Fluid Flow Equations

The fluid flow equations that are used to describe the flow behavior in a reservoir can take many forms depending upon the combination of variables such as, (types of flow, types of fluids, etc.). By combining the conservation of mass equation with the transport equation (Darcy's equation) and various equations-of-state, the necessary flow equations can be developed. Since all flow equations to be considered depend on Darcy's Law, it is important to consider this transport relationship first.

Darcy's Law

The fundamental law of fluid motion in porous media is Darcy's Law. The mathematical expression developed by Henry Darcy in 1856 states the velocity of a homogeneous fluid in a porous medium is proportional to the pressure gradient and inversely proportional to the fluid viscosity. For a horizontal linear system, this relationship is:

$$\nu = \frac{q}{A} \alpha \left[-\frac{1}{\mu}\frac{dp}{dx} \right]$$

Darcy defined the proportionality constant "α" as the permeability of the rock and represented by "k"

$$\nu = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx}$$

Where:

v = apparent velocity, cm/sec

q = volumetric flow rate, cm3 /sec

A =total cross-sectional area of the porous medium, cm2

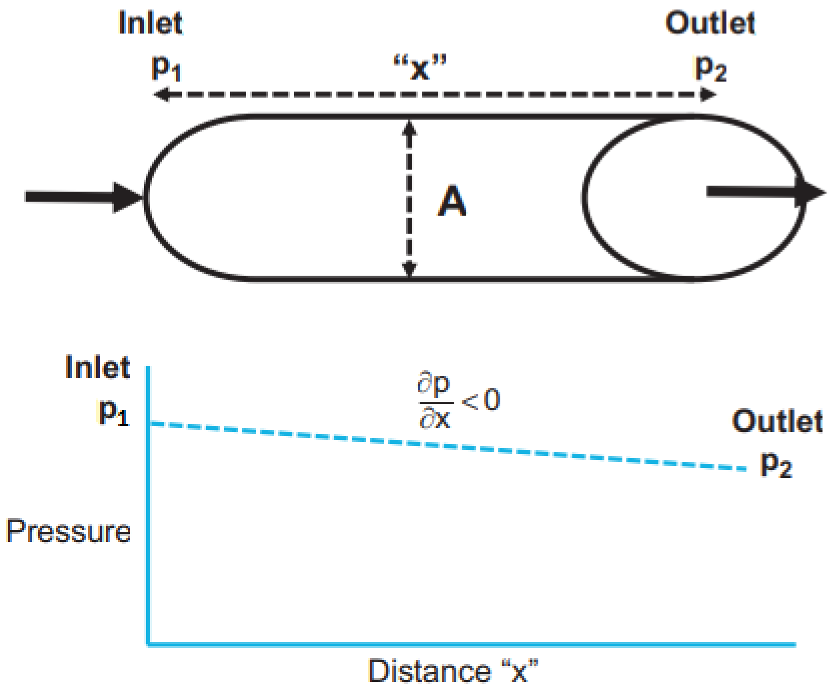
μ = The fluid viscosity, centipoise units

dp/dx = and the pressure gradient, atm/cm, taken in the same direction as ν and q.

k = the permeability of the rock expressed in Darcy units.

