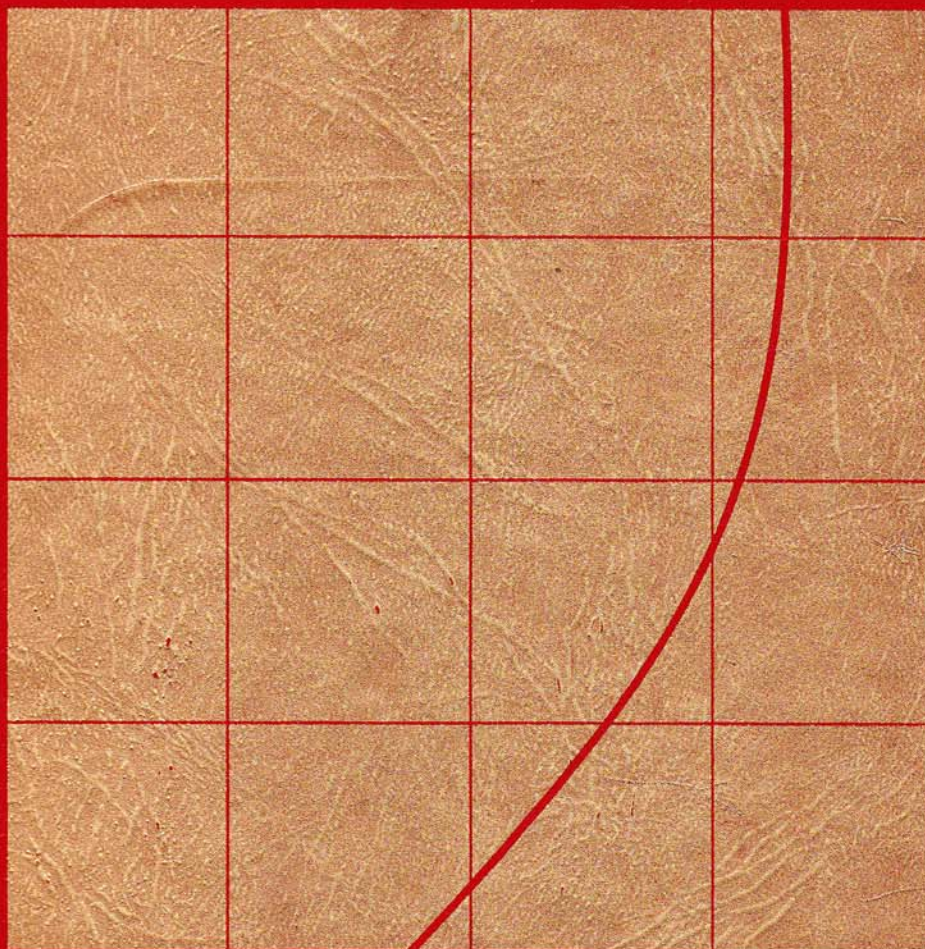


WELL TESTING

JOHN LEE



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Well Testing

John Lee

Professor of Petroleum Engineering
Texas A&M University

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Introduction

This textbook explains how to use well pressures and flow rates to evaluate the formation surrounding a tested well. Basic to this discussion is an understanding of the theory of fluid flow in porous media and of pressure-volume-temperature (PVT) relations for fluid systems of practical interest. This book contains a review of these fundamental concepts, largely in summary form.

One major purpose of well testing is to determine the ability of a formation to produce reservoir fluids. Further, it is important to determine the underlying reason for a well's productivity. A properly designed, executed, and analyzed well test usually can provide information about formation permeability, extent of wellbore damage or stimulation, reservoir pressure, and (perhaps) reservoir boundaries and heterogeneities.

The basic test method is to create a pressure drawdown in the wellbore; this causes formation fluids to enter the wellbore. If we measure the flow rate and the pressure in the wellbore during production or the pressure during a shut-in period following production, we usually will have sufficient information to characterize the tested well.

This book begins with a discussion of basic equations that describe the unsteady-state flow of fluids in porous media. It then moves into

discussions of pressure buildup tests; pressure drawdown tests; other flow tests; type-curve analysis; gas well tests; interference and pulse tests; and drillstem and wireline formation tests. Fundamental principles are emphasized in this discussion, and little effort is made to bring the intended audience—undergraduate petroleum engineering students—to the frontiers of the subject. This role is filled much better by other publications, such as the Society of Petroleum Engineers' monographs on well testing^{1,2} and Alberta Energy Resources and Conservation Board's gas well testing manual.³

Basic equations and examples use engineering units. However, to smooth the expected transition to the Intl. System of Units (SI) in the petroleum industry, Appendix F discusses this unit system and restates major equations in SI units. In addition, answers to examples worked out in the text are given in SI units in Appendix F.

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1. Matthews, C.S. and Russell, D.G.: *Pressure Buildup and Flow Tests in Wells*, Monograph Series, SPE, Dallas (1967) 1.
2. Earlougher, R.C. Jr.: *Advances in Well Test Analysis*, Monograph Series, SPE, Dallas (1977) 5.
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Chapter 1

Fluid Flow in Porous Media

1.1 Introduction

In this initial chapter on fluid flow in porous media, we begin with a discussion of the differential equations that are used most often to model unsteady-state flow. Simple statements of these equations are provided in the text; the more tedious mathematical details are given in Appendix A for the instructor or student who wishes to develop greater understanding. The equations are followed by a discussion of some of the most useful solutions to these equations, with emphasis on the exponential-integral solution describing radial, unsteady-state flow. An appended discussion (Appendix B) of dimensionless variables may be useful to some readers at this point.

The chapter concludes with a discussion of the radius-of-investigation concept and of the principle of superposition. Superposition, illustrated in multiwell infinite reservoirs, is used to simulate simple reservoir boundaries and to simulate variable rate production histories. An approximate alternative to superposition, Horner's "pseudoproduction time," completes this discussion.

1.2 The Ideal Reservoir Model

To develop analysis and design techniques for well testing, we first must make several simplifying assumptions about the well and reservoir that we are modeling. We naturally make no more simplifying assumptions than are absolutely necessary to obtain simple, useful solutions to equations describing our situation—but we obviously can make no fewer assumptions. These assumptions are introduced as needed, to combine (1) the law of conservation of mass, (2) Darcy's law, and (3) equations of state to achieve our objectives. This work is only outlined in this chapter; detail is provided in Appendix A and the References.

Consider radial flow toward a well in a circular reservoir. If we combine the law of conservation of mass and Darcy's law for the isothermal flow of fluids of small and constant compressibility (a highly satisfactory model for single-phase flow of reservoir

oil), we obtain a partial differential equation that simplifies to

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi \mu c}{0.000264 k} \frac{\partial p}{\partial t}, \quad \dots \dots \dots (1.1)$$

if we assume that compressibility, c , is small and independent of pressure; permeability, k , is constant and isotropic; viscosity, μ , is independent of pressure; porosity, ϕ , is constant; and that certain terms in the basic differential equation (involving pressure gradients squared) are negligible. This equation is called the *diffusivity equation*; the term $0.000264k/\phi\mu c$ is called the *hydraulic diffusivity* and frequently is given the symbol η .

Eq. 1.1 is written in terms of field units. Pressure, p , is in pounds per square inch (psi); distance, r , is in feet; porosity, ϕ , is a fraction; viscosity, μ , is in centipoise; compressibility, c , is in volume per volume per psi [$c = (1/\rho) (d\rho/dp)$]; permeability, k , is in millidarcies; time, t , is in hours; and hydraulic diffusivity, η , has units of square feet per hour.

A similar equation can be developed for the radial flow of a nonideal gas:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{p}{\mu z} r \frac{\partial p}{\partial r} \right) = \frac{\phi}{0.000264 k} \frac{\partial}{\partial t} \left(\frac{p}{z} \right), \quad \dots \dots (1.2)$$

where z is the gas-law deviation factor.

For simultaneous flow of oil, gas, and water,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi c_t}{0.000264 \lambda_t} \frac{\partial p}{\partial t}, \quad \dots \dots \dots (1.3)$$

where c_t is the total system compressibility,

$$c_t = S_o c_o + S_w c_w + S_g c_g + c_f, \quad \dots \dots \dots (1.4)$$

and the total mobility λ_t is the sum of the mobilities of the individual phases:

$$\lambda_t = \left(\frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w} \right). \quad \dots \dots \dots (1.5)$$

In Eq. 1.4, S_o refers to oil-phase saturation, c_o to oil-phase compressibility, S_w and c_w to water phase, S_g and c_g to gas phase; and c_f is the formation

compressibility. In Eq. 1.5, k_o is the effective permeability to oil in the presence of the other phases, and μ_o is the oil viscosity; k_g and μ_g refer to the gas phase; and k_w and μ_w refer to the water phase. Because the formation is considered compressible (i.e., pore volume decreases as pressure decreases), porosity is not a constant in Eq. 1.3 as it was assumed to be in Eqs. 1.1 and 1.2.

1.3 Solutions to Diffusivity Equation

This section deals with useful solutions to the diffusivity equation (Section 1.2) describing the flow of a slightly compressible liquid in a porous medium. We also have some comments on solutions to Eqs. 1.2 and 1.3.

There are four solutions to Eq. 1.1 that are particularly useful in well testing: the solution for a bounded cylindrical reservoir; the solution for an infinite reservoir with a well considered to be a line source with zero wellbore radius; the pseudosteady-state solution; and the solution that includes wellbore storage for a well in an infinite reservoir. Before we discuss these solutions, however, we should summarize the assumptions that were necessary to develop Eq. 1.1: homogeneous and isotropic porous medium of uniform thickness; pressure-independent rock and fluid properties; small pressure gradients; radial flow; applicability of Darcy's law (sometimes called laminar flow); and negligible gravity forces. We will introduce further assumptions to obtain solutions to Eq. 1.1.

Bounded Cylindrical Reservoir

Solution of Eq. 1.1 requires that we specify two boundary conditions and an initial condition. A realistic and practical solution is obtained if we assume that (1) a well produces at constant rate, qB , into the wellbore (q refers to flow rate in STB/D at surface conditions, and B is the formation volume factor in RB/STB); (2) the well, with wellbore radius r_w , is centered in a cylindrical reservoir of radius r_e , and that there is no flow across this outer boundary; and (3) before production begins, the reservoir is at uniform pressure, p_i . The most useful form of the desired solution relates flowing pressure, p_{wf} , at the sandface to time and to reservoir rock and fluid properties. The solution is¹

$$p_{wf} = p_i - 141.2 \frac{qB\mu}{kh} \left\{ \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4} + 2 \sum_{n=1}^{\infty} \frac{e^{-\alpha_n^2 t_D} J_1^2(\alpha_n r_{eD})}{\alpha_n^2 [J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)]} \right\}, \quad \dots (1.6)$$

where, for efficiency and convenience, we have introduced the dimensionless variables

$$r_{eD} = r_e / r_w$$

and

$$t_D = 0.000264 kt / \phi \mu c_t r_w^2,$$

where the α_n are the roots of

$$J_1(\alpha_n r_{eD}) Y_1(\alpha_n) - J_1(\alpha_n) Y_1(\alpha_n r_{eD}) = 0;$$

and where J_1 and Y_1 are Bessel functions. (Total compressibility, c_t , is used in all equations in this chapter because even formations that produce a single-phase oil contain an immobile water phase and have formation compressibility.)

The reader unfamiliar with Bessel functions should not be alarmed at this equation. It will not be necessary to use Eq. 1.6 in its complete form to calculate numerical values of p_{wf} ; instead, we will use limiting forms of the solution in most computations. The most important fact about Eq. 1.6 is that, under the assumptions made in its development, it is an *exact solution* to Eq. 1.1. It sometimes is called the van Everdingen-Hurst constant-terminal-rate solution.² Appendix C discusses this solution more completely. Because it is exact, it serves as a standard with which we may compare more useful (but more approximate) solutions. One such approximate solution follows.

Infinite Cylindrical Reservoir With Line-Source Well

Assume that (1) a well produces at a constant rate, qB ; (2) the well has zero radius; (3) the reservoir is at uniform pressure, p_i , before production begins; and (4) the well drains an infinite area (i.e., that $p = p_i$ as $r \rightarrow \infty$). Under those conditions, the solution to Eq. 1.1 is

$$p = p_i + 70.6 \frac{qB\mu}{kh} Ei \left(\frac{-948 \phi \mu c_t r^2}{kt} \right), \quad \dots (1.7)$$

where the new symbols are p , the pressure (psi) at distance r (feet) from the well at time t (hours), and

$$Ei(-x) = - \int_x^{\infty} \frac{e^{-u}}{u} du,$$

the Ei function or exponential integral.

Before we examine the properties and implications of Eq. 1.7, we must answer a logical question: Since Eq. 1.6 is an exact solution and since Eq. 1.7 clearly is based on idealized boundary conditions, when (if ever) are pressures calculated at radius r_w from Eq. 1.7 satisfactory approximations to pressures calculated from Eq. 1.6? Analysis of these solutions shows³ that the Ei -function solution is an accurate approximation to the more exact solution for time $3.79 \times 10^5 \phi \mu c_t r_w^2 / k < t < 948 \phi \mu c_t r_e^2 / k$. For times less than $3.79 \times 10^5 \phi \mu c_t r_w^2 / k$, the assumption of zero well size (i.e., assuming the well to be a line source or sink) limits the accuracy of the equation; at times greater than $948 \phi \mu c_t r_e^2 / k$, the reservoir's boundaries begin to affect the pressure distribution in the reservoir, so that the reservoir is no longer infinite acting.

A further simplification of the solution to the flow equation is possible: for $x < 0.02$, $Ei(-x)$ can be approximated with an error less than 0.6% by

$$Ei(-x) = \ln(1.781x). \quad \dots (1.8)$$

To evaluate the Ei function, we can use Table 1.1 for $0.02 < x \leq 10.9$. For $x \leq 0.02$, we use Eq. 1.8; and for $x > 10.9$, $Ei(-x)$ can be considered zero for applications in well testing.

In practice, we find that most wells have reduced permeability (damage) near the wellbore resulting

from drilling or completion operations. Many other wells are stimulated by acidization or hydraulic fracturing. Eq. 1.7 fails to model such wells properly; its derivation holds the explicit assumption of uniform permeability throughout the drainage area of the well up to the wellbore. Hawkins⁴ pointed out that if the damaged or stimulated zone is considered equivalent to an altered zone of uniform permeability (k_s) and outer radius (r_s), the *additional* pressure drop across this zone (Δp_s) can be modeled by the steady-state radial flow equation (see Fig. 1.1). Thus,

$$\begin{aligned}\Delta p_s &= 141.2 \frac{qB\mu}{k_s h} \ln(r_s/r_w) \\ &\quad - 141.2 \frac{qB\mu}{kh} \ln(r_s/r_w) \\ &= 141.2 \frac{qB\mu}{kh} \left(\frac{k}{k_s} - 1 \right) \ln(r_s/r_w). \quad \dots (1.9)\end{aligned}$$

TABLE 1.1* – VALUES OF THE EXPONENTIAL INTEGRAL, $-Ei(-x)$

$-Ei(-x)$, $0.000 < 0.209$, interval = 0.001

x	0	1	2	3	4	5	6	7	8	9
0.00	$+\infty$	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
0.01	4.038	3.944	3.858	3.779	3.705	3.637	3.574	3.514	3.458	3.405
0.02	3.355	3.307	3.261	3.218	3.176	3.137	3.098	3.062	3.026	2.992
0.03	2.959	2.927	2.897	2.867	2.838	2.810	2.783	2.756	2.731	2.706
0.04	2.681	2.658	2.634	2.612	2.590	2.568	2.547	2.527	2.507	2.487
0.05	2.468	2.449	2.431	2.413	2.395	2.377	2.360	2.344	2.327	2.311
0.06	2.295	2.279	2.264	2.249	2.235	2.220	2.206	2.192	2.178	2.164
0.07	2.151	2.138	2.125	2.112	2.099	2.087	2.074	2.062	2.050	2.039
0.08	2.027	2.015	2.004	1.993	1.982	1.971	1.960	1.950	1.939	1.929
0.09	1.919	1.909	1.899	1.889	1.879	1.869	1.860	1.850	1.841	1.832
0.10	1.823	1.814	1.805	1.796	1.788	1.779	1.770	1.762	1.754	1.745
0.11	1.737	1.729	1.721	1.713	1.705	1.697	1.689	1.682	1.674	1.667
0.12	1.660	1.652	1.645	1.638	1.631	1.623	1.616	1.609	1.603	1.596
0.13	1.589	1.582	1.576	1.569	1.562	1.556	1.549	1.543	1.537	1.530
0.14	1.524	1.518	1.512	1.506	1.500	1.494	1.488	1.482	1.476	1.470
0.15	1.464	1.459	1.453	1.447	1.442	1.436	1.431	1.425	1.420	1.415
0.16	1.409	1.404	1.399	1.393	1.388	1.383	1.378	1.373	1.368	1.363
0.17	1.358	1.353	1.348	1.343	1.338	1.333	1.329	1.324	1.319	1.314
0.18	1.310	1.305	1.301	1.296	1.291	1.287	1.282	1.278	1.274	1.269
0.19	1.265	1.261	1.256	1.252	1.248	1.243	1.239	1.235	1.231	1.227
0.20	1.223	1.219	1.215	1.210	1.206	1.202	1.198	1.195	1.191	1.187

$-Ei(-x)$, $0.00 < x < 2.09$, interval = 0.01

0.0	$+\infty$	4.038	3.335	2.959	2.681	2.468	2.295	2.151	2.027	1.919
0.1	1.823	1.737	1.660	1.589	1.524	1.464	1.409	1.358	1.309	1.265
0.2	1.223	1.183	1.145	1.110	1.076	1.044	1.014	0.985	0.957	0.931
0.3	0.906	0.882	0.858	0.836	0.815	0.794	0.774	0.755	0.737	0.719
0.4	0.702	0.686	0.670	0.655	0.640	0.625	0.611	0.598	0.585	0.572
0.5	0.560	0.548	0.536	0.525	0.514	0.503	0.493	0.483	0.473	0.464
0.6	0.454	0.445	0.437	0.428	0.420	0.412	0.404	0.396	0.388	0.381
0.7	0.374	0.367	0.360	0.353	0.347	0.340	0.334	0.328	0.322	0.316
0.8	0.311	0.305	0.300	0.295	0.289	0.284	0.279	0.274	0.269	0.265
0.9	0.260	0.256	0.251	0.247	0.243	0.239	0.235	0.231	0.227	0.223
1.0	0.219	0.216	0.212	0.209	0.205	0.202	0.198	0.195	0.192	0.189
1.1	0.186	0.183	0.180	0.177	0.174	0.172	0.169	0.166	0.164	0.161
1.2	0.158	0.156	0.153	0.151	0.149	0.146	0.144	0.142	0.140	0.138
1.3	0.135	0.133	0.131	0.129	0.127	0.125	0.124	0.122	0.120	0.118
1.4	0.116	0.114	0.113	0.111	0.109	0.108	0.106	0.105	0.103	0.102
1.5	0.1000	0.0985	0.0971	0.0957	0.0943	0.0929	0.0915	0.0902	0.0889	0.0876
1.6	0.0863	0.0851	0.0838	0.0826	0.0814	0.0802	0.0791	0.0780	0.0768	0.0757
1.7	0.0747	0.0736	0.0725	0.0715	0.0705	0.0695	0.0685	0.0675	0.0666	0.0656
1.8	0.0647	0.0638	0.0629	0.0620	0.0612	0.0603	0.0595	0.0586	0.0578	0.0570
1.9	0.0562	0.0554	0.0546	0.0539	0.0531	0.0524	0.0517	0.0510	0.0503	0.0496
2.0	0.0489	0.0482	0.0476	0.0469	0.0463	0.0456	0.0450	0.0444	0.0438	0.0432

$2.0 < x < 10.9$, interval = 0.1

x	0	1	2	3	4	5	6	7	8	9
2	4.89×10^{-2}	4.26×10^{-2}	3.72×10^{-2}	3.25×10^{-2}	2.84×10^{-2}	2.49×10^{-2}	2.19×10^{-2}	1.92×10^{-2}	1.69×10^{-2}	1.48×10^{-2}
3	1.30×10^{-2}	1.15×10^{-2}	1.01×10^{-2}	8.94×10^{-3}	7.89×10^{-3}	6.87×10^{-3}	6.16×10^{-3}	5.45×10^{-3}	4.82×10^{-3}	4.27×10^{-3}
4	3.78×10^{-3}	3.35×10^{-3}	2.97×10^{-3}	2.64×10^{-3}	2.34×10^{-3}	2.07×10^{-3}	1.84×10^{-3}	1.64×10^{-3}	1.45×10^{-3}	1.29×10^{-3}
5	1.15×10^{-3}	1.02×10^{-3}	9.08×10^{-4}	8.09×10^{-4}	7.19×10^{-4}	6.41×10^{-4}	5.71×10^{-4}	5.09×10^{-4}	4.53×10^{-4}	4.04×10^{-4}
6	3.60×10^{-4}	3.21×10^{-4}	2.86×10^{-4}	2.55×10^{-4}	2.28×10^{-4}	2.03×10^{-4}	1.82×10^{-4}	1.62×10^{-4}	1.45×10^{-4}	1.29×10^{-4}
7	1.15×10^{-4}	1.03×10^{-4}	9.22×10^{-5}	8.24×10^{-5}	7.36×10^{-5}	6.58×10^{-5}	5.89×10^{-5}	5.26×10^{-5}	4.71×10^{-5}	4.21×10^{-5}
8	3.77×10^{-5}	3.37×10^{-5}	3.02×10^{-5}	2.70×10^{-5}	2.42×10^{-5}	2.16×10^{-5}	1.94×10^{-5}	1.73×10^{-5}	1.55×10^{-5}	1.39×10^{-5}
9	1.24×10^{-5}	1.11×10^{-5}	9.99×10^{-6}	8.95×10^{-6}	8.02×10^{-6}	7.18×10^{-6}	6.44×10^{-6}	5.77×10^{-6}	5.17×10^{-6}	4.64×10^{-6}
10	4.15×10^{-6}	3.73×10^{-6}	3.34×10^{-6}	3.00×10^{-6}	2.68×10^{-6}	2.41×10^{-6}	2.16×10^{-6}	1.94×10^{-6}	1.74×10^{-6}	1.56×10^{-6}

*Adapted from Nisle, R.G.: "How To Use The Exponential Integral," *Pet. Eng.* (Aug. 1956) B171-173.

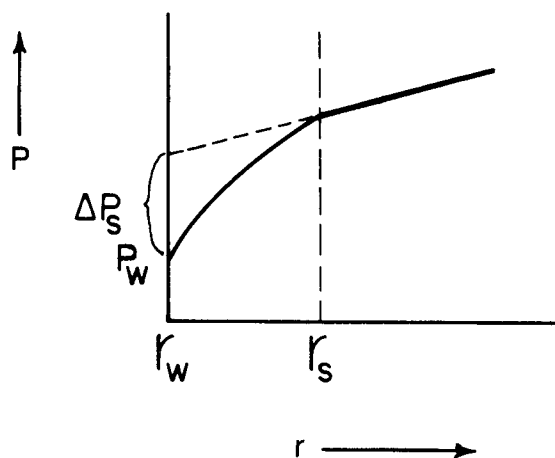


Fig. 1.1—Schematic of pressure distribution near wellbore.

Eq. 1.9 simply states that the pressure drop in the altered zone is inversely proportional to k_s rather than to k and that a correction to the pressure drop in this region (which assumed the same permeability, k , as in the rest of the reservoir) must be made. Combining Eqs. 1.7 and 1.9, we find that the total pressure drop at the wellbore is

$$\begin{aligned} p_i - p_{wf} &= -70.6 \frac{qB\mu}{kh} Ei\left(-\frac{948 \phi \mu c_t r_w^2}{kt}\right) + \Delta p_s \\ &= -70.6 \frac{qB\mu}{kh} \left[Ei\left(-\frac{948 \phi \mu c_t r_w^2}{kt}\right) \right. \\ &\quad \left. - 2\left(\frac{k}{k_s} - 1\right) \ln\left(\frac{r_s}{r_w}\right) \right]. \end{aligned}$$

For $r = r_w$, the argument of the Ei function is sufficiently small after a short time that we can use the logarithmic approximation; thus, the drawdown is

$$\begin{aligned} p_i - p_{wf} &= -70.6 \frac{qB\mu}{kh} \left[\ln\left(\frac{1,688 \phi \mu c_t r_w^2}{kt}\right) \right. \\ &\quad \left. - 2\left(\frac{k}{k_s} - 1\right) \ln\left(\frac{r_s}{r_w}\right) \right]. \end{aligned}$$

It is convenient to define a skin factor, s , in terms of the properties of the equivalent altered zone:

$$s = \left(\frac{k}{k_s} - 1\right) \ln\left(\frac{r_s}{r_w}\right). \quad (1.10)$$

Thus, the drawdown is

$$\begin{aligned} p_i - p_{wf} &= -70.6 \frac{qB\mu}{kh} \left[\ln\left(\frac{1,688 \phi \mu c_t r_w^2}{kt}\right) - 2s \right]. \\ &\dots\dots\dots (1.11) \end{aligned}$$

Eq. 1.10 provides some insight into the physical significance of the sign of the skin factor. If a well is damaged ($k_s < k$), s will be positive, and the greater the contrast between k_s and k and the deeper into the

formation the damage extends, the larger the numerical value of s . There is no upper limit for s . Some newly drilled wells will not flow at all before stimulation; for these wells, $k_s \approx 0$ and $s \rightarrow \infty$. If a well is stimulated ($k_s > k$), s will be negative, and the deeper the stimulation, the greater the numerical value of s . Rarely does a stimulated well have a skin factor less than -7 or -8 , and such skin factors arise only for wells with deeply penetrating, highly conductive hydraulic fractures. We should note finally that, if a well is neither damaged nor stimulated ($k = k_s$), $s = 0$. We caution the reader that Eq. 1.10 is best applied qualitatively; actual wells rarely can be characterized *exactly* by such a simplified model.

Before leaving the discussion of skin factor, we should point out that an altered zone near a particular well affects only the pressure near that well — i.e., the pressure in the unaltered formation away from the well is not affected by the existence of the altered zone. Said another way, we use Eq. 1.11 to calculate pressures at the sandface of a well with an altered zone, but we use Eq. 1.7 to calculate pressures beyond the altered zone in the formation surrounding the well. We have presented *no* simple equations that can be used to calculate pressures for radius, r , such that $r_w < r < r_s$, but this will offer no difficulties in well test analysis.

Example 1.1—Calculation of Pressures Beyond the Wellbore Using the Ei -Function Solution

Problem. A well and reservoir have the following characteristics: The well is producing only oil; it is producing at a constant rate of 20 STB/D. Data describing the well and formation are

$$\begin{aligned} \mu &= 0.72 \text{ cp,} \\ k &= 0.1 \text{ md,} \\ c_t &= 1.5 \times 10^{-5} \text{ psi}^{-1} \\ p_i &= 3,000 \text{ psi,} \\ r_e &= 3,000 \text{ ft,} \\ r_w &= 0.5 \text{ ft,} \\ B_o &= 1.475 \text{ RB/STB,} \\ h &= 150 \text{ ft,} \\ \phi &= 0.23, \text{ and} \\ s &= 0. \end{aligned}$$

Calculate the reservoir pressure at a radius of 1 ft after 3 hours of production; then, calculate the pressure at radii of 10 and 100 ft after 3 hours of production.

Solution. The Ei function is not an accurate solution to flow equations until $t > 3.79 \times 10^5 \phi \mu c_t r_w^2 / k$. Here,

$$\begin{aligned} \frac{3.79 \times 10^5 \phi \mu c_t r_w^2}{k} &= \left[(3.79 \times 10^5)(0.23)(0.72) \right. \\ &\quad \left. \cdot (1.5 \times 10^{-5})(0.5)^2 \right] / (0.1) \\ &= 2.35 < t = 3 \text{ hours.} \end{aligned}$$

Thus, we can use Eq. 1.7 with satisfactory accuracy if

the reservoir is still infinite acting at this time. The reservoir will act as an infinite reservoir until $t > 948 \phi \mu c_r^2 / k$.

Here,

$$\frac{948 \phi \mu c_r^2}{k} = \left[(948)(0.23)(0.72) \cdot (1.5 \times 10^{-5})(3,000)^2 \right] / 0.3 = 211,900 \text{ hours.}$$

Thus, for times less than 211,900 hours, we can use Eq. 1.7. At a radius of 1 ft,

$$\begin{aligned} p &= p_i + 70.6 \frac{qB\mu}{kh} Ei \left(\frac{-948 \phi \mu c_r^2}{kt} \right) \\ &= 3,000 + \frac{(70.6)(20)(1.475)(0.72)}{(0.1)(150)} \\ &\quad \cdot Ei \left[\frac{-(948)(0.23)(0.72)(1.5 \times 10^{-5})(1)^2}{(0.1)(3)} \right] \\ &= 3,000 + (100)Ei(-0.007849) \\ &= 3,000 + 100 \ln[(1.781)(0.007849)] \\ &= 3,000 + (100)(-4.27) \\ &= 2,573 \text{ psi.} \end{aligned}$$

At a radius of 10 ft,

$$\begin{aligned} p &= 3,000 + 100 \\ &\quad \cdot Ei \left[\frac{-(948)(0.23)(0.72)(1.5 \times 10^{-5})(10)^2}{(0.1)(3)} \right] \\ &= 3,000 + 100 Ei(-0.7849) \\ &= 3,000 + (100)(-0.318) \\ &= 2,968 \text{ psi.} \end{aligned}$$

In this calculation, we find the value of the Ei function from Table 1.1. Note, as indicated in the table, that it is a negative quantity.

At a radius of 100 ft,

$$\begin{aligned} p &= 3,000 + 100 \\ &\quad \cdot Ei \left[\frac{-(948)(0.23)(0.72)(1.5 \times 10^{-5})(100)^2}{(0.1)(3)} \right] \\ &= 3,000 + 100 Ei(-78.49) \\ &= 3,000 \text{ psi.} \end{aligned}$$

Here we note that for an argument of 78.49, the Ei function is essentially zero.

Pseudosteady-State Solution. We now discuss the next solution to the radial diffusivity equation that we will use extensively in this introduction to well test analysis. Actually, this solution (the pseudosteady-state solution) is not new. It is simply a limiting form

of Eq. 1.6, which describes pressure behavior with time for a well centered in a cylindrical reservoir of radius r_e . The limiting form of interest is that which is valid for large times, so that the summation involving exponentials and Bessel functions is negligible; after this time ($t > 948 \phi \mu c_r^2 / k$),

$$p_{wf} = p_i - 141.2 \frac{qB\mu}{kh} \left(\frac{2t_D}{r_e^2} + \ln r_{eD} - \frac{3}{4} \right),$$

or

$$\begin{aligned} p_{wf} &= p_i - 141.2 \frac{qB\mu}{kh} \left[\frac{0.000527kt}{\phi \mu c_r^2} \right. \\ &\quad \left. + \ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} \right]. \quad \dots \dots \dots (1.12) \end{aligned}$$

Note that during this time period we find, by differentiating Eq. 1.12,

$$\frac{\partial p_{wf}}{\partial t} = - \frac{0.0744 qB}{\phi c_r h r_e^2}.$$

Since the liquid-filled pore volume of the reservoir, V_p (cubic feet), is

$$V_p = \pi r_e^2 h \phi,$$

then

$$\frac{\partial p_{wf}}{\partial t} = - \frac{0.234 qB}{c_r V_p}. \quad \dots \dots \dots (1.13)$$

Thus, during this time period, the rate of pressure decline is inversely proportional to the liquid-filled pore volume V_p . This result leads to a form of well testing sometimes called reservoir limits testing, which seeks to determine reservoir size from the rate of pressure decline in a wellbore with time.

Another form of Eq. 1.12 is useful for some applications. It involves replacing original reservoir pressure, p_i , with average pressure, $\bar{p}$, within the drainage volume of the well.

The volumetric average pressure within the drainage volume of the well can be found from material balance. The pressure decrease ($p_i - \bar{p}$) resulting from removal of qB RB/D of fluid for t hours [a total volume removed of $5.615 qB(t/24)$ cu ft] is

$$\begin{aligned} p_i - \bar{p} &= \frac{\Delta V}{c_r V} = \frac{5.615 qB(t/24)}{c_r (\pi r_e^2 h \phi)} \\ &= \frac{0.0744 qBt}{\phi c_r h r_e^2}. \quad \dots \dots \dots (1.14) \end{aligned}$$

Substituting in Eq. 1.12,

$$\begin{aligned} p_{wf} &= \bar{p} + \frac{0.0744 qBt}{\phi c_r h r_e^2} - \frac{0.0744 qBt}{\phi c_r h r_e^2} \\ &\quad - 141.2 \frac{qB\mu}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} \right], \end{aligned}$$

or

$$\bar{p} - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} \right]. \quad \dots \dots \dots (1.15)$$

Eqs. 1.12 and 1.15 become more useful in practice if they include a skin factor to account for the fact that most wells are either damaged or stimulated. For example, in Eq. 1.15,

$$\bar{p} - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right] + (\Delta p)_s,$$

$$\bar{p} - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s \right], \quad \dots (1.16)$$

and

$$p_i - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\frac{0.000527 kt}{\phi\mu c_t r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s \right]. \quad \dots (1.17)$$

Further, we can define an average permeability, k_J , such that

$$\begin{aligned} \bar{p} - p_{wf} &= 141.2 \frac{qB\mu}{k_J h} \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right] \\ &= 141.2 \frac{qB\mu}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s \right], \end{aligned}$$

from which,

$$k_J = k \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right] / \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s \right]. \quad \dots (1.18)$$

This average permeability, k_J , proves to have considerable value in well test analysis, as we shall see later. Note that for a damaged well, the average permeability k_J is lower than the true, bulk formation permeability k ; in fact, these quantities are equal only when the skin factor s is zero. Since we sometimes estimate the permeability of a well from productivity-index (PI) measurements, and since the productivity index J (STB/D/psi), of an oil well is defined as

$$J \equiv \frac{q}{\bar{p} - p_{wf}} = \frac{k_J h}{141.2 B\mu \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right]}, \quad \dots (1.19)$$

this method *does not* necessarily provide a good estimate of formation permeability, k . Thus, there is a need for a more complete means of characterizing a producing well than exclusive use of PI information.

Example 1.2 – Analysis of Well From PI Test

Problem. A well produces 100 STB/D oil at a measured flowing bottomhole pressure (BHP) of 1,500 psi. A recent pressure survey showed that average reservoir pressure is 2,000 psi. Logs indicate a net sand thickness of 10 ft. The well drains an area with drainage radius, r_e , of 1,000 ft; the borehole radius is 0.25 ft. Fluid samples indicate that, at current reservoir pressure, oil viscosity is 0.5 cp and

formation volume factor is 1.5 RB/STB.

1. Estimate the productivity index for the tested well.

2. Estimate formation permeability from these data.

3. Core data from the well indicate an effective permeability to oil of 50 md. Does this imply that the well is either damaged or stimulated? What is the apparent skin factor?

Solution.

