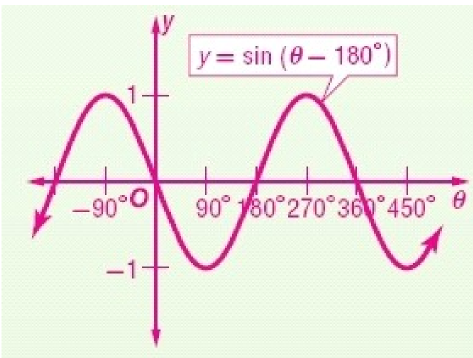
Translations of Trigonometric Graphs

State the amplitude, period, and phase shift for each function. Then graph the function.

1. $y = \sin(\theta - 180°)$

ANSWER:

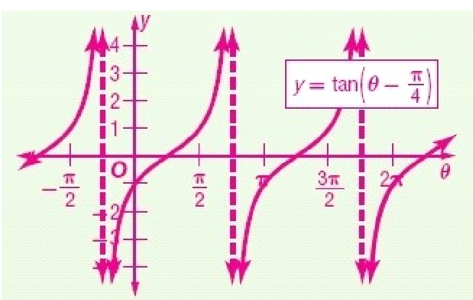
1; 360°; $h = 180°$



2. $y = \tan\left(\theta - \frac{\pi}{4}\right)$

ANSWER:

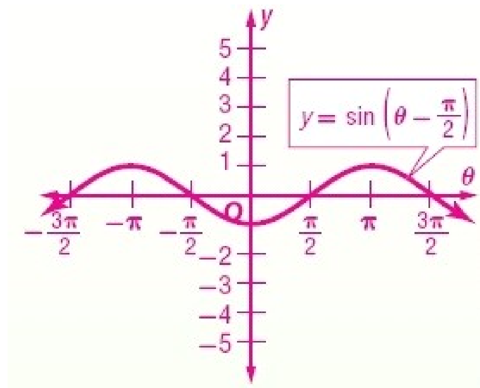
no amplitude; 180°; $h = \frac{\pi}{4}$



3. $y = \sin\left(\theta - \frac{\pi}{2}\right)$

ANSWER:

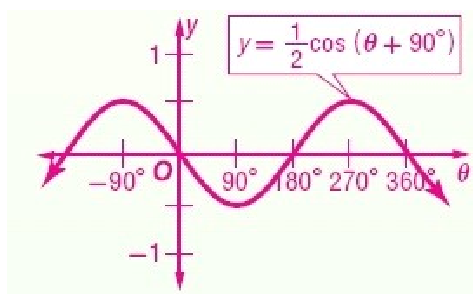
1; 2π; $h = \frac{\pi}{2}$



4. $y = \frac{1}{2}\cos(\theta + 90°)$

ANSWER:

$\frac{1}{2}$; 360°; $h = -90°$

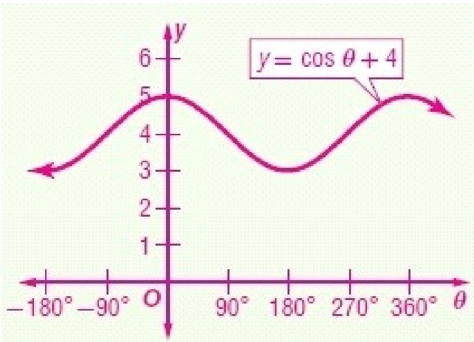


Translations of Trigonometric Graphs

State the amplitude, period, vertical shift, and equation of the midline for each function. Then graph the function.

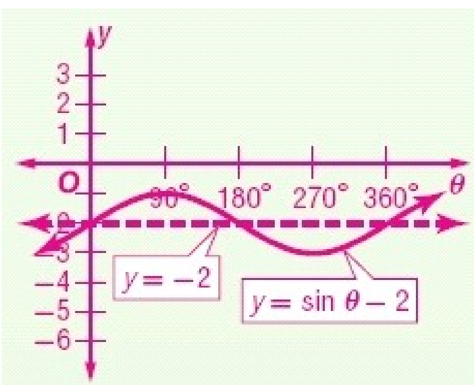
5. $y = \cos\theta + 4$

ANSWER:
1; 360°; $k = 4$; $y = 4$



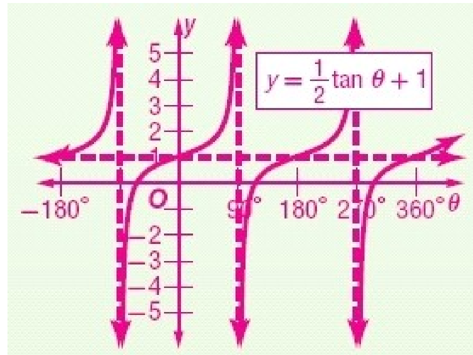
6. $y = \sin\theta - 2$

ANSWER:
1; 360°; $k = –2$; $y = –2$



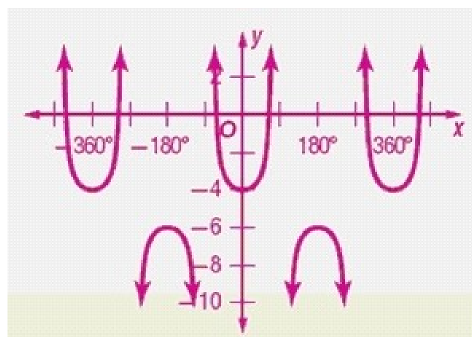
7. $y = \frac{1}{2}\tan\theta + 1$

ANSWER:
no amplitude; 180°; $k = 1$; $y = 1$



8. $y = \sec\theta - 5$

ANSWER:
no amplitude; 360°; $k = –5$; $y = –5$



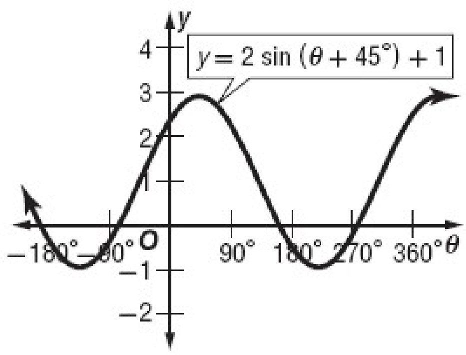
Translations of Trigonometric Graphs

CCSS REGULARITY State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

9. $y = 2\sin(\theta + 45°) + 1$

ANSWER:

