— University of Mosul —
College of Petroleum & Mining Engineering

Mathematics I

Lecture (4)

Integrating

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LECTURE CONTENTS

Definite Integral

Indefinite Integral

Integration

There are two types of integration

1. **Definite integral** $\int_a^b f(x)dx$
   - $\int$: Integral sign
   - Given numerical values
   - No constant of integrals
2. **Indefinite integral** $\int f(x)dx$
   - General solution (constant of integration C)
   - Particular solution (applying boundary condition)
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1. **Definite integral**

Rules for definite integral

1. $\int_a^a f(x)dx = 0$
2. $\int_a^b f(x)dx = -\int_a^b f(x)dx$
3. $\int_a^b Kf(x)dx = K\int_a^b f(x)dx$
4. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
5. $\int_a^c f(x)dx = \int_a^b f(x)dx \pm \int_b^c f(x)dx$
6. $\int_b^c f(x)dx = \int_a^c f(x)dx - \int_a^b f(x)dx$
7. If $g(x) \geq f(x)$ $\int_a^b g(x)dx \geq \int_a^b f(x)dx$
8. If $f(x) \geq 0$ $\int_a^b f(x)dx \geq 0$

1. **Indefinite integral**

Rules for Indefinite integrals

1. $\int du = u(x) + c$
2. $\int a.u(x)\,dx = a\int u(x)dx$
3. $\int u(x) \pm v(x)dx = \int u(x)dx \pm \int v(x)dx$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + c$ When $n \neq -1$
   & $\int u^{-1}du = \int \frac{1}{u}du = \ln u + c$
5. $\int a^u du = \frac{a^u}{\ln a} + c \implies e^u du = e^u + c$

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Integrals of Trigonometric Function

1. $\int \sin\theta \, d\theta = -\cos\theta + c$
2. $\int \cos\theta \, d\theta = \sin\theta + c$
3. $\int sec^2\theta \, d\theta = tan + c$
4. $\int csc^2\theta = -cot^2\theta + c$
5. $\int \sec\theta \tan\theta \, d\theta = \sec\theta + c$
6. $\int \csc\theta \cot\theta \, d\theta = -\csc\theta + c$

Examples

1. $\int \frac{x+1}{\sqrt{x^2+2x+5}} \, dx$

   $= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+5}} \, dx$

   $= \frac{1}{2}\left(\frac{\sqrt{x^2+2x+5}}{1/2}\right) + c$

   $= \sqrt{x^2+2x+5} + c$

2. $\int \frac{(x^2+1)^2}{\sqrt{x}} \, dx$

   $= \int \frac{x^4+2x^2+1}{\sqrt{x}} \, dx$

   $= \int x^{7/2} + 2x^{3/2} + x^{-1/2} dx$

   $= \frac{2}{9}x^{9/2} + \frac{4}{5}x^{5/2} + 2x^{1/2} + c$

   $= \frac{2}{9}\sqrt{x^9} + \frac{4}{5}\sqrt{x^5} + 2\sqrt{x} + c$

3. $\int 3x^2 \, dx$

   $= 3\int x^2 dx \quad = 3 * \frac{x^3}{3} + c$

   $= x^3 + c$

4. $\int (x^{-2} + x) dx$

   $= \int x^{-2} dx + \int x \, dx \quad = \frac{x^{-1}}{-1} + \frac{x^2}{2} + c$

$$= -\frac{1}{x} + \frac{x^2}{2} + c$$

5. $\int \frac{x^2+x+1}{x^3-1}\, dx$

$$= \int \frac{(x^2+x+1)}{(x-1)(x^2+x+1)}\, dx$$

$$= \int \frac{dx}{(x-1)} = \ln|x-1| + c$$

6. $\int \sqrt{(z^2 - z^{-2})^2 + 4}\, dz$

$$= \int \sqrt{z^4 - 2z^2 z^{-2} + z^{-4} + 4}\, dz$$

$$= \int \sqrt{z^4 - 2 + z^{-4} + 4}\, dz$$

$$= \int \sqrt{z^4 - 2 + z^{-4}}\, dz$$

$$= \int (z^2 + z^{-2})^2 dz$$

$$= \int (z^2 + z^{-2})\, dz$$

$$= \frac{z^3}{3} + \frac{z^{-1}}{1-1} + c$$

$$= \frac{1}{3}z^3 - \frac{1}{z} + c$$

7. $\int \sqrt{z^4 + z^2}\, dz$

$$= \int \sqrt{z^2(z^2+1)}\, dz$$

$$= \int z\sqrt{z^2+1}\, dz$$

$$= \frac{1}{3}\sqrt{(z^2+1)^3} + c$$

8. $\int_0^4 \frac{y-1}{\sqrt{y}-1}\, dy$

$$= \int_0^4 \frac{(\sqrt{y}-1)(\sqrt{y}+1)}{(\sqrt{y}-1)} dy$$

$$= \int_0^4 \sqrt{y}\, dy + \int_0^4 dy$$

$$= \frac{2}{3}\sqrt{y^3}\Big|_0^4 + y|_0^4$$

$$= \frac{2}{3}\sqrt{(4)^3} + 4 = \frac{16}{3} + 4 = \frac{28}{3}$$

9. $\int \frac{x+3}{\sqrt{x^2+6x}}\, dx = \frac{1}{2}\int (2x+6)(x^2+6x)^{-\frac{1}{2}}\, dx$

$$= \frac{1}{2}\frac{(x^2+6x)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \sqrt{x^2+6x} + c$$

10. $\int \frac{x+2}{x^2}\, dx$

$$= \int \left(\frac{x}{x^2} + \frac{2}{x^2}\right) dx$$

$$= \int (x^{-1} + 2x^{-2}) dx$$

$$= \ln x + \frac{2x^{-1}}{-1} + c$$

$$= \ln x - \frac{2}{x} + c$$

11. $\int \csc(3\theta - 1) d\theta$

$$= \frac{1}{3}\int 3\csc(3\theta - 1) d\theta$$

$$= \frac{1}{3}\sin(3\theta - 1) + c$$

12. $\int x.\sin(2x^2) dx$

$$= \frac{1}{4}\int 4\, x\, .\sin(2x^2)\, dx$$

$$= -\frac{1}{4}\cos(2x^2) + c$$

13. $\int sec^3 x\, .\tan x\, dx$

$$= \int sec^2 x\, .(\sec x\, .\tan x\,)\, dx$$

$$= \frac{sec^3 x}{3} + c$$

14. $\int cos^2(2y).\sin(2y) dy$

$$= -\frac{1}{2}\int (cos2y)^2.(-2sin2y\, dy)$$

$$= -\frac{1}{2}.\frac{(\cos 2y)^3}{3} + c$$