1. To estimate productivity index, we use Eq. 1.19:

$$\begin{aligned} J &= \frac{q}{\bar{p} - p_{wf}} = \frac{100}{(2,000 - 1,500)} \\ &= 0.2 \text{ STB/psi-D.} \end{aligned}$$

2. We do not have sufficient information to estimate formation permeability; we can calculate average permeability, k_J , only, which is not necessarily a good approximation of formation permeability, k . From Eq. 1.19,

$$\begin{aligned} k_J &= \frac{141.2 JB\mu \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right]}{h} \\ &= \frac{(141.2)(0.2)(1.5)(0.5) \left[\ln\left(\frac{1,000}{0.25}\right) - 0.75 \right]}{10} \\ &= 16 \text{ md.} \end{aligned}$$

3. Core data frequently provide a better estimate of formation permeability than do permeabilities derived from the productivity index, particularly for a well that is badly damaged. Since cores indicate a permeability of 50 md, we conclude that this well is damaged. Eq. 1.18 provides a method for estimating the skin factor s :

$$\begin{aligned} s &= \left(\frac{k}{k_J} - 1 \right) \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right] \\ &= \left(\frac{50}{16} - 1 \right) \left[\ln\left(\frac{1,000}{0.25}\right) - 0.75 \right] \\ &= 16. \end{aligned}$$

Flow Equations for Generalized Reservoir Geometry

Eq. 1.16 is limited to a well centered in a circular drainage area. A similar equation⁵ models pseudo-steady-state flow in more general reservoir shapes:

$$\begin{aligned} \bar{p} - p_{wf} &= 141.2 \frac{qB\mu}{kh} \left[\frac{1}{2} \ln\left(\frac{10.06 A}{C_A r_w^2}\right) - \frac{3}{4} + s \right], \\ &\dots (1.20) \end{aligned}$$

where

A = drainage area, sq ft, and

C_A = shape factor for specific drainage-area shape and well location, dimensionless.

Values of C_A are given in Table 1.2; further explanation of the source of these C_A values is given in

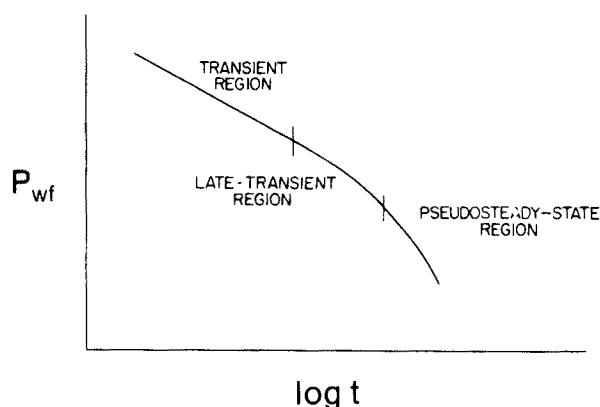


Fig. 1.2—Flow regions on semilogarithmic paper.

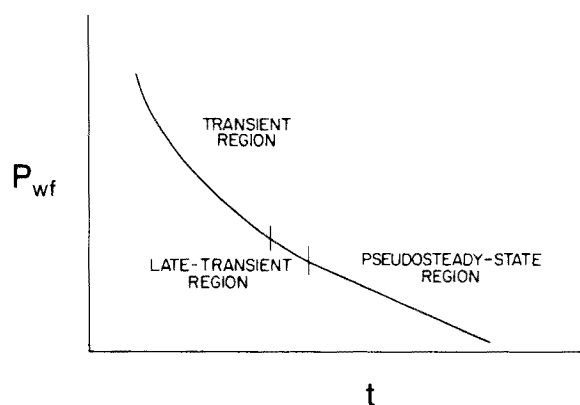


Fig. 1.3—Flow regions on Cartesian-coordinate graph.

Chap. 2.

Productivity index, J , can be expressed for general drainage-area geometry as

$$J = \frac{q}{\bar{p} - p_{wf}} = \frac{0.00708 kh}{B\mu \left[\frac{1}{2} \ln \left(\frac{10.06 A}{C_A r_w^2} \right) - \frac{3}{4} + s \right]} \quad (1.21)$$

Other numerical constants tabulated in Table 1.2 allow us to calculate (1) the maximum elapsed time during which a reservoir is infinite acting (so that the Ei -function solution can be used); (2) the time required for the pseudosteady-state solution to predict pressure drawdown within 1% accuracy; and (3) time required for the pseudosteady-state solution to be exact.

For a given reservoir geometry, the maximum time a reservoir is infinite acting can be determined using the entry in the column "Use Infinite-System Solution With Less Than 1% Error for $t_{DA} < .$ " Since $t_{DA} = 0.000264 kt / \phi \mu c_t A$, this means that the time in hours is calculated from

$$t < \frac{\phi \mu c_t A t_{DA}}{0.000264 k}$$

Time required for the pseudosteady-state equation to be accurate within 1% can be found from the entry in the column headed "Less Than 1% Error for $t_{DA} > .$ " and the relationship

$$t > \frac{\phi \mu c_t A t_{DA}}{0.000264 k}$$

Finally, time required for the pseudosteady-state equation to be exact is found from the entry in the column "Exact for $t_{DA} > .$ "

At this point, it is helpful to depict graphically the flow regimes that occur in different time ranges. Figs. 1.2 and 1.3 show BHP, p_{wf} , in a well flowing at constant rate, plotted as a function of time on both logarithmic and linear scales.

In the transient region, the reservoir is infinite acting, and is modeled by Eq. 1.11, which implies that p_{wf} is a linear function of $\log t$. In the

pseudosteady-state region, the reservoir is modeled by Eq. 1.20 in the general case or Eqs. 1.15 and 1.12 for the special case of a well centered in a cylindrical reservoir. Eq. 1.12 shows the linear relationship between p_{wf} and t during pseudosteady-state. This linear relationship also exists in generalized reservoir geometries.

At times between the end of the transient region and the beginning of the pseudosteady-state region, this is a transition region, sometimes called the late-transient region, as in Figs. 1.2 and 1.3. No simple equation is available to predict the relationship between BHP and time in this region. This region is small (or, for practical purposes nonexistent) for a well centered in a circular, square, or hexagonal drainage area, as Table 1.2 indicates. However, for a well off-center in its drainage area, the late-transient region can span a significant time region, as Table 1.2 also indicates.

Note that the determination of when the transient region ends or when the pseudosteady-state region begins is somewhat subjective. For example, the limits on applicability of Eqs. 1.7 and 1.12 (stated in the text earlier) are not exactly the same as given in Table 1.2—but the difference is slight. Other authors¹ consider the deviation from Eq. 1.7 to be sufficient for $t > 379 \phi \mu c_t r_e^2 / k$ that a late-transient region exists even for a well centered in a cylindrical reservoir between this lower limit and an upper limit of $1,136 \phi \mu c_t r_e^2 / k$. These apparently contradictory opinions are nothing more than different judgments about when the slightly approximate solutions, Eqs. 1.7 and 1.12, can be considered to be identical to the exact solution, Eq. 1.6.

These concepts are illustrated in Example 1.3.

Example 1.3—Flow Analysis in Generalized Reservoir Geometry

Problem. 1. For each of the following reservoir geometries, calculate the time in hours for which (a) the reservoir is infinite acting; (b) the pseudosteady state

TABLE 1.2 – SHAPE FACTORS FOR VARIOUS SINGLE-WELL DRAINAGE AREAS¹⁰

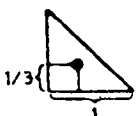
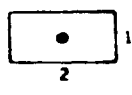
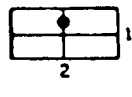
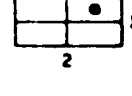
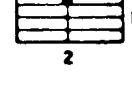
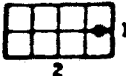
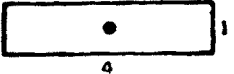
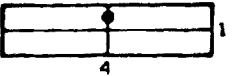
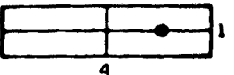
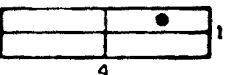
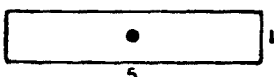
In Bounded Reservoirs	C_A	$\ln C_A$	$0.5 \ln \left(\frac{2.2458}{C_A} \right)$	Exact for $t_{DA} >$	Less Than 1% Error for $t_{DA} >$	Use Infinite System Solution With Less Than 1% Error for $t_{DA} <$
	31.62	3.4538	1.3224	0.1	0.06	0.10
	31.6	3.4532	- 1.3220	0.1	0.06	0.10
	27.6	3.3178	- 1.2544	0.2	0.07	0.09
	27.1	3.2995	- 1.2452	0.2	0.07	0.09
	21.9	3.0865	- 1.1387	0.4	0.12	0.08
	0.098	- 2.3227	1.5659	0.9	0.60	0.015
	30.8828	3.4302	- 1.3106	0.1	0.05	0.09
	12.9851	2.5638	- 0.8774	0.7	0.25	0.03
	4.5132	1.5070	- 0.3490	0.6	0.30	0.025
	3.3351	1.2045	- 0.1977	0.7	0.25	0.01
	21.8369	3.0836	- 1.1373	0.3	0.15	0.025
	10.8374	2.3830	- 0.7870	0.4	0.15	0.025
	4.5141	1.5072	- 0.3491	1.5	0.50	0.06
	2.0769	0.7309	0.0391	1.7	0.50	0.02
	3.1573	1.1497	- 0.1703	0.4	0.15	0.005

TABLE 1.2 – SHAPE FACTORS FOR VARIOUS SINGLE-WELL DRAINAGE AREAS¹⁰

In Bounded Reservoirs	C_A	$\ln C_A$	$0.5 \ln \left(\frac{2.2458}{C_A} \right)$	Exact for $t_{DA} >$	Less Than 1% Error for $t_{DA} >$	Use Infinite System Solution With Less Than 1% Error for $t_{DA} <$
	0.5813	-0.5425	0.6758	2.0	0.60	0.02
	0.1109	-2.1991	1.5041	3.0	0.60	0.005
	5.3790	1.6825	-0.4367	0.8	0.30	0.01
	2.6896	0.9894	-0.0902	0.8	0.30	0.01
	0.2318	-1.4619	1.1355	4.0	2.00	0.03
	0.1155	-2.1585	1.4838	4.0	2.00	0.01
	2.3606	0.8589	-0.0249	1.0	0.40	0.025
In vertically fractured reservoirs: use $(r_e/L_f)^2$ in place of A/r_w^2 for fractured systems						
	2.6541	0.9761	-0.0835	0.175	0.08	cannot use
	2.0348	0.7104	0.0493	0.175	0.09	cannot use
	1.9886	0.6924	0.0583	0.175	0.09	cannot use
	1.6620	0.5080	0.1505	0.175	0.09	cannot use
	1.3127	0.2721	0.2685	0.175	0.09	cannot use
	0.7887	-0.2374	0.5232	0.175	0.09	cannot use
In water-drive reservoirs						
	19.1	2.95	-1.07	-	-	-
In reservoirs of unknown production character						
	25.0	3.22	-1.20	-	-	-

is exact; and (c) the pseudosteady-state equation is accurate to within 1%. (1) Well centered in circular drainage area, (2) well centered in square drainage area, and (3) well centered in one quadrant of square drainage area.

In each case,

$$\begin{aligned} A &= 17.42 \times 10^6 \text{ sq ft (40 acres)}, \\ \phi &= 0.2, \\ \mu &= 1 \text{ cp}, \\ c_f &= 1 \times 10^{-5} \text{ psi}^{-1}, \text{ and} \\ k &= 100 \text{ md}. \end{aligned}$$

2. For each of the wells in Part 1, estimate PI and stabilized production rate with $\bar{p} - p_{wf} = 500$ psi, if

$$\begin{aligned} h &= 10 \text{ ft}, \\ s &= 3.0, \\ r_w &= 0.3 \text{ ft, and} \\ B &= 1.2 \text{ RB/STB}. \end{aligned}$$

3. For the well centered in one of the quadrants of a square, write equations relating constant flow rate and wellbore pressure drops at elapsed times of 30, 200, and 400 hours.

Solution. 1. We first calculate the group $\phi\mu c_f A / 0.000264 k$.

$$\begin{aligned} \frac{\phi\mu c_f A}{0.000264 k} &= \frac{(0.2)(1)(1 \times 10^{-5})(17.42 \times 10^6)}{(0.000264)(100)} \\ &= 1,320. \end{aligned}$$

We then prepare the following table (values from Table 1.2):

Geometry	Infinite Solution		Pseudosteady-State (Approximate)		Pseudosteady-State (Exact)	
	t_{DA}	t (hours)	t_{DA}	t (hours)	t_{DA}	t (hours)
Circular	0.1	132	0.06	79.2	0.1	132
Square-centered	0.09	119	0.05	66.0	0.1	132
Square-quadrant	0.025	33	0.3	396	0.6	792

For the off-center well, the reduction in time for which the infinite-system solution is accurate and the increase in time required to achieve pseudosteady-state flow are noteworthy.

2. To calculate PI and stabilized production rate, we use the equations

$$\begin{aligned} J &= \frac{0.00708 kh}{B\mu \left[\frac{1}{2} \ln \left(\frac{10.06 A}{C_A r_w^2} \right) - \frac{3}{4} + s \right]} \\ &= \frac{(0.00708)(100)(10)}{(1.2)(1) \left[\frac{1}{2} \ln \left(\frac{(10.06)(17.42 \times 10^6)}{C_A (0.3)^2} \right) - \frac{3}{4} + 3.0 \right]}, \\ J &= 5.9 / (12.94 - \frac{1}{2} \ln C_A), \end{aligned}$$

and

$$q = J(\bar{p} - p_{wf}) = 500 J.$$

Thus, we can prepare the following table.

Geometry	C_A	$\frac{J}{(\text{STB/D-psi})}$	$\frac{q}{(\text{STB/D})}$
Circular	31.62	0.526	263
Square-centered	30.88	0.526	263
Square-quadrant	4.513	0.484	242

3. (a) For $t = 30$ hours, the reservoir is infinite acting, and

$$p_i - p_{wf} = -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi\mu c_f r_w^2}{kt} \right) - 2s \right].$$

(b) For $t = 200$ hours, the reservoir is no longer infinite acting and the pseudosteady-state equation is not yet accurate. Accordingly, no simple equation can be written.

(c) For $t = 400$ hours, the pseudosteady-state equation is accurate, and

$$\bar{p} - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\frac{1}{2} \ln \left(\frac{10.06 A}{C_A r_w^2} \right) - \frac{3}{4} + s \right].$$

Radial Flow in Infinite Reservoir With Wellbore Storage

The next solution to the radial diffusivity equation includes a phenomenon causing variable flow rates after production begins. This phenomenon is called wellbore storage, as shown in Fig. 1.4, and assume there is of wellbore storage before we can deal with the solution to the flow equations that includes its effect.

Consider a shut-in oil well in a reservoir with uniform and unchanging pressure. Reservoir pressure will support a column of liquid to some equilibrium height in the wellbore. If we open a valve at the surface and initiate flow, the first oil produced will be that stored in the wellbore, and the initial flow rate from the formation to the well will be zero. With increasing flow time, at constant surface producing rate, the downhole flow rate will approach the surface rate, and the amount of liquid stored in the wellbore will approach a constant value.

We now develop a mathematical relationship between sandface (formation) and surface flow rates. Consider a well with a liquid/gas interface in the wellbore, as shown in Table 1.4, and assume there is some mechanism (a pump or gas lift) to deliver liquid to the surface. Let the surface rate q be variable in the general case.

From a mass balance in the wellbore, the rate of liquid in is $q_{sf}B$ in RB/D; the rate of liquid out is qB in RB/D; and the rate of liquid accumulation in the well is

$$\frac{d}{dt} \left(\frac{24 V_{wb}}{5.615} \right) = \frac{24 A_{wb}}{5.615} \frac{dz}{dt}.$$

Then, assuming constant wellbore area, A_{wb} , and constant oil formation volume factor, B , the same at the sandface and at the surface, the balance becomes

$$\frac{24}{5.615} A_{wb} \frac{dz}{dt} = (q_{sf} - q)B. \quad \dots \dots \dots (1.22)$$

For a well with surface pressure p_t ,

$$p_w = p_t + \frac{\rho z}{144} \frac{g}{g_c}, \quad \dots \dots \dots (1.23)$$

Where ρ is the density of the liquid in the wellbore (lbm/cu ft) and $g/g_c = \text{lbf/lbm}$. Then,

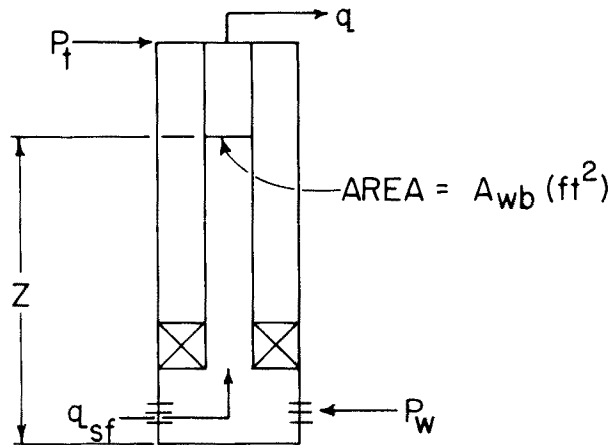


Fig. 1.4—Schematic of wellbore with moving liquid/gas interface.

$$\frac{d(p_w - p_t)}{dt} = \frac{\rho}{144} \frac{g}{g_c} \frac{dz}{dt} \quad (1.24)$$

Thus,

$$\begin{aligned} \frac{(24)(144)}{5.615\rho} \frac{g_c}{g} A_{wb} \frac{d(p_w - p_t)}{dt} \\ = (q_{sf} - q)B. \quad (1.25) \end{aligned}$$

Define a wellbore storage constant, C_s :

$$C_s = \frac{144}{5.615\rho} \frac{A_{wb}}{g} \quad (1.26)$$

Then,

$$q_{sf} = q + \frac{24 C_s}{B} \frac{d(p_w - p_t)}{dt} \quad (1.27)$$

For zero or unchanging surface pressure, p_t (a major and not necessarily valid assumption),

$$q_{sf} = q + \frac{24 C_s}{B} \frac{dp_w}{dt} \quad (1.28)$$

To understand the solution to flow problems that include wellbore storage, it is necessary to introduce dimensionless variables, similar to those discussed in Appendix B. Let q_i be the surface rate at $t=0$ and introduce the definitions of dimensionless time and dimensionless pressure:

$$p_D = \frac{0.00708 kh(p_i - p_w)}{q_i B \mu} \quad (1.29)$$

$$t_D = \frac{0.000264 kt}{\phi \mu c_t r_w^2} \quad (1.30)$$

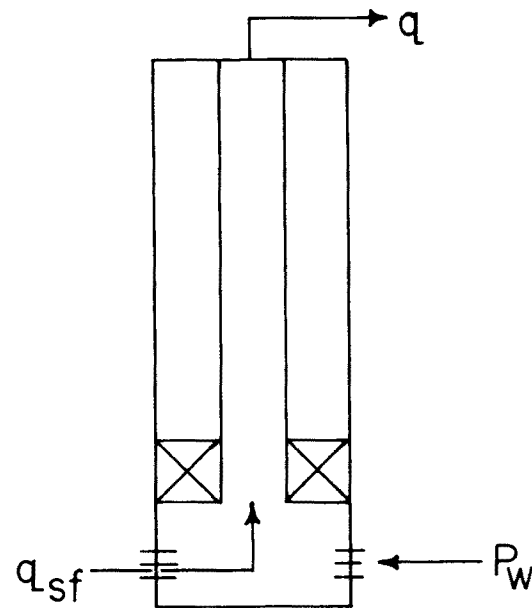


Fig. 1.5—Schematic of wellbore containing single-phase liquid or gas.

Substituting,

$$\begin{aligned} \frac{dp_w}{dt} &= - \frac{q_i B \mu}{0.00708 kh} \times \frac{0.000264 k}{\phi \mu c_t r_w^2} \frac{dp_D}{dt_D} \\ &= - \frac{0.0373 q_i B}{\phi \mu c_t h r_w^2} \frac{dp_D}{dt_D} \quad (1.31) \end{aligned}$$

Thus,

$$q_{sf} = q - \frac{0.894 q_i C_s}{\phi c_t h r_w^2} \frac{dp_D}{dt_D} \quad (1.32)$$

If we define a dimensionless wellbore storage constant, C_{sD} , as

$$C_{sD} = 0.894 C_s / \phi c_t h r_w^2 \quad (1.33)$$

then

$$q_{sf} = q_i \left[\frac{q}{q_i} - C_{sD} \frac{dp_D}{dt_D} \right] \quad (1.34)$$

For constant-rate production [$q(t) = q_i$], Eq. 1.34 becomes

$$\frac{q_{sf}}{q} = 1 - C_{sD} \frac{dp_D}{dt_D} \quad (1.35)$$

Eq. 1.35 is the inner boundary condition for the problem of constant-rate flow of a slightly compressible liquid with wellbore storage. Note that, for small C_{sD} or for small dp_D/dt_D , $q_{sf}/q \approx 1$ (i.e., the effect of wellbore storage or sandface rate will be negligible).

As a second example, consider a wellbore (Fig. 1.5) that contains a single-phase fluid (liquid or gas) and that is produced at some surface rate, q . If we let V_{wb} be the volume of wellbore open to formation (barrels) and c_{wb} be the compressibility of the fluid in the wellbore (evaluated at *wellbore conditions*), the mass-balance components are (1) rate of fluid

in = $q_{sf}B$, (2) rate of fluid out = qB , and (3) rate of fluid accumulation in wellbore = $24V_{wb}c_{wb} \times (dp_w/dt)$. The balance becomes

$$(q_{sf} - q)B = 24 V_{wb}c_{wb} \frac{dp_w}{dt}, \dots\dots\dots (1.36)$$

or

$$q_{sf} = q + \frac{24 V_{wb}c_{wb}}{B} \frac{dp_w}{dt} \dots\dots\dots (1.37)$$

In this case, let $C_s = c_{wb} V_{wb}$. Then,

$$q_{sf} = q + \frac{24 V_{wb}c_{wb}}{B} \frac{dp_w}{dt} \dots\dots\dots (1.37)$$

Eq. 1.38 is identical to Eq. 1.28; the wellbore storage constant C_s simply has a different definition. Note, however, that we are forced to make a significant simplifying assumption: when we apply Eq. 1.38 to a gas well, c_{wb} is the compressibility of the gas in the wellbore, and it is a strong function of pressure (as an approximation, $c_{wb} = 1/p_{wb}$). Thus, the wellbore storage “constant” for a gas well may be far from constant.

Since Eqs. 1.38 and 1.28 are identical, Eqs. 1.34 and 1.35 are also valid for a wellbore containing a single-phase fluid.

The radial diffusivity equation, with the wellbore storage equation (Eq. 1.35) as an inner boundary condition, infinite drainage radius, uniform initial formation pressure, and formation damage or stimulation (characterized by skin factor, s), has been solved both analytically and numerically in Refs. 6 and 7. The analytical solution is presented in Fig. 1.6. From this figure, values of p_D (and thus p_w) can be determined for a well in a formation with given values of t_D , C_{sD} , and s .

Two properties of this log-log graph require special mention at this point.

Presence of Unit-Slope Line

At earliest times for a given value of C_{sD} , and for most values of s , a “unit-slope line” (i.e., line with 45° slope) is present on the graph. This line appears and remains as long as *all* production comes from the wellbore and *none* comes from the formation. Eq. 1.35 leads us to expect this line. For $q_{sf}/q=0$, the equation becomes

$$1 - C_{sD} \frac{dp_D}{dt_D} = 0,$$

or

$$dt_D = C_{sD} dp_D \dots\dots\dots (1.39)$$

Integrating from $t_D = 0$ (where $p_D = 0$) to t_D and p_D , the result is

$$C_{sD} p_D = t_D \dots\dots\dots (1.40)$$

Taking logarithms of both sides of the equation,

$$\log C_{sD} + \log p_D = \log t_D \dots\dots\dots (1.41)$$

Thus, as long as $q_{sf}=0$, theory leads us to expect that a graph of $\log p_D$ vs. $\log t_D$ will have a slope of unity; it also leads us to expect that *any point* (p_D, t_D) on this unit-slope line must satisfy the relation

$$\frac{C_{sD} p_D}{t_D} = 1 \dots\dots\dots (1.42)$$

This observation is of major value in well test analysis.

End of Wellbore Storage Distortion

When wellbore storage has ceased (i.e., when $q_{sf} \cong q$), we would expect the solution to the flow equations to be the same as if there had never been any wellbore storage—i.e., the same as for $C_{sD}=0$. In Fig. 1.6, note that the solutions for finite C_{sD} and for $C_{sD}=0$ *do* become identical after sufficient elapsed time. One useful empirical observation is that this time (called the “end of wellbore storage distortion,” t_{wbs}) occurs approximately one and a half log cycles after the disappearance of the unit-slope line. Another useful observation is that the dimensionless time at which wellbore storage distortion ceases is given by

$$t_D = (60 + 3.5 s) C_{sD} \dots\dots\dots (1.43)$$

These observations are also very useful in well test analysis.

Linear Flow

Linear flow occurs in some petroleum reservoirs with long, highly conductive vertical fractures. For that reason, it is of interest to review one of the fundamental equations describing linear flow in a reservoir. Consider a situation with linear flow (in the x direction, for convenience) of a slightly compressible fluid in an infinite, homogeneous reservoir, initially at uniform pressure, p_i . Fluid is produced at constant rate qB over an area A_f (square feet). [If the area A_f represents a vertical fracture with two equal-length wings of length L_f (feet) and height h (feet), $A_f = 4 hL_f$, with flow entering both sides of each wing of the fracture.]

This situation is modeled by the diffusivity equation in the form

$$\frac{\partial^2 p}{\partial x^2} = \frac{\phi \mu c_t}{0.000264 k} \frac{\partial p}{\partial t} \dots\dots\dots (1.44)$$

For the conditions stated, the solution⁸ to this equation at $x=0$ is

$$p_i - p_{wf} = 16.26 \frac{qB}{A_f} \left(\frac{\mu t}{k \phi c_t} \right)^{1/2} \dots\dots\dots (1.45)$$

For linear flow into vertical fractures, $A_f = 4 hL_f$, and

$$p_i - p_{wf} = 4.064 \frac{qB}{hL_f} \left(\frac{\mu t}{k \phi c_t} \right)^{1/2} \dots\dots\dots (1.46)$$

1.4 Radius of Investigation

The radius-of-investigation concept is of both quantitative and qualitative value in well test design and analysis. By radius of investigation, r_i , we mean the distance that a pressure transient has moved into a formation following a rate change in a well. We will show that this distance is related to formation rock and fluid properties and time elapsed since the rate change.

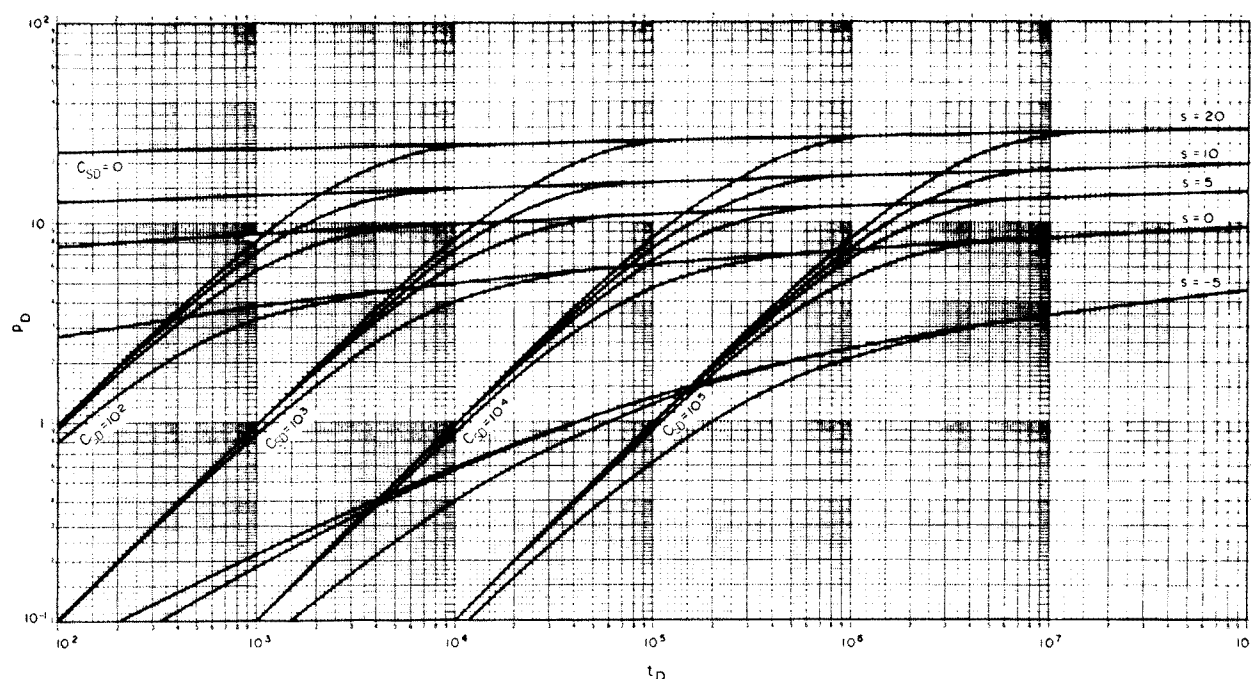


Fig. 1.6—Dimensionless pressure solutions with wellbore storage, skin factor. (After Earlougher.¹⁰)

Before we develop a quantitative means of calculating r_i , however, we will examine pressure distributions at ever-increasing times to develop a feel for the movement of transients into a formation. Fig. 1.7 shows pressure as a function of radius for 0.1, 1.0, 10, and 100 hours after a well begins to produce from a formation originally at 2,000 psi. These pressure distributions were calculated using the *Ei*-function solution to the diffusivity equation for a well and a formation with these characteristics.

$$\begin{aligned} q &= 177 \text{ STB/D,} \\ \mu &= 1 \text{ cp,} \\ B &= 1.2 \text{ RB/STB,} \\ k &= 10 \text{ md,} \\ h &= 150 \text{ ft,} \\ \phi &= 0.15, \\ c_t &= 70.3 \times 10^{-6} \text{ psi}^{-1}, \\ r_e &= 3,000 \text{ ft,} \\ r_w &= 0.1 \text{ ft, and} \\ s &= 0. \end{aligned}$$

Two observations are particularly important:

1. The pressure in the wellbore (at $r=r_w$) decreases steadily with increasing flow time; likewise, pressures at other fixed values of r also decrease with increasing flow time.

2. The pressure disturbance (or pressure transient) caused by producing the well moves further into the reservoir as flow time increases. For the range of flow times shown, there is always a point beyond which the drawdown in pressure from the original value is negligible.

Now consider a well into which we instantaneously inject a volume of liquid. This injection introduces a pressure disturbance into the formation; the

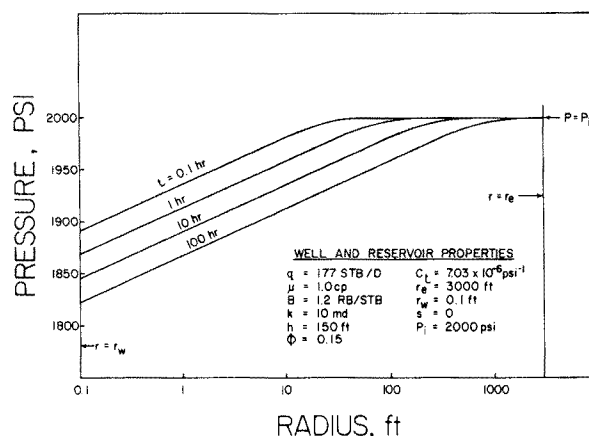


Fig. 1.7—Pressure distribution in formation near producing well.

disturbance at radius r_i will reach its maximum at time t_m after introduction of the fluid volume. We seek the relationship between r_i and t_m . From the solution to the diffusivity equation for an instantaneous line source in an infinite medium,⁹

$$p - p_i = \frac{c_1}{t} e^{-r^2/4\eta t},$$

where c_1 is a constant, related to the strength of the instantaneous source. We find the time, t_m , at which the pressure disturbance is a maximum at r_i by differentiating and setting equal to zero:

$$\begin{aligned} \frac{dp}{dt} &= \frac{-c_1}{t^2} e^{-r^2/4\eta t} + \frac{c_1 r^2}{4\eta t^3} \\ &\cdot e^{-r^2/4\eta t} = 0. \end{aligned}$$

Thus,

$$t_m = r_i^2 / 4\eta = 948 \phi \mu c_t r_i^2 / k.$$

Stated another way, in time t , a pressure disturbance reaches a distance r_i , which we shall call radius of investigation, as given by the equation

$$r_i = \left(\frac{kt}{948 \phi \mu c_t} \right)^{1/2} \quad (1.47)$$

The radius of investigation given by Eq. 1.47 also proves to be the distance a significant pressure disturbance is propagated by production or injection at a constant rate. For example, for the formation with pressure distributions shown in Fig. 1.7, application of Eq. 1.47 yields the following results.

t (hours)	r_i (ft)
0.1	32
1.0	100
10.0	316
100.0	1,000

Comparison of these results with the pressure distributions plotted shows that r_i as calculated from Eq. 1.47 is near the point at which the drawdown in reservoir pressure caused by producing the well becomes negligible.

We also use Eq. 1.47 to calculate the radius of investigation achieved at any time after any rate change in a well. This is significant because the distance a transient has moved into a formation is approximately the distance from the well at which formation properties are being investigated at a particular time in a well test.

The radius of investigation has several uses in pressure transient test analysis and design. A qualitative use is to help explain the *shape* of a pressure buildup or pressure drawdown curve. For example, a buildup curve may have a difficult-to-interpret shape or slope at earliest times when the radius of investigation is in the zone of altered permeability, k_s , nearest the wellbore. Or, more commonly, a pressure buildup curve may change shape at long times when the radius of investigation reaches the general vicinity of a reservoir boundary (such as a sealing fault) or some massive reservoir heterogeneity. (In practice, we find that a heterogeneity or boundary influences pressure response in a well when the calculated radius of investigation is of the order of twice the distance to the heterogeneity.)

The radius-of-investigation concept provides a guide for well test design. For example, we may want to sample reservoir properties at least 500 ft from a tested well. How long a test shall be run? Six hours? Twenty-four hours? We are not forced to guess—or to run a test for an arbitrary length of time that could be either too short or too long. Instead, we can use the radius-of-investigation concept to estimate the time required to test to the desired depth in the formation.

The radius-of-investigation equation also provides a means of estimating the length of time required to achieve “stabilized” flow (i.e., the time required for

a pressure transient to reach the boundaries of a tested reservoir). For example, if a well is centered in a cylindrical drainage area of radius r_e , then, setting $r_i = r_e$, the time required for stabilization, t_s , is found to be

$$t_s = 948 \phi \mu c_t r_e^2 / k. \quad (1.48)$$

It is no coincidence that this is the time at which pseudosteady-state flow begins (i.e., the time at which Eq. 1.12 becomes an accurate approximation of the exact solution to the diffusivity equation). A word of caution: For other drainage-area shapes, time to stabilize can be quite different, as illustrated in Example 1.3.

Useful as the radius-of-investigation concept is, we must caution the reader that it is no panacea. First, we note that it is exactly correct only for a homogeneous, isotropic, cylindrical reservoir—reservoir heterogeneities will decrease the accuracy of Eq. 1.47. Further, Eq. 1.47 is exact only for describing the time the maximum pressure disturbance reaches radius r_i following an instantaneous burst of injection into or production from a well. Exact location of the radius of investigation becomes less well defined for continuous injection or production at constant rate following a change in rate. Limitations kept in mind, though, the radius-of-investigation concept can serve us well.

Example 1.4—Calculation of Radius of Investigation

Problem. We wish to run a flow test on an exploratory well for sufficiently long to ensure that the well will drain a cylinder of more than 1,000-ft radius. Preliminary well and fluid data analysis suggests that $k = 100$ md, $\phi = 0.2$, $c_t = 2 \times 10^{-5}$ psi⁻¹, and $\mu = 0.5$ cp. What length flow test appears advisable? What flow rate do you suggest?

Solution. The minimum length flow test would propagate a pressure transient approximately 2,000 ft from the well (twice the minimum radius of investigation for safety). Time required is

$$\begin{aligned} t &= 948 \phi \mu c_t r_i^2 / k \\ &= \frac{(948)(0.2)(0.5)(2 \times 10^{-5})(2,000)^2}{100} \\ &= 75.8 \text{ hours.} \end{aligned}$$

In principle, *any* flow rate would suffice—time required to achieve a particular radius of investigation is independent of flow rate. In practice, we require a flow rate sufficiently large that pressure change with time can be recorded with sufficient precision to be useful for analysis. What constitutes sufficient precision depends on the particular pressure gauge used in the test.