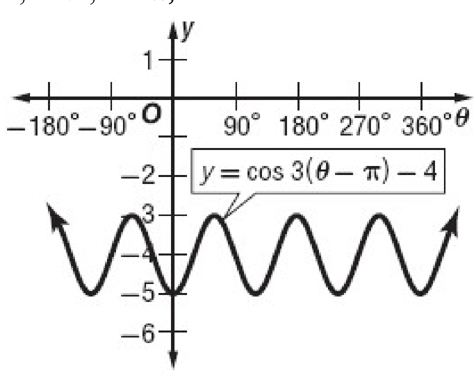
2; 360°; $h = -45°$; $k = 1$



10. $y = \cos 3(\theta - \pi) - 4$

ANSWER:

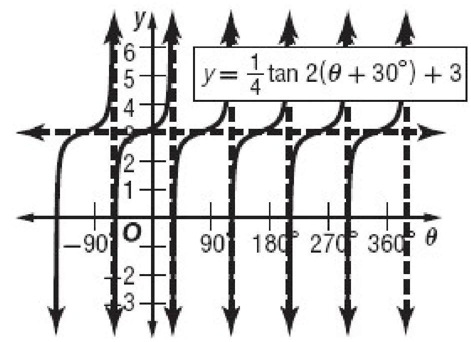
1; 120°; $h = \pi$; $k = -4$



11. $y = \frac{1}{4}\tan 2(\theta + 30°) + 3$

ANSWER:

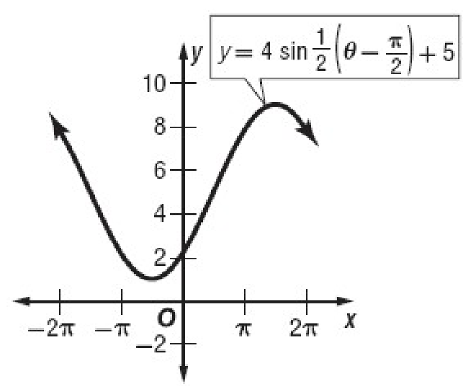
no amplitude; 90°; $h = -30°$; $k = 3$



12. $y = 4\sin\frac{1}{2}\left(\theta - \frac{\pi}{2}\right) + 5$

ANSWER:

4; 4π; $h = \frac{\pi}{2}$; $k = 5$

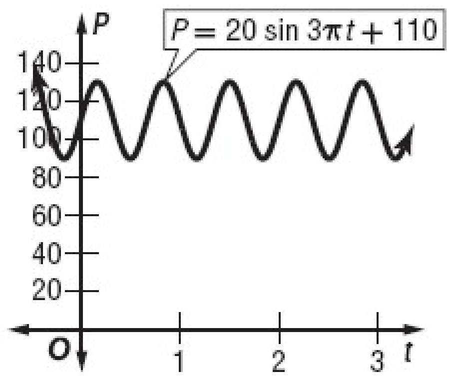


Translations of Trigonometric Graphs

13. **EXERCISE** While doing some moderate physical activity, a person's blood pressure oscillates between a maximum of 130 and a minimum of 90. The person's heart rate is 90 beats per minute. Write a sine function that represents the person's blood pressure P at time t seconds. Then graph the function.

ANSWER:

$P = 20 \sin 3\pi t + 110$

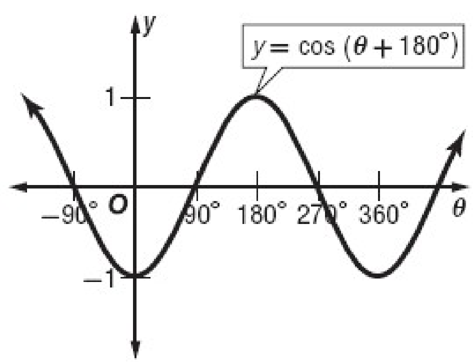


State the amplitude, period, and phase shift for each function. Then graph the function.

14. $y = \cos(\theta + 180°)$

ANSWER:

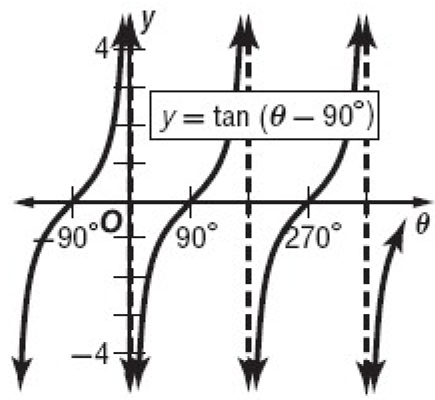
1; 360°; $h = -180°$



15. $y = \tan(\theta - 90°)$

ANSWER:

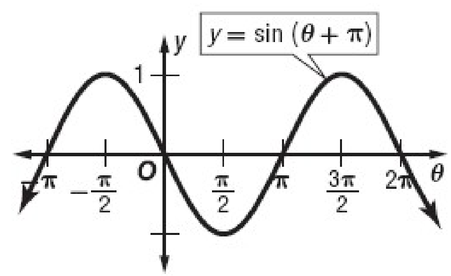
no amplitude; 180°; $h = 90°$



16. $y = \sin(\theta + \pi)$

ANSWER:

