— University of Mosul —
College of Petroleum & Mining Engineering

Mathematics II

Lecture (3)

Logarithmic Functions

Petroleum and Refining Engineering Department
Ass.L. Ghufran Faris Alrahhawi
Email: Ghufranalrahhawi@uomosul.edu.iq

LECTURE CONTENTS

Logarithmic Functions

Derivatives

Integration

Logarithmic Functions

If a is any positive real number other than 1, the base a exponential function $f(x) = a^x$ is one-to-one. It therefore has an inverse. It inverse is called the logarithm function with base a.

The logarithm function with base a, $y = \log_a x$, is the inverse of the base a exponential function $y = a^x$ $(a > 0, a \neq 1)$.

The domain of $\log_a x$ is $(0, \infty)$ = the range of a^x

The range of $\log_a x$ is $(-\infty, \infty)$ = the domain of a^x.

1- Common Logarithm

$$\operatorname{Log}_a u = \frac{\ln u}{\ln a} \qquad u > 0 \;, a > 0 \quad a \neq 1$$

2- Natural Logarithm

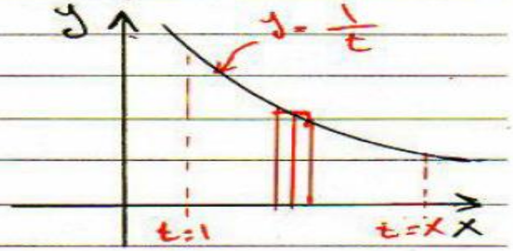
$$\operatorname{Log}_e u = \frac{\ln u}{\ln e} = \ln u \qquad e = 2.718 \quad \ln e = 1$$

Let $y = \frac{1}{t}$ $\quad 1 \leq t \leq x$

$t = 0 \longrightarrow y = \infty$

$t = \infty \longrightarrow y = 0$

$$A = \int_{t_1}^{t_2} y\,dt = \int_1^x \frac{1}{t}\,dt = \ln t \Big]_1^x$$



$$\ln x - \ln 1 = \ln x$$

$\therefore A = \ln x$

$$\ln x = \begin{cases} > 0 & \text{if } x > 1 \\ = 0 & \text{if } x = 1 \\ < 0 & \text{if } 0 < x < 1 \\ = \infty & \text{if } x = 0 \end{cases}$$

$$\ln(-x) = \begin{cases} > 0 & \text{if } x < -1 \\ = 0 & \text{if } x = -1 \\ < 0 & \text{if } 0 > x > -1 \\ = \infty & \text{if } x = 0 \end{cases}$$

Derivative of Natural Logarithm

$y = \ln u$

$$\frac{d(\ln u)}{dx} = \frac{1}{u} * \frac{du}{dx}$$

* مشتقة الـ ln هو مقلوب الكمية مابعد الـ ln * مشتقة تلك الكمية

Example

let $y = \ln \sin x$ Find dy/dx

Sol.

$$\frac{dy}{dx} = \frac{1}{\sin x} * \cos x = \cot x$$

Integral of Natural Logarithm (N.L)

$$\int \frac{du}{u} = \begin{cases} \ln u + c & \text{if } u > 0 \\ \ln(-u) + c & \text{if } u < 0 \\ \ln|u| + c & \text{if } |u| > 0 \end{cases}$$

properties of N.L

1 $\ln ax = \ln a + \ln x$

2- $\ln \frac{x}{a} = \ln x - \ln a$

3- $\ln x^n = n \ln x$

Examples

Here are examples of the natural logarithm properties

1- $\ln 4 + \ln \sin x = \ln(4 \sin x)$

2- $\ln \frac{x+1}{2x-3} = \ln(x+1) - \ln(2x-3)$

3- $\ln \frac{1}{8} = -\ln 8$

$= -\ln 2^3 = -3 \ln 2$

Example(1)

Find dy/dx for the function $y = x^2 \ln(4x)$

Sol.

$$\frac{dy}{dx} = x^2\left(\frac{1}{4x}\cdot 4\right) + \ln(4x)\cdot 2x$$

$$= x + 2x\ln 4x = x\,(1 + 2\ln 4x)$$

Example(2)

$y = \ln(\tan x + \sec x)$, find dy/dx

Sol.

$$\frac{dy}{dx} = \frac{1}{\tan x + \sec x}\left[\sec^2 x + \sec x \tan x\right]$$

$$= \sec x$$

$$\therefore \int \sec x \, dx = \ln|\sec x + \tan x| + c$$

Example(3)

Evaluate $\int \frac{x}{x^2+3}\, dx$

Sol.

let $u = x^2+3$, $du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$$\int \frac{1}{u}\cdot\frac{du}{2} = \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln|u| + c$$

$$= \frac{1}{2}\ln|x^2+3| + C$$

Example (4)

Evaluate $\int \frac{x}{x+1}\, dx$

Sol.

$$\frac{x}{x+1} = 1 - \frac{1}{x+1}$$

$$\int \frac{x}{x+1} dx = \int \left(1 - \frac{1}{x+1}\right) dx$$

$$= \int dx - \int \frac{dx}{x+1}$$

$$= x - \ln|x+1| + c$$

Note :- Sometimes we need $(\ln x)$ to find the derivative of functions that involve products, quotient and powers quickly.

Example (5)

If $y = (\sqrt{x+3})(\sin x \cos x)$, find dy/dx

Sol.

$$\ln y = \ln(\sqrt{x+3})(\sin x \cos x)$$

$$\ln y = \ln\sqrt{x+3} + \ln(\sin x \cos x)$$

$$\ln y = \ln(x+3)^{1/2} + \ln \sin x + \ln \cos x$$

$$\ln y = \frac{1}{2}\ln(x+3) + \ln \sin x + \ln \cos x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x+3}\right) + \frac{1}{\sin x}(\cos x) + \frac{1}{\cos x}(-\sin x)$$