1.5 Principle of Superposition

At this point, the most useful solution to the flow equation, the *Ei*-function solution, appears to be applicable only for describing the pressure

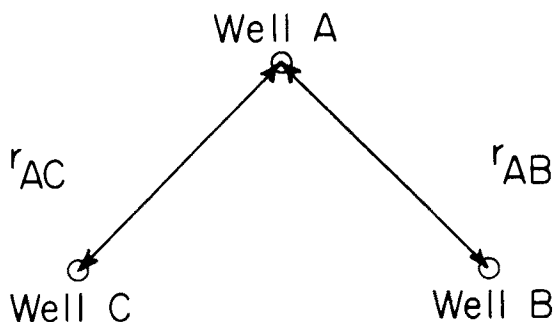


Fig. 1.8—Multiple-well system in infinite reservoir.

distribution in an infinite reservoir, caused by the production of a single well in the reservoir, and, most restrictive of all, production of the well at constant rate beginning at time zero. In this section, we demonstrate how application of the principle of superposition can remove some of these restrictions, and we conclude with examination of an approximation that greatly simplifies modeling a variable-rate well.

For our purposes, we state the principle of superposition in the following way: The total pressure drop at any point in a reservoir is the sum of the pressure drops at that point caused by flow in each of the wells in the reservoir. The simplest illustration of this principle is the case of more than one well in an infinite reservoir. As an example, consider three wells, Wells A, B, and C, that start to produce at the same time from an infinite reservoir (Fig. 1.8). Application of the principle of superposition shows that

$$\begin{aligned}(p_i - p_{wf})_{\text{total at Well A}} &= (p_i - p)_{\text{due to Well A}} \\ &+ (p_i - p)_{\text{due to Well B}} \\ &+ (p_i - p)_{\text{due to Well C}}.\end{aligned}$$

In terms of Ei functions and logarithmic approximations,

$$\begin{aligned}(p_i - p_{wf})_{\text{total at Well A}} &= -70.6 \frac{q_A B \mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_{wA}^2}{kt} \right) - 2s_A \right] \\ &- 70.6 \frac{q_B B \mu}{kh} Ei \left(\frac{-948 \phi \mu c_t r_{AB}^2}{kt} \right) \\ &- 70.6 \frac{q_C B \mu}{kh} Ei \left(\frac{-948 \phi \mu c_t r_{AC}^2}{kt} \right), \dots (1.49)\end{aligned}$$

where q_A refers to the rate at which Well A

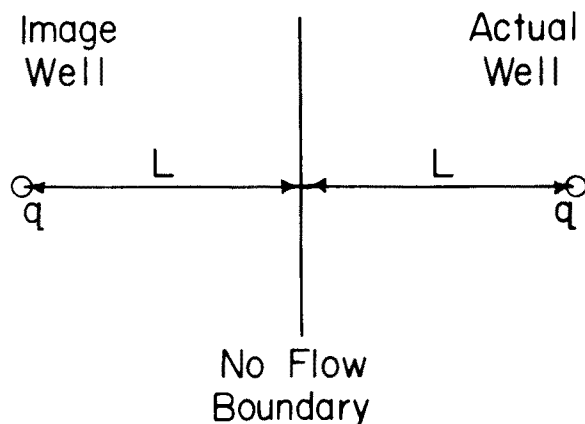


Fig. 1.9—Well near no-flow boundary illustrating use of imaging.

produces; q_B , Well B; and q_C , Well C. Note that this equation includes a skin factor for Well A, but does not include skin factors for Wells B and C. Because most wells have a nonzero skin factor and because we are modeling pressure inside the zone of altered permeability near Well A, we must include its skin factor. However, the presence of nonzero skin factors for Wells B and C affects pressure only inside their zones of altered permeability and has no influence on pressure at Well A if Well A is not within the altered zone of either Well B or Well C.

Using this method, we can treat any number of wells flowing at constant rate in an infinite-acting reservoir. Thus, we can model so-called interference tests, which basically are designed to determine reservoir properties from the observed response in one well (such as Well A) to production from one or more other wells (such as Well B or Well C) in a reservoir. A relatively modern method of conducting interference tests, called pulse testing, is based on these ideas.¹⁰

Our next application of the principle of superposition is to simulate pressure behavior in bounded reservoirs. Consider the well in Fig. 1.9 a distance, L , from a single no-flow boundary (such as a sealing fault). Mathematically, this problem is identical to the problem of a well a distance $2L$ from an "image" well (i.e., a well that has the same production history as the actual well). The reason this two-well system simulates the behavior of a well near a boundary is that a line equidistant between the two wells can be shown to be a no-flow boundary—i.e., along this line the pressure gradient is zero, which means that there can be no flow. Thus, this is a simple two-well-in-an-infinite-reservoir problem:

$$\begin{aligned}p_i - p_{wf} &= -70.6 \frac{q B \mu}{kh} \left(\ln \frac{1,688 \phi \mu c_t r_w^2}{kt} - 2s \right) \\ &- 70.6 \frac{q B \mu}{kh} Ei \left(\frac{-948 \phi \mu c_t (2L)^2}{kt} \right). \\ &\dots\dots\dots (1.50)\end{aligned}$$

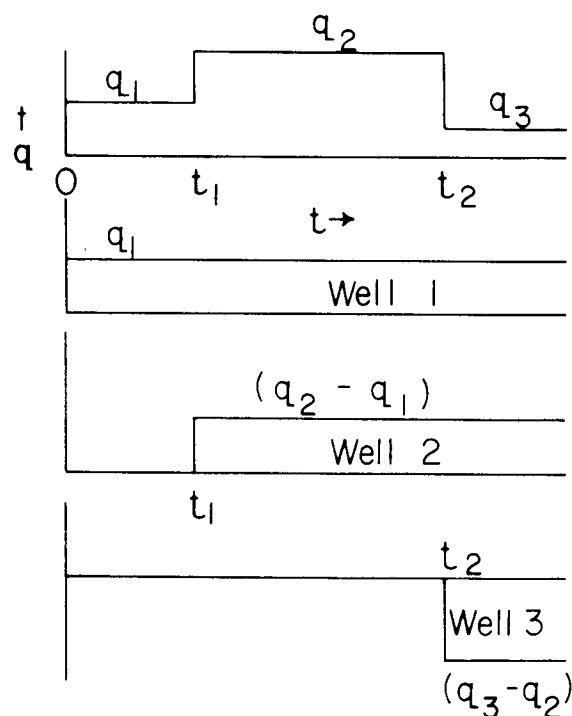


Fig. 1.10—Production schedule for variable-rate well.

Here again, note that whether the image well has a nonzero skin factor is immaterial. Its influence outside its zone of altered permeability is independent of whether this zone exists.

Extensions of the imaging technique also can be used, for example, to model (1) pressure distribution for a well between two boundaries intersecting at 90°; (2) the pressure behavior of a well between two parallel boundaries; and (3) pressure behavior for wells in various locations completely surrounded by no-flow boundaries in rectangular-shaped reservoirs. This last case has been studied quite completely; the study by Matthews *et al.*¹¹ is one of the methods most frequently used to estimate average drainage-area pressure from pressure buildup tests.

Our final and most important application of the superposition principle will be to model variable-rate producing wells. To illustrate this application, consider the case (Fig. 1.10) in which a well produces at rate q_1 from time 0 to time t_1 ; at t_1 , the rate is changed to q_2 ; and at time t_2 , the rate is changed to q_3 . The problem that we wish to solve is this: At some time $t > t_2$, what is the pressure at the sandface of the well? To solve this problem, we will use superposition as before, but, in this case, each well that contributes to the total pressure drawdown will be at the same position in the reservoir—the wells simply will be “turned on” at different times.

The first contribution to a drawdown in reservoir pressure is by a well producing at rate q_1 starting at $t=0$. This well, in general, will be inside a zone of altered permeability; thus, its contribution to drawdown of reservoir pressure is

$$(\Delta p)_1 = (p_i - p_{wf})_1 = -70.6 \frac{\mu q_1 B}{kh} \cdot \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) - 2s \right].$$

Starting at time t_1 , the new total rate is q_2 . We introduce a Well 2, producing at rate $(q_2 - q_1)$ starting at time t_1 , so that the total rate after t_1 is the required q_2 . Note that total elapsed time since this well started producing is $(t - t_1)$; note further that this well is still inside a zone of altered permeability. Thus, the contribution of Well 2 to drawdown of reservoir pressure is

$$(\Delta p)_2 = (p_i - p_{wf})_2 = -70.6 \frac{\mu (q_2 - q_1) B}{kh} \cdot \left\{ \ln \left[\frac{1,688 \phi \mu c_t r_w^2}{k(t - t_1)} \right] - 2s \right\}.$$

Similarly, the contribution of a third well is

$$(\Delta p)_3 = (p_i - p_{wf})_3 = -70.6 \frac{\mu (q_3 - q_2) B}{kh} \cdot \left\{ \ln \left[\frac{1,688 \phi \mu c_t r_w^2}{k(t - t_2)} \right] - 2s \right\}.$$

Thus, the total drawdown for the well with two changes in rate is

$$\begin{aligned} p_i - p_{wf} &= (\Delta p)_1 + (\Delta p)_2 + (\Delta p)_3 \\ &= -70.6 \frac{\mu q_1 B}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) - 2s \right] \\ &\quad - 70.6 \frac{\mu (q_2 - q_1) B}{kh} \cdot \left\{ \ln \left[\frac{1,688 \phi \mu c_t r_w^2}{k(t - t_1)} \right] - 2s \right\} \\ &\quad - 70.6 \frac{\mu (q_3 - q_2) B}{kh} \cdot \left\{ \ln \left[\frac{1,688 \phi \mu c_t r_w^2}{k(t - t_2)} \right] - 2s \right\}. \quad \dots (1.51) \end{aligned}$$

Proceeding in a similar way, we can model an actual well with dozens of rate changes in its history; we also can model the rate history for a well with a continuously changing rate (with a sequence of constant-rate periods at the average rate during the period)—but, in many such cases, this use of superposition yields a lengthy equation, tedious to use in hand calculations. Note, however, that such an equation is valid only if Eq. 1.11 is valid for the total time elapsed since the well began to flow at its initial rate—i.e., for time t , r_i must be $\leq r_e$.

Example 1.5 – Use of Superposition

Problem. A flowing well is completed in a reservoir that has the following properties.

$$\begin{aligned} p_i &= 2,500 \text{ psia,} \\ B &= 1.32 \text{ RB/STB,} \\ \mu &= 0.44 \text{ cp,} \\ k &= 25 \text{ md,} \\ h &= 43 \text{ ft.} \\ c_t &= 18 \times 10^{-6} \text{ psi}^{-1}, \text{ and} \\ \phi &= 0.16. \end{aligned}$$

What will the pressure drop be in a shut-in well 500 ft from the flowing well when the flowing well has been shut in for 1 day following a flow period of 5 days at 300 STB/D?

Solution. We must superimpose the contributions of two wells because of the rate change:

$$p_i - p = -\frac{70.6 \mu B}{kh} \left\{ q_1 Ei\left(\frac{-948 \phi \mu c_t r^2}{kt}\right) + (q_2 - q_1) Ei\left[\frac{-948 \phi \mu c_t r^2}{k(t-t_1)}\right] \right\}.$$

Now,

$$\frac{948 \phi \mu c_t r^2}{k} = [(948)(0.16)(0.44)(1.8 \times 10^{-5}) \cdot (500)^2] / 25 = 12.01.$$

Then,

$$\begin{aligned} p_i - p &= -\frac{(70.6)(0.44)(1.32)}{(25)(43)} \\ &\quad \cdot \left\{ (300) Ei\left[\frac{-12.01}{(6)(24)}\right] + (0 - 300) Ei\left[\frac{-12.01}{(1)(24)}\right] \right\} \\ &= 11.44 [-Ei(-0.0834) + Ei(-0.5)] \\ &= 11.44 (1.989 - 0.560) \\ &= 16.35 \text{ psi.} \end{aligned}$$

1.6 Horner's Approximation

In 1951, Horner¹² reported an approximation that can be used in many cases to avoid the use of superposition in modeling the production history of a variable-rate well. With this approximation, we can replace the sequence of *Ei* functions, reflecting rate changes, with a single *Ei* function that contains a single producing time and a single producing rate. The single rate is the most recent nonzero rate at which the well was produced; we call this rate q_{last} for now. The single producing time is found by dividing cumulative production from the well by the most recent rate; we call this producing time t_p , or

pseudoproducing time:

$$t_p (\text{hours}) = 24 \frac{\text{cumulative production from well, } N_p (\text{STB})}{\text{most recent rate, } q_{\text{last}} (\text{STB/D})}. \quad (1.52)$$

Then, to model pressure behavior at any point in a reservoir, we can use the simple equation

$$p_i - p = -\frac{70.6 \mu q_{\text{last}} B}{kh} Ei\left(\frac{-948 \phi \mu c_t r^2}{kt_p}\right). \quad (1.53)$$

Two questions arise logically at this point: (1) What is the basis for this approximation? (2) Under what conditions is it applicable?

The basis for the approximation is not rigorous, but intuitive, and is founded on two criteria: (1) If we use a single rate in the approximation, the clear choice is the most recent rate; such a rate, maintained for any significant period, determines the pressure distribution nearest the wellbore and approximately out to the radius of investigation achieved with that rate. (2) Given the single rate to use, intuition suggests that we choose an effective production time such that the product of the rate and the production time results in the correct cumulative production. In this way, material balances will be maintained accurately.

But when is the approximation adequate? If we maintain a most-recent rate for too brief a time interval, previous rates will play a more important role in determining the pressure distribution in a tested reservoir. We can offer two helpful guidelines. First, if the most recent rate is maintained sufficiently long for the radius of investigation achieved at this rate to reach the drainage radius of the tested well, then Horner's approximation is always sufficiently accurate. This rule is quite conservative, however. Second, we find that, for a new well that undergoes a series of rather rapid rate changes, it is usually sufficient to establish the last constant rate for at least twice as long as the previous rate. When there is any doubt about whether these guidelines are satisfied, the safe approach is to use superposition to model the production history of the well.

Example 1.6 – Application of Horner's Approximation

Problem. Following completion, a well is produced for a short time and then shut in for a buildup test. The production history was as follows.

Production Time (hours)	Total Production (STB)
25	52
12	0
26	46
72	68

1. Calculate the pseudoproducing time, t_p .
2. Is Horner's approximation adequate for this case? If not, how should the production history for this well be simulated?

Solution.

1.

$$q_{\text{last}} = \frac{68 \text{ STB}}{72 \text{ hours}} \times \frac{24 \text{ hours}}{\text{day}} = 22.7 \text{ STB/D.}$$

Then,

$$t_p = \frac{24 (\text{cumulative production, STB})}{q_{\text{last, STB/D}}}$$

$$= \frac{(24)(166)}{(22.7)} = 176 \text{ hours.}$$

2. In this case,

$$\frac{\Delta t_{\text{last}}}{\Delta t_{\text{next-to-last}}} = \frac{72}{26} = 2.77 > 2.$$

Thus, Horner's approximation is probably adequate for this case. It should not be necessary to use superposition, which is required when Horner's approximation is not adequate.

Exercises

1.1 Compare values of $Ei(-x)$ and $\ln(1.781x)$ for the following values of x : 0.01, 0.02, 0.1, and 1. What do you conclude about the accuracy of the logarithmic approximation? About its range of applicability?

1.2 A well has flowed for 10 days at a rate of 350 STB/D. Rock and fluid properties include $B = 1.13$ RB/STB; $p_i = 3,000$ psia; $\mu = 0.5$ cp; $k = 25$ md (uniform to wellbore—i.e., $s = 0$); $h = 50$ ft; $c_t = 2 \times 10^{-5}$ psi $^{-1}$; $\phi = 0.16$; and $r_w = 0.333$ ft. Calculate pressures at radii of 0.333, 1, 10, 100, 1,000, and 3,160 ft, and plot the results as pressure vs. the logarithm of radius. What minimum drainage radius have you assumed in this calculation?

1.3 For the well described in Exercise 1.2, plot pressure in the wellbore vs. logarithm of time at times of 0.1, 1, and 10 days. What minimum drainage radius have you assumed in this calculation?

1.4 Calculate (a) elapsed time required for the Ei -function solution to be valid for the conditions described in Exercise 1.2; (b) time required for the logarithmic approximation of the Ei function to apply for calculations at the wellbore; and (c) time required for the logarithmic approximation to apply for calculations at a radius of 1,000 ft. Is the logarithmic approximation valid by the time the Ei function itself is a valid solution to the flow equation at the wellbore? At a radius of 1,000 ft?

1.5 Estimate the radius of investigation achieved after 10 days flow time for the reservoir described in Exercise 1.2. Compare this estimate with the extrapolation to 3,000 psi of the straight line passing through radii of 0.333 and 100 ft on the plot of pressure vs. logarithm of radius.

On this plot, how far into the formation has a significant pressure disturbance been propagated? What is the size of the pressure disturbance at the radius of investigation calculated from Eq. 1.47?

1.6 If the drainage radius of the well described in Exercise 1.2 were 3,160 ft, and if the flow rate at the well suddenly was changed from 350 to 500 STB/D,

how long would it take for the well to stabilize at the new rate?

1.7 Suppose the well described in Exercise 1.2 flowed at a rate of 700 STB/D for 10 days. Prepare a plot of pressure vs. logarithm of radius for this situation on the same graph as the plot developed for a rate of 350 STB/D. Is the radius of investigation calculated from Eq. 1.47 affected by change in flow rate? Does the extrapolation of the straight line referred to in Exercise 1.5 change? What is the effect of increased rate?

1.8 Write an equation similar to Eq. 1.49 for the case in which Wells, A, B, and C begin to produce at different times from one another. What do you assume about the location of reservoir boundaries when you write this equation?

1.9 (a) Suppose a well is 250 ft due west of a north-south trending fault. From pressure transient tests, the skin factor, s , of this well has been found to be 5.0. Suppose further that the well has been flowing for 8 days at 350 B/D; reservoir and well properties are those given in Exercise 1.2. Calculate pressure at the flowing well.

(b) Suppose there is a shut-in well 500 ft due north of the producing well. Calculate the pressure at the shut-in well at the end of 8 days.

1.10 A reservoir has the following properties.

$$\begin{aligned} p_i &= 2,500 \text{ psia,} \\ B &= 1.32 \text{ RB/STB,} \\ \mu &= 0.44 \text{ cp,} \\ k &= 25 \text{ md,} \\ h &= 43 \text{ ft,} \\ c_t &= 18 \times 10^{-6} \text{ psi}^{-1}, \text{ and} \\ \phi &= 0.16. \end{aligned}$$

In this reservoir, a well is opened to flow at 250 STB/D for 1 day. The second day its flow is increased to 450 B/D and the third to 500 B/D. What is the pressure in a shut-in well 660 ft away after the third day?

1.11 In Example 1.6, Application of Horner's Approximation, what influence did the 12-hour shut-in time have on the calculation? How would the influence of this shut-in period have changed had the shut-in period been 120 hours? How do you suggest that the calculation procedure be modified to take into account long shut-in periods prior to producing at the final rate?

1.12 Consider a well and formation with the following properties.

$$\begin{aligned} B &= 1.0 \text{ RB/STB,} \\ \mu &= 1.0 \text{ cp,} \\ h &= 10 \text{ ft,} \\ k &= 25 \text{ md,} \\ \phi &= 0.2, \\ p_i &= 3,000 \text{ psig,} \\ c_t &= 10 \times 10^{-6} \text{ psi}^{-1}, \\ s &= 0, \text{ and} \\ r_w &= 1.0 \text{ ft.} \end{aligned}$$

The well produced 100 STB/D for 3.0 days, was shut-in for the next 1.0 day, produced 150 STB/D for the next 2.0 days, produced 50 STB/D for the next 1.0 day, and produced 200 STB/D for next 2.0 days.

(a) Calculate the pseudoproducing time, t_p . Compare this with the actual total producing time.

(b) Calculate and plot the pressure distribution in the reservoir at the end of 9 days using Horner's approximation.

(c) On the same graph, plot the pressure distribution at the end of 9 days using superposition.

(d) What do you conclude about the adequacy of Horner's approximation in this particular case?

1.13 A well and reservoir have the following properties.

$$A = 17.42 \times 10^6 \text{ sq ft (40 acres),}$$

$$\phi = 0.2,$$

$$\mu = 1 \text{ cp,}$$

$$c_t = 10 \times 10^{-6} \text{ psi}^{-1},$$

$$k = 100 \text{ md,}$$

$$h = 10 \text{ ft,}$$

$$s = 3.0,$$

$$r_w = 0.3 \text{ ft, and}$$

$$B = 1.2 \text{ RB/STB.}$$

For each of the drainage areas in Table 1.2, determine (a) the time (hours) up to which the reservoir is infinite-acting; (b) the time (hours) beyond which the pseudosteady-state solution is an adequate approximation; (c) PI of the well; and (d) stabilized production rate with 500-psi drawdown.

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Chapter 2

Pressure Buildup Tests

2.1 Introduction

This chapter discusses the most frequently used pressure transient test, the pressure buildup test. Basically, the test is conducted by producing a well at constant rate for some time, shutting the well in (usually at the surface), allowing the pressure to build up in the wellbore, and recording the pressure (usually downhole) in the wellbore as a function of time. From these data, it is frequently possible to estimate formation permeability and current drainage-area pressure, and to characterize damage or stimulation and reservoir heterogeneities or boundaries.

The analysis method discussed in this chapter is based largely on a plotting procedure suggested by Horner.¹ While this procedure is strictly correct only for infinite-acting reservoirs, these plots also can be interpreted correctly for finite reservoirs,² so only this plotting method is emphasized. Another important analysis technique for buildup tests, using type curves, is discussed in Chap. 4.

The chapter begins with a derivation of the Horner plotting technique and the equation for calculating skin factor. Differences in actual and idealized test behavior then are discussed, followed by comments on dealing with deviations from assumptions made in developing the Horner plotting technique. We then examine qualitatively the behavior of actual tests in common reservoir situations. The chapter next develops in detail a systematic analysis procedure for buildup tests: (1) effects and duration of afterflow (continued production into the wellbore following surface shut-in), (2) determination of permeability, (3) well damage and stimulation, (4) determination of pressure level in the surrounding formation, and (5) reservoir limits tests.

Up to this point, the analysis procedure discussed is applicable only to single-phase flow of a slightly compressible liquid. The chapter concludes with a discussion of how the procedure can be modified to analyze tests in gas wells and in wells with two or three phases flowing simultaneously.

2.2 The Ideal Buildup Test

In this section we derive an equation describing an ideal pressure buildup test. By ideal test we mean a

test in an infinite, homogeneous, isotropic reservoir containing a slightly compressible, single-phase fluid with constant fluid properties. Any wellbore damage or stimulation is considered to be concentrated in a skin of zero thickness at the wellbore; at the instant of shut-in, flow into the wellbore ceases totally. No actual buildup test is modeled exactly by this idealized description, but the analysis methods developed for this case prove useful for more realistic situations if we recognize the effect of deviation from some of these assumptions on actual test behavior.

Assume that (1) a well is producing from an infinite-acting reservoir (one in which no boundary effects are felt during the entire flow and later shut-in period), (2) the formation and fluids have uniform properties, so that the Ei function (and, thus, its logarithmic approximation) applies, and (3) that Horner's pseudoproducing time approximation is applicable. If the well has produced for a time t_p at rate q before shut-in, and if we call time elapsed since shut-in Δt , then, using superposition (Fig. 2.1), we find that following shut-in

$$p_i - p_{ws} = -70.6 \frac{qB\mu}{kh} \left\{ \ln \left[\frac{1,688 \phi \mu c_t r_w^2}{k(t_p + \Delta t)} \right] - 2s \right\} \\ - 70.6 \frac{(-q) B\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2}{k\Delta t} \right) - 2s \right],$$

which becomes

$$p_{ws} = p_i - 70.6 \frac{qB\mu}{kh} \ln \left[(t_p + \Delta t) / \Delta t \right],$$

or

$$p_{ws} = p_i - 162.6 \frac{qB\mu}{kh} \log \left[(t_p + \Delta t) / \Delta t \right]. \quad \dots (2.1)$$

The form of Eq. 2.1 suggests that shut-in BHP, p_{ws} , recorded during a pressure buildup test should plot as a straight-line function of $\log [(t_p + \Delta t) / \Delta t]$. Further, the slope m of this straight line should be

$$m = -162.6 \frac{qB\mu}{kh}.$$

It is convenient to use the absolute value of m in test

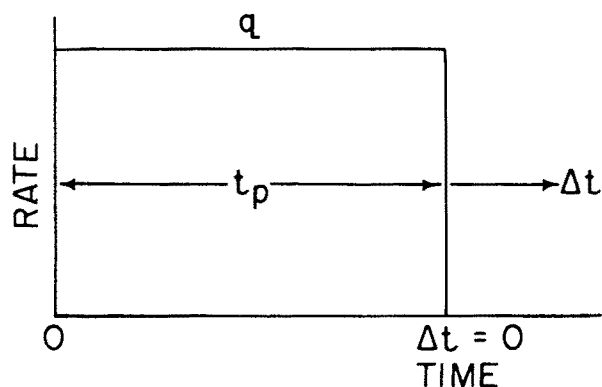


Fig. 2.1 – Rate history for ideal pressure buildup test.

analysis; accordingly, in this text we will use the convention that m is considered a positive number and that

$$m = 162.6 \frac{qB\mu}{kh} \quad \dots\dots\dots (2.2)$$

Thus, formation permeability, k , can be determined from a buildup test by measuring the slope m . In addition, if we extrapolate this straight line to infinite shut-in time [i.e., $(t_p + \Delta t)/\Delta t = 1$] the pressure at this time will be the original formation pressure p_i .

Conventional practice in the industry is to plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$ (Fig. 2.2) on semilogarithmic paper with values of $(t_p + \Delta t)/\Delta t$ decreasing from left to right. The slope m on such a plot is found by simply subtracting the pressures at any two points on the straight line that are one cycle (i.e., a factor of 10) apart on the semilog paper.

We also can determine skin factor s from the data available in the idealized pressure buildup test. At the instant a well is shut in, the flowing BHP, p_{wf} , is

$$p_{wf} = p_i + 70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2}{kt_p} \right) - 2s \right]$$

$$= p_i + 162.6 \frac{qB\mu}{kh}$$

$$\cdot \left[\log \left(\frac{1,688 \phi \mu c_t r_w^2}{kt_p} \right) - 0.869 s \right]$$

$$= p_i + m \left[\log \left(\frac{1,688 \phi \mu c_t r_w^2}{kt_p} \right) - 0.869 s \right].$$

At shut-in time Δt in the buildup test,

$$p_{ws} = p_i - m \log \left[(t_p + \Delta t) / \Delta t \right].$$

Combining these equations and solving for the skin factor s , we have

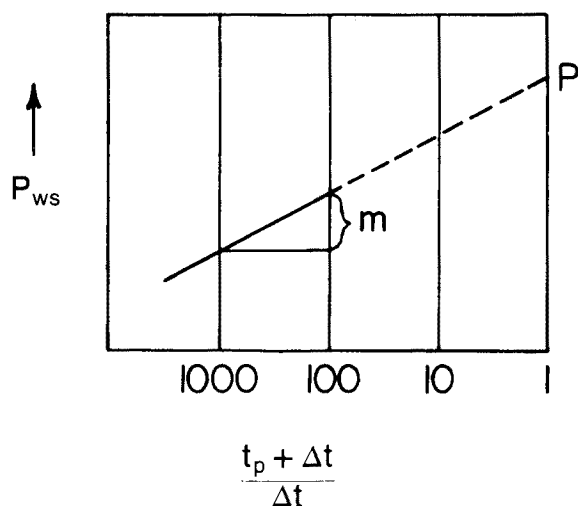


Fig. 2.2 – Plotting technique for pressure buildup test.

$$s = 1.151 \left(\frac{p_{ws} - p_{wf}}{m} \right) + 1.151 \log \left(\frac{1,688 \phi \mu c_t r_w^2}{k \Delta t} \right) + 1.151 \log \left(\frac{t_p + \Delta t}{t_p} \right) \quad \dots\dots\dots (2.3)$$

It is conventional practice in the petroleum industry to choose a fixed shut-in time, Δt , of 1 hour and the corresponding shut-in pressure, $p_{1 \text{ hr}}$, to use in this equation (although *any* shut-in time and the corresponding pressure would work just as well). The pressure, $p_{1 \text{ hr}}$, must be *on* the straight line or its extrapolation. We usually can assume further that $\log (t_p + \Delta t)/t_p$ is negligible. With these simplifications,

$$s = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \quad \dots\dots\dots (2.4)$$

Note again that the slope m is considered to be a positive number in this equation.

In summary, from the ideal buildup test, we can determine formation permeability (from the slope m of the plotted test results), original reservoir pressure, p_i , and skin factor, s , which is a measure of damage or stimulation.

Example 2.1 – Analysis of Ideal Pressure Buildup Test

Problem. A new oil well produced 500 STB/D for 3 days; it then was shut in for a pressure buildup test, during which the data in Table 2.1 were recorded.

For this well, net sand thickness, h , is 22 ft; formation volume factor, B_o , is 1.3 RB/STB; porosity, ϕ , is 0.2; total compressibility, c_t , is 20×10^{-6} ; oil viscosity, μ_o , is 1.0 cp; and wellbore radius, r_w , is 0.3 ft. From these data, estimate formation permeability, k , initial reservoir pressure, p_i , and skin factor, s .

TABLE 2.1 – IDEAL PRESSURE BUILDUP DATA

Time After Shut-In Δt (hours)	p_{ws} (psig)
0	1,150
2	1,794
4	1,823
8	1,850
16	1,876
24	1,890
48	1,910

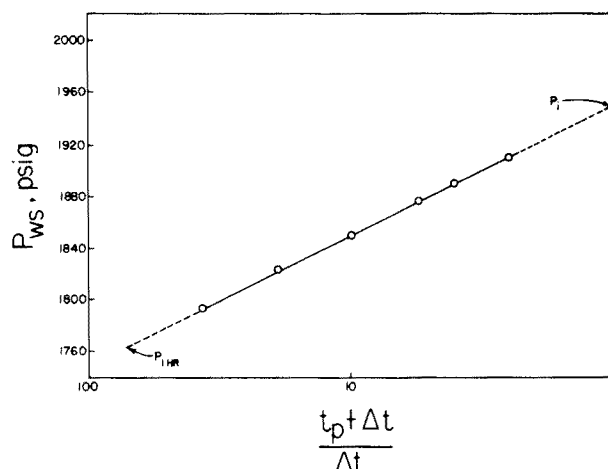


Fig. 2.3 – Ideal pressure buildup test graph.

TABLE 2.2 – BUILDUP TEST DATA FOR HORNER PLOT

Δt (hours)	$\frac{t_p + \Delta t}{\Delta t}$	p_{ws} (psig)
2	37.0	1,794
4	19.0	1,823
8	10.0	1,850
16	5.5	1,876
24	4.0	1,890
48	2.5	1,910

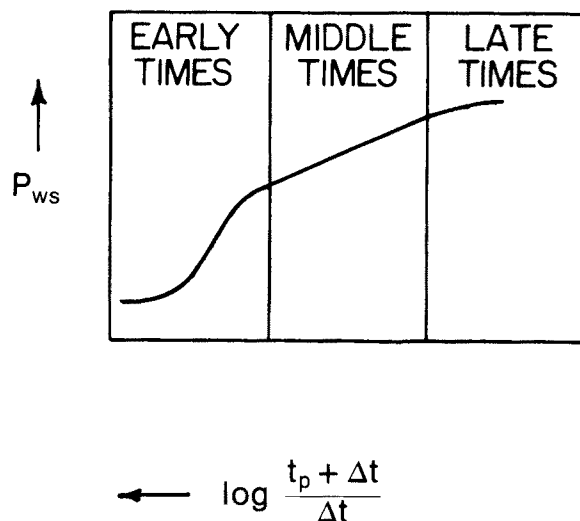


Fig. 2.4 – Actual buildup test graph.

Solution. To estimate permeability, original reservoir pressure, and skin factor, we must plot shut-in BHP, p_{ws} , vs. $\log (t_p + \Delta t) / \Delta t$; measure the slope m and use Eq. 2.2 to calculate formation permeability, k ; extrapolate the curve to $[(t_p + \Delta t) / \Delta t] = 1$ and read original reservoir pressure, p_i ; and use Eq. 2.4 to calculate the skin factor s . (See Fig. 2.3.)

Producing time, t_p , is given to be 3 days, or 72 hours (in this case, Horner's approximation is *exact* because the well was produced at constant rate since time zero). Thus, we develop Table 2.2.

We plot these data, and they fall along a straight line suggested by ideal theory. The slope m of the straight line is $1,950 - 1,850 = 100$ psi (units are actually psi/cycle). The formation permeability k is

$$k = 162.6 \frac{qB\mu}{mh} = \frac{(162.6)(500)(1.3)(1.0)}{(100)(22)} = 48 \text{ md.}$$

From extrapolation of the buildup curve to $[(t_p + \Delta t) / \Delta t] = 1$, $p_i = 1,950$ psig. The skin factor s is found from Eq. 2.4:

$$s = 1.151 \left[\frac{(p_{1hr} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_f r_w^2} \right) + 3.23 \right].$$

The value for p_{ws} is p_{1hr} on the ideal straight line at $(t_p + \Delta t) / \Delta t = (72 + 1) / 1 = 73$; this value is $p_{1hr} = 1,764$ psig. Thus,

$$s = 1.151 \left[\frac{(1,764 - 1,150)}{(100)} \right]$$

$$= -\log \left(\frac{48}{(0.2)(1.0)(2 \times 10^{-5})(0.3)^2} \right) + 3.23 \Big] = 1.43.$$

This means the well has a flow restriction.

2.3 Actual Buildup Tests

Encouraged by the simplicity and ease of application of the ideal buildup test theory, we may test an actual well and obtain a most discouraging result: Instead of a single straight line for all times, we obtain a curve with a complicated shape. To explain what went wrong, the radius-of-investigation concept is useful. Based on this concept, we logically can divide a buildup curve into three regions (Fig. 2.4): (1) an early-time region during which a pressure transient is moving through the formation nearest the wellbore; (2) a middle-time region during which the pressure transient has moved away from the wellbore and into the bulk formation; and (3) a late-time region, in which the radius of investigation has reached the well's drainage boundaries. Let us examine each region in more detail.

Early-Time Region

As we have noted, most wells have altered permeability near the wellbore. Until the pressure transient caused by shutting in the well for the buildup test moves through this region of altered

permeability, there is no reason to expect a straight-line slope that is related to formation permeability. (We should note that the ideal buildup curve—i.e., one with a single straight line over virtually all time—is possible for a damaged well only when the damage is concentrated in a very thin skin at the sandface.)

There is another complication at earliest times in a pressure buildup test. Continued movement of fluid into a wellbore (afterflow, a form of wellbore storage) following the usual surface shut-in compresses the fluids (gas, oil, and water) in the wellbore. Why should this affect the character of a buildup curve at earliest times? Perhaps the clearest answer lies in the observation that the idealized theory leading to the equation $p_{ws} = p_i - m \log [(t_p + \Delta t) / \Delta t]$ explicitly assumed that, at $\Delta t = 0$, flow rate abruptly changed from q to zero. In practice, q declines toward zero but, at the instant of surface shut-in, the downhole rate is, in fact, still q . (See Fig. 2.5.) Thus, one of the assumptions we made in deriving the buildup equation is violated in the actual test, and another question arises. Does afterflow *ever* diminish to such an extent that data obtained in a pressure buildup test can be analyzed as in the ideal test? The answer is yes, fortunately, but the important problem of finding the point at which afterflow ceases distorting buildup data remains. This is the point at which the early-time region usually ends, because afterflow frequently lasts longer than the time required for a transient to move through the altered zone near a well. We will deal with this problem more completely when we discuss a systematic analysis procedure for pressure buildup tests.