1; 2π; $h = -\pi$

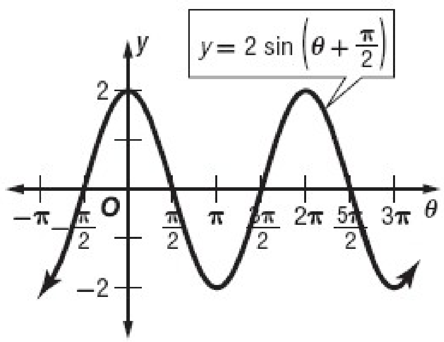


Translations of Trigonometric Graphs

17. $y = 2\sin\left(\theta + \frac{\pi}{2}\right)$

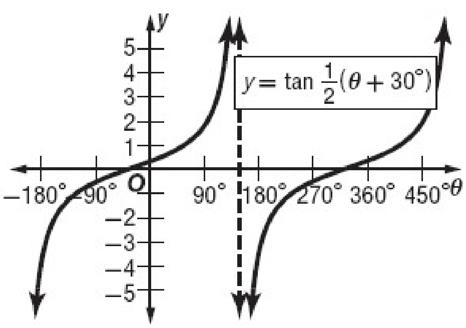
ANSWER:

2; 2π; $h = -\frac{\pi}{2}$



18. $y = \tan\frac{1}{2}(\theta + 30°)$

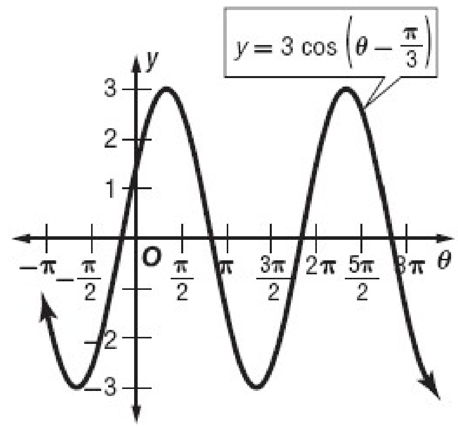
ANSWER:
no amplitude; 360°; $h = -30°$



19. $y = 3\cos\left(\theta - \frac{\pi}{3}\right)$

ANSWER:

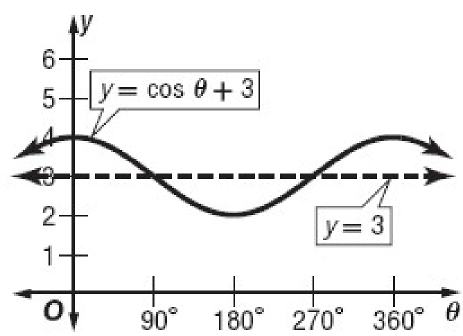
3; 2π; $h = \frac{\pi}{3}$



State the amplitude, period, vertical shift, and equation of the midline for each function. Then graph the function.

20. $y = \cos\theta + 3$

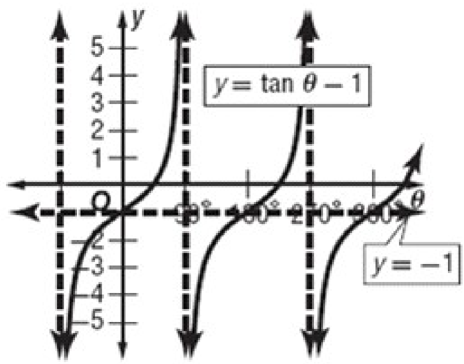
ANSWER:
1; 360°; $k = 3$; $y = 3$



Translations of Trigonometric Graphs

21. $y = \tan \theta - 1$

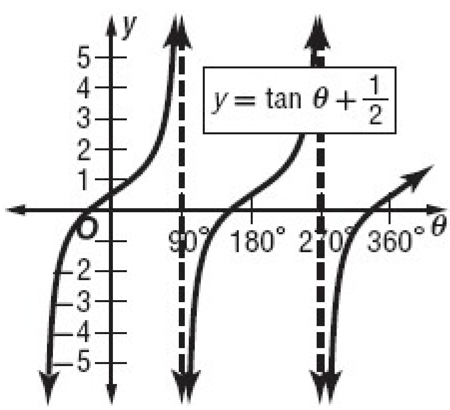
ANSWER:
no amplitude; 180°; $k = -1$; $y = -1$



22. $y = \tan \theta + \frac{1}{2}$

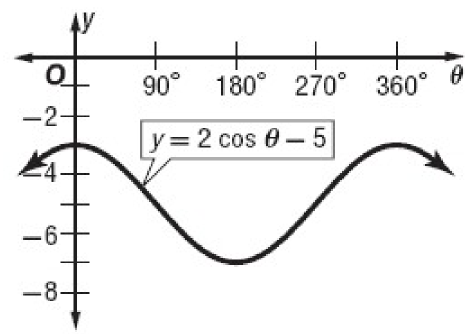
ANSWER:

no amplitude; 180°; $k = \frac{1}{2}$; $y = \frac{1}{2}$



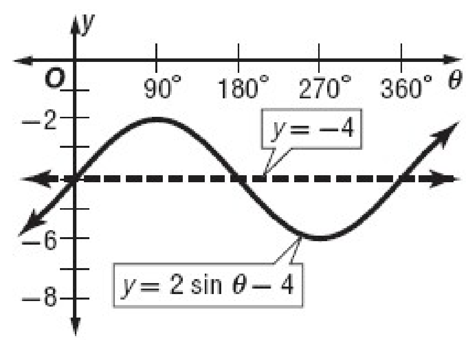
23. $y = 2 \cos \theta - 5$

ANSWER:
2; 360°; $k = -5$; $y = -5$



24. $y = 2 \sin \theta - 4$

ANSWER:
2; 360°; $k = -4$; $y = -4$

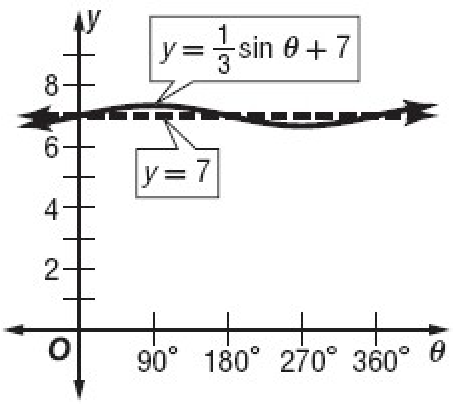


Translations of Trigonometric Graphs

25. $y = \frac{1}{3}\sin\theta + 7$

ANSWER:

$\frac{1}{3}$; 360°; k = 7; y = 7

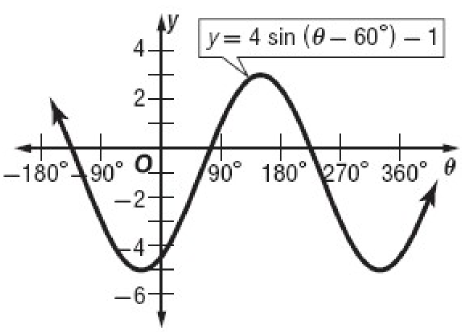


State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

26. $y = 4\sin(\theta - 60^\circ) - 1$

ANSWER:

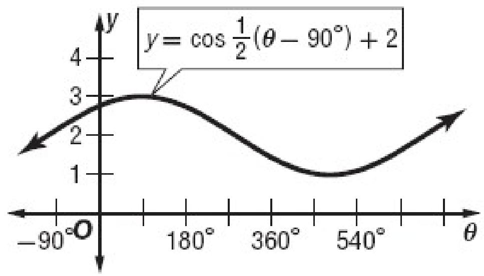
4; 360°; h = 60°; k = –1



27. $y = \cos\frac{1}{2}(\theta - 90^\circ) + 2$

ANSWER:

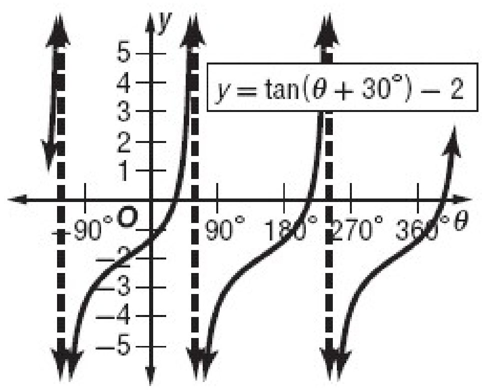
1; 720°; h = 90°; k = 2



28. $y = \tan(\theta + 30^\circ) - 2$

ANSWER:

no amplitude; 180°; h = –30°; k = –2

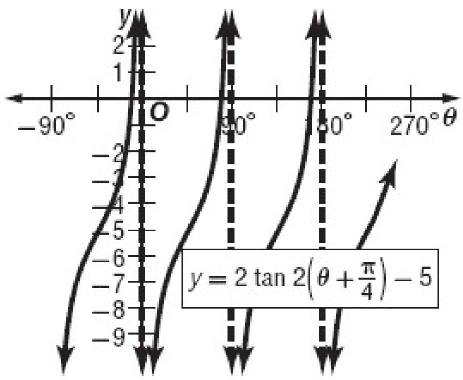


Translations of Trigonometric Graphs

29. $y = 2\tan 2\left(\theta + \frac{\pi}{4}\right) - 5$

ANSWER:

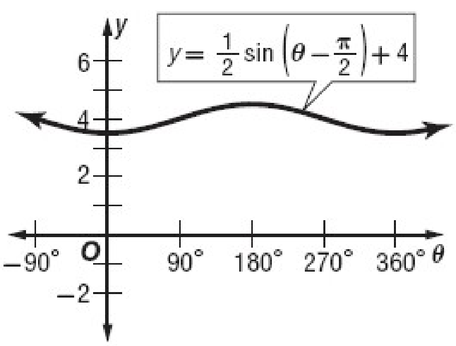
no amplitude; $\frac{\pi}{2}$; $h = -\frac{\pi}{4}$; $k = -5$



30. $y = \frac{1}{2}\sin\left(\theta - \frac{\pi}{2}\right) + 4$

ANSWER:

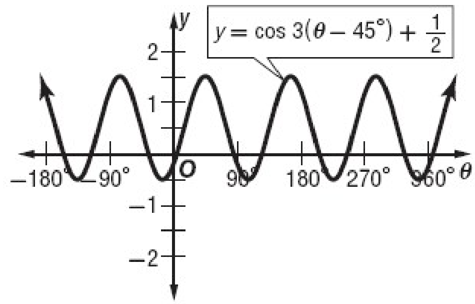
$\frac{1}{2}$; 2π; $h = \frac{\pi}{2}$; $k = 4$



31. $y = \cos 3(\theta - 45°) + \frac{1}{2}$

ANSWER:

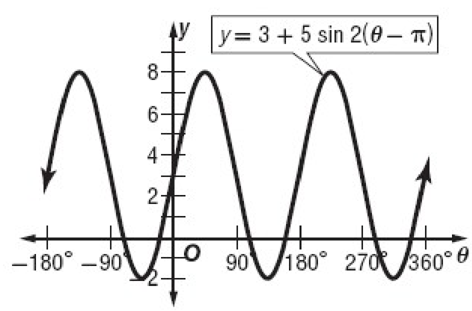
1; 120°; $h = 45°$; $k = \frac{1}{2}$



32. $y = 3 + 5 \sin 2(\theta - \pi)$

ANSWER:

5; π; $h = \pi$; $k = 3$

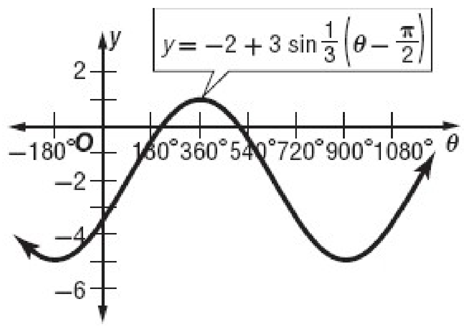


Translations of Trigonometric Graphs

33. $y = -2 + 3\sin\frac{1}{3}\left(\theta - \frac{\pi}{2}\right)$

ANSWER:

3; 6π; $h = \frac{\pi}{2}$; $k = -2$



34. **TIDES** The height of the water in a harbor rose to a maximum height of 15 feet at 6:00 p.m. and then dropped to a minimum level of 3 feet by 3:00 a.m. The water level can be modeled by the sine function. Write an equation that represents the height *h* of the water *t* hours after noon on the first day.

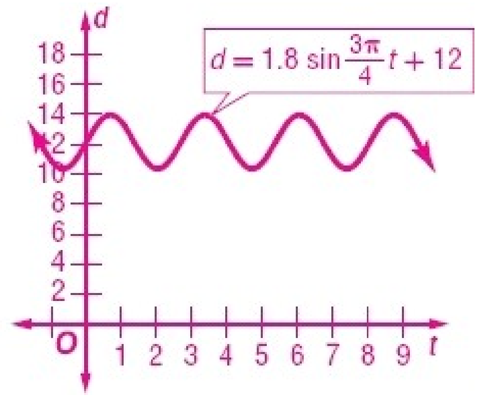
ANSWER:

$h = 9 + 6\sin\left[\frac{\pi}{9}(t - 1.5)\right]$

35. **LAKES** A buoy marking the swimming area in a lake oscillates each time a speed boat goes by. Its distance *d* in feet from the bottom of the lake is given by $d = 1.8\sin\frac{3\pi}{4}t + 12$, where *t* is the time in seconds.

Graph the function. Describe the minimum and maximum distances of the buoy from the bottom of the lake when a boat passes by.

ANSWER:



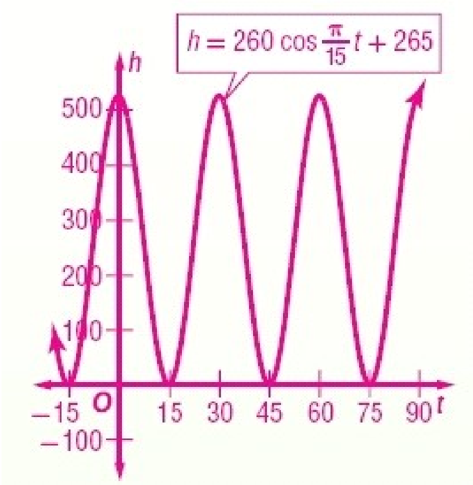
min: 10.2 ft; max: 13.8 ft

Translations of Trigonometric Graphs

36. **FERRIS WHEEL** Suppose a Ferris wheel has a diameter of approximately 520 feet and makes one complete revolution in 30 minutes. Suppose the lowest car on the Ferris wheel is 5 feet from the ground. Let the height at the top of the wheel represent the height at time 0. Write an equation for the height of a car h as a function of time t. Then graph the function.

ANSWER:

$$h = 260\cos\frac{\pi}{15}t + 265$$



Write an equation for each translation.

37. $y = \sin x$, 4 units to the right and 3 units up

ANSWER:

$y = \sin(x - 4) + 3$

38. $y = \cos x$, 5 units to the left and 2 units down

ANSWER:

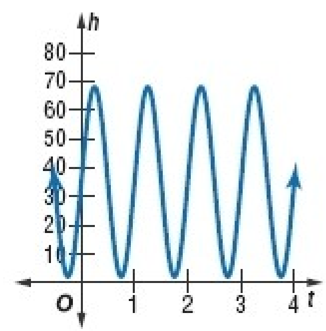
$y = \cos(x + 5) - 2$

39. $y = \tan x$, π units to the right and 2.5 units up

ANSWER:

$y = \tan(x - \pi) + 2.5$

40. **JUMP ROPE** The graph approximates the height of a jump rope h in inches as a function of time t in seconds. A maximum point on the graph is (1.25, 68), and a minimum point is (2.75, 2).



a. Describe what the maximum and minimum points mean in the context of the situation.

b. What is the equation for the midline, the amplitude, and the period of the function?

c. Write an equation for the function.

ANSWER:

a. At 1.25 seconds, the height of the rope is 68 inches; at 2.75 seconds, the height of the rope is 2 inches.

b. $y = 35$; 33, 1

c. $h = 33\sin 2\pi t + 35$

Translations of Trigonometric Graphs

41. **CAROUSEL** A horse on a carousel goes up and down 3 times as the carousel makes one complete rotation. The maximum height of the horse is 55 inches, and the minimum height is 37 inches. The carousel rotates once every 21 seconds. Assume that the horse starts and stops at its median height.

a. Write an equation to represent the height of the horse h as a function of time t seconds.

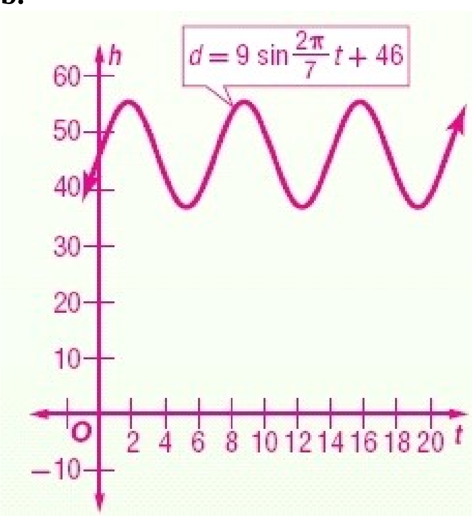
b. Graph the function.

c. Use your graph to estimate the height of the horse after 8 seconds. Then use a calculator to find the height to the nearest tenth.

