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-2nd-Class
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Mathematics III

Lecture 1

INFINITE SEQUENCES AND SERIES (PART I)

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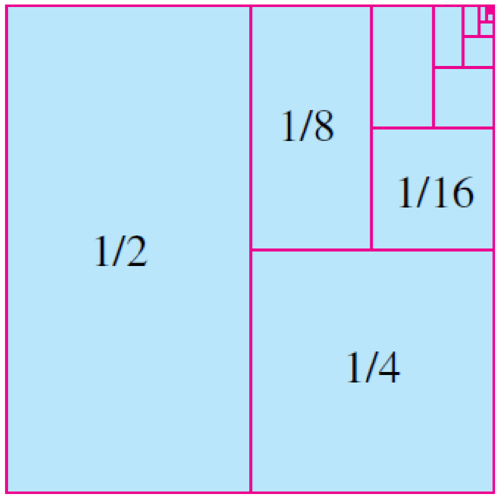
INFINITE SEQUENCES AND SERIES

OVERVIEW While everyone knows how to add together two numbers, or even several, how to add together infinitely many numbers is not so clear. In this lecture we study such questions, the subject of the theory of infinite series. Infinite series sometimes have finite sum, as in

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1 .$$



This sum is represented geometrically by the areas of the repeatedly halved unit square shown here. The areas of the small rectangles add together to give the area of the unit square, which they fill. Adding together more and more terms gets us closer and closer to the total.

Other infinite series do not have a finite sum, as with

$$1 + 2 + 3 + 4 + 5 + \cdots .$$

The sum of the first few terms gets larger and larger as we add more and more terms.

A sequence is a list of numbers

$$a_1, a_2, a_3, \ldots, a_n, \ldots$$

in a given order. Each of a_1, a_2, a_3 and so on represents a number. These are the **terms** of the sequence. For example the sequence

$$2, 4, 6, 8, 10, 12, \ldots, 2n, \ldots$$

has first term $a_1 = 2$, second term $a_2 = 4$ and nth term $a_n = 2n$. The integer n is called the **index** of a_n, and indicates where a_n occurs in the list. We can think of the sequence

$$a_1, a_2, a_3, \ldots, a_n, \ldots$$

as a function that sends 1 to a_1, 2 to a_2, 3 to a_3, and in general sends the positive integer n to the nth term a_n. This leads to the formal definition of a sequence.

DEFINITION   Infinite Sequence

An **infinite sequence** of numbers is a function whose domain is the set of positive integers.

The function associated to the sequence

$$2, 4, 6, 8, 10, 12, \ldots, 2n, \ldots$$

sends 1 to $a_1 = 2$, 2 to $a_2 = 4$, and so on. The general behavior of this sequence is described by the formula

$$a_n = 2n.$$

We can equally well make the domain the integers larger than a given number n_0, and we allow sequences of this type also.

The sequence

$$12, 14, 16, 18, 20, 22 \ldots$$

is described by the formula $a_n = 10 + 2n$.