Middle-Time Region

When the radius of investigation has moved beyond the influence of the altered zone near the tested well, and when afterflow has ceased distorting the pressure buildup test data, we usually observe the ideal straight line whose slope is related to formation permeability. This straight line ordinarily will continue until the radius of investigation reaches one or more reservoir boundaries, massive heterogeneities, or a fluid/fluid contact.

Systematic analysis of a pressure buildup test using the Horner method of plotting $[p_{ws} \text{ vs. } \log (t_p + \Delta t) / \Delta t]$ requires that we recognize this middle-time line and that, in particular, we do not confuse it with false straight lines in the early- and late-time regions. As we have seen, determination of reservoir permeability and skin factor depends on recognition of the middle-time line; estimation of average drainage-area pressure for a well in a developed field also requires that this line be defined.

Late-Time Region

Given enough time, the radius of investigation eventually will reach the drainage boundaries of a well. In this late-time region pressure behavior is influenced by boundary configuration, interference from nearby wells, significant reservoir heterogeneities, and fluid/fluid contacts.

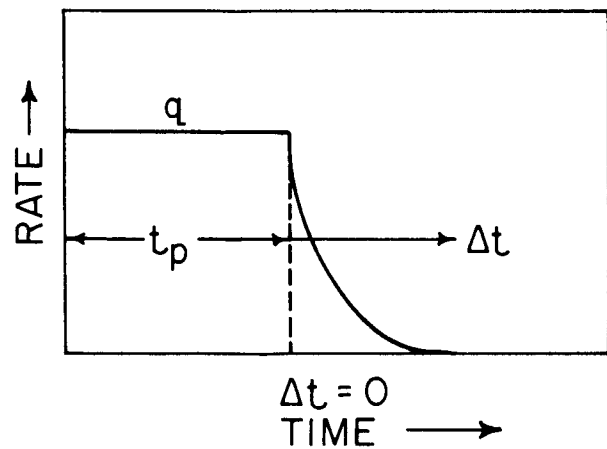


Fig. 2.5 – Rate history for actual pressure buildup test.

2.4 Deviations From Assumptions in Ideal Test Theory

In suggesting that tests logically can be divided into early-, middle-, and late-time regions, we have recognized that several assumptions made in developing the theory of ideal buildup test behavior are not valid for actual tests. In this section, we examine further the implications of three over-idealized assumptions: (1) the infinite-reservoir assumption; (2) the single-phase liquid assumption; and (3) the homogeneous reservoir assumption.

Infinite Reservoir Assumption

In developing the equation suggesting the Horner plot, we assumed that the reservoir was infinite acting during both the production period preceding the buildup test and the buildup test itself. Frequently, the reservoir is at pseudosteady-state before shut-in; if so, neither the Ei -function solution nor its logarithmic approximation should be used to describe the pressure drawdown caused by the producing well:

$$(p_i - p_{wf})_{\text{prod well}} \neq -70.6 \frac{qB\mu}{kh}$$

$$\cdot \left\{ \ln \left[\frac{1,688 \phi \mu c_t r_w^2}{k(t_p + \Delta t)} \right] - 2 \right\}.$$

Instead, if the well is centered in a cylindrical reservoir,

$$(p_i - p_{wf})_{\text{prod well}} = 141.2 \frac{qB\mu}{kh}$$

$$\cdot \left[\frac{0.000527 k(t_p + \Delta t)}{\phi \mu c_t r_e^2} + \ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} \right].$$

Thus, we must conclude that in principle, the Horner plot is *incorrect* when the reservoir is not infinite acting during the flow period preceding the buildup test. Boundaries become important as $r_i \rightarrow r_e$.

The problem is compounded when $r_i \rightarrow r_e$ after shut-in. Then, too, the Horner plot is incorrect in principle.

This difficulty is resolved in different ways by

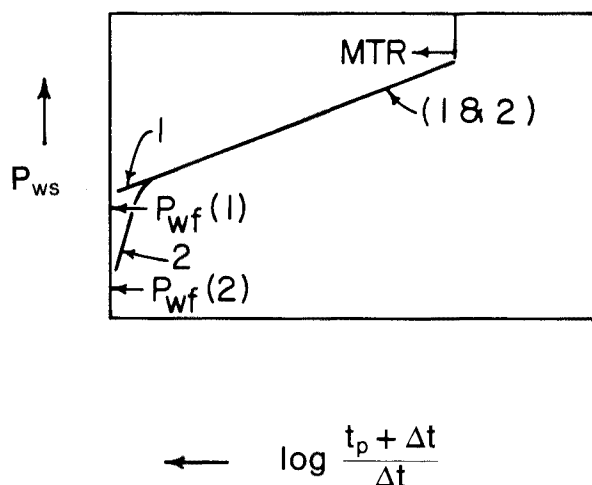


Fig. 2.6 – Buildup test with no afterflow: (1) without wellbore damage and (2) with wellbore damage.

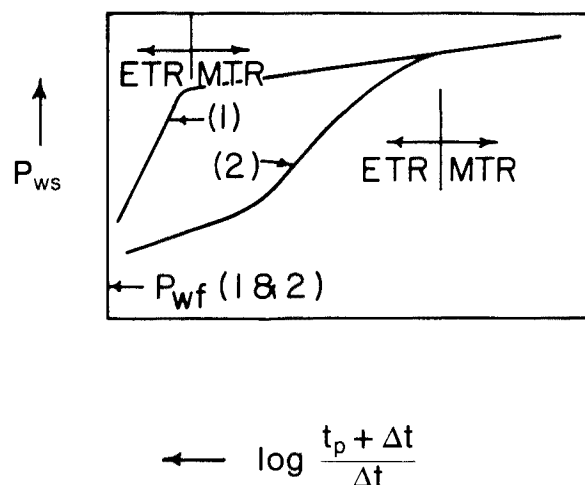


Fig. 2.7 – Buildup test with formation damage: (1) without afterflow and (2) with afterflow.

different analysts. In this text, we will use a method supported by the research of Cobb and Smith.² We will use the Horner plot for all tests (even when the reservoir has reached pseudosteady-state during the production period preceding the test) for the following reasons.

1. This method of plotting is correct *theoretically* for an infinite-acting reservoir (i.e., one for which, at time $t_p + \Delta t$, $r_i < r_e$).

2. The Horner plot offers a convenient means of extrapolating to $\Delta t \rightarrow \infty$ not found in some other plots; the pressure at this shut-in time is a useful checkpoint for the test analyst.

3. For finite-acting reservoirs, formation permeability can be determined accurately from the slope of the Horner plot at even greater shut-in times than from a plotting method developed specifically for reservoirs at pseudosteady state at shut-in.² The curve *will* begin to deviate from the ideal slope before r_i during shut-in reaches reservoir boundaries. However, the middle-time region still can be identified, except for long early-time regions.

Other analysis methods for finite-acting reservoirs are discussed by Miller, Dyes, and Hutchinson³ (MDH) and Slider.⁴ Many analysts use the data-plotting method suggested by MDH because it is simpler than the Horner method. Consider a buildup test with a middle-time region described by Eq. 2.1:

$$\begin{aligned} p_{ws} &= p_i - m \log (t_p + \Delta t) / \Delta t \\ &= p_i - m \log (t_p + \Delta t) + m \log \Delta t. \end{aligned}$$

If $t_p \gg \Delta t$ during the range of shut-in time values examined, then $\log (t_p + \Delta t) \approx \log t_p = \text{constant}$, and

$$p_{ws} = \text{constant} + m \log \Delta t.$$

This leads to the plotting technique suggested by MDH: p_{ws} vs. $\log \Delta t$. It has the same slope m as the Horner plot (in the time range of applicability). Further insight into this plotting technique is provided by Exercise 2.2.

Single-Phase Liquid Assumption

The assumption that a petroleum reservoir contains only a single-phase liquid must be modified. Even reservoirs in which only oil *flows* contain an immobile water saturation; many also contain an immobile gas saturation. Also, in many cases, compressibility of the formation cannot be ignored. These factors are taken into account if we use total compressibility, c_t , in solutions to flow equations:

$$c_t = c_o S_o + c_w S_w + c_g S_g + c_f. \quad (1.4)$$

Even in single-phase flow, when $S_g \neq 0$, evaluation of oil compressibility, c_o , and water compressibility, c_w , are somewhat complicated:

$$c_o = \frac{-1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_s}{dp}. \quad (2.5)$$

$$c_w = \frac{-1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp}. \quad (2.6)$$

These compressibility relationships allow us to model single-phase flow of oil. Later sections in this chapter discuss modifications to the flow equations required for (1) single-phase gas flow and (2) simultaneous flow of two or three phases. Appendix D summarizes correlations useful for calculating compressibilities and other fluid properties needed in analysis of well tests.

Homogeneous Reservoir Assumption

No reservoir is homogeneous, yet solutions to the flow equations are valid only for homogeneous reservoirs. The solutions prove to be adequate for most real reservoirs, particularly early in time while conditions nearest the tested well dominate test behavior. Rate of pressure change is dominated by *average* rock and fluid properties. When massive heterogeneities are encountered (particularly in a localized portion of the reservoir), the simple solutions to flow equations lose accuracy. Examples include changes of depositional environment, with

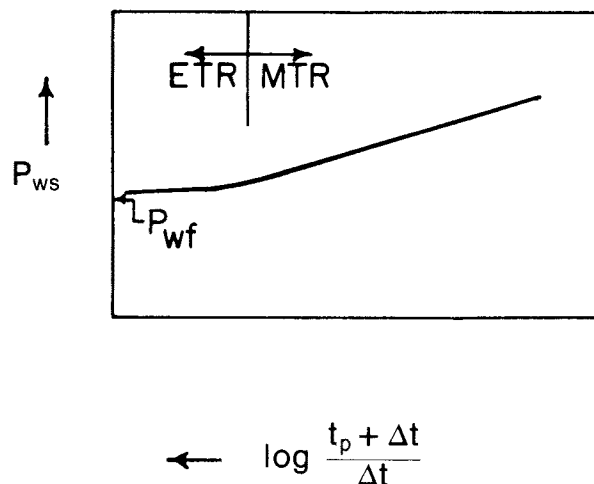


Fig. 2.8 – Buildup test in hydraulically fractured well.

resultant changes in permeability or thickness, and some fluid/fluid contacts. The longer a test is run, the higher the probability that a significant heterogeneity will be encompassed within the radius of investigation and thus influence the test.

Modifications to the simple reservoir models have been developed for some important reservoir heterogeneities. Still, in actual heterogeneous reservoirs, the test analyst must be aware constantly of the possibility of an unknown or improperly modeled heterogeneity. These heterogeneities make analysis of late-time data in transient tests more difficult – reservoirs are rarely uniform cylinders or parallelepipeds, and the analysis technique that is based on these assumptions for treatment of late-time data can be difficult to apply without ambiguity.

What is the test analyst to do with late-time data? Opinions vary. One frequent approach is to use analysis techniques suggested by published simple models – but to try to find other models that also fit the observed data. One then chooses the most probable reservoir description, and recognizes that the analysis may be *absolutely incorrect*.

2.5 Qualitative Behavior of Field Tests

We now have developed the background required to understand the qualitative behavior of commonly occurring pressure buildup curves. There is an important reason for this examination of behavior. It provides a convenient means of introducing some factors that influence these curves and that can obscure interpretation unless they are recognized. In the figures that follow, the early-, middle-, and late-time regions are designated by ETR, MTR, and LTR, respectively. In these curves, the most important region is the MTR. Interpretation of the test using the Horner plot [p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$] is usually impossible unless the MTR can be recognized.

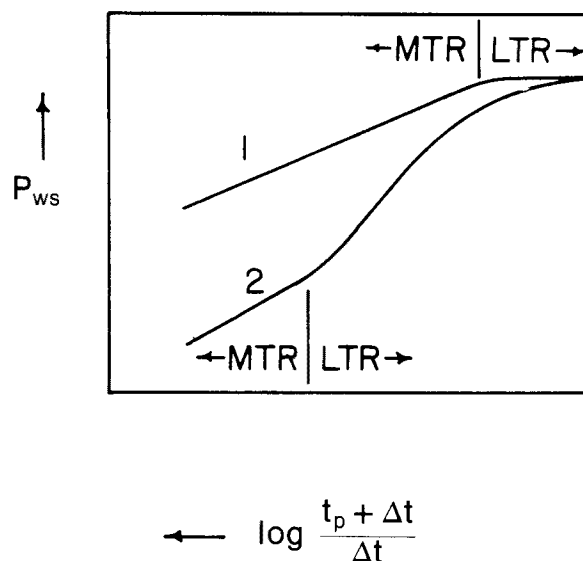


Fig. 2.9 – Boundary effects in pressure buildup test: (1) well centered in drainage area and (2) well off-center in drainage area.

Fig. 2.6 illustrates the ideal buildup test, in which the MTR spans almost the entire range of the plotted data. Such a curve is possible for an undamaged well (Curve 1, with the level of p_{wf} , the flowing pressure at shut-in, is shown for reference) and for a damaged well with an altered zone concentrated at the wellbore. This latter situation, shown in Curve 2, is indicated by a rapid rise in pressure from flowing pressure at shut-in to the pressures along the MTR. Neither case is observed often in practice with a surface shut-in because afterflow usually distorts the early data that would fall on the straight line.

Fig. 2.7 illustrates the pressure buildup test obtained for a damaged well. Curve 1 would be obtained with a shut-in near the perforations (minimizing the duration of afterflow); Curve 2 would be obtained with the more conventional surface shut-in. Note in this figure that the flowing BHP at shut-in, p_{wf} , is the same for either case, but that the afterflow that appears with the surface shut-in (1) completely obscures information reflecting near-well conditions in the ETR and (2) delays the beginning of the MTR. A further complication introduced by afterflow is that several apparent straight lines appear on the buildup curve. The question arises, how do we find the straight line (the MTR line) whose slope is related to formation permeability? We will deal with this question shortly.

Fig. 2.8 shows characteristic behavior of a buildup test for a fractured well without afterflow. For such a well, the pressure builds up slowly at first; the MTR develops only when the pressure transient has moved beyond the region influenced by the fracture. In a buildup test for a fractured well, there is a possibility that boundary effects will appear before the ETR has ended (i.e., that there will be no MTR at all).

Fig. 2.9 illustrates two different types of behavior in the LTR of a buildup test plot. Curve 1 illustrates middle- and late-time behavior for a well reasonably

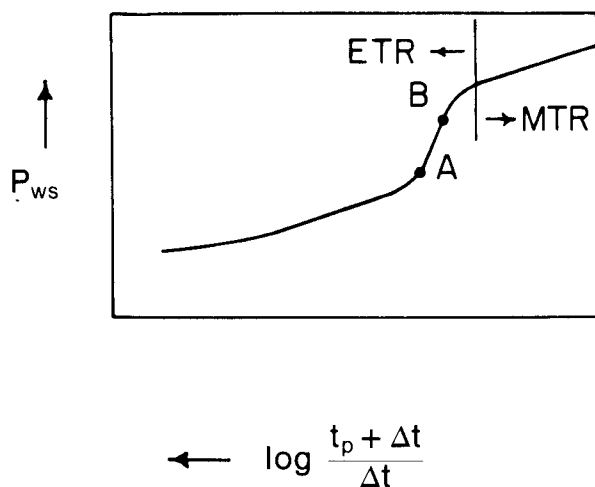


Fig. 2.10 – Characteristic influence of afterflow on Horner graph.

centered in its drainage area; Curve 2 illustrates behavior for a well highly off center in its drainage area. For simplicity, the ETR is not shown in either case.

Many curve shapes other than those discussed above appear in practice, of course. Still, these few examples illustrate the need for a systematic analysis procedure that allows us to determine the end of the ETR (usually, the time at which afterflow ceases distorting the test data) and the beginning of the LTR.

Without this procedure, there is a high probability of choosing the incorrect straight-line segment and using it to estimate permeability and skin factor.

2.6 Effects and Duration of Afterflow

In our discussion thus far, we have noted several problems that afterflow causes the buildup test analyst. Summarizing, these problems include (1) delay in the beginning of the MTR, making its recognition more difficult; (2) total lack of development of the MTR in some cases, with relatively long periods of afterflow and relatively early onset of boundary effects; and (3) development of several false straight lines, any one of which could be mistaken for the MTR line. We note further that recognition of the middle-time line is essential for successful buildup curve analysis based on the Horner plotting method $\{p_{ws} \text{ vs. } \log [(t_p + \Delta t) / \Delta t]\}$, because the line must be identified to estimate reservoir permeability, to calculate skin factor, and to estimate static drainage-area pressure. The need for methods to determine when (if ever) afterflow ceased distorting a buildup test is clear; this section fills that need.

The characteristic influence of afterflow on a pressure buildup test plot is a lazy S-shape at early times, as shown in Fig. 2.10. In some tests, parts of the S-shape may be missing in the time range during which data have been recorded – e.g., data before Time A may be missing, or data for times greater than Time B may be absent. Thus, the shape of the

buildup test alone is not sufficient to indicate the presence or absence of afterflow – it is merely a clue that sometimes indicates presence of afterflow.

A log-log graph of pressure change, $p_{ws} - p_{wf}$, in a buildup test vs. shut-in time, Δt , is an even more diagnostic indicator of the end of afterflow distortion. Fig. 1.6, based on solutions to the flow equations for constant-rate production with wellbore storage distortion, describes pressure buildup tests, as we discuss in some detail in Chap. 4. For use of this figure for buildup tests, dimensionless pressure, p_D , is defined as

$$p_D = \frac{0.00708 kh(p_{ws} - p_{wf})}{qB\mu} \quad (2.7)$$

Dimensionless time, t_D , and dimensionless wellbore storage constant, C_{sD} , are defined essentially as for constant-rate production:

$$t_D = \frac{0.000264 k \Delta t_e}{\phi \mu c_t r_w^2}, \quad (2.8)$$

$$C_{sD} = \frac{0.894 C_s}{\phi \mu c_t r_w^2}, \quad (2.9)$$

where

$$C_s = 25.65 \frac{A_{wb}}{\rho}$$

for a well with a rising liquid/gas interface in the wellbore, and

$$C_s = c_{wb} V_{wb}$$

for a wellbore containing only single-phase fluid (liquid or gas).

We define⁵

$$\Delta t_e = \Delta t / (1 + \Delta t / t_p) \quad (2.10)$$

As noted in Chap. 1, wellbore storage distortion (afterflow in the case of a buildup test) has ceased when the graphed solutions for finite C_{sD} become identical to those for $C_{sD} = 0$. Also, a line with unit slope (45° line) appears at early times for most values of C_{sD} and s . The meaning of this line in a buildup test is that the rate of afterflow is identical to the flow rate just before shut-in.

If the unit-slope line is present, the end of afterflow distortion occurs at approximately one and a half log cycles after the disappearance of the unit-slope line. Regardless of whether the unit-slope line is present, the end of afterflow distortion can be determined by overlaying the log-log plot of the test data onto the Ramey solution (Fig. 1.6) – plotted on graph or tracing paper with a scale identical in size to the Ramey graph – finding *any* preplotted curve that matches the test data, and noting when the preplotted curve for finite value of C_{sD} becomes identical to the curve for $C_{sD} = 0$. This point, on the actual data plot, is the end of afterflow or wellbore storage distortion.

If the unit-slope line is present, we can use a relationship developed in Chap. 1 to establish the value of C_{sD} that characterizes the actual test. There,

TABLE 2.3 – OILWELL PRESSURE BUILDUP TEST DATA

Δt (hours)	$t_p + \Delta t$ Δt	$\Delta t_e = \Delta t / \left(1 + \frac{\Delta t}{t_p}\right)$ (hours)	p_{ws} (psia)	$p_{ws} - p_{wf}$ (psia)
0	—	—	3,534	0
0.15	90,900	0.15	3,680	146
0.2	68,200	0.2	3,723	189
0.3	45,400	0.3	3,800	266
0.4	34,100	0.4	3,866	332
0.5	27,300	0.5	3,920	386
1	13,600	1	4,103	569
2	6,860	2	4,250	716
4	3,410	4	4,320	786
6	2,270	6	4,340	806
7	1,950	7	4,344	810
8	1,710	8	4,350	816
12	1,140	12	4,364	830
16	853	16	4,373	839
20	683	20	4,379	845
24	569	24	4,384	850
30	455	29.9	4,393	859
40	342	39.9	4,398	864
50	274	49.8	4,402	868
60	228	59.7	4,405	871
72	190	71.6	4,407	873

we noted that any point on the unit-slope line must satisfy the relationship

$$\frac{C_s D p_D}{t_D} = 1, \dots \dots \dots (1.42)$$

which, in terms of variables with dimensions, leads to

$$C_s = \frac{qB}{24} \frac{\Delta t}{\Delta p}, \dots \dots \dots (2.11)$$

where Δt and Δp are values read from a point on the unit-slope line. If we can establish C_s in this way (a less acceptable alternative is to use the actual mechanical properties of the well—e.g., $C_s = 25.65 A_{wb}/\rho_{wb}$ for a well with a rising liquid/gas interface), we then can establish $C_s D$ from Eq. 2.9 and thus determine the proper curve on Fig. 1.6 on which to attempt a curve match. (It is difficult to interpolate between values of $C_s D$ on this curve; accordingly, many test analysts prefer to find a match with the preplotted value of $C_s D$ closest in value to the calculated value.) With $C_s D$ established, and permeability, k , and skin factor, s , determined from complete analysis of the test, we can use the empirical relationships below to verify the time, t_{wbs} , marking the end of wellbore storage distortion.

$$t_D \cong 50 C_s D e^{0.14s}, \dots \dots \dots (2.12)$$

or

$$t_{wbs} \cong \frac{170,000 C_s e^{0.14s}}{(kh/\mu)}. \dots \dots \dots (2.13)$$

We will illustrate (1) the application of the basic curve-matching procedure in Example 2.2; (2) the check provided by Eq. 2.13 in Example 2.4; and (3) complete, quantitative curve-matching procedures in Chap. 4.

In this discussion of the qualitative application of curve matching, we also should note that appearance of boundary effects or the effects of heterogeneities frequently can be verified from the curves. Fig. 1.6 is

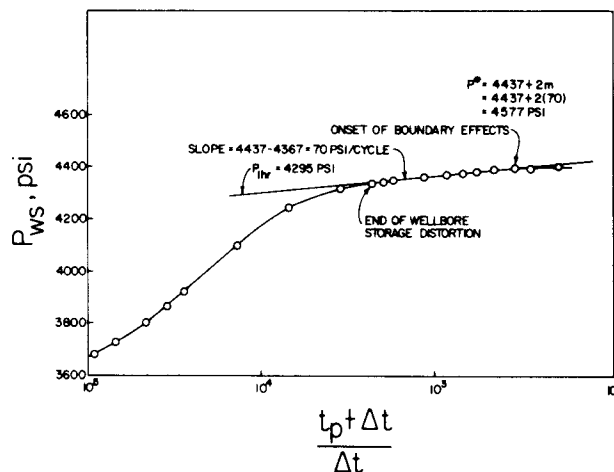


Fig. 2.11 – Semilog graph of example buildup test data.

a solution to the flow equations for an infinite-acting, homogeneous reservoir; when, in an actual reservoir, a pressure transient reaches a boundary or important heterogeneity, the actual test data plot will deviate from Fig. 1.6. This characteristic of curve matching is illustrated in Example 2.2.

Example 2.2 Finding the End of Wellbore Storage Distortion

Problem. The data in Table 2.3 were obtained in a pressure buildup test on an oil well producing above the bubble point.

The well was produced for an effective time of 13,630 hours at the final rate (i.e., $t_p = 13,630$ hours). Other data include the following.

$$\begin{aligned} q_o &= 250 \text{ STB/D,} \\ \mu_o &= 0.8 \text{ cp,} \\ \phi &= 0.039, \\ B &= 1.136 \text{ RB/STB,} \end{aligned}$$

$$\begin{aligned}
c_t &= 17 \times 10^{-6} \text{ psi}^{-1}, \\
r_w &= 0.198 \text{ ft}, \\
r_e &= 1,489 \text{ ft (well is centered in square} \\
&\quad \text{drainage area, } 2,640 \times 2,640 \text{ ft; } r_e \text{ is} \\
&\quad \text{radius of circle with same area),} \\
\rho_o &= 53 \text{ lbm/cu ft}, \\
A_{wb} &= 0.0218 \text{ sq ft}, \\
h &= 69 \text{ ft, and} \\
&\text{rising liquid level in well during shut-in.}
\end{aligned}$$

A semilog graph [p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$] of these data is shown in Fig. 2.11, and a log-log graph ($p_{ws} - p_{wf}$ vs. Δt_e) in Fig. 2.12. From these graphs, answer the following questions.

1. At what shut-in time (Δt) does afterflow cease distorting the pressure buildup test data?

2. At what shut-in time (Δt) do boundary effects appear?

Solution. From the semilog graph (Fig. 2.11), it seems plausible that afterflow distortion disappears at $(t_p + \Delta t)/\Delta t \cong 2,270$ or $\Delta t = 6$ hours because of the end of the characteristic lazy-S-shaped curve. However, other reservoir features can lead to this same shape, so we confirm the result with the log-log graph. After plotting $\Delta p = p_{ws} - p_{wf}$ vs. $\Delta t_e = \Delta t/(1 + \Delta t/t_p)$ on log-log paper, we find that the actual data fit well* curves for $s=5$ for several values of C_{sD} (e.g., $C_{sD} = 10^3$, 10^4 , and 10^5). In each case, the curve fitting the earliest data coincides with the $C_{sD} = 0$ curve for $s=5$ at $\Delta t_e = \Delta t = 4$ to 6 hours. This, then, is the end of wellbore effects: $t_{wbs} \cong 6$ hours. The data begin to deviate from the semilog straight line at $(t_p + \Delta t)/\Delta t \cong 274$ or $\Delta t = 50$ hours. On the log-log graph, data begin falling below the fitting curve at $\Delta t = \Delta t_e \cong 40$ hours, consistent with the semilog graph.

In summary, basing our quantitative judgment on the more sensitive semilog graph, we say that the MTR spans the time range of $\Delta t = 6$ hours to $\Delta t = 50$ hours. This judgment is verified qualitatively by the log-log graph curve matching. Even though the semilog graph is more sensitive (i.e., can be read with greater accuracy), it alone is not sufficient to determine the beginning and end of the MTR: matching Ramey's solution is a critically important part of the analysis.

The log-log curve-matching analysis was performed without knowledge of C_{sD} . Note that C_{sD} can be established in this case, at least approximately: from the curve match, we note that the data are near the unit-slope line on the graph of Ramey's solution; the point $\Delta p = 100$, $\Delta t = 0.1$ is essentially on this line. Thus, from Eq. 2.11,

$$\begin{aligned}
C_s &\cong \frac{qB}{24} \frac{\Delta t}{\Delta p} = \frac{(250)(1.136)}{24} \frac{(0.1)}{(100)} \\
&= 0.0118 \text{ bbl/psi.}
\end{aligned}$$

Alternatively (and, in general, less accurately),

$$\begin{aligned}
C_s &= \frac{25.65 A_{wb}}{\rho} = \frac{(25.65)(0.0218)}{53} \\
&= 0.0106 \text{ bbl/psi.}
\end{aligned}$$

*Data are plotted on 3 × 5 cycle log-log graph paper (11 × 16½ in.) and matched with the Ramey solution (such as provided in the SPE type-curve package) plotted on the same size scale.

Then, from Eq. 2.9,

$$\begin{aligned}
C_{sD} &= \frac{0.894 C_s}{\phi c_t h r_w^2} \\
&= \frac{(0.894)(0.0118)}{(0.039)(1.7 \times 10^{-5})(69)(0.198)^2} = 5,880.
\end{aligned}$$

Thus, matching should be attempted in the range $10^3 < C_{sD} < 10^4$.

2.7 Determination of Permeability

In this section, we examine techniques for the next step in the systematic analysis of a pressure buildup or falloff test: determining bulk-formation permeability. Because bulk-formation permeability is obtained from the slope of the MTR line, correct selection of this region is critical. Average permeability, k_f , also can be estimated from information available in buildup tests.

The first problem is identification of the MTR. This region *cannot* begin until afterflow ceases distorting the data; indeed, cessation of afterflow effects *usually* determines the beginning of the MTR. If the altered zone is unusually deep (as with a hydraulic fracture), passage of the transient through the region of the drainage area influenced by the fracture will determine the beginning of the MTR.

Predicting the time at which the MTR ends is more difficult than predicting when it begins. Basically, the middle-time line ends when the radius of investigation begins to detect drainage boundaries of the tested well; at this time, the pressure buildup curve begins to bend. The problem is that the time at which the middle region ends depends on (1) the distance from the tested well to the reservoir boundaries, (2) the geometry of the area drained by the well, and (3) the duration of the flow period as well as the shut-in period. Cobb and Smith² present charts that allow the analyst to predict the shut-in time Δt at which the MTR should end if drainage-area geometry and producing time are known. If this information is available and if the reservoir is sufficiently homogeneous that, until the LTR begins, it behaves in the ideal way required by Cobb and Smith's theory, their charts can be used to check results.

One useful generalization can be made from their results. If a well was at pseudosteady-state before shut-in, the time Δt at which the LTR begins* is approximately $\Delta t_{lt} \cong 38 \phi \mu c_t A/k$ for a well centered in a square or circular drainage area. In the equation, A (sq ft) is the drainage area of the tested well. If the well was not at pseudosteady-state, Δt_{lt} is larger than calculated by the rule above. In many cases, we simply assume that the straight line spanning the times between the end of afterflow distortion and a later bend of the Horner plot constitutes the MTR. Use of the log-log graph and curve matching, as in Example 2.2, can help confirm this assumption.

The calculated radius of investigation (r_i) at the assumed end of the MTR provides a *qualitative*

*Choice of time at which LTR begins is somewhat arbitrary. The rule stated is based on a 10% deviation in slope of the Horner plot from the true MTR.

estimate only of the radius of the infinite-acting drainage area in the reservoir.

In summary, the procedure for determining bulk-formation permeability is as follows.

1. Determine the probable beginning of the MTR by estimating when afterflow effects disappear.

2. Assume that the probable end of the MTR occurs when the Horner plot becomes nonlinear, verified by a deviation from a curve fitting early- and middle-time data on a log-log graph using the curve-matching technique.

3. If there is an apparent linear MTR, calculate the slope of the middle-time line, and estimate permeability from

$$k = 162.6 \frac{qB\mu}{mh}$$

4. Radius-of-investigation estimates at the beginning and end of the assumed MTR may help establish its plausibility but should be viewed as qualitative only.

5. If there is no clear-cut MTR or if it is so short that its slope cannot be determined with confidence, bulk-formation permeability estimates from quantitative type-curve analysis (i.e., curve-matching) can be used and probably are essential.

It can be helpful in MTR analysis to calculate average permeability, k_J , from data obtained in a buildup test. From Eq. 1.19, which is valid only if pseudosteady-state is reached during the production period,

$$k_J = \frac{141.2 qB\mu \left[\ln(r_e/r_w) - \frac{3}{4} \right]}{h(\bar{p} - p_{wf})}$$

For a well that is neither damaged nor stimulated, k_J should equal bulk-formation permeability, k , determined from the slope of the MTR; for a damaged well, $k_J < k$; and for a stimulated well, $k_J > k$. Average permeability is valuable in checking consistency in buildup test analysis. If k and k_J do not bear the proper relationship to each other, something is wrong—possibly an incorrect analysis due to incorrect choice of MTR.

Example 2.3—Estimating Formation Permeability

Problem. For the test discussed in Example 2.2, determine formation permeability.

Solution. In Example 2.2, we established that the MTR spans the time range of $\Delta t \approx 6$ hours to $\Delta t \approx 50$ hours [$2,270 \geq (t_p + \Delta t)/\Delta t \geq 274$]. From Fig. 2.11, note that the slope m of this straight line is

$$m = 4,437 - 4,367 = 70 \text{ psi/cycle.}$$

Then,

$$k = 162.6 \frac{qB\mu}{mh} = \frac{(162.6)(250)(1.136)(0.8)}{(70)(69)} = 7.65 \text{ md.}$$

It is of interest to determine the portion of the

reservoir sampled during the MTR; that region is given roughly by the radius of investigation achieved by the shut-in transient at the start and end of the MTR. From Eq. 1.47,

$$r_i = \left(\frac{kt}{948 \phi \mu c_t} \right)^{1/2}$$

Thus, at $\Delta t = 6$ hours,

$$r_i = \left[\frac{(7.65)(6)}{(948)(0.039)(0.8)(1.7 \times 10^{-5})} \right]^{1/2} = 302 \text{ ft,}$$

and, at $\Delta t = 50$ hours,

$$r_i = 302 \left(\frac{50}{6} \right)^{1/2} = 872 \text{ ft.}$$

Thus, a significant fraction of the well's drainage area has been sampled; its permeability is 7.65 md.

2.8 Well Damage and Stimulation

This section shows how to use data available in a buildup or falloff test to estimate damage or stimulation quantitatively. The basic technique when analyzing data from a plot of p_{ws} vs. $(t_p + \Delta t)/\Delta t$ is called the skin factor method. It involves calculating the skin factor and translating it into a more easily visualized characterization of a well. We will deal with another method of analyzing damage or stimulation in a later chapter—a method using type-curve analysis.

Before we examine the skin factor method, it is useful to consider briefly the physical reasons for damage or stimulation of a well. Wellbore damage is a descriptive term applied when permeability is reduced near a wellbore. This permeability reduction usually occurs during drilling or completion operations. Causes include plugging of pores with fine material in the drilling fluid and reaction of the formation with filtrate from the drilling fluid (e.g., swelling of clays in the formation resulting from contact with low-salinity filtrate). Completion fluids can cause similar permeability reduction as they enter the formation.

Stimulation usually results from deliberate attempts to improve a well's productivity. Common techniques include acidization and hydraulic fracturing. Acidization is dissolving plugging materials and the formation near the wellbore with acid injection through the perforations. Hydraulic fracturing is creating seams in the formation with high-pressure injection of special fluids, usually accompanied by sand or some other agent that will prop the fracture open when the pressure creating the fracture is removed.

Eq. 2.4 shows how we can calculate skin factor, s , once the MTR is identified and bulk-formation permeability is estimated:

$$s = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \quad (2.4)$$

We recall that $p_{1 \text{ hr}}$ is the value of p_{ws} at shut-in

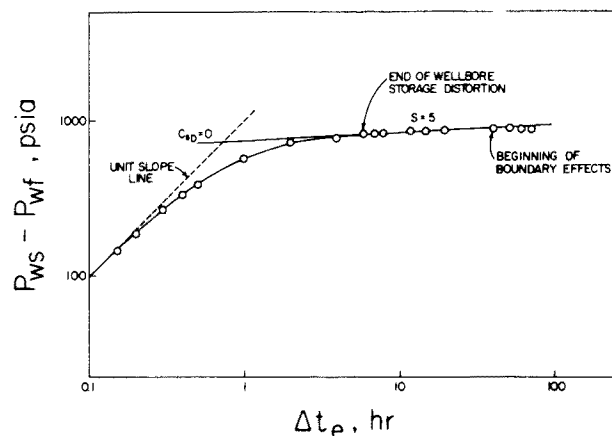


Fig. 2.12 – Log-log graph of example buildup test data

time Δt of 1 hour on the middle-time line, or its extrapolation as shown in Fig. 2.13. It is not possible to calculate the skin factor until the middle-time line has been established because values of k , m , and $p_{1 \text{ hr}}$ are found from this line. If an accurate skin factor is to be calculated from a buildup test, the flowing pressure p_{wf} must be measured before shut-in.