ANSWER:

a. $h = 9\sin\frac{2\pi}{7}t + 46$

b.



c. Sample answer: 8 s; 53.0 in.

42. **CCSS REASONING** During one month, the outside temperature fluctuates between 40°F and 50°F. A cosine curve approximates the change in temperature, with a high of 50°F being reached every four days.

a. Describe the amplitude, period, and midline of the function that approximates the temperature y on day d.

b. Write a cosine function to estimate the temperature y on day d.

c. Sketch a graph of the function.

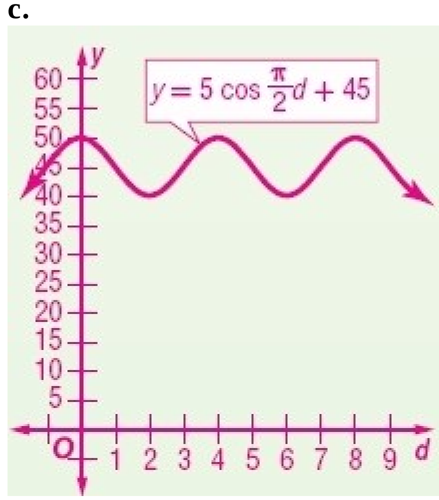
d. Estimate the temperature on the 7th day of the month.

ANSWER:

a. 5; 4; $y = 45$

b. $y = 5\cos\frac{\pi}{2}d + 45$

c.



d. about 45°F

Translations of Trigonometric Graphs

Find a coordinate that represents a maximum for each graph.

43. $y = -2\cos\left(x - \frac{\pi}{2}\right)$

ANSWER:

Sample answer: $\left(\frac{3\pi}{2}, 2\right)$

44. $y = 4\sin\left(x + \frac{\pi}{3}\right)$

ANSWER:

Sample answer: $\left(\frac{\pi}{6}, 4\right)$

45. $y = 3\tan\left(x + \frac{\pi}{2}\right) + 2$

ANSWER:
no maximum values

46. $y = -3\sin\left(x - \frac{\pi}{4}\right) - 4$

ANSWER:

Sample answer: $\left(\frac{7\pi}{4}, -1\right)$

Compare each pair of graphs.

47. $y = -\cos 3\theta$ and $y = \sin 3(\theta - 90°)$

ANSWER:
The graphs are reflections of each other over the x-axis.

48. $y = 2 + 0.5 \tan \theta$ and $y = 2 + 0.5 \tan (\theta + \pi)$

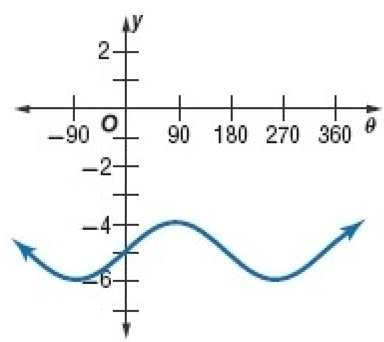
ANSWER:
The graphs are identical.

49. $y = 2\sin\left(\theta - \frac{\pi}{6}\right)$ and $y = -2\sin\left(\theta + \frac{5\pi}{6}\right)$

ANSWER:
The graphs are identical.

Identify the period of each function. Then write an equation for the graph using the given trigonometric function.

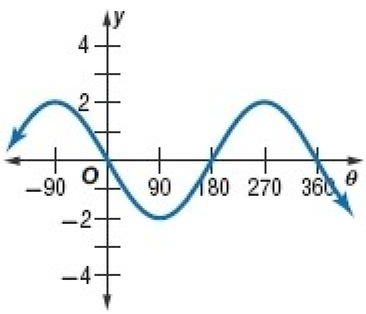
50. sine



ANSWER:
360°; Sample answer: $y = \sin \theta - 5$

Translations of Trigonometric Graphs

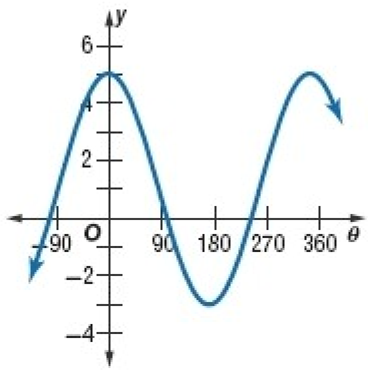
51. cosine



ANSWER:

360°; Sample answer: $y = 2\cos(\theta + 90^o)$

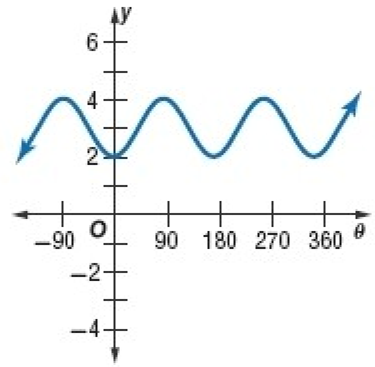
52. cosine



ANSWER:

360°; Sample answer: $y = 4\cos(\theta) + 1$

53. sine



ANSWER:

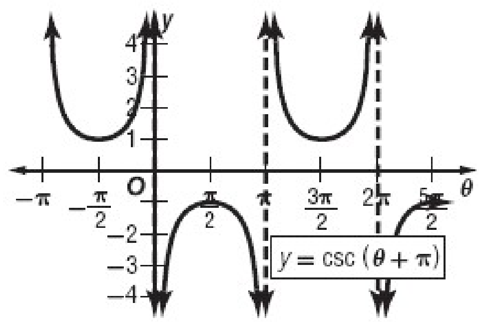
180°; Sample answer: $y = \sin 2(\theta - 45°) + 3$

State the period, phase shift, and vertical shift. Then graph the function.