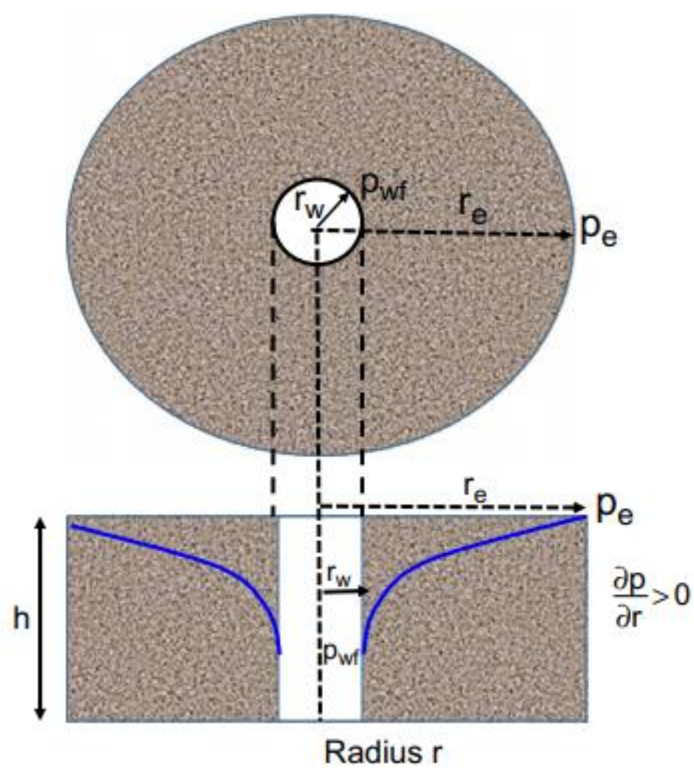
The negative sign in Equation above is added because the pressure gradient is negative in the direction of flow as shown in Figure 1. For a horizontal-radial system, the pressure gradient is positive (see Figure 2) and Darcy's equation can be expressed in the following generalized radial form:

$$\nu = \frac{q_r}{A_r} = \frac{k}{\mu}\left(\frac{\partial p}{\partial r}\right)_r$$



$\frac{\partial p}{\partial x}$<0 meaning negative signal (-)

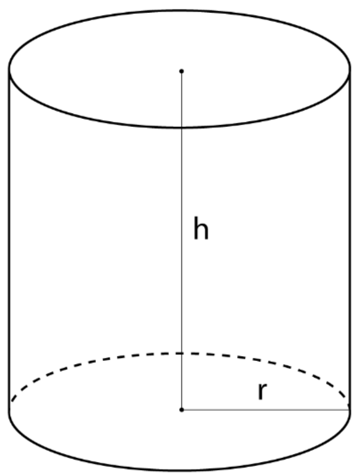
Figure 1 Pressure vs. distance in a linear flow.



Fgure 2 Pressure profile and gradient in a radial flow.

The cross-sectional area at radius r is essentially the surface area of a cylinder. For a fully penetrated well with a net thickness of h, the cross-sectional area Ar is given by:

$$A_r = 2\pi rh$$



Steady-State Flow

As defined previously, steady-state flow represents the condition that exists when the pressure throughout the reservoir does not change with time. The applications of the steady-state flow to describe the flow behavior of several types of fluid in different reservoir geometries are presented below. These include:

- Linear flow of incompressible fluids
- Linear flow of slightly compressible fluids
- Linear flow of compressible fluids
- Radial flow of incompressible fluids
- Radial flow of slightly compressible fluids

- Radial flow of compressible fluids
- Multiphase flow

Linear Flow of Incompressible Fluids

Fluids In the linear system, it is assumed the flow occurs through a constant crosssectional area A, where both ends are entirely open to flow. It is also assumed that no flow crosses the sides, top, or bottom as shown in Figure 3.

$$\nu = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx}$$

If an incompressible fluid is flowing across the element dx, then the fluid velocity v and the flow rate q are constants at all points. The flow behavior in this system can be expressed by the differential form of Darcy’s equation, i.e., Equation above. Separating the variables of Equation above and integrating over the length of the linear system gives:

$$\frac{q}{A}\int_{0}^{L} dx = -\frac{k}{\mu}\int_{p_1}^{p_2} dp$$

or:

$$q = \frac{kA(p_1 - p_2)}{\mu L}$$

It is desirable to express the above relationship in customary field units, or:

$$q = \frac{0.001127\, kA(p_1 - p_2)}{\mu L}$$

the apparent velocity:

$$v = \frac{q}{A}$$

the actual fluid velocity:

$$v = \frac{q}{\phi A}$$

where:

q=flow rate, bbl/day

k=absolute permeability, md

p=pressure, psia

μ= viscosity, cp

L= distance, ft

A= cross-sectional area, ft2

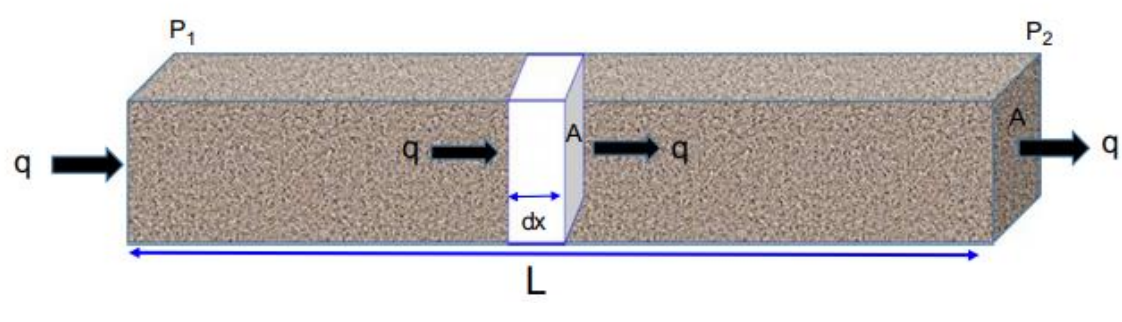


Figure 3 Darcy’s linear flow model

The difference in the pressure (p1–p2) is not the only driving force in a tilted reservoir. **The gravitational force** is the other important driving force that must be accounted for to determine the **direction and rate of flow**. The fluid gradient force (gravitational force) is always directed vertically downward while the force that results from an applied pressure drop may be in any direction. The force causing flow would then be the vector sum of these two. In practice, we obtain this result by introducing a new parameter, called **fluid potential**, which has the same dimensions as pressure, e.g., **psi**. Its symbol is **Φ**.

Letting **Δzi be the vertical distance from a point (i) in the reservoir to this datum level.**

$$\Phi_i = p_i - \left(\frac{\rho}{144}\right)\Delta z_i$$

where ρ is the density in lb/ft3.

Expressing the fluid density in gm/cc in Equation gives:

$$\Phi_i = p_i - 0.433\ \gamma\ \Delta z_i$$

Where:

Φ_i = fluid potential at point i, psi

p_i = pressure at point i, psi

Δz_i = vertical distance from point i to the selected datum level

ρ = fluid density, lb/ft^3

γ = fluid density, gm/cm^3.

The datum is usually selected at the **gas-oil contact, oil-water contact, or at the highest point in formation**. In using Equations above to calculate the fluid potential Φi at location i, the vertical distance Δzi is assigned as a negative value when the point i is below the datum level and as a positive when it is above the datum level, i.e.:

If point i is above the datum level:

$$\Phi_i = p_i + \left(\frac{\rho}{144}\right)\Delta z_i$$

and

$$\Phi_i = p_i + 0.433\ \gamma\ \Delta z_i$$

If point i is below the datum level:

$$\Phi_i = p_i - \left(\frac{\rho}{144}\right)\Delta z_i$$

and

$$\Phi_i = p_i - 0.433\ \gamma\ \Delta z_i$$

Applying the above-generalized concept to Darcy's equation gives:

$$q = \frac{0.001127\, kA(\Phi_1 - \Phi_2)}{\mu L}$$

It should be pointed out that the fluid potential drop (Φ1 – Φ2) is equal to the pressure drop (P1 – P2) **only when the flow system is horizontal**.

Example: An incompressible fluid flows in a linear porous media with the following properties:

L = 2000 ft   h = 20 ft   width = 300 ft

K = 100 md ϕ = 15% μ = 2 cp

P1 = 2000 psi   P2 = 1990 psi

Assume that the porous media with these properties is tilted with a dip angle of 5° as shown in Figure (6.2). The incompressible fluid has a density of 42 lb/ft3.