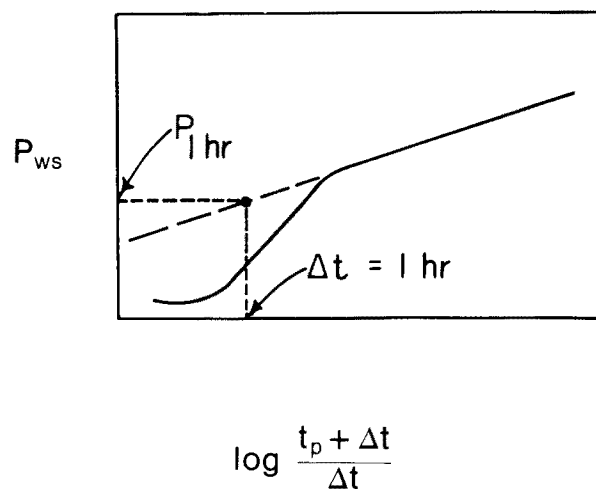
Interpretation of a given numerical value of the skin factor can be summarized as follows.

1. A positive skin factor indicates a flow restriction (e.g., wellbore damage); the larger the skin factor, the more severe the restriction.

2. A negative skin factor indicates stimulation; the larger the absolute value of the skin factor, the more effective the stimulation.

3. Conditions other than wellbore damage can cause an apparent skin factor. The reason is that any deviation from purely radial flow near a well, which results in total well production squeezing through a smaller vertical thickness near the well than away from the well, increases the pressure drop near the well. This is precisely the same effect that wellbore damage has; damage also results in an increased pressure drop near the well. The basic equation used in constructing our theory of pressure buildup and falloff test behavior, Eq. 1.7, is based on the assumption that flow is radial throughout the drainage area of the well up to the sandface; a deviation from this assumption invalidates the equation, but Eqs. 1.11 and 1.16 are usually excellent approximations when the nonradial flow occurs near the wellbore only.

Conditions leading to nonradial flow near the wellbore include (1) when the well does not completely penetrate the productive interval, and (2) when the well is perforated only in a portion of the interval (e.g., the top 10 ft of a 50-ft sand). In these cases, the analyst will calculate a positive skin factor even for an undamaged well. (In addition, the perforations themselves—their size, spacing, and depth—also can affect the skin factor.) We will examine results of this nonradial flow after com-

Fig. 2.13 – Determination of $p_{1 \text{ hr}}$.

pleting our consideration of skin factor solely caused by formation damage or stimulation.

We now turn our attention to methods for translating values of s into less abstract characterizations of the wellbore. We consider three methods: estimation of effective wellbore radius, r_{wa} ; calculation of additional pressure drop near the wellbore; and calculation of flow efficiency.

Estimation of Effective Wellbore Radius

The effective wellbore radius r_{wa} is defined as

$$r_{wa} = r_w e^{-s} \quad (2.14)$$

To understand the significance of this quantity, note that from Eq. 1.11,

$$\begin{aligned} p_i - p_{wf} &= -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) - 2s \right] \\ &= -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) + \ln(e^{-2s}) \right] \\ &= -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_t r_w^2 e^{-2s}}{kt} \right) \right] \\ &= -70.6 \frac{qB\mu}{kh} \ln \left(\frac{1,688 \phi \mu c_t r_{wa}^2}{kt} \right). \end{aligned}$$

This shows that the effect of s on total pressure drawdown is the same as that of a well with no altered zone but with a wellbore radius of r_{wa} .

Calculation of effective wellbore radius is of special value for analyzing wells with vertical fractures. Model studies have shown that for highly conductive vertical fractures with two equal-length wings of length L_f ,

$$L_f \approx 2 r_{wa} \quad (2.15)$$

Thus, calculation of skin factor from a pressure

buildup or falloff test can lead to an estimate of fracture length—useful in a postfracture analysis. However, this analysis technique for a fractured well is frequently oversimplified; more complete methods are discussed later.

Calculation of Additional Pressure Drop Near Wellbore

We defined additional pressure drop $(\Delta p)_s$ across the altered zone in Eq. 1.9 in terms of the skin factor s :

$$(\Delta p)_s = 141.2 \frac{qB\mu}{kh} s.$$

In terms of the slope m of the middle-time line,

$$(\Delta p)_s = 0.869 m(s). \quad (2.16)$$

Calculation of this additional pressure drop across the altered zone can be a meaningful way of translating the abstract skin factor into a concrete characterization of the tested well. For example, a well may be producing 100 STB/D oil with a drawdown of 1,000 psi. Analysis of a buildup test might show that $(\Delta p)_s$ is 900 psi and, thus, that 900 psi of the total drawdown occurs across the altered zone. This implies that if the damage were removed, the well could produce much more fluid with the same drawdown or, alternatively, could produce the same 100 STB/D with a much smaller drawdown.

Calculation of Flow Efficiency

The final method that we will examine for translating s into a physically meaningful characterization of a well is by calculation of the flow efficiency, E . We define flow efficiency as the ratio of actual or observed PI of a tested well to its ideal PI (i.e., the PI it would have if the permeability were unaltered all the way to the sandface of the well). In mathematical terms,

$$E = \frac{J_{\text{actual}}}{J_{\text{ideal}}} = \frac{\bar{p} - p_{wf} - (\Delta p)_s}{\bar{p} - p_{wf}}. \quad (2.17)$$

For rapid analysis of a pressure buildup or falloff test, Eq. 2.17 can be written in approximate form as

$$E \approx \frac{p^* - p_{wf} - (\Delta p)_s}{p^* - p_{wf}}, \quad (2.18)$$

where p^* , the extrapolation of the middle-time line to $(t_p + \Delta t)/\Delta t = 1$, is found more readily than $\bar{p}$, which can require lengthy analysis. Flow efficiency is actually time dependent unless a well reaches pseudosteady state during the producing period (only then is $\bar{p} - p_{wf}$ in Eq. 2.17 constant).

Flow efficiency is unity for a well that is neither damaged nor stimulated. For a damaged well, flow efficiency is less than one; for a stimulated well, flow efficiency is greater than one. A damaged well with a calculated flow efficiency of 0.1 is producing about 10% as much fluid with a given pressure drawdown as it would if the damage were removed; a stimulated well with a calculated flow efficiency of two is

producing about twice as much fluid with a given pressure drawdown as it would had the well not been stimulated.

Use of the skin factor method is illustrated in Example 2.4.

Example 2.4—Damage Analysis

Problem. Consider the buildup test described in Examples 2.2 and 2.3. Make the following calculations with those data.

1. Calculate the skin factor for the tested well.
2. Calculate the effective wellbore radius r_{wa} .
3. Calculate the additional pressure drop near the wellbore caused by the damage that is present.
4. Calculate the flow efficiency.
5. Verify the end of wellbore storage distortion using Eq. 2.13.

Solution.

1. *Skin Factor.* In the skin factor equation, we need $p_{1 \text{ hr}}$ from extrapolation of the middle-time line to a shut-in time of 1 hour. At $\Delta t = 1$ hour, $(t_p + \Delta t)/\Delta t = 13,631$. From an extrapolation (Fig. 2.11) of the middle-time line to this time $p_{1 \text{ hr}} = 4,295$ psi. (Note how different this is from the actual pressure at $\Delta t = 1$ hour: 4,103 psi.) Then, because $k/\phi\mu c_t = 1.442 \times 10^7$,

$$s = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log \left(\frac{k}{\phi\mu c_t r_w^2} \right) + 3.23 \right] \\ = 1.151 \left\{ \frac{(4,295 - 3,534)}{70} - \log \left[\frac{1.442 \times 10^7}{(0.198)^2} \right] + 3.23 \right\} = 6.37.$$

In Example 2.2, we found from curve matching that $s \approx 5$, which is good agreement.

2. *Effective Wellbore Radius.* From Eq. 2.14,

$$r_{wa} = r_w e^{-s} \\ = (0.198) e^{-6.37} \\ = 0.00034 \text{ ft.}$$

The physical interpretation of this result is that the tested well is producing 250 STB/D oil with the same pressure drawdown as would a well with a wellbore radius of 0.00034 ft and permeability unaltered up to the sandface.

3. *Additional Pressure Drop Near the Wellbore.* From Eq. 2.16,

$$(\Delta p)_s = 0.869 m(s) \\ = (0.869)(70)(6.37) \\ = 387 \text{ psi.}$$

Thus, of the total drawdown of approximately $4,420 - 3,534 = 886$ psi, about 387 psi is caused by damage. Much of this additional drawdown could be

avoided if the skin resulted from formation damage (rather than from partial penetration, for example) and if the well were stimulated.

4. *Flow Efficiency.* To calculate flow efficiency, we need p^* , the value of p_{ws} on the middle-time line at $(t_p + \Delta t)/\Delta t = 1$. We cannot extrapolate directly on our plot because there are no values of $(t_p + \Delta t)/\Delta t$ less than 100, but we note that the pressure increases by 70 psi over each cycle; thus, we can add 2 (70) psi to the value of p at $(t_p + \Delta t)/\Delta t = 100$:

$$p^* = 4,437 + 2(70) = 4,577 \text{ psi.}$$

Then, from Eq. 2.18,

$$E = \frac{p^* - p_{wf} - (\Delta p)_s}{p^* - p_{wf}}$$

$$= \frac{4,577 - 3,534 - 387}{4,577 - 3,534}$$

$$= 0.629.$$

This means that the well is producing about 62% as much fluid with the given drawdown as an undamaged well in a completely perforated interval would produce.

5. *End of Wellbore Storage Distortion.* From Eq. 2.13 and Examples 2.2 and 2.3,

$$t_{wbs} = \frac{170,000 C_s e^{0.14s}}{kh/\mu}$$

$$= \frac{(170,000)(0.0118)e^{(0.14)(6.37)}}{(7.65)(69)/0.8}$$

$$= 7.42 \text{ hours.}$$

This agrees closely with the results of Example 2.2.

Effect of Incompletely Perforated Interval

When the completed interval is less than total formation thickness, the pressure drop near the well is increased and the apparent skin factor becomes increasingly positive. In a review of technology in this area, Saidikowski⁶ found that total skin factor, s , determined from a pressure transient test is related to true skin factor, s_d , caused by formation damage and apparent skin factor, s_p , caused by an incompletely perforated interval. The relationship between these skin factors is

$$s = \frac{h_t}{h_p} s_d + s_p, \dots \dots \dots (2.19)$$

where h_t is total interval height (ft) and h_p is the perforated interval (ft).

Saidikowski also verified that s_p can be estimated from the equation

$$s_p = \left(\frac{h_t}{h_p} - 1 \right) \left[\ln \left(\frac{h_t}{r_w} \sqrt{\frac{k_H}{k_V}} \right) - 2 \right], \dots \dots (2.20)$$

where k_H is horizontal permeability (md) and k_V is vertical permeability (md). Use of these equations is best illustrated with an example.

Example 2.5 – Incompletely Perforated Interval

Problem. A well with disappointing productivity is perforated in 10 ft of a total formation thickness of 50 ft. Vertical and horizontal permeabilities are believed to be equal. A pressure buildup test was run on the well; results and basic properties are summarized as follows.

$$p_{wf} = 1,190 \text{ psi,}$$

$$p_{1 \text{ hr}} = 1,940 \text{ psi,}$$

$$\phi = 0.20,$$

$$m = 50 \text{ psi/cycle,}$$

$$\mu = 0.5 \text{ cp,}$$

$$r_w = 0.25 \text{ ft,}$$

$$c_t = 15 \times 10^{-6} \text{ psi}^{-1}, \text{ and}$$

$$k = 3.35 \text{ md.}$$

Calculate s , s_d , and s_p ; on the basis of these results, determine whether the productivity problem results from formation damage or from other causes.

Solution. From Eq. 2.4,

$$s = 1.151 \left[\frac{p_{1 \text{ hr}} - p_{wf}}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]$$

$$= 1.151 \left[\frac{(1,940 - 1,190)}{50} \right.$$

$$\left. - \log \left(\frac{3.35}{(0.2)(0.5)(15 \times 10^{-6})(0.25)^2} \right) + 3.23 \right]$$

$$= 12.3.$$

The contribution of an incompletely perforated interval to the total skin factor is, from Eq. 2.20,

$$s_p = \left(\frac{h_t}{h_p} - 1 \right) \left[\ln \left(\frac{h_t}{r_w} \sqrt{\frac{k_H}{k_V}} \right) - 2 \right]$$

$$= \left(\frac{50}{10} - 1 \right) \left[\ln \left(\frac{50}{0.25} \sqrt{\frac{3.35}{3.35}} \right) - 2 \right]$$

$$= 13.2.$$

Rearranging Eq. 2.19, skin factor, s_d , resulting from formation damage is

$$s_d = \frac{h_p}{h_t} (s - s_p)$$

$$= \frac{10}{50} (12.3 - 13.2)$$

$$= -0.18.$$

As a practical matter, the well is neither damaged nor stimulated. The observed productivity problem is caused entirely by the effects of an incompletely perforated interval.

Analysis of Hydraulically Fractured Wells

Type curves provide a general method of analyzing hydraulically fractured wells—particularly because

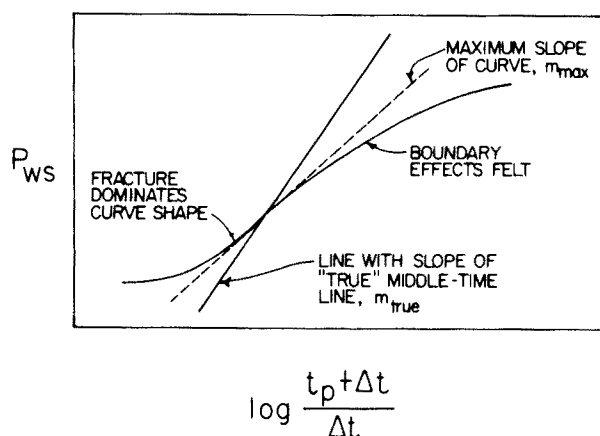


Fig. 2.14 – Buildup curve for hydraulically fractured well, bounded reservoir.

finite conductivity can be considered. Some conventional methods are also of value for infinite-conductivity fractures. This section summarizes some of the useful conventional methods.

When fractures are highly conductive (i.e., when there is little pressure drop in the fracture itself) and when there is uniform flux of fluid into the fracture, linear flow theory describes well behavior at earliest times in a buildup test. (Uniform flux means identical flow rates of formation fluid into the fracture per unit cross-sectional area at all points along the fracture.) From Eq. 1.46, for constant-rate production,

$$p_i - p_{wf} = 4.064 \frac{qB}{hL_f} \left(\frac{\mu t}{k\phi c_t} \right)^{1/2}.$$

For a buildup test, for $t_p \gg \Delta t$,

$$p_{ws} - p_{wf} = 4.064 \frac{qB}{hL_f} \left(\frac{\mu \Delta t}{k\phi c_t} \right)^{1/2}.$$

Thus, the slope m_L of a p_{ws} vs. $\sqrt{\Delta t}$ plot is

$$m_L = 4.064 \frac{qB}{hL_f} \left(\frac{\mu}{k\phi c_t} \right)^{1/2}. \quad (2.21)$$

From measurements of this slope, fracture length, L_f , can be estimated. This procedure requires that an independent estimate of permeability be available – from a prefracture pressure buildup test on the well, for example.

When linear flow cannot be recognized (i.e., when there is no early straight-line relationship between p_{ws} and $\sqrt{\Delta t}$), we can use the observation that $L_f = 2r_{wa}$ for infinitely conductive fractures to estimate fracture length. Rather than calculate s directly, we can note that

$$s = 1.151 \left[\frac{(p_{1hr} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right];$$

and, because

$$r_{wa} = \frac{L_f}{2} = r_w e^{-s},$$

then

$$s = -\ln \left(\frac{L_f}{2r_w} \right) = -2.303 \log \left(\frac{L_f}{2r_w} \right).$$

TABLE 2.4 – BUILDUP TEST SLOPES FOR HYDRAULICALLY FRACTURED WELLS

L_f/r_e	m_{max}/m_{true}
0.1	0.87
0.2	0.70
0.4	0.46
0.6	0.32
1.0	0.28

Thus,

$$\begin{aligned} -2 \log \left(\frac{L_f}{2r_w} \right) &= -2 \log \frac{L_f}{2} - 2 \log \left(\frac{1}{r_w} \right) \\ &= \frac{(p_{1hr} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_t} \right) \\ &\quad - 2 \log \left(\frac{1}{r_w} \right) + 3.23. \end{aligned}$$

This simplifies to

$$\begin{aligned} \log(L_f) &= \frac{1}{2} \left[\left(\frac{p_{wf} - p_{1hr}}{m} \right) \right. \\ &\quad \left. + \log \frac{k}{\phi \mu c_t} - 2.63 \right]. \quad (2.22) \end{aligned}$$

Using Eq. 2.22, fracture length, L_f , can be estimated if the MTR can be recognized, which allows m , p_{1hr} , and k to be determined.

In buildup tests from some hydraulically fractured wells, the true middle-time line does not appear, as illustrated in Fig. 2.14. (Afterflow can cause the same curve shape.) This situation arises because, at earliest times, the depth of investigation is in a region dominated by the fracture; at later times, the depth of investigation reaches a point dominated by boundary effects. (See Fig. 2.14.) When the length L_f of a vertical fracture is greater than one-tenth of the drainage radius r_e of a well centered in its drainage area, it has been found⁷ that boundary effects begin before the influence of the fracture disappears. For a given drainage radius, the greater the fracture length, the greater the discrepancy between the maximum slope achieved on a buildup test and the slope of the true middle-time line. Table 2.4 summarizes the ratio of the maximum slope attained in a buildup test to the slope of the true middle-time line (from the work of Russell and Truitt⁷) for infinitely conductive fractures.

The implication is that if the test analyst simply does the best he can, and finds the maximum slope on a buildup test from a hydraulically fractured well and assumes that this maximum slope is an adequate approximation to the slope of the true middle-time

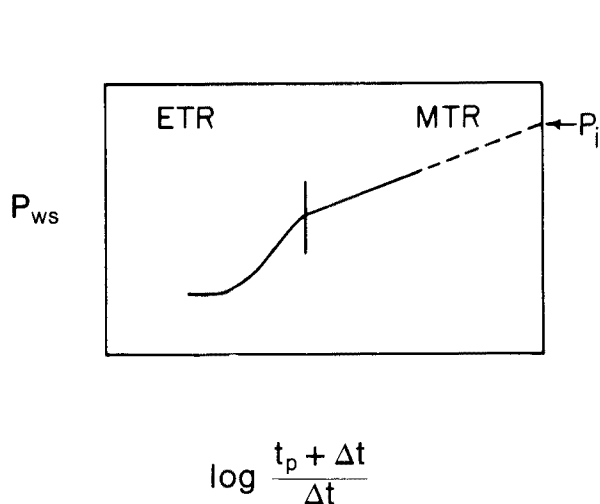


Fig. 2.15 – Buildup test graph for infinite-acting reservoir.

line, then the permeability, skin factor, and fracture estimates will be in error, with the error growing as fracture length increases.

Correlation of reservoir model results by Russell and Truitt⁷ showed that an equation similar to Eq. 2.22 can be used to estimate true fracture length even when $L_f > 0.1 r_e$.

We again emphasize that all methods in this section assume highly conductive, vertical fractures with two equal-length wings. When fracture conductivity is not high, fracture length estimated by these methods will be too small.

2.9 Pressure Level in Surrounding Formation

A pressure buildup test can be used to determine average drainage-area pressure in the formation surrounding a tested well. We have seen that ideal pressure buildup theory suggests a method for estimating *original* reservoir pressure in an infinite-acting reservoir – that is, extrapolating the buildup test to infinite shut-in time $[(t_p + \Delta t)/\Delta t = 1]$ and reading the pressure there. For wells with partial pressure depletion, extrapolation of a buildup test to infinite shut-in time provides an estimate of p^* , which is related to, but *is not equal to*, current average drainage-area pressure. In this section, we will examine methods for estimating original and current average drainage-area pressures.

Original Reservoir Pressure

For a well with an uncomplicated drainage area, original reservoir pressure, p_i , is found as suggested by ideal theory. We simply identify the middle-time line, extrapolate it to infinite shut-in time, and read the pressure, which is original reservoir pressure (Fig. 2.15). This technique is possible only for a well in a new reservoir (i.e., one in which there has been negligible pressure depletion). Strictly speaking, this is true only for tests in which the radius of investigation does not encounter any reservoir boundary during production.

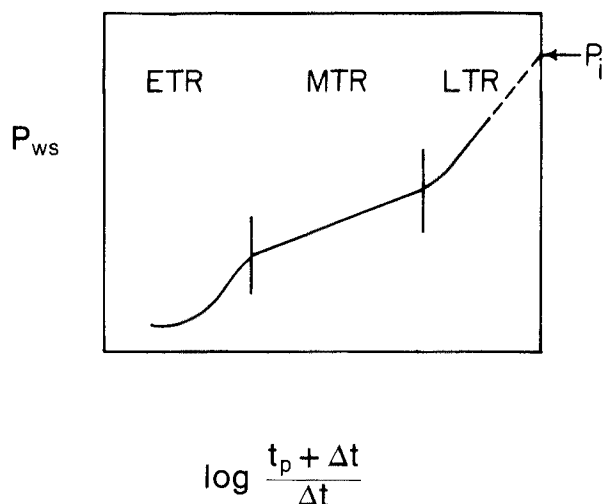


Fig. 2.16 – Buildup test graph for well near reservoir limit(s).

For a reservoir with one or more boundaries relatively near a tested well (and encountered by the radius of investigation during the production period), the late-time line must be extrapolated (Fig. 2.16). (This can be quite complex for multiple boundaries near a well.) Note that our discussion is still restricted to reservoirs in which there has been negligible pressure depletion. Thus, even in the case under consideration, the well must be relatively far from boundaries in at least one direction.

Static Drainage-Area Pressure

For a well in a reservoir in which there has been some pressure depletion, we do not obtain an estimate of original reservoir pressure from extrapolation of a buildup curve. Our usual objective is to estimate the average pressure in the drainage area of the well; we will call this pressure static drainage-area pressure. We will examine two useful methods for making these estimates: (1) the Matthews-Brons-Hazebroek (MBH)⁸ p^* method and (2) the modified Muskat method.⁹

The p^* method was developed by Matthews *et al.* by computing buildup curves for wells at various positions in drainage areas of various shapes and then, from the plotted buildup curves, comparing the pressure (p^*) on an extrapolated middle-time line with the static drainage-area pressure ($\bar{p}$), which is the value at which the pressure will stabilize given sufficient shut-in time. The buildup curves were computed using imaging techniques and the principle of superposition. The results of the investigation are summarized in a series of plots of $kh(p^* - \bar{p})/70.6 q\mu B$ vs. $0.000264 kt_p/\phi\mu c_i A$. [Note that $kh(p^* - \bar{p})/70.6 q\mu B$ can be written more compactly as $2.303(p^* - \bar{p})/m$. Also, the group $0.000264 kt_p/\phi\mu c_i A$ is a dimensionless time and is symbolized by t_{DA} . The group $kh(p^* - \bar{p})/70.6 q\mu B$ is a dimensionless pressure and is given the symbol p_{DMBH}]. The only new symbol in these expressions is the drainage area, A , of the tested well expressed in square feet. Figs. 2.17A through 2.17G (reproduced from the Mat-

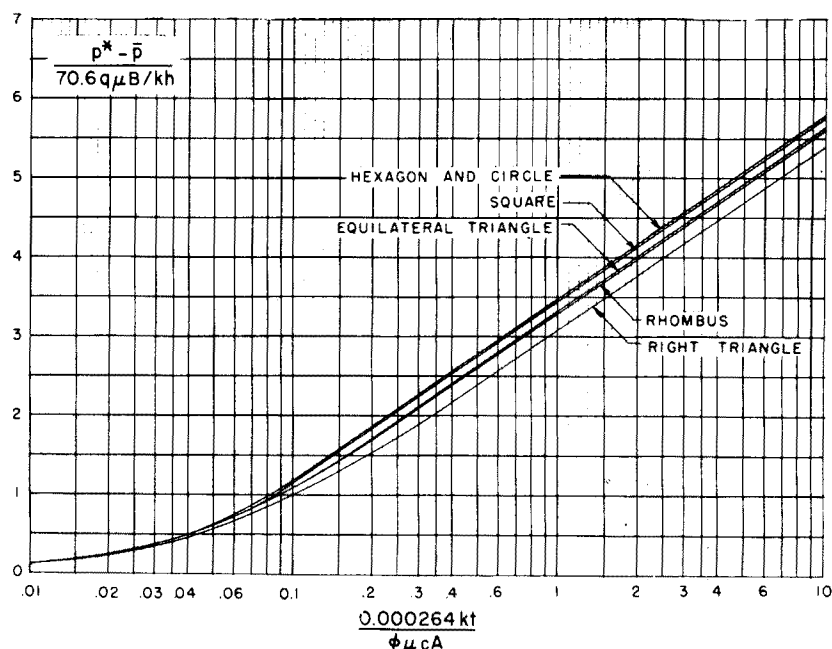


Fig. 2.17A – MBH pressure function for well in center of equilateral figures.

thews and Russell monograph¹⁰) summarize the results.

The steps in application of this method are as follows.

- (1) Extrapolate the middle-time line to $(t_p + \Delta t) / \Delta t = 1$, and read the extrapolated pressure, p^* .
- (2) Estimate the drainage area shape, which is frequently reasonably symmetrical, particularly for a well in a field developed on regular spacing.*

- (3) Choose the proper curve from Figs. 2.17A through 2.17 G for the drainage-area shape of the tested well.

- (4) Estimate $0.000264 kt_p / \phi \mu c_i A$, and find $2.303(p^* - \bar{p}) / m = p_{DMBH}$. (It is essential that t_p used in this calculation be the same used in the Horner plot.)

- (5) Calculate $\bar{p} = p^* - m p_{DMBH} / 2.303$.

The advantages of the p^* method for estimating static drainage-area pressure are that it does not require data beyond the MTR and that it is applicable to a wide variety of drainage-area shapes. Its disadvantages are that it requires knowledge of drainage-area size and shape and estimates of reservoir and fluid properties such as c_i and ϕ , which are not always known with great accuracy. We will compare these advantages and disadvantages with those of the modified Muskat method. First, we present an example illustrating application of the p^* method.

Example 2.6 – p^* Method To Calculate Average Pressure in Drainage Area

Problem. Consider the buildup test described in Examples 2.2 and 2.3. Estimate the average pressure

*Ref. 10 explains a method for estimating drainage-area size and shape for use when the symmetrical-drainage-area assumption is inadequate. The method can be tedious because it can involve verifying by trial-and-error planimetry that drainage volumes are proportional to flow rates in individual wells.

in the well's drainage area using the p^* method.

Solution. In our use of the p^* method, we assume that the well is centered in a square drainage area; thus, Fig. 2.17A applies. To use this figure, we first calculate the group

$$\frac{0.000264 kt_p}{\phi \mu c_i A} = \frac{[(0.000264)(1.442 \times 10^7) \cdot (1.363 \times 10^4)]}{(2 \times 1,320)^2} = 7.44.$$

From Fig. 2.17A,

$$\frac{2.303(p^* - \bar{p})}{m} = 5.45.$$

Therefore,

$$\bar{p} = p^* - \frac{5.45 m}{2.303} = 4,577 - \frac{(5.45)(70)}{2.303} = 4,411 \text{ psi.}$$

Accuracy of the p^* method may be improved by using t_{pss} , producing time required to achieve pseudosteady state, in the Horner plot and in the abscissa in the MBH figures.¹¹ In principle, results should be identical for any $t_p > t_{pss}$; in practice, use of the smallest possible t_p may reduce the error arising from a lengthy extrapolation. Time to reach pseudosteady state can be calculated after formation permeability, k , has been established, given drainage-area size and shape. Values are given in Table 1.2 for common drainage areas. As an example, for conditions in Example 2.6, in the column "Exact for t_{DA} ," we read 0.1 for a well centered in a square drainage area. This means that

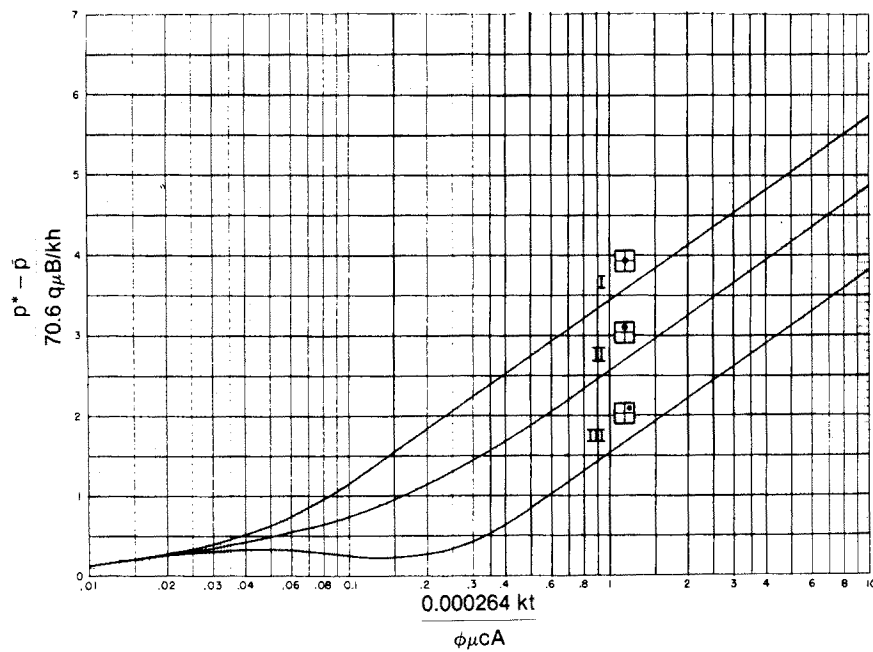


Fig. 2.17B – MBH pressure function for different well locations in a square boundary.

$$t_{DA} = \frac{0.000264 kt_{pss}}{\phi \mu c_A A} = 0.1,$$

or, for this case (with $A = 160$ acres $= 6.97 \times 10^6$ sq ft),

$$t_{pss} = 183 \text{ hours.}$$

The reader can verify that use of t_{pss} in the Horner plot and in the p^* method leads to the same results as in Examples 2.3 and 2.6.

Shape Factors for Reservoirs

Brons and Miller¹² observed that reservoirs with

general drainage-area shapes follow equations of the form

$$\bar{p} - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\frac{1}{2} \ln \left(\frac{10.06 A}{C_A r_w^2} \right) - \frac{3}{4} + s \right], \quad \dots (1.20)$$

after pseudosteady state flow is achieved. Values for the shape factor C_A can be derived from the $p_{D,MBH}$ vs. t_{DA} charts for the various reservoir geometries in Figs. 2.17A through 2.17G. Note that the definition of p^* implies that

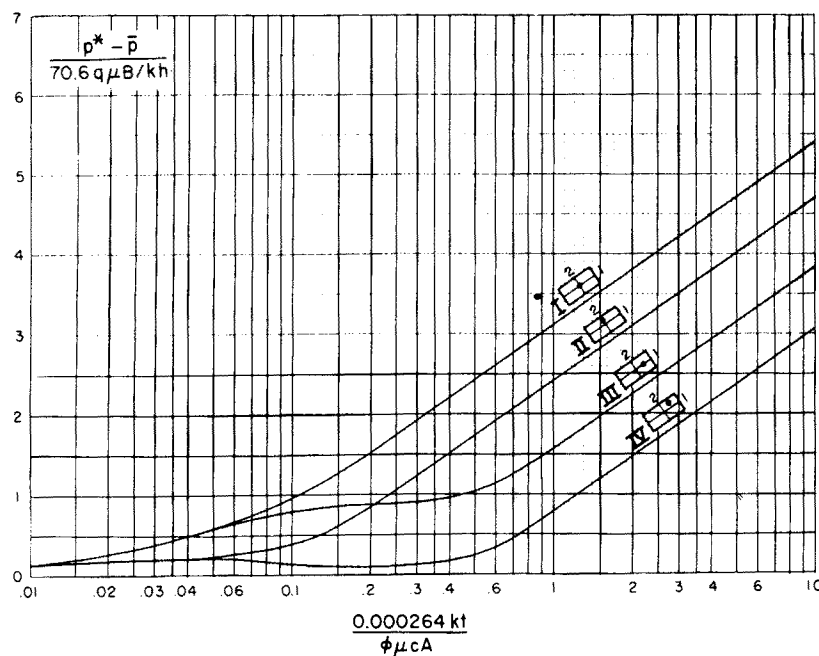


Fig. 2.17C – MBH pressure function for different well locations in a 2:1 rectangular boundary

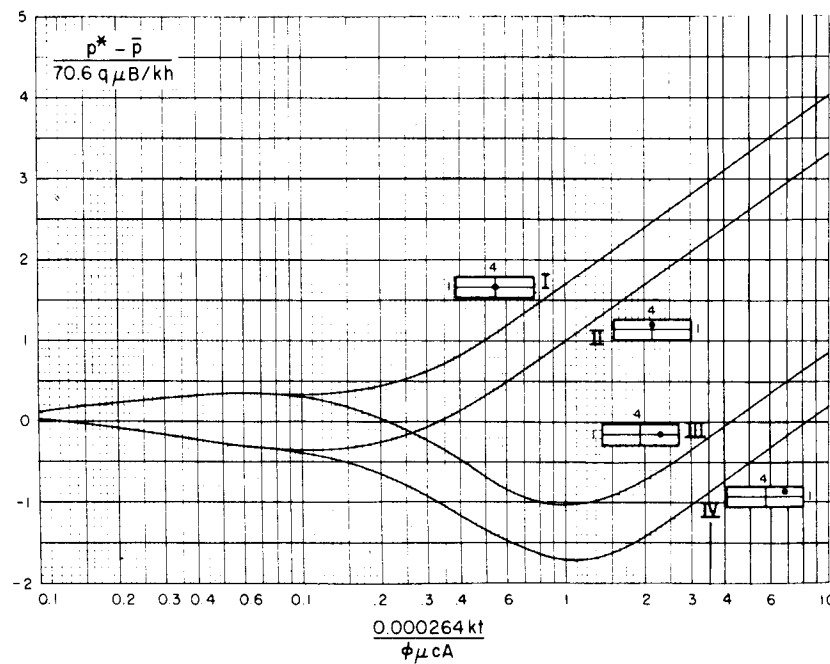


Fig. 2.17D – MBH pressure function for different well locations in a 4:1 rectangular boundary.

$$p^* - p_{ws} = 70.6 \frac{qB\mu}{kh} \ln(t_p + \Delta t) / \Delta t,$$

and that, at the instant of shut-in ($\Delta t = 0$),

$$p^* - p_{wf} = 70.6 \frac{qB\mu}{kh} \left[\ln\left(\frac{kt_p}{1,688 \phi \mu c_t r_w^2}\right) + 2.5 \right].$$

These relationships result from replacing p_i with p^* in Eqs. 1.11 and 2.1, which are valid for infinite-acting reservoirs only.