54. $y = \csc(\theta + \pi)$

ANSWER:

360°; $h = -\pi$; no vertical shift

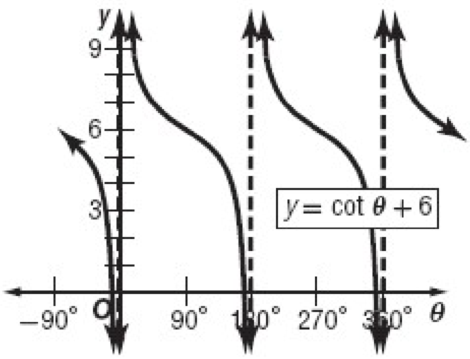


Translations of Trigonometric Graphs

55. $y = \cot \theta + 6$

ANSWER:

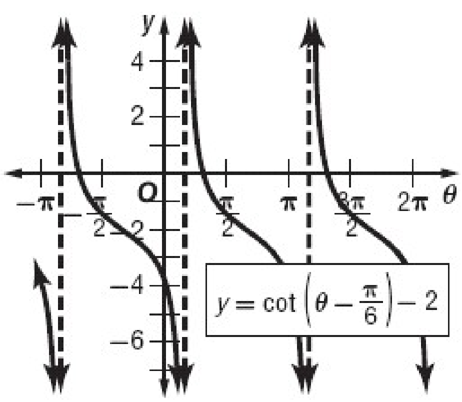
180°; no phase shift; $k = 6$



56. $y = \cot\left(\theta - \frac{\pi}{6}\right) - 2$

ANSWER:

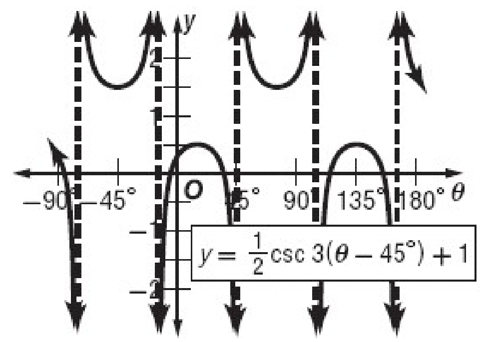
π; $h = \frac{\pi}{6}$; $k = -2$



57. $y = \frac{1}{2}\csc 3(\theta - 45°) + 1$

ANSWER:

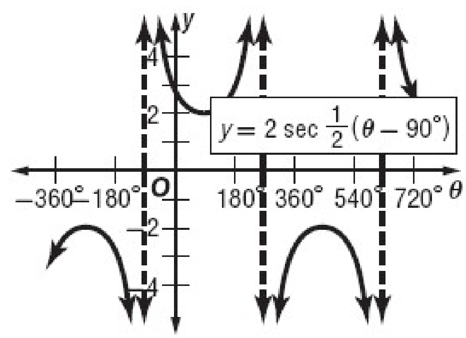
120°; $h = 45°$; $k = 1$



58. $y = 2\sec\frac{1}{2}(\theta - 90°)$

ANSWER:

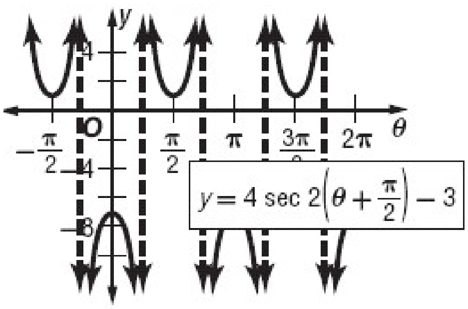
720°; $h = 90°$; no vertical shift



59. $y = 4\sec 2\left(\theta + \frac{\pi}{2}\right) - 3$

ANSWER:

π; $h = -\frac{\pi}{2}$; $k = -3$



60. **CCSS ARGUMENTS** If you are given the amplitude and period of a cosine function, is it *sometimes*, *always*, or *never* possible to find the maximum and minimum values of the function? Explain your reasoning.

ANSWER:

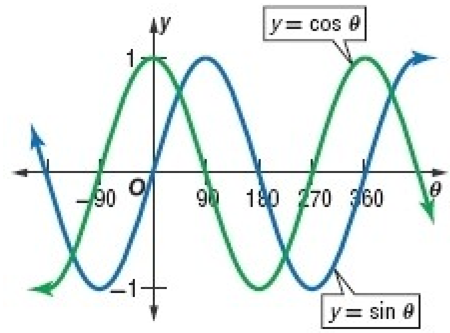
Sometimes; if the function is shifted vertically, then you also need to know the value of the midline. The maximum value is the value of the midline plus the amplitude. The minimum value is the midline value minus the amplitude.

61. **REASONING** Describe how the graph of $y = 3\sin 2\theta + 1$ is different from $y = \sin \theta$.

ANSWER:

The graph of $y = 3\sin 2\theta + 1$ has an amplitude of 3 rather than an amplitude of 1. It is shifted up 1 unit from the parent graph and is compressed so that it has a period of 180°.

62. **WRITING IN MATH** Describe two different phase shifts that will translate the sine curve onto the cosine curve shown at the right. Then write an equation for the new sine curve using each phase shift.



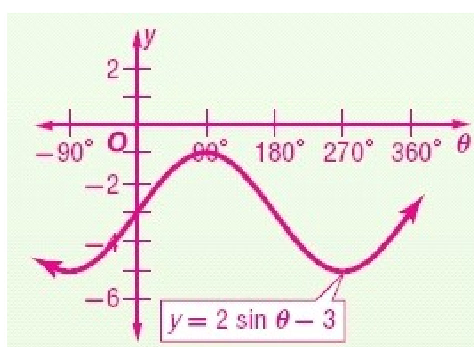
ANSWER:

Sample answer: a phase shift 90° left, $y = \sin(\theta + 90°)$; a phase shift 270° right, $y = \sin(\theta - 270°)$

63. **OPEN ENDED** Write a periodic function that has an amplitude of 2 and midline at $y = -3$. Then graph the function.

