometric Function Properties

1) $\sin^{-1}(-x) = -\sin^{-1}(x)$

2) $\tan^{-1}(-x) = -\tan^{-1}(x)$

3) $\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$

4) $\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$

5) $\sec^{-1}(x) = \cos^{-1}(\frac{1}{x})$

6) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

7) $\cot^{-1}(x) = \tan^{-1}(\frac{1}{x})$

8) $\csc^{-1}(x) = \sin^{-1}(\frac{1}{x})$

PROOF :

1) $let \quad y = \sin^{-1}(-x)$

$\sin y = \cancel{\sin}\ \cancel{\sin^{-1}}(-x) = -x \Rightarrow -\sin y = x$

$\sin(-y) = x \Rightarrow -y = \sin^{-1} x \Rightarrow y = -\sin^{-1} x$

$\therefore \sin^{-1}(-x) = -\sin^{-1} x$

3) $let \quad y = \frac{\pi}{2} - \sin^{-1}(x)$

$\sin^{-1}(x) = \frac{\pi}{2} - y \Rightarrow x = \sin(\frac{\pi}{2} - y) = \cos y \Rightarrow y = \cos^{-1} x$

$\therefore \cos^{-1} x = \frac{\pi}{2} - \sin^{-1}(x)$

8) $let \quad y = \sin^{-1}(\frac{1}{x})$

$\sin y = \frac{1}{x} \Rightarrow x = \frac{1}{\sin y} = \csc y \Rightarrow y = \csc^{-1} x$

$\therefore \csc^{-1} x = \sin^{-1}(\frac{1}{x})$

مشتقات الدوال المثلثية العكسية :

$\boxed{1}$ $\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\boxed{2}$ $\frac{d}{dx}\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\boxed{3}$ $\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$

$\boxed{4}$ $\frac{d}{dx}\cot^{-1}u = \frac{-1}{1+u^2}\frac{du}{dx}$

$\boxed{5}$ $\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

$\boxed{6}$ $\frac{d}{dx}\csc^{-1}u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

EXAM : Find $\frac{dy}{dx}$ to :

$$y = \sin^{-1} 2x \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

$$y = \tan^{-1} 3x + e^{\tan^{-1} x} \Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} + e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$y = \cos^{-1} \cos x \Rightarrow \frac{dy}{dx} = \frac{-(-\sin x)}{\sqrt{1-\cos^2 x}} = \frac{\sin x}{\sin x} = 1$$

$$OR \quad \cancel{\cos^{-1}} \cancel{\cos} x = x \Rightarrow \frac{dy}{dx} = 1$$

$$y = e^x \sec^{-1} x \Rightarrow \frac{dy}{dx} = e^x \cdot \frac{1}{x\sqrt{x^2-1}} + \sec^{-1} x \cdot e^x$$

تكاملات الدوال المثلثية العكسية :

$$\int \frac{du}{\sqrt{a^2-u^2}} = \begin{cases} \frac{1}{a}\sin^{-1}\frac{u}{a} \\ \\ \frac{-1}{a}\cos^{-1}\frac{u}{a} \end{cases} + C$$

$$\int \frac{du}{a^2+u^2} = \begin{cases} \frac{1}{a}\tan^{-1}\frac{u}{a} \\ \\ \frac{-1}{a}\cot^{-1}\frac{u}{a} \end{cases} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \begin{cases} \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| \\ \\ \frac{-1}{a}\csc^{-1}\left|\frac{u}{a}\right| \end{cases} + C$$

EXAM

$$\int \frac{dx}{1+16x^2} = \frac{1}{4}\int \frac{4dx}{1+(4x)^2} = \frac{1}{4}\tan^{-1}(4x) + C$$

$$\int \frac{dx}{9+4x^2} = \frac{1}{2}\cdot\frac{1}{3}\tan^{-1}\frac{2x}{3} + C$$

$$\int \frac{e^x dx}{1+e^{2x}} = \int \frac{e^x dx}{1+(e^x)^2} = \tan^{-1}(e^x) + C$$

$$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{e^x dx}{e^x \sqrt{e^{2x}-1}} = \sec^{-1}\left|e^x\right| + C$$

$$\int \frac{dx}{\sqrt{x}\sqrt{4-x}} = 2\int \frac{dx}{2\sqrt{x}\sqrt{4-x}} = 2 \cdot \frac{1}{2}\sin^{-1}\frac{\sqrt{x}}{2} = \sin^{-1}\frac{\sqrt{x}}{2} + C$$

$$\int \frac{x}{3+x^4} dx = \frac{1}{2}\int \frac{2x}{3+(x^2)^2} dx = \frac{1}{2}\frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} + C$$

HOME WORK

1) Find $\frac{dy}{dx}$ to

$y = Ln(\cos^{-1} x)$, $y = \sqrt{\cot^{-1} x}$, $y = (\tan x)^{-1}$, $y = \cot^{-1}\sqrt{x}$

2) Find $\frac{dy}{dx}$ to

$x^3 + x \tan^{-1} y = e^y$, $\sin^{-1}(xy) = cos^{-1}(x-y)$

3) Find

$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$, $\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx$, $\int \frac{dx}{x\sqrt{1-Ln^2 x}}$

$\int \frac{\sin x}{\cos^2 x + 1} dx$, $\int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}} dx$, $\int \frac{\sin x \cos x}{1+\cos^2(2x)} dx$

4) Find the area of the reigon bounded by the curve $x^2 y + 4y - 12 = 0$ **and x-axis .**

5) Find the volume generated by rotation about the x-axis , the reigon bounded by the curve $y = 5(x^2+1)^{-1/2}$, $y > 0$, *from* $x = 0 , x = 4$

6) Find the length of the curve :

$y = \frac{1}{\sqrt{1-x^2}}$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$