Eliminating p_{wf} between equations,

$$p^* - \bar{p} = 70.6 \frac{qB\mu}{kh} \left[\ln\left(\frac{kt_p}{1,688 \phi \mu c_t r_w^2}\right) \right.$$

or

$$\begin{aligned} & - \ln\left(\frac{10.06 A}{C_A r_w^2}\right) + 1.5 \Big] \\ & = 70.6 \frac{qB\mu}{kh} \ln\left(\frac{0.000264 kt_p C_A}{\phi \mu c_t A}\right) \\ & = 70.6 \frac{qB\mu}{kh} \ln(C_A t_{DA}), \end{aligned}$$

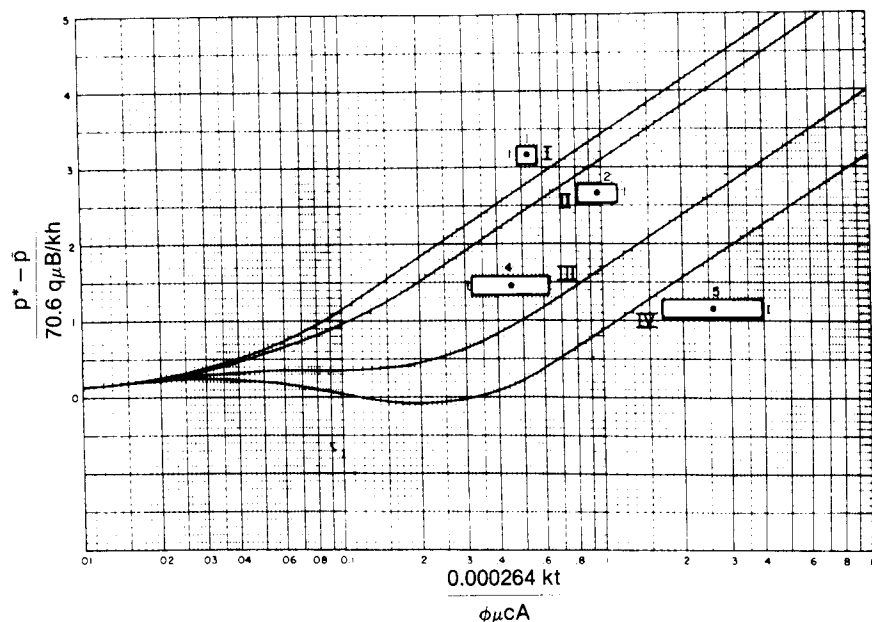


Fig. 2.17E – MBH pressure function for rectangles of various shapes.

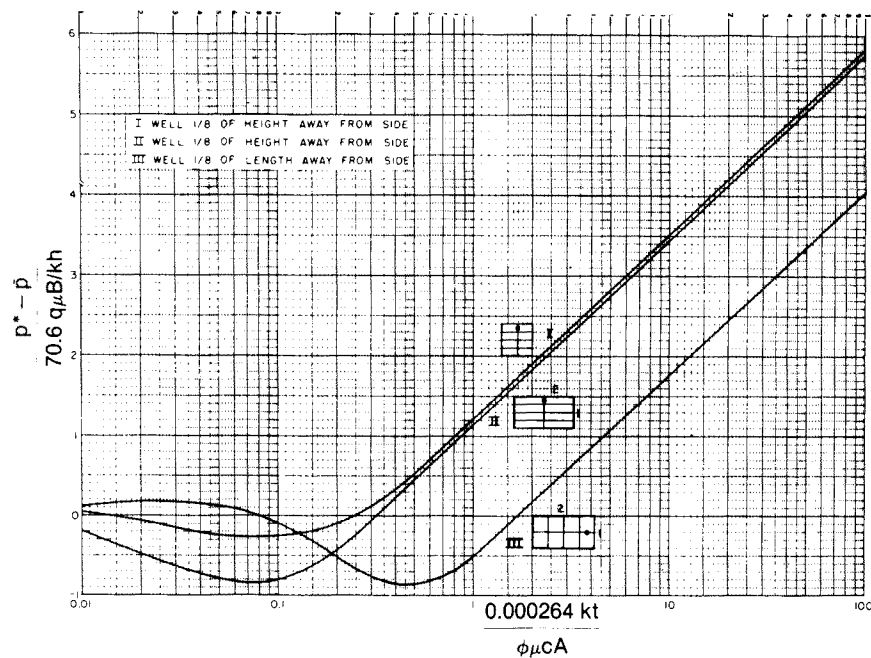


Fig. 2.17F – MBH pressure function in a square and in 2:1 rectangles.

$$p_{DMBH} = \frac{p^* - \bar{p}}{70.6 qB\mu/kh}$$

$$= \ln(C_A t_{DA}).$$

This equation implies a linear relationship between p_{DMBH} and t_{DA} after pseudosteady state flow has been achieved. Indeed, inspection of the curves in Figs. 2.17A through 2.17G shows that, for sufficiently large t_{DA} , a linear relation *does* develop. Further, any point on this straight line can be used to

determine C_A . For example, consider a circular drainage area with a centered well.

From Fig. 2.17A, for

$$t_{DA} = \frac{0.000264 kt_p}{\phi\mu c_t A} = 1,$$

$$p_{DMBH} = \frac{p^* - \bar{p}}{70.6 qB\mu/kh} = 3.454.$$

Thus, $\ln [C_A(1)] = 3.454$ or $C_A = 31.6$ – essentially

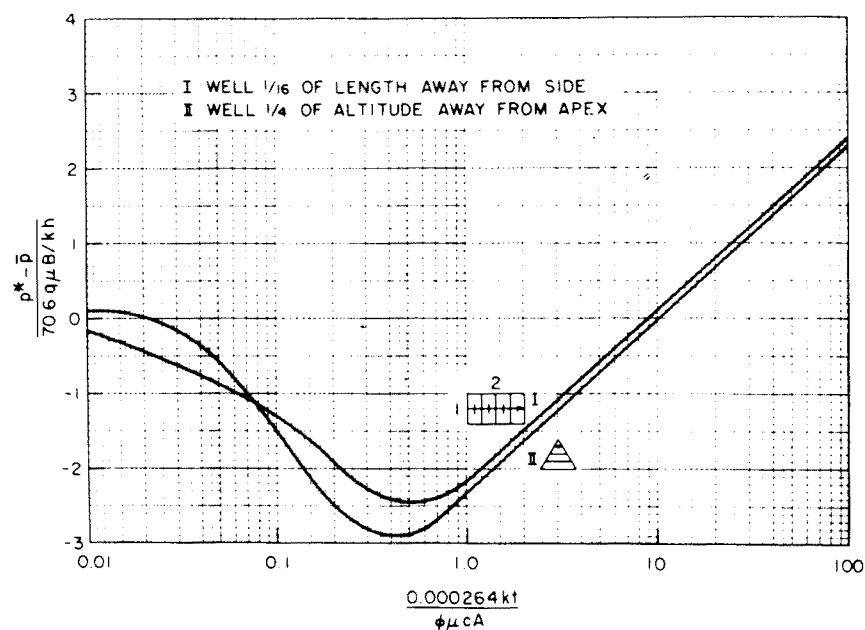


Fig. 2.17G – MBH pressure function on a 2:1 rectangle and equilateral triangle.

TABLE 2.5 – APPLICATION OF MODIFIED MUSKAT METHOD

Δt (hours)	p_{ws} (psi)	$\bar{p} - p_{ws}$ (psi)		
		$\bar{p} = 4,408$	$\bar{p} = 4,412$	$\bar{p} = 4,422$
30	4,393	15	19	29
40	4,398	10	14	24
50	4,402	6	10	20
60	4,405	3	7	17
72	4,407	1	5	15

the value given in Table 1.2. Note also that the linear relationship between p_{DMBH} and t_{DA} begins at $t_{DA} = 0.1$; in Table 1.2, this is the value at which the pseudosteady-state flow equation becomes exact (in the column “Exact for $t_{DA} >$ ”).

Modified Muskat Method

The modified Muskat method is based on a limiting form of Eq. 1.6, which is a solution to the flow equations for a well producing from a closed, cylindrical reservoir at constant rate. Using superposition to simulate a buildup following stabilized flow (depth of investigation has reached reservoir boundaries) and noting that, once boundary effects are felt in the buildup, the equation can be approximated as

$$\bar{p} - p_{ws} = 118.6 \frac{qB\mu}{kh} \exp(-0.00388 k\Delta t / \phi\mu c_t r_e^2). \quad (2.23)$$

For analysis of buildup tests, we usually express this equation as

$$\log(\bar{p} - p_{ws}) = \log\left(118.6 \frac{qB\mu}{kh}\right) - \frac{0.00168 k\Delta t}{\phi\mu c_t r_e^2}. \quad (2.24)$$

Approximations used in developing this equation are valid in the shut-in time range

$$\frac{250 \phi\mu c_t r_e^2}{k} \leq \Delta t \leq \frac{750 \phi\mu c_t r_e^2}{k}.$$

Note that Eq. 2.24 has the form

$$\log(\bar{p} - p_{ws}) = A + B\Delta t,$$

where A and B are constants. This form of the equation suggests how it is applied. We assume a value for $\bar{p}$ and plot $\log(\bar{p} - p_{ws})$ vs. Δt until a straight line results; when it does, the correct value of $\bar{p}$ (the static drainage-area pressure of the tested well) has been found. Experience with the method indicates that it is quite sensitive. An assumed value of $\bar{p}$ that is too low produces a plot of $\log(\bar{p} - p_{ws})$ vs. Δt that has a noticeable downward curvature for data in the time range

$$\frac{250 \phi\mu c_t r_e^2}{k} \leq \Delta t \leq \frac{750 \phi\mu c_t r_e^2}{k}.$$

An assumed value of $\bar{p}$ that is too high produces a noticeable upward curvature in this same time range. The correct assumed value of $\bar{p}$ produces a straight line in (and only in) the proper time range. (See Fig. 2.18.)

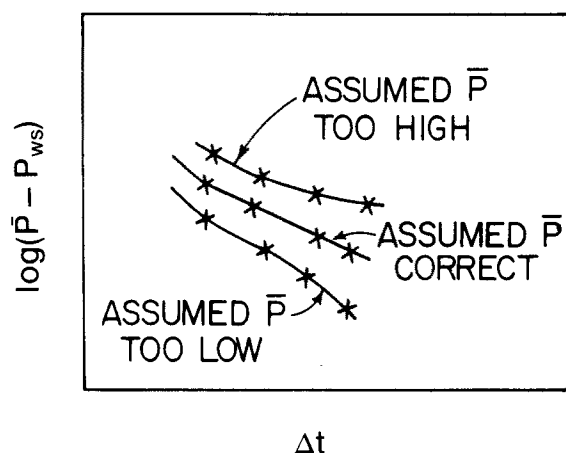


Fig. 2.18 – Schematic graph for modified Muskat method.

The modified Muskat method has two important advantages over the p^* method: (1) it requires no estimates of reservoir properties when it is used to establish $\bar{p}$ (except to choose the correct portion of data for analysis); and (2) it has been found to provide satisfactory $\bar{p}$ estimates for hydraulically fractured wells and for wells with layers of different permeability that communicate only at the wellbore. In these cases, the p^* method fails. The Muskat method has serious disadvantages, too: (1) it fails when the tested well is not reasonably centered in its drainage area (although the drainage area need not be cylindrical, as implied in derivation of this method); and (2) the required shut-in times of $(250 \phi\mu c_t r_e^2)/k$ to $(750 \phi\mu c_t r_e^2)/k$ are frequently impractically long, particularly in low-permeability reservoirs.

We now illustrate this method with an example.

Example 2.7 – Modified Muskat Method To Calculate Average Pressure in Drainage Area

Problem. Consider the buildup test described in Examples 2.2 and 2.3. Estimate the average pressure in the well's drainage area by using the modified Muskat method.

Solution. The data that can be examined by this method are those in the range $\Delta t = (250 \phi\mu c_t r_e^2)/k$ to $\Delta t = (750 \phi\mu c_t r_e^2)/k$. In this case we are fortunate to have estimates of k and r_e , so we can eliminate data outside the time range of interest. Often, of course, we do not have these estimates – a situation that does not limit the applicability of the method, but one that slightly complicates the trial-and-error nature of the calculations. Here,

$$\frac{250 \phi\mu c_t r_e^2}{k} = \frac{(250)(1,320)^2}{(1.442 \times 10^7)} = 30.2 \text{ hours,}$$

and

$$\frac{750 \phi\mu c_t r_e^2}{k} = 90.6 \text{ hours.}$$

Thus, we can examine data from $\Delta t = 30$ hours until

TABLE 2.6 – BUILDUP DATA FOR WELL NEAR BOUNDARY

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	3,103	8	4,085	30	4,614
1	3,488	10	4,172	36	4,700
2	3,673	12	4,240	42	4,770
3	3,780	14	4,298	48	4,827
4	3,861	16	4,353	54	4,882
5	3,936	20	4,435	60	4,931
6	3,996	24	4,520	66	4,975

we stop recording pressures at $\Delta t = 72$ hours.

We make Table 2.5 for three trial values of $\bar{p}$. From plots of $(\bar{p} - p_{ws})$, we see that $\bar{p} = 4,412$ psi is clearly the best choice; we also note the sensitivity of this method. There is a noticeable curvature for $\bar{p} = 4,408$ psi and $\bar{p} = 4,422$ psi (Fig. 2.19).

This example application of the modified Muskat method is intended to illustrate only the mechanics of using the method. The pressure had built up to within 5 psi of its static value at a shut-in time of 72 hours; in such a case, there is little value in applying either this method or the MBH p^* method. Both methods are of value only when the final pressure is a significant distance from stabilization.

2.10 Reservoir Limits Test

In this section, we deal briefly with techniques for estimating reservoir size and distance to boundaries. These comments are introductory only, and deal only with the simplest cases. The intent is to illustrate an approach to these problems rather than the state of the art. The techniques presented are based on *buildup*-test data analysis. Much technology based on *drawdown* test analysis has been developed and is summarized by Earlougher.¹³

We demonstrate that a single boundary near a well causes the slope of a buildup curve to double, and then develop a method for estimating distance from a tested well to a single boundary.

In Chap. 1, when we illustrated application of the superposition principle, we showed that the flowing pressure in a well a distance L from a no-flow boundary (such as a sealing fault) is given by Eq. 1.50. Rearranged, it becomes

$$p_i - p_{wf} = -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_i r_w^2}{kt_p} \right) - 2s \right] - 70.6 \frac{qB\mu}{kh} Ei \left[\frac{-948 \phi \mu c_i (2L)^2}{kt_p} \right].$$

We can develop an equation describing a buildup test for such a well. Note that

$$p_i - p_{ws} = -70.6 \frac{qB\mu}{kh} \left\{ \ln \left[\frac{1,688 \phi \mu c_i r_w^2}{k(t_p + \Delta t)} \right] - 2s \right\} - 70.6 (-q) \frac{B\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_i r_w^2}{k\Delta t} \right) - 2s \right]$$

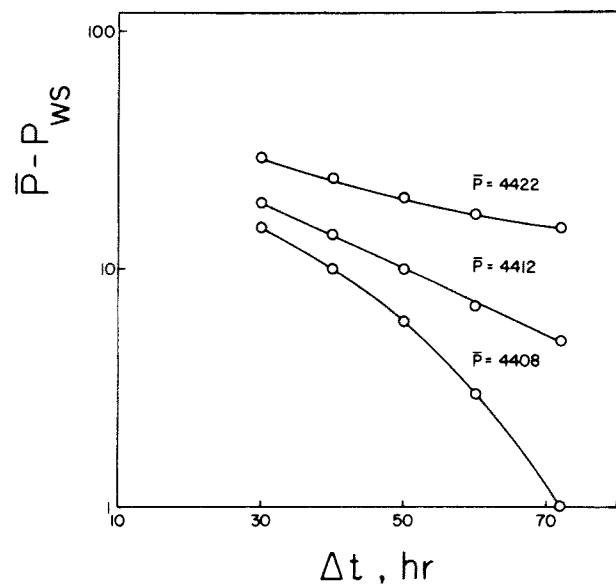


Fig. 2.19 – Modified Muskat method applied to example buildup test.

$$-70.6 \frac{qB\mu}{kh} Ei \left[\frac{-3,792 \phi \mu c_i L^2}{k(t_p + \Delta t)} \right] - 70.6 (-q) \frac{B\mu}{kh} Ei \left(\frac{-3,792 \phi \mu c_i L^2}{k\Delta t} \right). \quad \dots \dots \dots (2.25)$$

For a shut-in time sufficiently large that the logarithmic approximation is accurate for the Ei functions, the equation becomes

$$p_i - p_{ws} = 70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{t_p + \Delta t}{\Delta t} \right) + \ln \left(\frac{t_p + \Delta t}{\Delta t} \right) \right] = 141.2 \frac{qB\mu}{kh} \ln \left(\frac{t_p + \Delta t}{\Delta t} \right).$$

This can be written as

$$p_{ws} = p_i - 325.2 \frac{qB\mu}{kh} \log [(t_p + \Delta t) / \Delta t]. \quad \dots (2.26)$$

Two observations can be made: (1) for a well near a single boundary, such as a sealing fault, Eq. 2.26 shows that the slope of a buildup curve will eventually double (compare Eq. 2.26 with Eq. 2.1) and (2) the time required for the slope to double can be long – specifically, $3,792 \phi \mu c_i L^2 / k\Delta t < 0.02$, or $\Delta t > 1.9 \times 10^5 \phi \mu c_i L^2 / k$. For large values of L or small values of permeability, shut-in time required for the logarithmic approximation to be valid can be longer than the time ordinarily available for a buildup test. For this reason, awaiting a doubling in slope on a buildup test is not necessarily a

satisfactory method of identifying a no-flow boundary near a well and estimating distance from the well to the boundary. Consequently, some analysts prefer to use Eq. 2.25 more directly, noting that for $t_p \gg \Delta t$ it may be rearranged as

$$p_{ws} \approx p_i - 162.6 \frac{qB\mu}{kh} \left[\log \left(\frac{t_p + \Delta t}{\Delta t} \right) - 0.434 Ei \left(\frac{-3,792 \phi \mu c_i L^2}{kt_p} \right) \right] - 70.6 \frac{qB\mu}{kh} Ei \left(\frac{-3,792 \phi \mu c_i L^2}{k\Delta t} \right). \dots (2.27)$$

Reasons for arranging the equation in this form are as follows.

1. The term $162.6 (qB\mu/kh) \{ \log [(t_p + \Delta t)/\Delta t] - 0.434 Ei(-3,792 \phi \mu c_i L^2/kt_p) \}$ determines the position of the middle-time line. Note that the Ei function is a constant; thus, it affects only the position of the MTR and has no effect on slope.

2. At earliest shut-in times in a buildup test, $Ei(-3,792 \phi \mu c_i L^2/k\Delta t)$ is negligible. Physically, this means that the radius of investigation has not yet encountered the no-flow boundary and, mathematically, that the late-time region in the buildup test has not yet begun.

These observations suggest a method for analyzing the buildup test (Fig. 2.20):

1. Plot p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$.
2. Establish the middle-time region.
3. Extrapolate the MTR into the LTR.
4. Tabulate the differences, Δp_{ws}^* , between the buildup curve and extrapolated MTR for several points ($\Delta p_{ws}^* = p_{ws} - p_{MT}$).
5. Estimate L from the relationship implied by Eq. 2.28:

$$\Delta p_{ws}^* = 70.6 \frac{qB\mu}{kh} \left[-Ei \left(\frac{-3,792 \phi \mu c_i L^2}{k\Delta t} \right) \right]. \dots (2.28)$$

L is the only unknown in this equation, so it can be solved directly. Remember, though, that accuracy of this equation requires that $\Delta t \ll t_p$; when this condition is not satisfied, a computer history match using Eq. 2.25 in its complete form is required to determine L .

This calculation implied in Eq. 2.28 should be made for several values of Δt . If the apparent value of L tends to increase or to decrease systematically with time, there is a strong indication that the model does not describe the reservoir adequately (i.e., the well is not behaving as if it were in a reservoir of uniform thickness and porosity, and much nearer one boundary than any others).

The following example illustrates this computational technique.

Example 2.8 – Estimating Distance to a No-Flow Boundary

Problem. Geologists suspect a fault near a newly

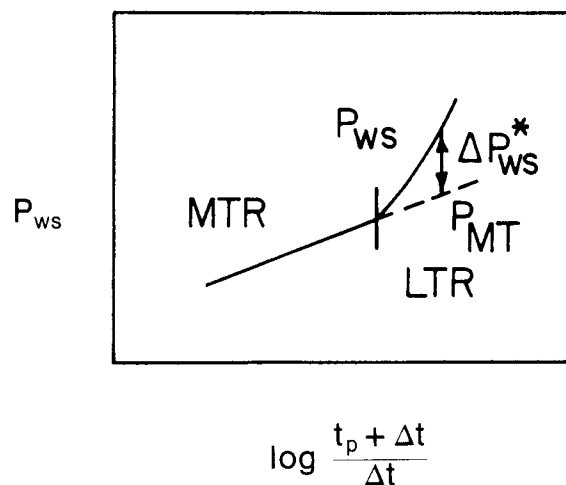


Fig. 2.20 – Buildup test graph for well near reservoir boundary.

drilled well. To confirm this fault and to estimate distance from it, we run a pressure buildup test. Data from the test are given in Table 2.6. Well and reservoir data include the following.

$$\begin{aligned} \phi &= 0.15, \\ \mu_o &= 0.6 \text{ cp}, \\ c_i &= 17 \times 10^{-6} \text{ psi}^{-1}, \\ r_w &= 0.5 \text{ ft}, \\ A_{wb} &= 0.00545 \text{ sq ft}, \\ \rho_o &= 54.8 \text{ lbm/cu ft}, \\ q_o &= 1,221 \text{ STB/D}, \\ B_o &= 1.310 \text{ RB/STB, and} \\ h &= 8 \text{ ft.} \end{aligned}$$

The well produced only oil and dissolved gas. Before shut-in, a total of 14,206 STB oil had been produced.

Analysis of these data show that afterflow distorts none of the data recorded at shut-in times of 1 hour or more. Based on the slope (650 psi/cycle) of the earliest straight line, permeability, k , appears to be 30 md. Depth of investigation at a shut-in time of 1 hour is 144 ft, lending confidence to the choice of the middle-time line. Pseudoproducing time, t_p , is 279.2 hours.

From these data, determine whether the buildup test data indicate that the well is behaving as if it were near a single fault and estimate distance to an apparent fault from buildup data at several times in the LTR.

Solution. Our attack is to plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$; extrapolate the middle-time line on into the LTR; read pressures, p_{MT} , from this extrapolated line; subtract those pressures from observed values of p_{ws} in the LTR ($\Delta p_{ws}^* = p_{ws} - p_{MT}$); estimate values of L from Eq. 2.28; and assume that, if calculated values of L are fairly constant, the well is indeed near a single sealing fault.

From Fig. 2.21, we obtain the data in Table 2.7. We now estimate L from Eq. 2.28. Note that

$$\frac{3,792 \phi \mu c_t}{k} = \frac{(3,792)(0.15)(0.6)(1.7 \times 10^{-5})}{30}$$

$$= 1.934 \times 10^{-4}.$$

We first estimate L at $\Delta t = 10$ hours, which assumes that the approximation $t_p \approx t_p + \Delta t$ is adequate in this case.

$$\Delta p_{ws}^* = 52 = -70.6 \frac{qB\mu}{kh} Ei\left(\frac{-3,792 \phi \mu c_t L^2}{k\Delta t}\right)$$

$$= \frac{-(70.6)(1,221)(1.310)(0.6)}{(30)(8)}$$

$$\cdot Ei\left(\frac{-3,792 \phi \mu c_t L^2}{k\Delta t}\right).$$

$$-Ei\left(\frac{-1.934 \times 10^{-4} L^2}{10}\right) = 0.184.$$

$$L^2 = \frac{(1.107)(10)}{1.934 \times 10^{-4}} = 5.72 \times 10^4,$$

or

$$L = 239 \text{ ft.}$$

For larger values of shut-in time, the approximation $t_p \approx t_p + \Delta t$ becomes decreasingly accurate, and no terms in Eq. 2.25 can be neglected, but $L \approx 240$ ft satisfies the equation for all values of shut-in time.

For the case in which the slope of the buildup test has time to double, estimation of distance from well to boundary is easier. From the buildup tests plot, we find the time, Δt_x , at which the two straight-line sections intersect (Fig. 2.22). Gray¹⁴ suggests that the distance L from the well to the fault can be calculated from

$$L = \sqrt{\frac{0.000148 k \Delta t_x}{\phi \mu c_t}} \dots \dots \dots (2.29)$$

In Fig. 2.21, the slope did double, and the figure shows that $(t_p + \Delta t_x) / \Delta t_x = 17$, from which $\Delta t_x = 17.45$ hours. Eq. 2.29 then shows that $L \approx 225$ ft, in reasonable agreement with our previous calculation.

The results of pressure buildup tests sometimes can be used to estimate reservoir size. The basic idea is to compare average static reservoir pressure before and after production of a known quantity of fluid from a closed, volumetric reservoir, with constant compressibility, c_t . If V_R is the reservoir volume (barrels), ΔN_p is the stock-tank barrels of oil produced between Times 1 and 2, and $\bar{p}_1$ and $\bar{p}_2$ are the average reservoir pressures before and after oil production, then a material balance on the reservoir shows that

$$\bar{p}_2 = \bar{p}_1 - \frac{(\Delta N_p)(B_o)}{V_R c_t \phi},$$

TABLE 2.7 – ANALYSIS OF DATA FROM WELL NEAR BOUNDARY

Δt (hours)	$(t_p + \Delta t) / \Delta t$	p_{ws} (psi)	p_{MT} (psi)	$(p_{ws} - p_{MT}) = \Delta p_{ws}^*$ (psi)
6	47.5	3,996	3,980	16
8	35.9	4,085	4,051	34
10	28.9	4,172	4,120	52
12	24.3	4,240	4,170	70
14	20.9	4,298	4,210	88
16	18.5	4,353	4,250	103
20	15.0	4,435	4,300	135
24	12.6	4,520	4,355	165
30	10.3	4,614	4,410	204
36	8.76	4,700	4,455	245
42	7.65	4,770	4,495	275
48	6.82	4,827	4,525	302
54	6.17	4,882	4,552	330
60	5.65	4,931	4,578	353
66	5.23	4,975	4,600	375

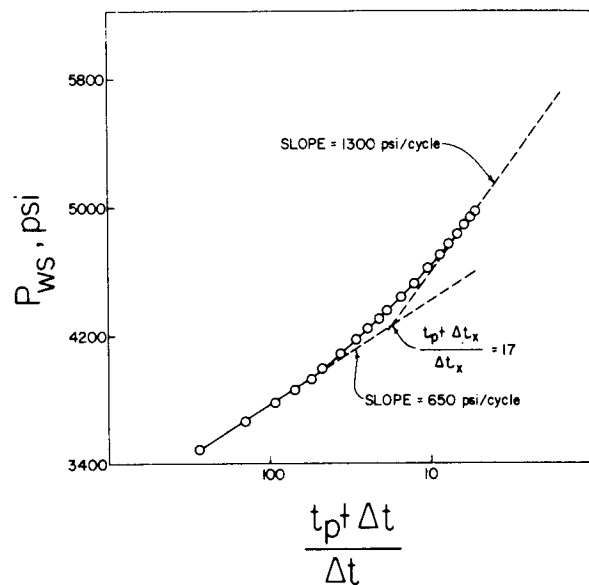


Fig. 2.21 – Estimating distance to a no-flow boundary.

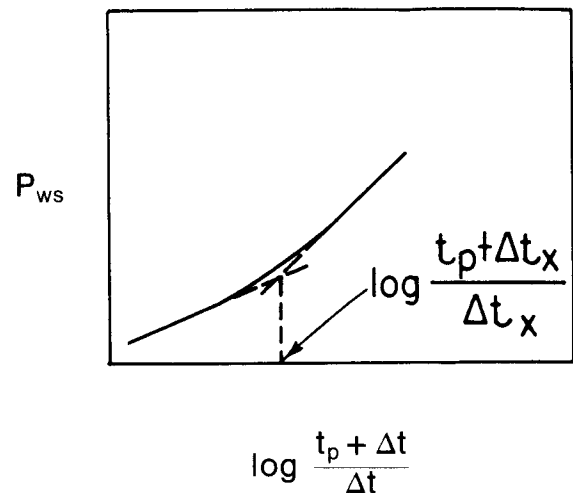


Fig. 2.22 – Distance to boundary from slope doubling.

or

$$V_R = \frac{(\Delta N_p)(B_o)}{(\bar{p}_1 - \bar{p}_2)c_t\phi} \quad (2.30)$$

Example 2.9—Estimating Reservoir Size

Problem. Two pressure buildup tests are run on the only well in a closed reservoir. The first test indicates an average pressure of 3,000 psi, the second indicates 2,100 psi. The well produced an average of 150 STB/D of oil in the year between tests. Average oil formation volume factor, B_o , is 1.3 RB/STB; total compressibility, c_t , is 10×10^{-6} psi $^{-1}$; porosity, ϕ , is 22%; and average sand thickness, h , is 10 ft. Estimate area, A_R , of the reservoir in acres.

Solution. From Eq. 2.30,

$$\begin{aligned} V_R &= \frac{\Delta N_p B_o}{(\bar{p}_1 - \bar{p}_2)c_t\phi} \\ &= \frac{(qt)B_o}{(\bar{p}_1 - \bar{p}_2)(c_t\phi)} \\ &= \frac{(150 \text{ STB/D})(365 \text{ days})(1.3 \text{ RB/STB})}{(3,000 - 2,100) \text{ psi} (10 \times 10^{-6} \text{ psi}^{-1})(0.22)} \end{aligned}$$

Thus,

$$\begin{aligned} V_R &= 35.9 \times 10^6 \text{ bbl} \\ &= \frac{43,560 A_R h}{5.615}, \end{aligned}$$

and

$$\begin{aligned} A_R &= \frac{(35.9 \times 10^6 \text{ bbl})(5.615 \text{ cu ft/bbl})}{(10 \text{ ft})(43.56 \times 10^3 \text{ sq ft/acre})} \\ &= 463 \text{ acres.} \end{aligned}$$

2.11 Modifications for Gases

This section presents modifications of the basic drawdown and buildup equations so that they can be applied to analysis of gas reservoirs. These modifications are based on results obtained with the gas pseudopressure,¹⁵ although a more complete discussion of that subject is left to Chap. 5.

Wattenbarger and Ramey¹⁶ have shown that for some gases at pressures above 3,000 psi, flow in an infinite-acting reservoir can be modeled accurately by the equation

$$\begin{aligned} p_{wf} &= p_i + \frac{162.6 q_g B_{gi} \mu_i}{kh} \left[\log \left(\frac{1,688 \phi \mu_i c_{ti}}{kt_p} \right) \right. \\ &\quad \left. - \frac{(s + Dq_g)}{1.151} \right] \quad (2.31) \end{aligned}$$

This equation has the same form as the equation for a slightly compressible liquid, but there are some important differences:

1. Gas production rate, q_g , is conveniently ex-

pressed in thousands of standard cubic feet per day (Mscf/D), and gas formation volume factor, B_g , is then expressed in reservoir barrels per thousand standard cubic feet (RB/Mscf), so that the product $q_g B_g$ is in reservoir barrels per day (RB/D) as in the analogous equation for slightly compressible liquids.

2. All gas properties (B_g , μ , and c_g) are evaluated at original reservoir pressure, p_i . (More generally, these properties should be evaluated at the uniform pressure in the reservoir before initiation of flow.) In Eq. 2.31,

$$B_{gi} = \frac{178.1 z_i T p_{sc}}{p_i T_{sc}} \text{ (RB/Mscf),}$$

$$c_{ti} = c_{gi} S_g + c_w S_w + c_f \approx c_{gi} S_g.$$

3. The factor D is a measure of non-Darcy or turbulent pressure loss (i.e., a pressure drop in addition to that predicted by Darcy's law). It cannot be calculated separately from the skin factor from a single buildup or drawdown test; thus, the concept of apparent skin factor, $s' = s + Dq_g$, is sometimes convenient since it can be determined from a single test.

For many cases at pressures below 2,000 psi, flow in an infinite acting reservoir can be modeled by

$$\begin{aligned} p_{wf}^2 &= p_i^2 + \frac{1,637 q_g \mu_i z_i T}{kh} \\ &\quad \cdot \left[\log \left(\frac{1,688 \phi \mu c_{ti}}{kt_p} \right) - \frac{(s + Dq_g)}{1.151} \right] \quad (2.32) \end{aligned}$$

Using these basic drawdown equations, we can use superposition to develop equations describing a buildup test for gas wells.

For $p > 3,000$ psi,

$$p_{ws} = p_i - 162.6 \frac{q_g B_{gi} \mu_i}{kh} \left[\log \left(\frac{t_p + \Delta t}{\Delta t} \right) \right], \quad (2.33)$$

and

$$\begin{aligned} s' &= s + D(q_g) = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} \right. \\ &\quad \left. - \log \left(\frac{k}{\phi \mu_i c_{ti} r_w^2} \right) + 3.23 \right] \quad (2.34) \end{aligned}$$

For $p < 2,000$ psi,

$$p_{ws}^2 = p_i^2 - 1,637 \frac{q_g \mu_i z_i T}{kh} \log \left(\frac{t_p + \Delta t}{\Delta t} \right), \quad (2.35)$$

and

$$\begin{aligned} s' &= s + D(q_g) = 1.151 \left[\frac{(p_{1 \text{ hr}}^2 - p_{wf}^2)}{m''} \right. \\ &\quad \left. - \log \left(\frac{k}{\phi \mu_i c_{ti} r_w^2} \right) + 3.23 \right] \quad (2.36) \end{aligned}$$

where m'' is the slope of the plot p_{ws}^2 vs. $\log[(t_p + \Delta t)/\Delta t]$, which is $1,637 q_g \mu_i z_i T/kh$.

An obvious question is, what technique should be used to analyze gas reservoirs with pressures in the range $2,000 < p < 3,000$ psi? One approach is to use equations written in terms of the gas pseudopressure instead of either pressure or pressure squared. This is at least somewhat inconvenient, so an alternative approach is to use equations written in terms of either p_{ws} or p_{ws}^2 and accept the resultant inaccuracies, which, in real, heterogeneous reservoirs, may be far from the most significant oversimplification on which the test analysis procedure is based. The smaller the pressure drawdown during the test, the less the inaccuracy in this approach.

Example 2.10 – Gas Well Buildup Test Analysis

Problem. A gas well is shut in for a pressure buildup test. Test data include the following.

$$\begin{aligned} q_g &= 5,256 \text{ Mscf/D,} \\ T &= 181^\circ\text{F} = 641^\circ\text{R,} \\ h &= 28 \text{ ft,} \\ \mu_i &= 0.028 \text{ cp,} \\ S_w &= 0.3, \\ \phi &= 0.18, \\ z_i &= 0.85, \\ r_w &= 0.3 \text{ ft,} \\ c_{gi} &= 0.344 \times 10^{-3} \text{ psi}^{-1}, \\ p_i &= 2,906 \text{ psia, and} \\ p_{wf} &= 1,801 \text{ psia.} \end{aligned}$$

Most of the test data fall in the intermediate pressure range, $2,000 < p < 3,000$ psia. On a plot of p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$, the MTR had a slope, m , of 81 psi/cycle; on this plot, $p_{1 \text{ hr}}$ was found to be 2,525 psia. Alternatively, on a plot of p_{ws}^2 vs. $\log(t_p + \Delta t)/\Delta t$, the MTR had a slope of 0.48×10^6 psi²/cycle and $p_{1 \text{ hr}}^2$ of 7.29×10^6 psi².

From these data, estimate apparent values of k and s' (1) based on characteristics of the p_{ws} plot and (2) based on characteristics of the p_{ws}^2 plot.