ANSWER:

Sample answer: $y = 2\sin\theta - 3$



64. **REASONING** How many different sine graphs pass through the origin $(n\pi, 0)$? Explain your reasoning.

ANSWER:
Sample answer: Infinitely many; any change in amplitude will create a different graph that has the same θ-intercepts.

65. **GRIDDED RESPONSE** The expression $\frac{3x-1}{4} + \frac{x+6}{4}$ is how much greater than x?

ANSWER:
1.25

66. Expand $(a - b)^4$.

A $a^4 - b^4$

B $a^4 - 4ab + b^4$

C $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

D $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

ANSWER:
D

67. Solve $\sqrt{x-3} + \sqrt{x+2} = 5$.

F 7

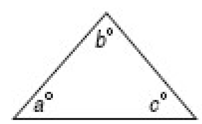
G 0,7

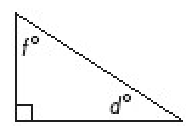
H 7, 13

J no solution

ANSWER:
F

68. **GEOMETRY** Using the figures below, what is the average of a, b, c, d, and f?





A 21

B 45

C 50

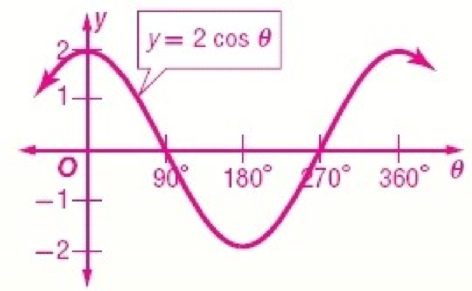
D 54

ANSWER:
D

Find the amplitude and period of each function. Then graph the function.

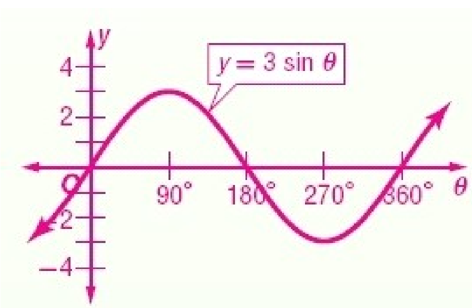
69. $y = 2\cos\theta$

ANSWER:
amplitude: 2; period: 360°



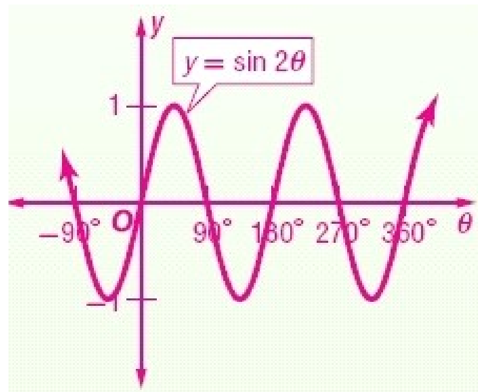
70. $y = 3\sin\theta$

ANSWER:
amplitude: 3; period: 360°



71. $y = \sin 2\theta$

ANSWER:
amplitude: 1; period: 180°



Find the exact value of each expression.

72. $\sin\frac{4\pi}{3}$

ANSWER:
$-\frac{\sqrt{3}}{2}$

73. sin(−30°)

ANSWER:
$-\frac{1}{2}$

74. cos 405°

ANSWER:
$\frac{\sqrt{2}}{2}$

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

75. A group of 220 adults is randomly split into two groups. One group exercises for an hour a day and the other group does not. The body mass indexes are then compared.

ANSWER:

experiment; sample: people that exercise for an hour a day; population: all adults

76. A soccer coach randomly selects some of his players and gives them a questionnaire asking about their daily sleeping habits.

ANSWER:

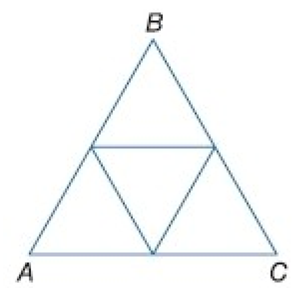
survey; sample: players that received the questionnaire; population: all soccer players

77. A teacher randomly selects 100 students who have part-time jobs and compares their grades.

ANSWER:

observational study; sample: 100 students selected; population: all students that have part-time jobs

78. **GEOMETRY** Equilateral triangle *ABC* has a perimeter of 39 centimeters. If the midpoints of the sides are connected, a smaller equilateral triangle results. Suppose the process of connecting midpoints of sides and drawing new triangles is continued indefinitely.



a. Write an infinite geometric series to represent the sum of the perimeters of all of the triangles.

b. Find the sum of the perimeters of all of the triangles.

ANSWER:

a. 39 + 19.5 + 9.75 + …

b. 78 cm

79. **CONSTRUCTION** A construction company will be fined for each day it is late completing a bridge. The daily fine will be $4000 for the first day and will increase by $1000 each day. Based on its budget, the company can only afford $60,000 in total fines. What is the maximum number of days it can be late?

ANSWER:

8 days

Find each value of θ. Round to the nearest degree.

80. $\sin\theta = \frac{7}{8}$

ANSWER:
61°

81. $\tan\theta = \frac{9}{10}$

ANSWER:
42°

82. $\cos\theta = \frac{1}{4}$

ANSWER:
76°

83. $\cos\theta = \frac{4}{5}$

ANSWER:
37°

84. $\sin\theta = \frac{5}{6}$

ANSWER:
56°

85. $\tan\theta = \frac{2}{7}$

ANSWER:
16°