Calculate:

a. Flow rate in bbl/day

b. Apparent fluid velocity in ft/day

c. Actual fluid velocity in ft/day

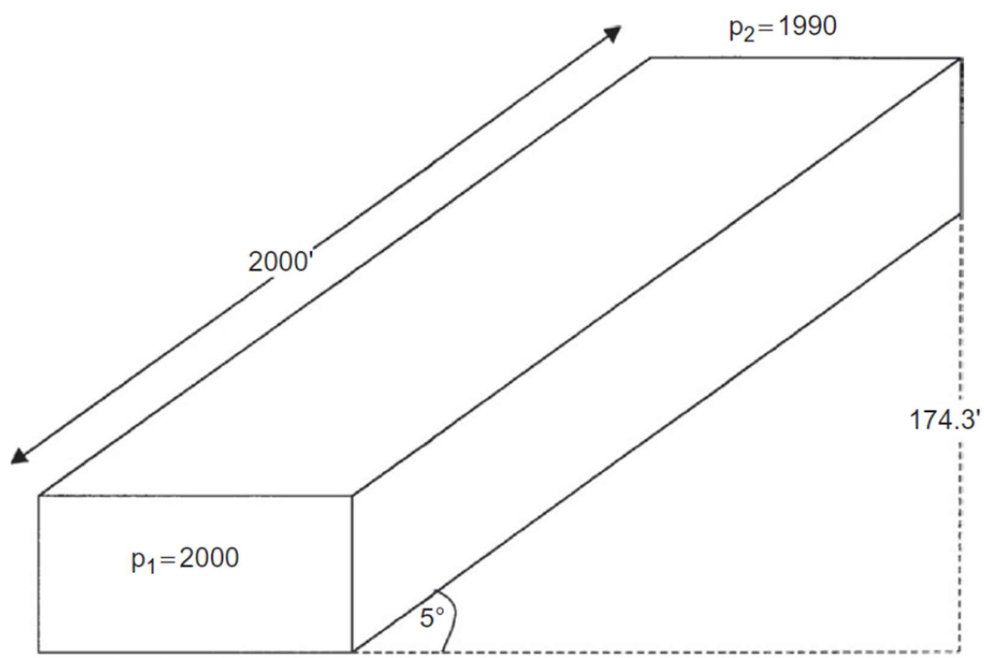


Figure (6.2) Example of a tilted layer.

Solution:

Step 1. For the purpose of illustrating the concept of fluid potential, select the datum level at half the vertical distance between the two points, **(at 87.15 ft)** as shown in Figure 6.2.

Step 2. Calculate the fluid potential at Points 1 and 2.

Since Point 1 is below the datum level, then:

$$\Phi_1 = p_1 - \left(\frac{\rho}{144}\right)\Delta Z_1 = 2000 - \left(\frac{42}{144}\right)(87.15) = 1974.58\, psi$$

Since Point 2 is above the datum level, then:

$$\Phi_2 = p_2 + \left(\frac{\rho}{144}\right)\Delta Z_2 = 1990 + \left(\frac{42}{144}\right)(87.15) = 2015.42\, psi$$

Because $\Phi_2 > \Phi_1$, the fluid flows downward from Point 2 to Point 1. The difference in the fluid potential is:

$$\Delta\Phi = 2015.42 - 1974.58 = 40.48\, psi$$

A = h* width

= 20* 300 = 6000 ft^2

Step 3. Calculate the flow rate:

$$q = \frac{(0.001127)(100)(6000)(40.84\,)}{(2)(2000)} = 6.9\ \ bbl/day$$

Step 4. Calculate the velocity:

$$apparent\ velocity = v = \frac{q}{A} = \frac{(6.9)(5.615)}{(6000)} = 0.0065\, ft/day$$

$$actual\ velocity = v = \frac{q}{\phi A} = \frac{(6.9)(5.615)}{(0.15)(6000)} = 0.043\, ft/day$$

