— University of Mosul —
College of Petroleum & Mining Engineering

Mathematics II

Lecture (5)

Method of Integration

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LECTURE CONTENTS

- Trigonometric Functions

Method of Integration

1- Trigonometric Functions

Trigonometric integrals involve algebraic combinations of the six basic trigonometric functions. In principle, we can always express such integrals terms of sines and cosines, but it is often simpler to work with other functions, as in the integral.

$$\int \sec^2 x\,dx = \tan x + C$$

The general idea is to use identities to transform the integrals we have to find into integrals that are easier to work with

Case-1-

$$\int \sin^n\theta\,d\theta \quad \text{or} \quad \int \cos^n\theta\,d\theta$$

n :- is odd number

حيث (n) عدد صحيح فردي موجب اكبر من واحد بمقدار ثلاثة على الاقل

$$\int \sin^{n-1}\theta \sin\theta\,d\theta \qquad \int \cos^{n-1}\theta\,.\cos\theta\,.d\theta$$

but $\sin^2\theta + \cos^2\theta = 1$,

$$\int \sin^{n-1}\theta\,.\sin\theta\,d\theta \quad , \quad \int \cos^{n-1}\theta\,.\cos\theta\,.d\theta$$

$$= \int (1-\cos^2\theta)^{\frac{n-1}{2}} \sin\theta\,.\,d\theta \quad , \quad = \int (1-\sin^2\theta)^{\frac{n-1}{2}}\,.\cos\theta\,d\theta$$

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Example(1)

$$\int \sin^5\theta \, d\theta$$

Sol.

$$\int \sin^4\theta \sin\theta \, d\theta$$

$$= \int (1-\cos^2\theta)^2 \sin\theta \, d\theta$$

$$= \int (1 - 2\cos^2\theta + \cos^4\theta) \sin\theta \, d\theta$$

$$= \int \sin\theta \, d\theta - 2\int \cos^2\theta \sin\theta \, d\theta + \int \cos^4\theta \sin\theta \, d\theta$$

$$\therefore \int \sin^5 d\theta = -\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta + C$$

Example (2)

$$\int \cos^3\theta \, d\theta$$

Sol.

$$\int \cos^2\theta \cos\theta \, d\theta = \int (1-\sin^2\theta)\cos\theta \, d\theta$$

$$= \int \cos\theta \, d\theta - \int \sin^2\theta \cos\theta \, d\theta$$

$$= \sin\theta - \frac{1}{3}\sin^3\theta + C$$

Home work

1) $\int \sin^7 2\theta \, d\theta$, 2) $\cos^9 3\theta \, d\theta$

Case -2-

$$\int \sin^m\theta \, d\theta \quad \text{or} \quad \int \cos^m\theta \, d\theta$$

m : is even number

حيث (m) عدد صحيح زوجي موجب

Use

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

} ..... (A) علاقة

$$\int \sin^m\theta \, d\theta = \int \left(\frac{1}{2}(1-\cos 2\theta)\right)^{\frac{m}{2}} d\theta$$

$$\int \cos^m\theta \, d\theta = \int \left(\frac{1}{2}(1+\cos 2\theta)\right)^{\frac{m}{2}} d\theta$$

ملاحظة

اذا احتوى التكامل بعد عملية التعويض الاولى (Case-2) على دوال جديدة مرفوعة الى اس زوجي او فردي نعيد تطبيق حالة (2) على الدوال المرفوعة الى اس زوجي باستخدام العلاقة (A) والحالة (1) على الدوال المرفوعة الى اس فردي كما موضح في المثال

Example (3)

$$\int \cos^6 3\theta \, d\theta$$

Sol.

$$= \int \left(\frac{1}{2}(1+\cos 6\theta)\right)^3 d\theta$$

$$= \frac{1}{8}\int (1+\cos 6\theta)^2 (1+\cos 6\theta)\, d\theta$$

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$$= \frac{1}{8}\int (1+2\cos 6\theta + \cos^2 6\theta)(1+\cos 6\theta)\,d\theta$$

$$= \frac{1}{8}\int (1+2\cos 6\theta + \cos^2 6\theta + \cos 6\theta + 2\cos^2 6\theta + \cos^3 6\theta)\,d\theta$$

$$= \frac{1}{8}\int (1+3\cos 6\theta + 3\cos^2 6\theta + \cos^3 6\theta)\,d\theta$$

$$= \frac{1}{8}\left[\theta + \frac{1}{2}\sin 6\theta + 3\int \frac{1}{2}(1+\cos 12\theta)\,d\theta + \int (1-\sin^2 6\theta)\cos 6\theta\,d\theta\right]$$

$$= \frac{1}{8}\left[\theta + \frac{1}{2}\sin 6\theta + \frac{3}{2}\theta + \frac{1}{8}\sin 12\theta + \frac{1}{6}\sin 6\theta - \frac{1}{18}\sin^3 6\theta\right] + c$$

$$= \frac{1}{8}\left[\frac{5\theta}{2} + \frac{2}{3}\sin 6\theta + \frac{1}{8}\sin 12\theta - \frac{1}{15}\sin^3 6\theta\right] + c$$

Case -3-

$$\int \sin^n\theta \cos^m\theta\,d\theta$$

n or m :- is odd number

على الأقل عدد الدالتين مرفوع الى اس فردي، في هذه الحالة نطبق حالة رقم (1) على الدوال المرفوعة الى اس فردي مع تثبيت الدالة الثانية. حيث أن طريقة الاس المرفوعة لتلك الدالة.

Example (4)

Evaluate $\int \sin^3\theta \cos^{10}\theta \, d\theta$

Sol.

$$\int \sin^2\theta \sin\theta \cos^{10}\theta \, d\theta$$

$$= \int (1-\cos^2\theta) \sin\theta \cos^{10}\theta \, d\theta$$

$$= \int \cos^{10}\theta \sin\theta \, d\theta - \int \cos^{12}\theta \sin\theta \, d\theta$$

$$= -\frac{1}{11}\cos^{11}\theta + \frac{1}{13}\cos^{13}\theta + C$$

Example (5)

Evaluate $\int \sin^5\theta \cos^7\theta \, d\theta$

Sol.

$$\int \sin^4\theta \sin\theta \cos^7\theta \, d\theta$$

$$= \int (1-\cos^2\theta)^2 \sin\theta \cos^7\theta \, d\theta$$

$$= \int \cos^7\theta \sin\theta \, d\theta - 2\int \cos^9\theta \sin\theta \, d\theta + \int \cos^{11}\theta \sin\theta \, d\theta$$

$$= -\frac{1}{8}\cos^8\theta + \frac{1}{5}\cos^{10}\theta - \frac{1}{12}\cos^{12}\theta + C$$

ملاحظة:- اذا كانت الدالتين مرفوعة الى اس فردي نطبق الحالة على الدالة ذات الاس الاقل.