Sequences can be described by writing rules that specify their terms, such as

$$a_n = \sqrt{n},$$

$$b_n = (-1)^{n+1}\frac{1}{n},$$

$$c_n = \frac{n-1}{n},$$

$$d_n = (-1)^{n+1}$$

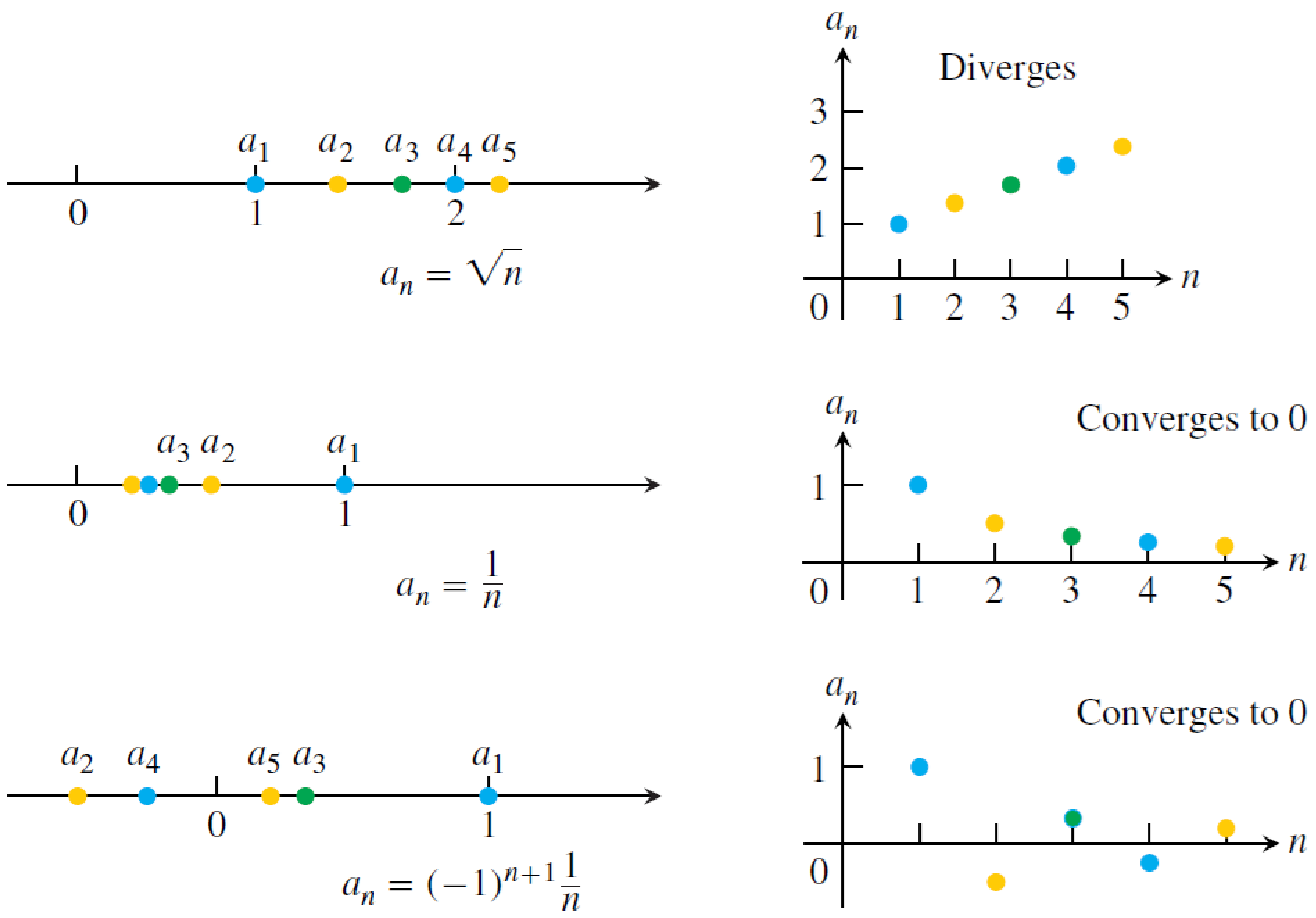
or by listing terms,

$$\{a_n\} = \left\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{n}, \ldots\right\}$$

$$\{b_n\} = \left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots, (-1)^{n+1}\frac{1}{n}, \ldots\right\}$$

$$\{c_n\} = \left\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots, \frac{n-1}{n}, \ldots\right\}$$

$$\{d_n\} = \{1, -1, 1, -1, 1, -1, \ldots, (-1)^{n+1}, \ldots\}.$$



Sequences can be represented as points on the real line or as points in the plane where the horizontal axis n is the index number of the term and the vertical axis a_n is its value.

Finding Terms of a Sequence

Each of Exercises 1–6 gives a formula for the nth term a_n of a sequence $\{a_n\}$. Find the values of a_1, a_2, a_3, and a_4.

1. $a_n = \dfrac{1 - n}{n^2}$
2. $a_n = \dfrac{1}{n!}$
3. $a_n = \dfrac{(-1)^{n+1}}{2n - 1}$
4. $a_n = 2 + (-1)^n$
5. $a_n = \dfrac{2^n}{2^{n+1}}$
6. $a_n = \dfrac{2^n - 1}{2^n}$

Each of Exercises 7–12 gives the first term or two of a sequence along with a recursion formula for the remaining terms. Write out the first ten terms of the sequence.

7. $a_1 = 1, \quad a_{n+1} = a_n + (1/2^n)$
8. $a_1 = 1, \quad a_{n+1} = a_n/(n + 1)$
9. $a_1 = 2, \quad a_{n+1} = (-1)^{n+1}a_n/2$
10. $a_1 = -2, \quad a_{n+1} = na_n/(n + 1)$
11. $a_1 = a_2 = 1, \quad a_{n+2} = a_{n+1} + a_n$
12. $a_1 = 2, \quad a_2 = -1, \quad a_{n+2} = a_{n+1}/a_n$

Finding a Sequence's Formula

In Exercises 13–22, find a formula for the nth term of the sequence.

13. The sequence $1, -1, 1, -1, 1, \ldots$	1's with alternating signs
14. The sequence $-1, 1, -1, 1, -1, \ldots$	1's with alternating signs
15. The sequence $1, -4, 9, -16, 25, \ldots$	Squares of the positive integers; with alternating signs
16. The sequence $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$	Reciprocals of squares of the positive integers, with alternating signs
17. The sequence $0, 3, 8, 15, 24, \ldots$	Squares of the positive integers diminished by 1
18. The sequence $-3, -2, -1, 0, 1, \ldots$	Integers beginning with -3
19. The sequence $1, 5, 9, 13, 17, \ldots$	Every other odd positive integer
20. The sequence $2, 6, 10, 14, 18, \ldots$	Every other even positive integer
21. The sequence $1, 0, 1, 0, 1, \ldots$	Alternating 1's and 0's
22. The sequence $0, 1, 1, 2, 2, 3, 3, 4, \ldots$	Each positive integer repeated