Solution. From results of the p_{ws} plot, for standard conditions of 14.7 psia and 60°F,

$$\begin{aligned} B_{gi} &= 178.1 \frac{z_i T p_{sc}}{p_i T_{sc}} \\ &= \frac{(178.1)(0.85)(641)(14.7)}{(2,906)(520)} \\ &= 0.944 \text{ RB/Mscf,} \end{aligned}$$

$$\begin{aligned} k &= 162.6 \frac{q_g B_{gi} \mu_i}{m h} \\ &= \frac{(162.6)(5,256)(0.944)(0.028)}{(81)(28)} \\ &= 9.96 \text{ md,} \end{aligned}$$

$$\begin{aligned} c_{ti} &\approx c_{gi} S_g = (3.44 \times 10^{-4})(0.7) \\ &= 2.41 \times 10^{-4} \text{ psi}^{-1}, \end{aligned}$$

and

$$\begin{aligned} s' &= s + D(q_g) = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} \right. \\ &\quad \left. - \log \left(\frac{k}{\phi \mu_i c_{ti} r_w^2} \right) + 3.23 \right] \\ &= 1.151 \left\{ \frac{(2,525 - 1,801)}{81} \right. \\ &\quad \left. - \log \left[\frac{9.96}{(0.18)(0.028)(2.41 \times 10^{-4})(0.3)^2} \right] \right. \\ &\quad \left. + 3.23 \right\} = 4.84. \end{aligned}$$

From results of the p_{ws}^2 plot,

$$\begin{aligned} k &= 1,637 \frac{q_g \mu_i z_i T}{m'' h} \\ &= \frac{(1,637)(5,256)(0.028)(0.85)(641)}{(4.8 \times 10^5)(28)} \\ &= 9.77 \text{ md,} \end{aligned}$$

and

$$\begin{aligned} s' &= 1.151 \left[\frac{(p_{1 \text{ hr}}^2 - p_{wf}^2)}{m''} - \log \frac{k}{\phi \mu_i c_{ti} r_w^2} + 3.23 \right] \\ &= 1.151 \left\{ \frac{[7.29 \times 10^6 - (1,801)^2]}{4.8 \times 10^5} \right. \\ &\quad \left. - \log \left[\frac{9.77}{(0.18)(0.028)(2.41 \times 10^{-4})(0.3)^2} \right] + 3.23 \right\} \\ &= 4.27. \end{aligned}$$

Neither set of results (k and s') is necessarily more accurate than the other in the *general* case; as in this *particular* case, use of an analysis procedure based on gas pseudopressure can be used to improve accuracy if disagreement in results from p_{ws} and p_{ws}^2 plots is unacceptably large.

2.12 Modifications for Multiphase Flow

Basic buildup and drawdown equations can be modified to model multiphase flow.^{17,18} For an infinite-acting reservoir, the drawdown equation becomes

$$\begin{aligned} p_{wf} &= p_i + 162.6 \frac{q_{Rt}}{\lambda_f h} \left[\log \left(\frac{1,688 \phi c_f r_w^2}{\lambda_f t} \right) \right. \\ &\quad \left. - \frac{s}{1.151} \right], \dots \dots \dots (2.37) \end{aligned}$$

and the buildup equation becomes

$$p_{ws} = p_i - 162.6 \frac{q_{Rt}}{\lambda_t h} \log\left(\frac{t_p + \Delta t}{\Delta t}\right). \quad (2.38)$$

In these equations, the total flow rate q_{Rt} is in reservoir barrels per day (neglecting solution gas liberated from produced water),

$$q_{Rt} = q_o B_o + \left(q_g - \frac{q_o R_s}{1,000}\right) B_g + q_w B_w, \quad (2.39)$$

and total mobility, λ_t , is

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} + \frac{k_g}{\mu_g}. \quad (2.40)$$

Total compressibility, c_t , was defined in Eq. 1.4.

These equations imply that it is possible to determine λ_t from the slope m of a buildup test run on a well that produces two or three phases simultaneously:

$$\lambda_t = 162.6 \frac{q_{Rt}}{mh}. \quad (2.41)$$

Perrine¹⁷ has shown that it is also possible to estimate the permeability to *each phase* flowing from the same slope, m :

$$k_o = 162.6 \frac{q_o B_o \mu_o}{mh}, \quad (2.42)$$

$$k_g = 162.6 \frac{\left(q_g - \frac{q_o R_s}{1,000}\right) B_g \mu_g}{mh}, \quad (2.43)$$

and

$$k_w = 162.6 \frac{q_w B_w \mu_w}{mh}. \quad (2.44)$$

The term $(q_g - q_o R_s/1,000) B_g$, which appears in Eqs. 2.39 and 2.43, is the *free* gas flow rate in the reservoir. It is found by subtracting the dissolved gas rate $(q_o R_s/1,000)$ from the total surface gas rate (q_g) and converting to a reservoir-condition basis.

Simultaneous solution of Eqs. 2.37 and 2.38 results in the following expression for the skin factor s .

$$s = 1.151 \left[\frac{p_{1 \text{ hr}} - p_{wf}}{m} - \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) + 3.23 \right]. \quad (2.45)$$

Static drainage-area pressure, $\bar{p}$, is calculated just as for a single-phase reservoir. In use of the MBH charts to determine $\bar{p}$ (and in the Horner plot itself), the effective production time t_p is best estimated by dividing cumulative oil production by the oil production rate just before shut-in.

An important assumption required for accurate use of these equations for multiphase flow analysis is that saturations of each phase remain essentially uniform throughout the drainage area of the tested well.

Example 2.12 – Multiphase Buildup Test Analysis

Problem. A buildup test is run in a well that produces oil, water, and gas simultaneously. Well, rock, and fluid properties evaluated at average reservoir pressure during the test include the following.

$$\begin{aligned} S_o &= 0.58, \\ S_g &= 0.08, \\ S_w &= 0.34, \\ c_w &= 3.6 \times 10^{-6} \text{ psi}^{-1}, \\ c_f &= 3.5 \times 10^{-6} \text{ psi}^{-1}, \\ c_g &= 0.39 \times 10^{-3} \text{ psi}^{-1}, \\ \mu_o &= 1.5 \text{ cp}, \\ \mu_w &= 0.7 \text{ cp}, \\ \mu_g &= 0.03 \text{ cp}, \\ B_o &= 1.3 \text{ RB/STB}, \\ B_w &= 1.02 \text{ RB/STB}, \\ B_g &= 1.480 \text{ RB/Mscf}, \\ R_s &= 685 \text{ scf/STB}, \\ \phi &= 0.17, \\ r_w &= 0.3 \text{ ft, and} \\ h &= 38 \text{ ft.} \end{aligned}$$

From plots of B_o vs. p and R_s vs. p at average pressure in the buildup test,

$$\frac{dR_s}{dp} = 0.0776 \text{ scf/STB/psi},$$

and

$$\frac{dB_o}{dp} = 2.48 \times 10^{-6} \text{ RB/STB/psi}.$$

The production rates prior to the buildup test were $q_o = 245$ STB/D, $q_w = 38$ STB/D, and $q_g = 489$ Mscf/D.

A plot of p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$ shows that the slope of the MTR, m , is 78 psi/cycle and that $p_{1 \text{ hr}} = 2,466$ psia. Flowing pressure, p_{wf} , at the instant of shut-in was 2,028 psia.

From these data, estimate λ_t , k_o , k_w , k_g , and s .

Solution. Permeabilities to each phase can be determined from the slope m of the MTR:

$$\begin{aligned} k_o &= 162.6 \frac{q_o B_o \mu_o}{mh} \\ &= \frac{(162.6)(245)(1.5)(1.3)}{(78)(38)} = 26.2 \text{ md}, \end{aligned}$$

$$\begin{aligned} k_w &= 162.6 \frac{q_w B_w \mu_w}{mh} \\ &= \frac{(162.6)(38)(0.7)(1.02)}{(78)(38)} = 1.49 \text{ md}, \end{aligned}$$

$$k_g = 162.6 \frac{\left(q_g - \frac{q_o R_s}{1,000}\right) B_g \mu_g}{mh}$$

$$= 162.6 \frac{\left[489 - \frac{(245)(685)}{(1,000)} \right] (1.480)(0.03)}{(78)(38)}$$

$$= 0.782 \text{ md.}$$

To calculate total mobility, λ_t , we first need total flow rate, q_{Rt} :

$$q_{Rt} = q_o B_o + \left(q_g - \frac{q_o R_s}{1,000} \right) B_g + q_w B_w$$

$$= (245)(1.3) + \left[489 - \frac{(245)(685)}{1,000} \right]$$

$$\cdot (1.480) + (38)(1.02)$$

$$= 833 \text{ RB/D.}$$

Then,

$$\lambda_t = 162.6 \frac{q_{Rt}}{mh} = \frac{(162.6)(833)}{(78)(38)}$$

$$= 45.7 \text{ md/cp.}$$

To calculate skin factor, s , we first need c_o and c_t :

$$c_o = \frac{B_g}{B_o} \frac{dR_s}{dp} - \frac{1}{B_o} \frac{dB_o}{dp}$$

$$= \frac{(1.480 \text{ RB/Mscf})(0.0776 \text{ scf})}{(1.3 \text{ RB/STB})(\text{STB-psi})}$$

$$\cdot \frac{1 \text{ Mscf}}{1,000 \text{ scf}} - \frac{(2.48 \times 10^{-6})}{1.3 \text{ (psi)}}$$

$$= 86.4 \times 10^{-6} \text{ psi}^{-1}.$$

Then,

$$c_t = S_o c_o + S_g c_g + S_w c_w + c_f$$

$$= (0.58)(86.4 \times 10^{-6}) + (0.08)(0.39 \times 10^{-3})$$

$$+ (0.34)(3.6 \times 10^{-6}) + 3.5 \times 10^{-6}$$

$$= 86.0 \times 10^{-6} \text{ psi}^{-1},$$

and

$$s = 1.151 \left[\frac{p_{1 \text{ hr}} - p_{wf}}{m} - \log \left(\frac{\lambda_t}{\phi c_t r_w^2} \right) + 3.23 \right]$$

$$= 1.151 \left\{ \frac{2,466 - 2,028}{78} \right.$$

$$\left. - \log \left[\frac{45.7}{(0.17)(86.0 \times 10^{-6})(0.3)^2} \right] + 3.23 \right\}$$

$$= 1.50.$$

Exercises

2.1. In Example 2.1, what error arises because we used Eq. 2.4 to calculate skin factor instead of the more exact Eq. 2.3? What difference would it have made in the value of s had we used a shut-in time of 10 hours in Eq. 2.3 and the corresponding value of p_{ws} ? What assumption have we made about distance from tested well to reservoir boundaries in Example 2.1?

2.2. Prove that the slope of a plot of shut-in BHP vs. $\log(t_p + \Delta t)/\Delta t$ is, as asserted in the text, the difference in pressure at two points one cycle apart. Also prove that, for $\Delta t \ll t_p$, we obtain the same slope on a plot of p_{ws} vs. $\log \Delta t$. Finally, prove that on a plot of p_{ws} vs. $\log \Delta t$, we obtain the same slope regardless of the units used for shut-in time, Δt , on the plot (i.e., that Δt can be expressed in minutes, hours, or days without affecting the slope of the plot).

2.3. A well producing only oil and dissolved gas has produced 12,173 STB. The well has not been stimulated, nor is there any reason to believe that there is a significant amount of formation damage. A pressure buildup test is run with the primary objective of estimating static drainage-area pressure. During buildup, there is a rising liquid level in the wellbore. Well and reservoir data are:

$$\phi = 0.14,$$

$$\mu = 0.55 \text{ cp},$$

$$c_t = 16 \times 10^{-6} \text{ psi}^{-1},$$

$$r_w = 0.5 \text{ ft},$$

$$A_{wb} = 0.0218 \text{ sq ft},$$

$$r_e = 1,320 \text{ ft (well centered in cylindrical drainage area),}$$

$$\rho = 54.8 \text{ lbm/cu ft},$$

$$q = 988 \text{ STB/D},$$

$$B = 1.126 \text{ RB/STB, and}$$

$$h = 7 \text{ ft.}$$

Data recorded during the buildup test are given in Table 2.8. Plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$ on semilog paper and $(p_{ws} - p_{wf})$ vs. Δt_e on log-log paper, and estimate the time at which afterflow ceased distorting the buildup test data.

2.4. Consider the buildup test described in Problem 2.3. Locate the MTR and estimate formation permeability.

2.5. Consider the buildup test described in Problems 2.3 and 2.4. Calculate skin factor, s ; pressure drop across the altered zone, $(\Delta p)_s$; flow efficiency, E ; and effective wellbore radius, r_{wa} .

2.6. Prove that in a buildup test for a well near a single fault, the technique suggested in the text (extrapolating the late-time line to infinite shut-in time) is the proper method for estimating original reservoir pressure. Comment on the possible errors in original reservoir pressure estimates in these cases: (1) some LTR data were obtained, but final straight line was not established; and (2) no LTR data were obtained.

2.7. Consider the buildup test described in Problems 2.3 and 2.4. Estimate static drainage-area pressure for this well (1) using the p^* method, and (2)

TABLE 2.8 – PRESSURE BUILDUP TEST DATA

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	709	19.7	4,198
1.97	3,169	24.6	4,245
2.95	3,508	29.6	4,279
3.94	3,672	34.5	4,306
4.92	3,772	39.4	4,327
5.91	3,873	44.4	4,343
7.88	3,963	49.3	4,356
9.86	4,026	59.1	4,375
14.8	4,133		

using the modified Muskat method.

2.8. In Example 2.7, explain how we could have applied the modified Muskat method to estimate static drainage-area pressure if we had not had estimates of $k/\phi\mu c_t$ or r_e .

2.9. Estimate formation permeability and skin factor from the following data available from a gas well pressure buildup test.

$$\begin{aligned} T &= 199^\circ\text{F} = 659^\circ\text{R}, \\ h &= 34 \text{ ft}, \\ \mu_i &= 0.023 \text{ cp}, \\ S_w &= 0.33 \text{ (water is immobile)}, \\ c_{gi} &= 0.000315 \text{ psi}^{-1}, \\ \phi &= 0.22, \\ z_i &= 0.87, \text{ and} \\ r_w &= 0.3 \text{ ft}. \end{aligned}$$

The well produced 6,068 Mcf/D before the test. A plot of BHP, p_{ws} , vs. $\log(t_p + \Delta t)/\Delta t$ gave a middle-time line with a slope of 66 psi/cycle. Analysis of the buildup curve showed that static drainage-area pressure, $\bar{p}$, was 3,171 psia. Pressure on the middle-time line at $\Delta t = 1$ hour, $p_{1 \text{ hr}}$, was 2,745 psia; flowing pressure at shut-in, p_{wf} , was 2,486 psia.

2.10. Estimate total mobility, λ_t , oil, water and gas permeabilities, and skin factor for a well that produced oil, water, and gas simultaneously before a pressure buildup test. Production rates before the test were $q_o = 276$ STB/D, $q_w = 68$ STB/D, and $q_g = 689$ Mcf/D. A plot of p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$ showed that the slope m of the middle-time line was 59 psi/cycle. Flowing pressure at shut-in, p_{wf} , was 1,581 psia; the pressure on the middle-time straight line at $\Delta t = 1$ hour, $p_{1 \text{ hr}}$, was 1,744 psia. Plots of B_o vs. p and R_s vs. p showed that $dR_s/dp = 0.263$ scf/STB/psi and that $dB_o/dp = 0.248 \times 10^{-3}$ RB/STB/psi. Rock, fluid, and well properties include the following.

$$\begin{aligned} S_o &= 0.56, \\ S_g &= 0.09, \\ S_w &= 0.35, \\ c_w &= 3.5 \times 10^{-6} \text{ psi}^{-1}, \\ c_f &= 3.5 \times 10^{-6} \text{ psi}^{-1}, \\ c_g &= 0.48 \times 10^{-3} \text{ psi}^{-1}, \\ \mu_o &= 1.1 \text{ cp}, \\ \mu_w &= 0.6 \text{ cp}, \\ \mu_g &= 0.026 \text{ cp}, \\ B_o &= 1.28 \text{ RB/STB}, \\ B_w &= 1.022 \text{ RB/STB}, \\ B_g &= 1.122 \text{ RB/Mscf}, \end{aligned}$$

TABLE 2.9 – BUILDUP TEST DATA FOR WELL NEAR FAULT

Δt (hours)	p_{ws} (psi)	Δt (hours)	p_{ws} (psi)
20	1,373	500	2,225
30	1,467	800	2,360
40	1,533	1,000	2,434
50	1,585	1,500	2,545
100	1,752	2,000	2,616
200	1,940		

$$R_s = 748 \text{ scf/STB},$$

$$\phi = 0.18,$$

$$r_w = 0.3 \text{ ft, and}$$

$$h = 33 \text{ ft}.$$

2.11. A pressure buildup test was run on an oil well believed to be near a sealing fault in an otherwise infinite-acting reservoir. Estimate the distance to the fault, given the well, rock, and fluid properties below and the buildup data in Table 2.9.

$$\begin{aligned} q &= 940 \text{ STB/D}, \\ \mu &= 50 \text{ cp}, \\ \phi &= 0.2, \\ c_t &= 78 \times 10^{-6} \text{ psi}^{-1}, \\ h &= 195 \text{ ft}, \\ p_i &= 2,945 \text{ psi}, \\ N_p &= 84,500 \text{ STB}, \\ B_o &= 1.11 \text{ RB/STB, and} \\ t_{wbs} &< 20 \text{ hours}. \end{aligned}$$

2.12. A well flowed for 10 days at 350 STB/D; it was then shut in for a pressure buildup test. Rock, fluid, and well properties include the following.

$$\begin{aligned} B_o &= 1.13 \text{ RB/STB}, \\ p_i &= 3,000 \text{ psi}, \\ \mu &= 0.5 \text{ cp}, \\ k &= 25 \text{ md}, \\ s &= 0, \\ h &= 50 \text{ ft}, \\ c_t &= 20 \times 10^{-6} \text{ psi}^{-1}, \\ \phi &= 0.16, \text{ and} \\ r_w &= 0.333 \text{ ft}. \end{aligned}$$

(a) Determine and plot the pressure distribution in the reservoir for shut-in times of 0, 0.1, 1, and 10 days. (Assume an infinite acting reservoir.)

(b) Calculate the radius of investigation at 0.1, 1, and 10 days. Compare r_i with the depth to which the transient appears to have moved on the plots prepared in Part a.

2.13. In Example 2.6, $\bar{p}$ was determined to be 4,411 psi. Both the Horner plot and the abscissa of the MBH chart used $t_p = 13,630$ hours. It can be shown that for a well centered in a square drainage area, the time required to reach semisteady state is $t_{pss} = (\phi\mu c_t A / 0.000264 k) (t_{DA})_{pss}$ and that $(t_{DA})_{pss} = 0.1$. Show that if t_{pss} is used instead of t_p in both the Horner plot and in the abscissa of the MBH chart, the resulting estimate of $\bar{p}$ is essentially unchanged. Buildup data (from the MTR only) are given in Table 2.10. Other data include:

TABLE 2.10 – MTR DATA FROM BUILDUP TEST

Δt (hours)	p_{ws} (psi)
8	4,354
12	4,366
16	4,376
20	4,382
24	4,388

$$\begin{aligned}
 q &= 250 \text{ STB/D,} \\
 B &= 1.136 \text{ RB/STB,} \\
 \mu &= 0.8 \text{ cp,} \\
 h &= 69 \text{ ft,} \\
 \phi &= 0.039, \\
 c_t &= 17 \times 10^{-6} \text{ psi}^{-1}, \\
 r_e &= 1,320 \text{ ft, and} \\
 k &= 7.65 \text{ md.}
 \end{aligned}$$

2.14. A well producing only oil and dissolved gas has produced 13,220 STB. To characterize the severe damage believed present, the well is shut in for a buildup test. Well and reservoir data are given below.

$$\begin{aligned}
 \phi &= 0.17, \\
 \mu &= 0.6 \text{ cp,} \\
 c_t &= 18 \times 10^{-6} \text{ psi}^{-1} \\
 r_e &= 1,320 \text{ ft, well centered in square drainage} \\
 &\quad \text{area (160 acres),} \\
 r_w &= 0.5 \text{ ft,} \\
 A_{wb} &= 0.036 \text{ sq ft,} \\
 \rho_o &= 54.8 \text{ lbm/cu ft,} \\
 q &= 1,135 \text{ STB/D (stabilized for several days),} \\
 B &= 1.214 \text{ bbl/STB, and} \\
 h &= 28 \text{ ft.}
 \end{aligned}$$

When the well was shut in for the buildup test, the liquid level rose in the wellbore as pressure increased. Data recorded during the buildup test are given in Table 2.11.

Determine (a) time at which afterflow distortion ceased; (b) time at which boundary effects begin; (c) formation permeability; (d) radius of investigation at beginning and end of MTR; (e) skin factor, Δp_s , and flow efficiency; (f) $\bar{p}$ using the MBH p^* method; and (g) $\bar{p}$ using the modified Muskat method.

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TABLE 2.11 – PRESSURE BUILDUP TEST DATA

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	2,752	10	4,272
0.3	3,464	12	4,280
0.5	3,640	14	4,287
1	3,852	16	4,297
2	4,055	20	4,303
3	4,153	24	4,308
4	4,207	30	4,313
5	4,244	36	4,317
6	4,251	42	4,320
8	4,263	50	4,322

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Chapter 3

Flow Tests

3.1 Introduction

This chapter discusses flow tests in wells, including constant-rate drawdown tests, continuously declining-rate drawdown tests, and multirate tests in infinite-acting reservoirs. The more general (and more complex) case of multirate tests in bounded reservoirs is discussed in Appendix E.

3.2 Pressure Drawdown Tests

A pressure drawdown test is conducted by producing a well, starting ideally with uniform pressure in the reservoir. Rate and pressure are recorded as functions of time.

The objectives of a drawdown test usually include estimates of permeability, skin factor, and, on occasion, reservoir volume. These tests are particularly applicable to (1) new wells, (2) wells that have been shut in sufficiently long to allow the pressure to stabilize, and (3) wells in which loss of revenue incurred in a buildup test would be difficult to accept. Exploratory wells are frequent candidates for lengthy drawdown tests, with a common objective of determining minimum or total volume being drained by the well.

An idealized constant-rate drawdown test in an infinite-acting reservoir is modeled by the logarithmic approximation to the *Ei*-function solution:

$$p_{wf} = p_i + 162.6 \frac{qB\mu}{kh} \left[\log \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) - 0.869 \right] \quad (3.1)$$

Like buildup tests, drawdown tests are more complex than suggested by simple equations such as Eq. 3.1. The usual test has an ETR, an MTR, and an LTR. The ETR usually is dominated by wellbore unloading: the rate at which fluid is removed from the wellbore exceeds the rate at which fluid enters the wellbore until, finally, equilibrium is established. Until that time, the constant flow rate at the sandface required by Eq. 3.1 is not achieved, and the straight-line plot of p_{wf} vs. $\log t$ suggested by Eq. 3.1 is not achieved. Duration of wellbore unloading can be

estimated by qualitative comparison of a log-log plot of $(p_i - p_{wf})$ vs. t with the solution of Fig. 1.6 or with the empirical equation based on that figure,

$$t_D \geq (60 + 3.5 s) C_{sD}, \quad (1.43)$$

or the equivalent form,

$$t_{wbs} = \frac{(200,000 + 12,000 s) C_s}{kh/\mu} \quad (3.2)$$

If the effective radius of the zone of altered permeability is unusually large (e.g., in a hydraulically fractured well), the duration of the ETR may depend on the time required for the radius of investigation to exceed the fracture half-length. (More exactly, for an infinite-conductivity vertical fracture with half-length L_f , shut-in time must exceed $1,260 \phi \mu c_t L_f^2/k$ or r_i must exceed $1.15 L_f$.)

The MTR begins when the ETR ends (unless boundaries or important heterogeneities are unusually near the well). In the MTR, a plot of p_{wf} vs. $\log t$ is a straight line with slope, m , given by

$$m = 162.6 \frac{qB\mu}{kh} \quad (3.3)$$

Thus, effective formation permeability, k , can be estimated from this slope:

$$k = 162.6 \frac{qB\mu}{mh} \quad (3.4)$$

After the MTR is identified, skin factor, s , can be determined. The usual equation results from solving Eq. 3.1 for s . Setting $t = 1$ hour, and letting $p_{wf} = p_{1 \text{ hr}}$ be the pressure on the MTR line at 1-hour flow time, the result is

$$s = 1.151 \left[\frac{(p_i - p_{1 \text{ hr}})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \quad (3.5)$$

The LTR begins when the radius of investigation reaches a portion of the reservoir influenced by

TABLE 3.1 – CONSTANT-RATE DRAWDOWN TEST DATA

t (hours)	p_{wf} (psia)	$p_i - p_{wf}$ (psia)	t (hours)	p_{wf} (psia)	$p_i - p_{wf}$ (psia)	t (hours)	p_{wf} (psia)	$p_i - p_{wf}$ (psia)
0	4,412	0	14.4	3,573	839	89.1	3,515	897
0.12	3,812	600	17.3	3,567	845	107	3,509	903
1.94	3,699	713	20.7	3,561	851	128	3,503	909
2.79	3,653	759	24.9	3,555	857	154	3,497	915
4.01	3,636	776	29.8	3,549	863	185	3,490	922
4.82	3,616	796	35.8	3,544	868	222	3,481	931
5.78	3,607	805	43.0	3,537	875	266	3,472	940
6.94	3,600	812	51.5	3,532	880	319	3,460	952
8.32	3,593	819	61.8	3,526	886	383	3,446	966
9.99	3,586	826	74.2	3,521	891	460	3,429	983

reservoir boundaries or massive heterogeneities. For a well centered in a square or circular drainage area, this occurs at a time given approximately by

$$t_{\ell t} \approx \frac{380 \phi \mu c_t A}{k}, \dots \dots \dots (3.6)$$

where A is the drainage area of the tested well. For more general drainage-area shapes, $t_{\ell t}$ can be calculated from the number in the column "Use Infinite System Solution With Less Than 1% Error for $t_{DA} <$ " in Table 1.2.¹ The dimensionless time t_{DA} is defined as

$$t_{DA} = \frac{0.000264 kt}{\phi \mu c_t A}.$$

For this more general case, then,

$$t_{\ell t} \approx \frac{3,800 \phi \mu c_t A t_{DA}}{k}. \dots \dots \dots (3.7)$$

Thus, the typical constant-rate drawdown test plot has the shape shown in Fig. 3.1. To analyze the typical test, the following steps are suggested.

1. Plot flowing BHP, p_{wf} , vs. flowing time, t , on semilog paper as shown in Fig. 3.1.
2. Estimate t_{wbs} from qualitative curve matching (with a full-size version of Fig. 1.6); this usually marks the beginning of the MTR (except for fractured wells).

3. Estimate the beginning of the LTR, $t_{\ell t}$, using deviation from a match with Fig. 1.6 to confirm deviation from an apparent semilog straight line. We must be cautious in drawdown test analysis, though. Even *small* rate changes can cause a drawdown curve to bend just as boundaries do (a method of analyzing this possibility is presented later).

4. Determine the slope m of the most probable MTR, and estimate formation permeability from Eq. 3.4.

5. Estimate the skin factor s from Eq. 3.5.

Example 3.1 – Constant-Rate Drawdown Test Analysis

Problem. The data in Table 3.1 were recorded during a constant-rate pressure drawdown test. The wellbore had a falling liquid/gas interface throughout the drawdown test. Other pertinent data include the following.

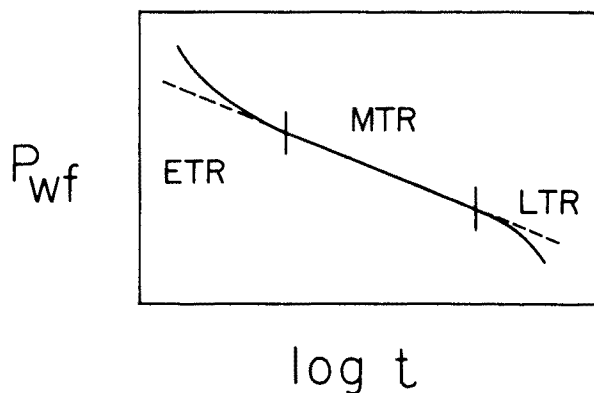


Fig. 3.1 – Typical constant-rate drawdown test graph.

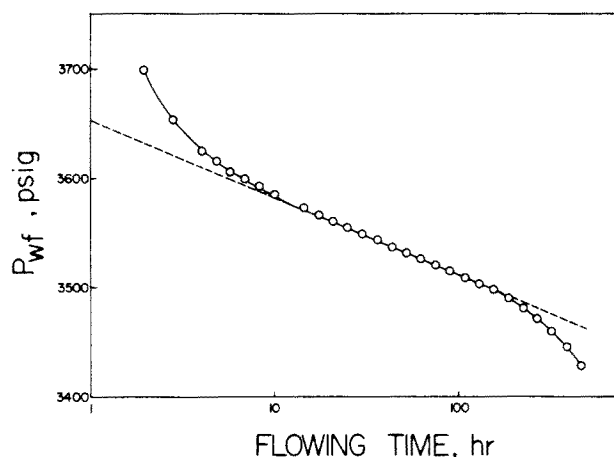


Fig. 3.2 – Semilog graph of example constant-rate drawdown test.

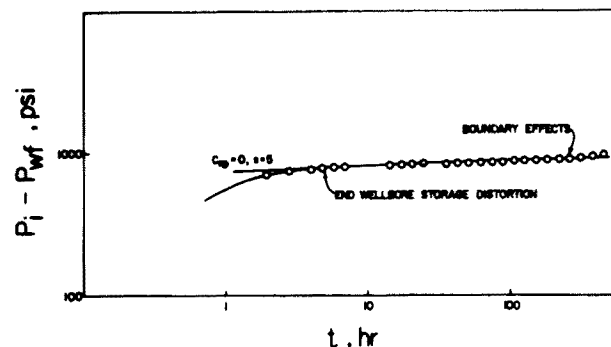


Fig. 3.3 – Log-log graph of example constant-rate drawdown test.

$$\begin{aligned}
 q &= 250 \text{ STB/D,} \\
 B &= 1.136 \text{ bbl/STB,} \\
 \mu &= 0.8 \text{ cp,} \\
 r_w &= 0.198 \text{ ft,} \\
 h &= 69 \text{ ft,} \\
 \phi &= 0.039, \text{ and} \\
 c_t &= 17 \times 10^{-6} \text{ psi}^{-1}.
 \end{aligned}$$

The tubing areas is 0.0218 sq ft; the density of the liquid in the wellbore is 53 lbm/cu ft. Determine the formation permeability and skin factor.

Solution. We first plot flowing BHP, p_{wf} , vs. time, t , on semilog paper and $(p_i - p_{wf})$ vs. t on log-log paper. Then we determine when wellbore effects ceased distorting the curve. From the shape of the semilog graph (Fig. 3.2), this appears to be at about 12 hours; however, we can check this assumption with the log-log graph, Fig. 3.3. For several values of C_D (e.g., 10^3 to 10^4), the graph shows a good fit with Fig. 1.6 for $s=5$; wellbore storage distortion ends at $\Delta t=5$ hours, in approximate agreement with the more sensitive semilog graph.

We have no information about the location of boundaries; therefore, we assume that boundary effects begin when the drawdown curve begins to deviate from the established straight line on the semilog graph at a flowing time of 150 hours. This is confirmed qualitatively on the less sensitive log-log graph by noticeable deviation beginning at $t \approx 260$ hours. The slope of the middle-time line is

$$\begin{aligned}
 m &= 3,652 - 3,582 \\
 &= 70 \text{ psi/cycle.}
 \end{aligned}$$

Thus, the permeability of the formation is

$$\begin{aligned}
 k &= 162.6 \frac{qB\mu}{mh} \\
 &= \frac{(162.6)(250)(1.136)(0.8)}{(70)(69)} \\
 &= 7.65 \text{ md.}
 \end{aligned}$$

We now check the radius of investigation at the beginning and end of the apparent middle-time line to ensure that we are sampling a representative portion of the formation.

At the beginning ($t = 12$ hours),

$$\begin{aligned}
 \frac{k}{948 \phi \mu c_t} &= \frac{(7.65)}{(948)(0.039)(0.8)(1.7 \times 10^{-5})} \\
 &= 1.521 \times 10^4,
 \end{aligned}$$

and, from Eq. 1.23,

$$\begin{aligned}
 r_i &= \sqrt{\frac{kt}{948 \phi \mu c_t}} \\
 &= \sqrt{(1.521 \times 10^4)(12)} \\
 &= 427 \text{ ft.}
 \end{aligned}$$

At the end of the MTR ($t \approx 150$ hours),

$$\begin{aligned}
 r_i &= \sqrt{(1.521 \times 10^4)(150)} \\
 &= 1,510 \text{ ft.}
 \end{aligned}$$

A substantial amount of formation has been sampled; thus, we can be more confident that the permeability of 7.65 md is representative.

We next calculate the skin factor s .

$$\begin{aligned}
 s &= 1.151 \left[\frac{p_i - p_{1 \text{ hr}}}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \\
 &= 1.151 \left[\frac{4,412 - 3,652}{70} \right. \\
 &\quad \left. - \log \frac{(1.442 \times 10^7)}{(0.198)^2} + 3.23 \right] \\
 &= 6.37.
 \end{aligned}$$

We now can verify more closely the expected end of wellbore storage distortion from Eq. 3.2, using

$$\begin{aligned}
 C_s &\cong \frac{25.65 A_{wb}}{\rho} \\
 &= 0.0106 \text{ bbl/psi.} \\
 t_{wbs} &= \frac{(200,000 + 12,000 s) C_s}{kh/\mu} \\
 &= \frac{[200,000 + (12,000)(6.37)](0.0106)}{(7.65)(69)10.8} \\
 &= 4.44 \text{ hours.}
 \end{aligned}$$

This closely agrees with the result from the log-log curve fit.

Another use of drawdown tests is to estimate reservoir pore volume, V_p . This is possible when the radius of investigation reaches all boundaries during a test so that pseudosteady-state flow is achieved. Eqs. 1.12 and 1.13 showed that, in pseudosteady-state flow, flowing BHP, p_{wf} , is related linearly to time and that the rate of change in p_{wf} with time is related to the reservoir pore volume. From Eq. 1.13, this relationship is

$$V_p = \frac{-0.234 qB}{c_t \left(\frac{\partial p_{wf}}{\partial t} \right)},$$

where $\partial p_{wf}/\partial t$ is simply the slope of the straight-line p_{wf} vs. t plot on ordinary Cartesian graph paper.

Even though Eqs. 1.12 and 1.13 were derived for a cylindrical reservoir centered at the wellbore of the tested well, the principles derived from them apply to all closed reservoir shapes. The graph of p_{wf} vs. t is a

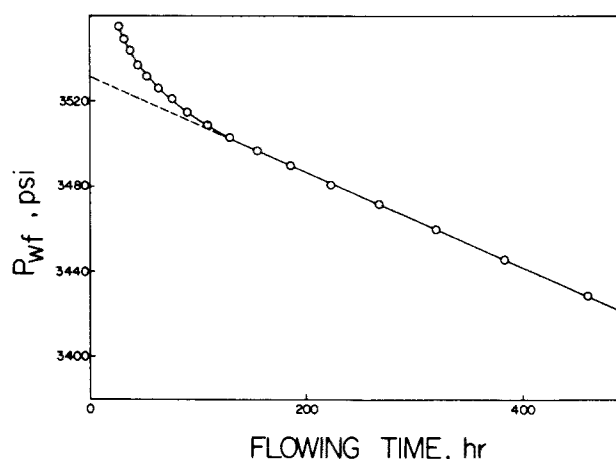


Fig. 3.4 – Cartesian-coordinate graph of example constant-rate drawdown test.

straight line once pseudosteady-state flow is achieved; the volume of the reservoir can be found from Eq. 1.13. It is important to remember, however, that these equations apply only to closed, or volumetric, reservoirs (i.e., they are not valid if there is water influx or gas-cap expansion). Further, they are limited to reservoirs in which total compressibility, c_t , is constant (and, specifically, independent of pressure).

We will illustrate pore-volume estimates with an example.

Example 3.2 – Estimation of Pore Volume

Problem. Estimate the pore volume of the reservoir with drawdown data reported in Example 3.1.