Radial Flow of Incompressible Fluid

In a radial flow system, all fluids move toward the producing well from all directions. Before flow can take place, however, a pressure differential must exist. Thus, if a well is to produce oil, which implies a flow of fluids through the formation to the wellbore, the pressure in the formation at the wellbore must be less than the pressure in the formation at some distance from the well. The pressure in the formation at the wellbore of a producing well is known as the bottom-hole flowing pressure (flowing BHP, pwf). Consider figure below, which schematically illustrates the radial flow of an incompressible fluid toward a vertical well. The formation is considered to a uniform thickness h and a constant permeability k. Because the fluid is incompressible, **the flow rate q must be constant at all radii. Due to the steady-state flowing condition**, the pressure profile around the wellbore is maintained constant with time.

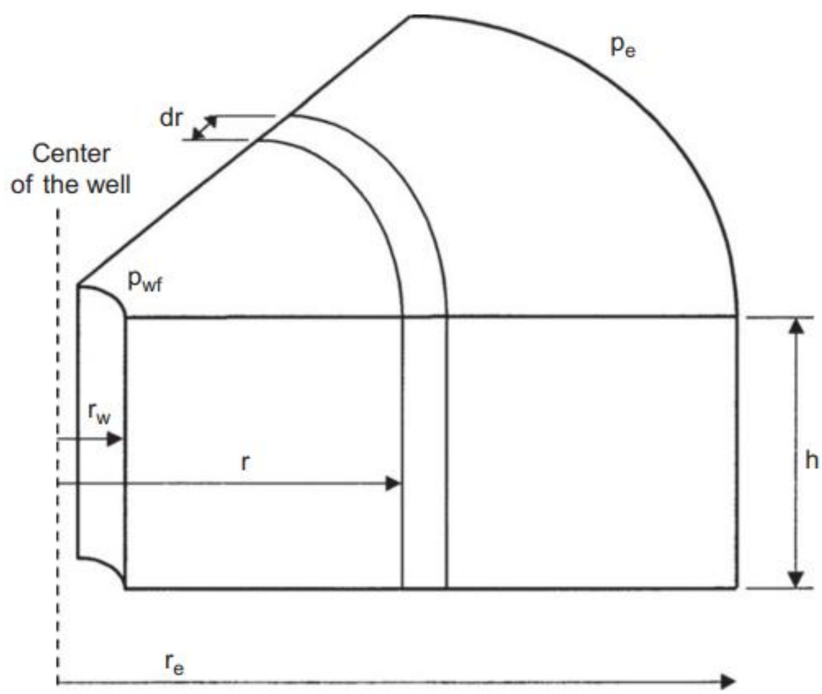


Figure showing radial flow model.

Let p_{wf} represent the maintained bottom-hole flowing pressure at the wellbore radius r_w and p_e denote the external pressure at the external or drainage radius. Darcy's equation as described by Equation below can be used to determine the flow rate at any radius r:

$$v = \frac{q}{A_r} = 0.001127 \frac{k}{\mu} \frac{dp}{dr}$$

where:

v = apparent fluid velocity, bbl/day-ft^2

q = flow rate at radius r, bbl/day

k = permeability, md

μ = viscosity, cp

0.001127 = conversion factor to express the equation in field units

Ar = cross-sectional area at radius r

The minus sign is no longer required for the radial system, whereas the radius increases in the same direction as the pressure. In other words, as the radius increases going away from the wellbore the pressure also increases. At any point in the reservoir the cross-sectional area across which flow occurs will be **the surface area of a cylinder, which is 2πrh,** or:

$$v = \frac{q}{A_r} = \frac{q}{2\pi rh} = 0.001127 \frac{k}{\mu} \frac{dp}{dr}$$

The flow rate for a crude oil system is customarily expressed in surface units, i.e., stock-tank barrels (STB), rather than reservoir units. Using the symbol Qo to represent the oil flow as expressed in STB/day, then:

$$q = B_o\ Q_o$$

where Bo is the oil formation volume factor bbl/STB. The flow rate in Darcy's equation can be expressed in STB/day to give:

$$\frac{Q_o\ B_o}{2\pi rh} = 0.001127 \frac{k}{\mu_o} \frac{dp}{dr}$$

Integrating the above equation between two radii, r_1 and r_2, when the pressures are p_1 and p_2 yields:

$$\int_{r_1}^{r_2} \left(\frac{Q_o}{2\pi h}\right) \frac{dr}{r} = 0.001127 \int_{P_1}^{P_2} \left(\frac{k}{\mu_o B_o}\right) dp$$

For incompressible system in a uniform formation, Equation above can be simplified to:

$$\frac{Q_o}{2\pi h} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{0.001127k}{\mu_o\ B_o} \int_{P_1}^{P_2} dp$$

Performing the integration, gives:

$$Q_o = \frac{0.00708\ k\ h\ (p_2 - p_1)}{\mu_o\ B_o\ \ln\ (r_2/r_1)}$$

Frequently the two radii of interest are the wellbore radius r_w and the external or drainage radius r_e. Then:

$$Q_o = \frac{0.00708\ k\,h\,(p_e - p_w)}{\mu_o\ B_o\ \ln(r_e/r_w)}$$

Where:

Q_o = oil, flow rate, STB/day

p_e = external pressure, psi

p_{wf} = bottom-hole flowing pressure, psi

k = permeability, md

μ_o = oil viscosity, cp

Bo = oil formation volume factor, bbl/STB

h = thickness, ft

r_e = external or drainage radius, ft

r_w = wellbore radius, ft

The external (drainage) radius re is usually determined from the well spacing by equating the area of the well spacing with that of a circle, i.e.,

$$\pi\, r_e^2 = 43,560\, A$$

or

$$r_e = \sqrt{\frac{43,560\, A}{\pi}}$$

The following equation

$$Q_o = \frac{0.00708\ k h\,(p_e - p_w)}{\mu_o\ B_o\ \ln(r_e/r_w)}$$

Can be arranged to solve for the pressure p at any radius r to give:

$$p = p_{wf} + \left[\frac{Q_o\, B_o\, \mu_o}{0.00708\, kh}\right] \ln\left(\frac{r}{r_w}\right)$$

Example: An oil well in the Nameless Field is producing at a stabilized rate of 600 STB / day at a stabilized bottom-hole flowing pressure of 1800 psi. Analysis of the pressure buildup test data indicates that the pay zone is characterized by a permeability of 120 md and a uniform thickness of 25 ft. The well drains an area of approximately 40 acres. The following additional data is available:

rw = 0.25ft

Bo = 1.25 bbl /STB $\mu_o = 2.5$cp

Calculate the pressure profile (distribution) and list the pressure drop across 1 ft intervals from rw to 1.25 ft, 4 to 5 ft, 19 to 20 ft, 99 to 100 ft, and 744 to 745 ft.

Solution:

$$p = p_{wf} + \left[\frac{\mu_o B_o Q_o}{0.00708\, kh}\right] \ln(r/r_w)$$

$$p = 1800 + \left[\frac{(2.5)(1.25)(600)}{(0.00708)(120)(25)}\right] \ln\left(\frac{r}{0.25}\right)$$

$$p = 1800 + 88.28 \ \ln\left(\frac{r}{0.25}\right)$$

r, ft	p, psi	Radius Interval	Pressure drop
0.25	1800		
1.25	1942	0.25–1.25	1942 − 1800 = 142 psi
4	2045		
5	2064	4–5	2064 − 2045 = 19 psi
19	2182		
20	2186	19–20	2186 − 2182 = 4 psi
99	2328		
100	2329	99–100	2329 − 2328 = 1 psi
744	2506.1		
745	2506.2	744–745	2506.2 − 2506.1 = 0.1 psi