University of Mosul
College of Petroleum & Mining Engineering

Mathematics II

Lecture (4)

Exponential Functions

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LECTURE CONTENTS

Exponential Functions

Derivatives

Integration

Exponential Function

let $x = \ln y \longrightarrow y = \ln^{-1} x = e^{x} \quad y > 0$

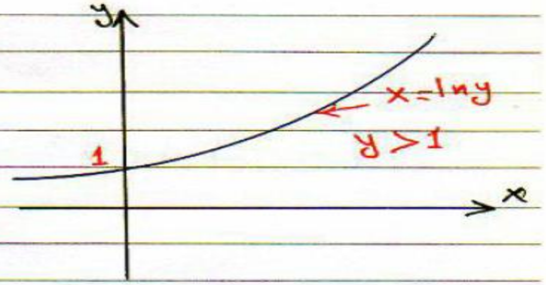
Where e^{x} is the inverse of the natural logarithm $(\ln y)$.

$y = e^{x}$ if and only if $x = \ln y \quad y > 0$

(e) :- is the number whose natural log. is equal to unity.

$\therefore \ln e = 1$ & $e = 2.718281828$

Dirivative and Integral of e^{u}



let $y = e^{x} \qquad y > 0$

$\therefore \quad x = \ln y$

$$1 = \frac{1}{y} * \frac{dy}{dx}$$

$$\frac{dy}{dx} = y \quad \text{but} \quad y = e^{x}$$

$$\therefore \frac{dy}{dx} = e^{x}$$

* الدوال الاسية هي معكوس للدوال اللوغارتمية

$$\therefore y = e^{u} \Rightarrow \frac{d(e^{u})}{dx} = e^{u} \cdot \frac{du}{dx} \Rightarrow d(e^{u}) = e^{u} * du$$

* مشتقة الدالة الاسية هي نفس الدالة * مشتقة الاس

$$\int e^{u}\, du = \int d(e^{u}) = e^{u} + c$$

* تكامل الدوال الاسية هو نفس الدالة بشرط توفر مشتقة الاس.

Properties of e^x

1- $e^x \cdot e^y = e^{x+y}$

Prove that

Sol.

let $x = \ln u_1$ & $y = \ln u_2$

$x + y = \ln u_1 + \ln u_2 = \ln u_1 . u_2$

$u_1 * u_2 = \ln^{-1}(x+y) = e^{x+y}$

but $u_1 = e^x$ & $u_2 = e^y$ $\therefore e^x . e^y = e^{x+y}$

2- $\dfrac{e^x}{e^y} = e^{(x-y)}$

Prove that

$e^x \cdot e^{-y} = e^{(x-y)}$ let $x = \ln u_1$ & $y = -\ln u_2$

$\therefore x - y = \ln u_1 + \ln u_2 = \ln u_1 . u_2$

$\therefore u_1 . u_2 = e^{(x-y)}$ but $u_1 = e^x$ and $u_2 = e^{-y}$

$\therefore e^x . e^{-y} = e^{(x-y)} = \dfrac{e^x}{e^y} = e^{(x-y)}$

3- $(e^x)^n = e^{nx}$

4- $y = \ln e^x = x \cancelto{1}{\ln e} = x$

5- $y = e^{\ln x} \longrightarrow \ln y = \ln e^{\ln x} \Rightarrow \ln y = \ln x \cancelto{1}{\ln e}$

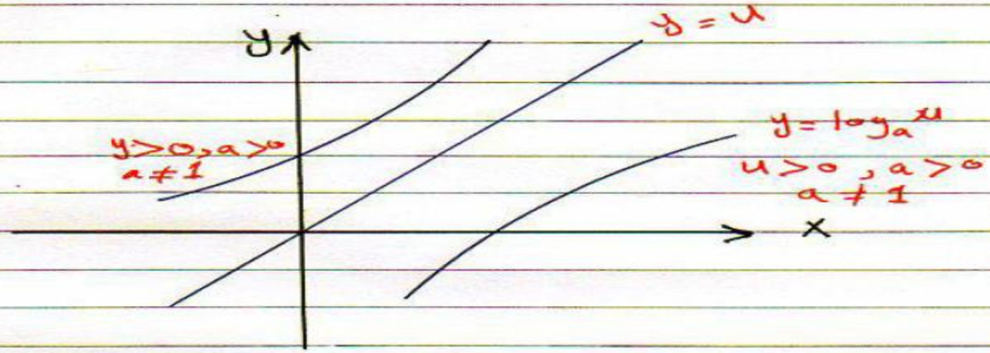
$\therefore \ln y = \ln x$

$y = x$

$y = e^{\ln x} = x$

Common Logarithm

$$y = \log_a u = \frac{\ln u}{\ln a} \qquad u > 0 \,,\, a > 0 \,,\, a \neq 1$$



Dirivatie

$$\frac{dy}{dx} = \frac{d(\log_a u)}{dx} = \frac{1}{\ln a} * \frac{1}{u} * \frac{du}{dx}$$

Properties

1- $\log_a x \cdot y = \dfrac{\ln xy}{\ln a} = \dfrac{\ln x + \ln y}{\ln a}$

$= \dfrac{\ln x}{\ln a} + \dfrac{\ln y}{\ln a} \quad \therefore \left[\log_a x \cdot y = \log_a x + \log_a y\right]$

2- $\log_a x/y = \dfrac{\ln \frac{x}{y}}{\ln a} = \dfrac{\ln x - \ln y}{\ln a} = \dfrac{\ln x}{\ln a} - \dfrac{\ln y}{\ln a}$

$\therefore \left[\log_a x/y = \log_a x - \log_a y\right]$

3- $\log_a^{x^n} = \frac{\ln x^n}{\ln a} = n \frac{\ln x}{\ln a}$

$$\boxed{\log_a^{x^n} = n \log_a^{x}}$$

4- $y = a^{\log_a^u} \Rightarrow \ln y = \ln a^{\log_a^u}$

$= \log_a^u \times \ln a = \frac{\ln u}{\cancel{\ln a}} \times \cancel{\ln a}$

$\therefore \ln y = \ln u \longrightarrow y = u$ but $y = a^{\log_a^u}$

$$\therefore \boxed{y = a^{\log_a^u} = u}$$

Example (1)

Find y for the following equations:

1- $\ln y = x^2$ 2- $\ln(y-2) = \ln(\sin x) - x$

Sol.

1- $\ln y = x^2$

$e^{\ln y} = e^{x^2}$

$y = e^{x^2}$

2- $\ln(y-2) = \ln(\sin x) - x$

$\ln(y-2) - \ln(\sin x) = -x$

$\ln \frac{y-2}{\sin x} = -x$

$e^{\ln \frac{y-2}{\sin x}} = e^{-x}$

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$$\frac{y-2}{\sin x} = e^{-x}$$

$$y-2 = e^{-x}\sin x$$

$$y = e^{-x}\sin x + 2$$

Example (2)

Find dy/dx for the function $y = e^{\tan x}$

Sol.

$$\frac{d}{dx} e^{\tan x} = e^{\tan x} \cdot \sec^2 x$$

Example (3)

Evaluate $\int_{-\ln x(a+1)}^{0} e^{-x}\, dx$

Sol.

$$\int_{-\ln x(a+1)}^{0} e^{-x} = -\int_{-\ln x(a+1)}^{0} e^{-x}(-dx)$$

$$= \left[-e^{-x}\right]_{-\ln x(a+1)}^{0}$$

$$= -\left[e^{0} - e^{-(-\ln x(a+1))}\right]$$

$$= -\left[1-(a+1)\right] =$$