Solution. The first step is to plot p_{wf} vs. t (Fig. 3.4). The slope of this curve is constant for $t > 130$ hours; this slope, $\partial p_{wf} / \partial t$, is

$$\begin{aligned} \frac{\partial p_{wf}}{\partial t} &= \frac{3,531 - 3,420}{0 - 500} \\ &= -0.222 \text{ psi/hr.} \end{aligned}$$

Thus,

$$\begin{aligned} V_p &= \frac{-0.234 qB}{c_t \left(\frac{\partial p_{wf}}{\partial t} \right)} \\ &= \frac{(-0.234)(250)(1.136)}{(1.7 \times 10^{-5})(-0.222)} \\ &= 17.61 \times 10^6 \text{ cu ft} \\ &= 3.14 \times 10^6 \text{ res bbl.} \end{aligned}$$

The method of permeability determination outlined above applies only to drawdown tests conducted at a *strictly constant rate*. If rate varies during a test – e.g., if rate declines slowly throughout

the test – results obtained using those techniques can lead to interpretations that are seriously in error. An analysis method that leads to proper interpretation is available, but it can be used only if the producing rate is changing *slowly* and *smoothly*. Abrupt rate changes will make the drawdown test data impossible to interpret using either the method discussed earlier or this new method.*

Winestock and Colpitts² show that when rate is changing slowly and smoothly, the equation modeling the MTR of the drawdown test becomes

$$\begin{aligned} \frac{p_i - p_{wf}}{q} &= 162.6 \frac{\mu B}{kh} \left[\log \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) \right. \\ &\quad \left. + 0.869 s \right] + \text{negligible terms}^2. \quad \dots (3.8) \end{aligned}$$

The analysis technique is to plot $(p_i - p_{wf})/q$ vs. t on semilog paper;** identify the middle-time straight line; measure the slope m' in psi/STB/D/cycle; calculate kh from

$$kh = 162.6 \frac{\mu B}{m'},$$

and, finally, calculate s from

$$\begin{aligned} s &= 1.151 \left[\left(\frac{p_i - p_{wf}}{q} \right)_{1 \text{ hr}} \frac{1}{m'} \right. \\ &\quad \left. - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]. \quad \dots (3.9) \end{aligned}$$

In Eq. 3.8, $[(p_i - p_{wf})/q]_{1 \text{ hr}}$ is the value of this quantity on the middle-time line or its extrapolation at a flowing time of 1 hour.

We will illustrate use of this method with an example.

Example 3.3 – Analysis of Drawdown Test With Varying Rate

Problem. The data in Table 3.2 were obtained in a drawdown test in which the rate q was measured as a function of time. Other data include the following

$$\begin{aligned} B &= 1.136 \text{ bbl/STB,} \\ \mu &\Rightarrow 0.8 \text{ cp,} \\ h &= 69 \text{ ft,} \\ \rho &= 53 \text{ lb/cu ft,} \\ A_{wb} &= 0.0218 \text{ sq ft,} \\ \phi &= 0.039, \\ c_t &= 17 \times 10^{-6} \text{ psi}^{-1}, \text{ and} \\ r_w &= 0.198 \text{ ft.} \end{aligned}$$

Determine formation permeability and skin factor.

Solution. We note immediately that conventional drawdown test analysis, using an average rate, would

*Verification of this method is incomplete in cases with severe wellbore storage effects. A nonexhaustive numerical simulation study by this author has shown that the method yields essentially correct permeability and skin factor even in these cases.

**The same analysis technique, but for a different application (analyzing wellbore-storage-dominated data) was suggested earlier by Gladfelter *et al.*³ and Ramey.⁴

TABLE 3.2 – VARIABLE-RATE DRAWDOWN TEST DATA

t (hours)	p_{wf} (psi)	q (STB/D)	t (hours)	p_{wf} (psi)	q (STB/D)
0	4,412	250	8.32	3,927	147
0.105	4,332	180	9.99	3,928	145
0.151	4,302	177	14.4	3,931	143
0.217	4,264	174	20.7	3,934	140
0.313	4,216	172	29.8	3,937	137
0.450	4,160	169	43.0	3,941	134
0.648	4,099	166	61.8	3,944	132
0.934	4,039	163	74.2	3,946	130
1.34	3,987	161	89.1	3,948	129
1.94	3,952	158	107	3,950	127
2.79	3,933	155	128	3,952	126
4.01	3,926	152	154	3,954	125
5.78	3,926	150	185	3,956	123

be futile. Pressures for flow times greater than about 6 hours are *increasing* even though production continues for another 179 hours and even though the rate decline from this time to the end of the test is only 27 STB/D (from 150 to 123 STB/D).

Thus, we must use the variable-rate analysis technique; the first step is to tabulate $(p_i - p_{wf})/q$, as in Table 3.3. These data are plotted in Fig. 3.5. On the basis of curve shape, wellbore storage appears to end at approximately 6 hours; we will check this assumption with Eq. 3.2 when k and s have been estimated.

There is no deviation from the straight line for $t > 6$ hours; accordingly, we assume the MTR spans the time range $6 \text{ hours} < t < 185 \text{ hours}$.

From the plot, $m' = 3.616 - 3.328 = 0.288$ psi/STB/D/cycle. Then,

$$k = 162.6 \frac{\mu B}{m' h}$$

$$= \frac{(162.6)(0.8)(1.136)}{(0.288)(69)}$$

$$= 7.44 \text{ md,}$$

and

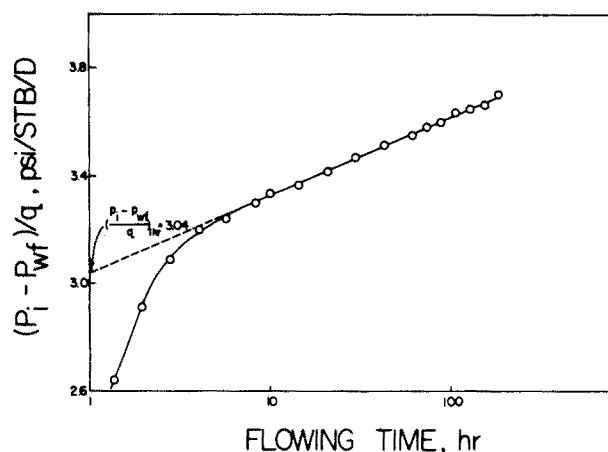


Fig. 3.5 – Example variable-rate drawdown test.

TABLE 3.3 – DATA FOR PLOTTING FROM VARIABLE-RATE DRAWDOWN TEST

t (hours)	$(p_i - p_{wf})/q$	t (hours)	$(p_i - p_{wf})/q$
0.105	0.444	8.32	3.299
0.151	0.621	9.99	3.338
0.217	0.851	14.4	3.364
0.313	1.140	20.7	3.414
0.450	1.491	29.8	3.467
0.648	1.886	43.0	3.515
0.934	2.288	61.8	3.545
1.34	2.640	74.2	3.585
1.94	2.911	89.1	3.597
2.79	3.090	107	3.638
4.01	3.197	128	3.651
5.78	3.240	154	3.664
		185	3.707

$$s = 1.151 \left[\left(\frac{p_i - p_{wf}}{q} \right)_{1 \text{ hr}} \left(\frac{1}{m'} \right) - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]$$

$$= 1.151 \left\{ \frac{3.04}{0.288} - \log \left[\frac{7.44}{(0.039)(0.8)(17 \times 10^{-6})(0.198)^2} \right] + 3.23 \right\}$$

$$= 6.02.$$

Since $C_s \cong 0.0106$ bbl/psi, as in Example 3.1,

$$t_{wbs} \cong \frac{(200,000 + 12,000 s) C_s}{kh/\mu}$$

$$= \frac{(200,000 + 12,000)(6.02)(0.0106)}{(7.44)(69)/0.8}$$

$$= 4.5 \text{ hours.}$$

This qualitatively confirms the choice of wellbore storage distortion end.

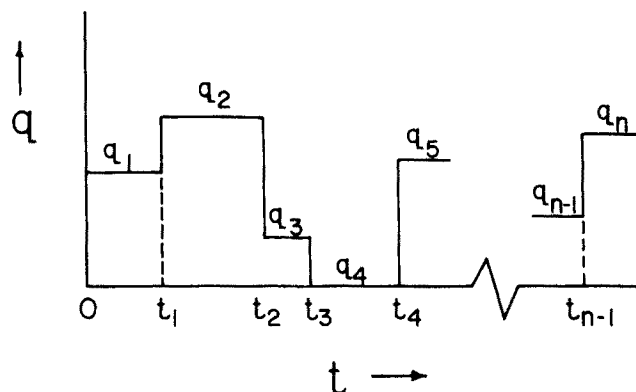


Fig. 3.6 – Rate history for multirate test.

3.3 Multirate Tests

We will develop a general theory for behavior of multirate tests in *infinite-acting* reservoirs for slightly compressible liquids. In Appendix E, we extend this general theory to reservoirs in which boundary effects may become important before the test ends.

Consider a well with n rate changes during its production history, as indicated in Fig. 3.6. Our objective is to determine the wellbore pressure of a well producing with this schedule. We will use superposition of the logarithmic approximation to the *Ei*-function solution; to simplify the algebra, we will write the solution as

$$\begin{aligned} p_i - p_{wf} &= 162.6 \frac{qB\mu}{kh} \left[\log \left(\frac{1,688 \phi \mu c_t r_w^2}{kt} \right) \right. \\ &\quad \left. - 0.869 s \right] \\ &= 162.6 \frac{qB\mu}{kh} \left(\log t + \log \frac{k}{\phi \mu c_t r_w^2} \right. \\ &\quad \left. - 3.23 + 0.869 s \right) \\ &= m' q (\log t + \bar{s}), \end{aligned}$$

where

$$m' = 162.6 \frac{\mu B}{kh},$$

and

$$\bar{s} = \log \frac{k}{\phi \mu c_t r_w^2} - 3.23 + 0.869 s.$$

With this nomenclature for n rates and for $t > t_{n-1}$, application of superposition (as in the discussion leading to Eq. 1.27) leads to

$$\begin{aligned} p_i - p_{wf} &= m' q_1 (\log t + \bar{s}) + m' (q_2 - q_1) \\ &\quad \cdot [\log (t - t_1) + \bar{s}] + m' (q_3 - q_2) \\ &\quad \cdot [\log (t - t_2) + \bar{s}] + \dots \\ &\quad + m' (q_n - q_{n-1}) [\log (t - t_{n-1}) + \bar{s}]. \end{aligned}$$

This can be written more compactly as

$$\begin{aligned} \frac{p_i - p_{wf}}{q_n} &= m' \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \\ &\quad \cdot \log (t - t_{j-1}) + m' \bar{s}, \quad q_n \neq 0 \quad \dots (3.10) \end{aligned}$$

In Eq. 3.10, we define $q_0 = 0$ and $t_0 = 0$. In terms of more fundamental quantities, Eq. 3.10 becomes

$$\begin{aligned} \frac{p_i - p_{wf}}{q_n} &= m' \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \\ &\quad \cdot \log (t - t_{j-1}) + m' \left[\log \left(\frac{k}{\phi \mu c_t r_w^2} \right) \right. \\ &\quad \left. - 3.23 + 0.869 s \right]. \quad \dots (3.11) \end{aligned}$$

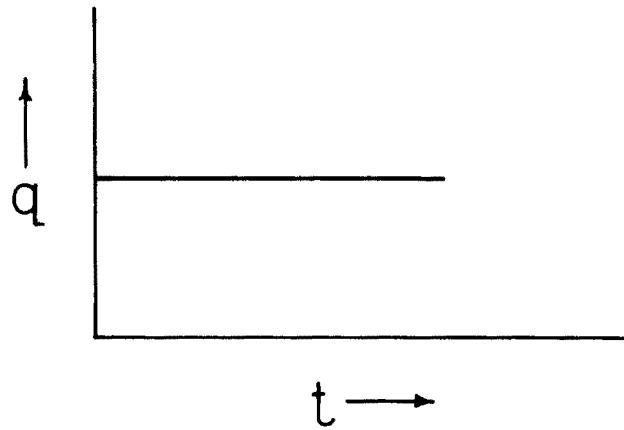


Fig. 3.7 – Rate history for single-rate drawdown test.

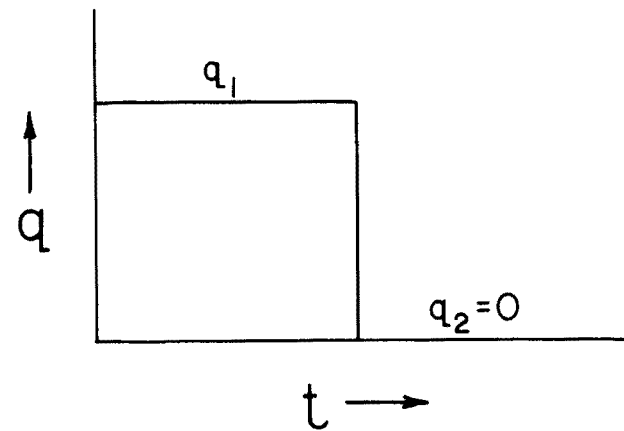


Fig. 3.8 – Rate history for buildup test following single flow rate.

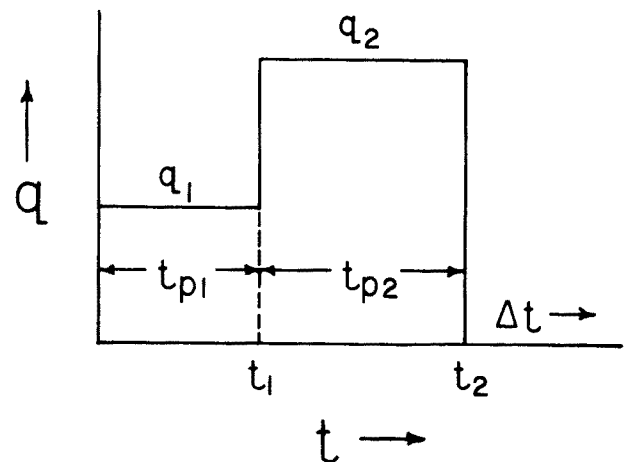


Fig. 3.9 – Rate history for buildup test following two different flow rates.

For the special case $q_n = 0$ (a pressure buildup test),

$$\begin{aligned}
 p_i - p_{ws} &= m' q_1 (\log t + \bar{s}) + m' (q_2 - q_1) \\
 &\quad \cdot [\log(t - t_1) + \bar{s}] + \dots + m' (q_{n-1} \\
 &\quad - q_{n-2}) [\log(t - t_{n-2}) + \bar{s}] - m' q_{n-1} \\
 &\quad \cdot [\log(t - t_{n-1}) + \bar{s}] \\
 &= 162.6 \frac{\mu B}{kh} \sum_{j=1}^n (q_j - q_{j-1}) \\
 &\quad \cdot \log(t - t_{j-1}). \dots \dots \dots (3.12)
 \end{aligned}$$

Eqs. 3.11 and 3.12 can be used to model several special cases of practical importance, but we must remember that they have an important limitation: the reservoir must be infinite acting for the total time elapsed t since the well began producing at rate q_1 . Appendix E suggests a more general method of modeling tests in which boundaries have been reached.

Pressure Drawdown Test

From Eq. 3.11, for $n = 1$ (Fig. 3.7),

$$\begin{aligned}
 \frac{p_i - p_{wf}}{q_1} &= 162.6 \frac{\mu B}{kh} \left(\log t + \log \frac{k}{\phi \mu c_t r_w^2} \right. \\
 &\quad \left. - 3.23 + 0.869 s \right). \dots \dots \dots (3.13)
 \end{aligned}$$

Pressure Buildup Test Preceded by Constant-Rate Production

From Eq. 3.12 for $n = 2$ (Fig. 3.8),

$$\begin{aligned}
 p_i - p_{ws} &= 162.6 \frac{\mu B}{kh} [q_1 \log t - q_1 \log(t - t_1)] \\
 &= 162.6 \frac{q_1 \mu B}{kh} \log\left(\frac{t}{t - t_1}\right). \dots \dots \dots (3.14)
 \end{aligned}$$

If we let $q_1 = q$, $t - t_1 = \Delta t$, and $t_1 = t_p$, the familiar equation that serves as the basis for the Horner plot results:

$$p_i - p_{ws} = 162.6 \frac{qB\mu}{kh} \log\left(\frac{t_p + \Delta t}{\Delta t}\right).$$

Pressure Buildup Test Preceded by Two Different Flow Rates

From Eq. 3.12 (Fig. 3.9),

$$\begin{aligned}
 p_i - p_{ws} &= m' [q_1 \log t + (q_2 - q_1) \log(t - t_1) \\
 &\quad - q_2 \log(t - t_2)].
 \end{aligned}$$

This can be written as

$$\begin{aligned}
 p_i - p_{ws} &= 162.6 \frac{q_2 B \mu}{kh} \left[\frac{q_1}{q_2} \log\left(\frac{t}{t - t_1}\right) \right. \\
 &\quad \left. + \log\left(\frac{t - t_1}{t - t_2}\right) \right].
 \end{aligned}$$

Let $t - t_2 = \Delta t$, $t_1 = t_{p1}$, $t_2 = t_{p1} + t_{p2}$, and $t - t_1 = t_{p2} + \Delta t$. Then,

$$\begin{aligned}
 p_i - p_{ws} &= 162.6 \frac{q_2 B \mu}{kh} \left[\frac{q_1}{q_2} \log\left(\frac{t_{p1} + t_{p2} + \Delta t}{t_{p2} + \Delta t}\right) \right. \\
 &\quad \left. + \log\left(\frac{t_{p2} + \Delta t}{\Delta t}\right) \right]. \dots \dots \dots (3.15)
 \end{aligned}$$

Eq. 3.15 has this application: when the producing rate is changed a short time before a buildup test begins, so that there is not sufficient time for Horner's approximation to be valid, we frequently can consider all production before time t_1 to have been at rate q_1 for time t_{p1} and production just before the test to have been at rate q_2 for time t_{p2} .

To analyze such a test, we plot

$$p_{ws} \text{ vs. } \left[\frac{q_1}{q_2} \log\left(\frac{t_{p1} + t_{p2} + \Delta t}{t_{p2} + \Delta t}\right) + \log\left(\frac{t_{p2} + \Delta t}{\Delta t}\right) \right].$$

The slope m of this plot is related to formation permeability by the equation

$$m = 162.6 \frac{q_2 B \mu}{kh}.$$

Extrapolation of the plot to $\Delta t = \infty$ gives $p_{ws} = p_i$ because, at $\Delta t = \infty$, the plotting function is zero. *Note that semilog paper is not to be used; instead, the sum of two logarithms is plotted on an ordinary Cartesian axis.*

To calculate skin factor, s , note that at the end of the flow period just before shut-in,

$$\begin{aligned}
 p_i - p_{ws} &= 162.6 \frac{q_2 B \mu}{kh} \left[\frac{q_1}{q_2} \log\left(\frac{t_{p1} + t_{p2}}{t_{p2}}\right) \right. \\
 &\quad \left. + \log(t_{p2}) + \bar{s} \right],
 \end{aligned}$$

or

$$p_i - p_{wf} = m \left[\frac{q_1}{q_2} \log\left(\frac{t_{p1} + t_{p2}}{t_{p2}}\right) + \log(t_{p2}) + \bar{s} \right],$$

The equation of the MTR line on the buildup test plot is

$$\begin{aligned}
 p_i - p_{ws} &= m \left[\frac{q_1}{q_2} \log\left(\frac{t_{p1} + t_{p2} + \Delta t}{t_{p2} + \Delta t}\right) \right. \\
 &\quad \left. + \log\left(\frac{t_{p2} + \Delta t}{\Delta t}\right) \right].
 \end{aligned}$$

Subtracting,

$$\begin{aligned}
 p_{ws} - p_{wf} &= m \left\{ \frac{q_1}{q_2} \log\left[\frac{(t_{p1} + t_{p2})(t_{p2} + \Delta t)}{(t_{p1} + t_{p2} + \Delta t)(t_{p2})}\right] \right. \\
 &\quad \left. + \log\left[\frac{(t_{p2})(\Delta t)}{t_{p2} + \Delta t}\right] + \bar{s} \right\}.
 \end{aligned}$$

Assume $t_{p1} + t_{p2} + \Delta t \approx t_{p1} + t_{p2}$ and $t_{p2} + \Delta t \approx t_{p2}$ for small Δt (e.g., $\Delta t = 1$ hour). Then,

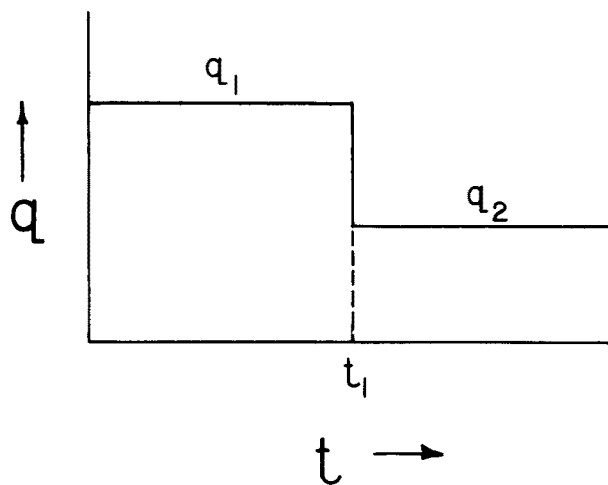


Fig. 3.10 – Rate history for two-rate flow test.

$$p_{ws} - p_{wf} = m(\log \Delta t + \bar{s}).$$

If we choose $\Delta t = 1$ hour, $p_{ws} = p_{1 \text{ hr}}$ (on the MTR line) and, for $t_{p2} \gg 1$,

$$\bar{s} = \frac{p_{1 \text{ hr}} - p_{wf}}{m}$$

$$= \log \frac{k}{\phi \mu c_t r_w^2} - 3.23,$$

or

$$s = 1.151 \left(\frac{p_{1 \text{ hr}} - p_{wf}}{m} - \log \frac{k}{\phi \mu c_t r_w^2} + 3.23 \right),$$

as before.

We also note that duration of wellbore storage distortion is calculated as in the previous analysis for buildup tests.

Pressure Buildup Test Preceded by $(n-1)$ Different Flow Rates

From Eq. 3.12,

$$\begin{aligned} p_i - p_{ws} = 162.6 \frac{q_{n-1} \mu B}{kh} & \left[\frac{q_1}{q_{n-1}} \log \left(\frac{t}{t-t_1} \right) \right. \\ & + \frac{q_2}{q_{n-1}} \log \left(\frac{t-t_1}{t-t_2} \right) + \dots + \frac{q_{n-2}}{q_{n-1}} \\ & \cdot \log \left(\frac{t-t_{n-3}}{t-t_{n-2}} \right) + \log \left(\frac{t-t_{n-2}}{t-t_{n-1}} \right) \Big] \\ & \dots \dots \dots (3.16) \end{aligned}$$

Although we introduce no specialized nomenclature for this situation, note that $t-t_{n-1} = \Delta t$ (time elapsed since shut-in) and that q_{n-1} is the production rate just before shut-in.

Applications of Eq. 3.16 in which more than three terms are needed are probably rare; sometimes, though, to satisfy precise legal contracts (e.g., gas

deliverability contracts), all significant rates are considered. Improvement in accuracy when using this approach is questionable; further, the fundamental assumption on which Eq. 3.16 is based (that for $t = t_{p1} + t_{p2} + \dots + t_{p,n-1} + \Delta t$ the reservoir is infinite acting) rarely will be valid for large values of t . Nevertheless, when Eq. 3.16 is used to model a buildup test, the following analysis procedure can be used.

1. Write a computer program to calculate the plotting function,

$$\left[\frac{q_1}{q_{n-1}} \log \left(\frac{t}{t-t_1} \right) + \dots + \log \left(\frac{t-t_{n-2}}{t-t_{n-1}} \right) \right] = X.$$

2. Plot p_{ws} vs. X on ordinary (Cartesian coordinate) graph paper.

3. Determine the slope m of the plot and relate to the formation permeability by the equation

$$m = 162.6 \frac{q_{n-1} B \mu}{kh}.$$

4. Calculate the skin factor s from the equation

$$s = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right].$$

(The derivation and assumptions implicit in this equation closely parallel those used for a buildup test preceded by two different flow rates.)

5. The original formation pressure p_i is the value of p_{ws} on the MTR line extrapolated to $X = 0$.

Two-Rate Flow Test

From Eq. 3.11, this test (Fig. 3.10) can be modeled⁵ by

$$\begin{aligned} p_i - p_{wf} = 162.6 \frac{q B_2 \mu}{kh} & \left[\frac{q_1}{q_2} \log t + \frac{(q_2 - q_1)}{q_2} \right. \\ & \cdot \log(t-t_1) + \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) \\ & \left. - 3.23 + 0.869 s \right] \dots \dots \dots (3.17) \end{aligned}$$

If we rearrange and introduce specialized nomenclature, $t_1 = t_{p1}$ and $t-t_{p1} = \Delta t'$, then Eq. 3.17 becomes

$$\begin{aligned} p_{wf} = p_i - 162.6 \frac{q_2 B \mu}{kh} & \left[\log \left(\frac{k}{\phi \mu c_t r_w^2} \right) - 3.23 \right. \\ & \left. + 0.869 s \right] - 162.6 \frac{q_1 B \mu}{kh} \left[\log \left(\frac{t_{p1} + \Delta t'}{\Delta t'} \right) \right. \\ & \left. + \frac{q_2}{q_1} \log(\Delta t') \right] \dots \dots \dots (3.18) \end{aligned}$$

This type of test can be used when estimates of permeability, skin factor, or reservoir pressure are needed but when the well cannot be shut in because loss of income cannot be tolerated. This test shares a

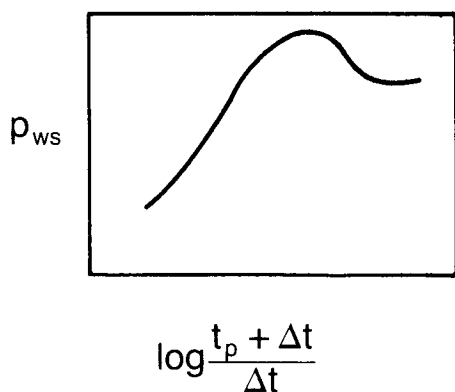


Fig. 3.11 – Buildup test with pressure humping.

fundamental analysis problem with the conventional drawdown test. The second rate must be kept *strictly constant* or the test interpretation may be substantially in error.

Eq. 3.18 is rigorously correct only when the reservoir is infinite acting for time $(t_{p1} + \Delta t')$ [just as the Horner plot is rigorous only when a reservoir is infinite acting for time $(t_p + \Delta t)$]. Nevertheless, application of the plotting and analysis technique suggested by Eq. 3.18 allows identification of the MTR and determination of formation permeability in a finite-acting reservoir.

The two-rate flow test does not reduce the duration of wellbore storage distortion – the duration of this distortion is essentially the same as in any buildup or drawdown test. However, the test procedure may minimize the effects of phase segregation in the wellbore, an extreme form of which is “pressure humping,”⁶ where high-pressure gas trapped in a wellbore in poor communication with a formation may lead to pressures in the wellbore higher than formation pressures (Fig. 3.11). A buildup test with this humping is, at best, difficult to interpret; thus, a test procedure that can minimize these phase-segregation effects can be of value.

The following method of analysis can be used for two-rate flow tests.

TABLE 3.4 – TWO-RATE FLOW TEST DATA

$\Delta t'$ (hours)	p_{wf} (psi)	$\Delta t'$ (hours)	p_{wf} (psi)
0	3,490	8.32	3,897
0.105	3,543	12.0	3,903
0.151	3,564	17.3	3,908
0.217	3,592	24.9	3,912
0.313	3,627	35.8	3,915
0.450	3,669	51.5	3,918
0.648	3,717	74.2	3,919
0.934	3,766	89.1	3,918
1.344	3,810	107	3,917
1.936	3,846	128	3,916
2.788	3,868	154	3,913
4.01	3,882	184.7	3,910
5.78	3,891		

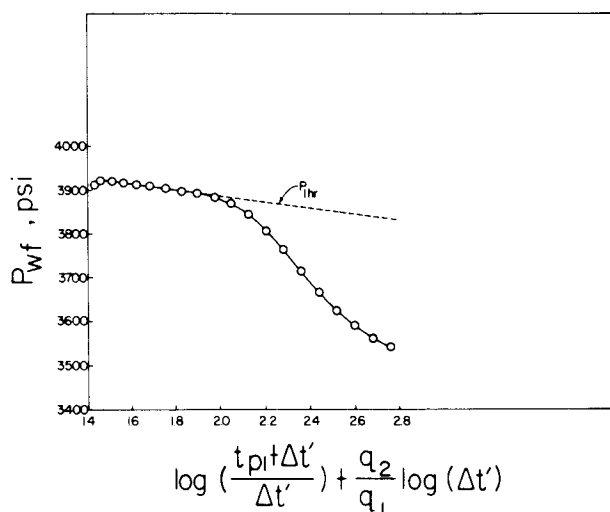


Fig. 3.12 – Example two-rate flow test.

$$1. \text{ Plot } p_{wf} \text{ vs. } \left[\log \left(\frac{t_{p1} + \Delta t'}{\Delta t'} \right) + \frac{q_2}{q_1} \log(\Delta t') \right].$$

2. Determine the slope m from the plot and use it to calculate permeability, k , from the relationship

$$k = 162.6 \frac{q_1 B \mu}{m h}.$$

3. Calculate the skin factor, s , from the equation

$$s = 1.151 \left[\frac{q_1}{(q_1 - q_2)} \left(\frac{p_{1 \text{ hr}} - p_{wf1}}{m} \right) - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]. \quad (3.19)$$

In Eq. 3.19, $p_{1 \text{ hr}}$ is the flowing pressure at $\Delta t' = 1$ hour on the MTR line and p_{wf1} is the flowing pressure at the time the rate is changed ($\Delta t' = 0$). (Eq. 3.19 was derived by simultaneous solution of Eqs. 3.18 and the drawdown equation for a single rate applied at $t = t_{p1}$, at which time $p_{wf} = p_{wf1}$.)

4. p_i (or, more generally, p^*) is obtained by solving for p_i (p^*) from the drawdown equation written to model conditions at the time of the rate change. (It is implied that s and m are known at this point.)

TABLE 3.5 – DATA FOR PLOTTING FROM TWO-RATE FLOW TEST

$\Delta t'$ (hours)	PF	p_{wf} (psi)	$\Delta t'$ (hours)	PF	p_{wf} (psi)
0	—	3,490	8.32	1.826	3,897
0.105	2.756	3,543	12.0	1.754	3,903
0.151	2.677	3,564	17.3	1.686	3,908
0.217	2.599	3,592	24.9	1.623	3,912
0.313	2.519	3,627	35.8	1.566	3,915
0.450	2.441	3,669	51.5	1.517	3,918
0.648	2.362	3,717	74.2	1.478	3,919
0.934	2.283	3,766	89.1	1.462	3,918
1.34	2.206	3,810	107	1.450	3,917
1.94	2.127	3,846	128	1.442	3,916
2.79	2.050	3,868	154	1.436	3,913
4.01	1.974	3,882	184.7	1.434	3,910
5.78	1.899	3,891			

$$p_i = p_{wf1} + m \left[\log \left(\frac{kt p_1}{\phi \mu c_t r_w^2} \right) - 3.23 + 0.869 s \right].$$

..... (3.20)

Example 3.4 – Two-Rate Flow Test

Problem. A two-rate flow test was run on a well with properties given below. From these properties and the data in Table 3.4, determine k , s , and p^* .

$$\begin{aligned} q_1 &= 250 \text{ STB/D,} \\ q_2 &= 125 \text{ STB/D,} \\ \mu &= 0.8 \text{ cp,} \\ B &= 1.136 \text{ RB/STB,} \\ p_i &= 4,412 \text{ psi,} \\ c_t &= 17 \times 10^{-6} \text{ psi}^{-1}, \\ A_{wb} &= 0.0218 \text{ sq ft,} \\ r_w &= 0.198 \text{ ft,} \\ h &= 69 \text{ ft,} \\ \rho &= 53 \text{ lb/cu ft,} \\ \phi &= 0.039, \text{ and} \\ t_{p1} &= 184.7 \text{ hours.} \end{aligned}$$

Solution.

1. We first tabulate the plotting function (PF),

$$PF = \left[\log \left(\frac{t_{p1} + \Delta t'}{\Delta t'} \right) + \frac{q_2}{q_1} \log(\Delta t') \right],$$

and plot p_{wf} vs. PF. Note that $t_{p1} = 184.7$ hours and $q_2/q_1 = 125/250 = 0.5$ (Table 3.5). The data are plotted in Fig. 3.12.

2. Next, we determine permeability. Assume that the MTR spans the time range $1.5 < PF < 1.9$ (50 hours $> \Delta t' > 6$ hours). Then the slope $m = (3,927 - 3,857)/(1.4 - 2.4) = 70$ psi/cycle and

$$\begin{aligned} k &= 162.6 \frac{q_1 B \mu}{m h} \\ &= \frac{(162.6)(250)(1.136)(0.8)}{(70)(69)} \end{aligned}$$

$$= 7.65 \text{ md.}$$

3. We determine skin factor,

$$\begin{aligned} s &= 1.151 \left[\frac{q_1}{(q_1 - q_2)} \frac{(p_{1 \text{ hr}} - p_{wf1})}{m} \right. \\ &\quad \left. - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right], \end{aligned}$$

at $\Delta t' = 1$ hour, $PF = \log(184.7 + 1)/1 + 0.5 \log(1) = 2.269$, and $p_{1 \text{ hr}} = 3,869$ psi (on the MTR line):

$$\begin{aligned} s &= 1.151 \left\{ \frac{250}{(250 - 125)} \frac{(3,869 - 3,490)}{70} \right. \\ &\quad \left. - \log \left[\frac{7.65}{(0.039)(0.8)(17 \times 10^{-6})(0.198)^2} \right] + 3.23 \right\} \\ &= 6.32. \end{aligned}$$

4. Determine p^* .

$$\begin{aligned} p^* &= p_{wf1} + m \left[\log \left(\frac{kt p_1}{\phi \mu c_t r_w^2} \right) - 3.23 + 0.869 s \right] \\ &= 3,490 + 70 \left\{ \log \left[(7.65)(184.7)/(0.039)(0.8) \right. \right. \\ &\quad \left. \left. \cdot (17 \times 10^{-6})(0.198)^2 \right] \right. \\ &\quad \left. - 3.23 + (0.869)(6.32) \right\} \\ &= 4,407 \text{ psi.} \end{aligned}$$

5. We then check on wellbore-storage duration. For this well, $C_s \cong 25.65 A_{wb}/p = (25.65)(0.0218)/53 = 0.0106$ bbl/psi. Then,

$$\begin{aligned} t_{wbs} &\cong \frac{(200,000 + 12,000 s) C_s}{kh/\mu} \\ &= \frac{[200,000 + (12,000)(6.32)](0.0106)}{(7.65)(69)/0.8} \\ &= 4.4 \text{ hours.} \end{aligned}$$

At this time, the plotting function is approximately 1.9; this confirms our choice of the start of the MTR.

n-Rate Flow Test

From Eq. 3.11, an n -rate flow test is modeled by

$$\begin{aligned} \frac{p_i - p_{wf}}{q_n} &= 162.6 \frac{\mu B}{kh} \left[\sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \right. \\ &\quad \left. \cdot \log(t_n - t_{j-1}) \right] + 162.6 \frac{\mu B}{kh} \\ &\quad \cdot \left[\log \left(\frac{k}{\phi \mu c_t r_w^2} \right) - 3.23 + 0.869 s \right]. \end{aligned}$$

This equation suggests a plot of

$$\frac{p_i - p_{wf}}{q_n} \text{ vs. } \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t_n - t_{j-1}).$$

Permeability is related to the slope m' of such a plot:

$$k = 162.6 \frac{\mu B}{m' h}.$$

If we let b' be the value of $(p_i - p_{wf})/q_n$ when the plotting function is zero, then

$$b' = m' \left[\log \left(\frac{k}{\phi \mu c_t r_w^2} \right) - 3.23 + 0.869 s \right],$$

or

$$s = 1.151 \left[\frac{b'}{m'} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]. \quad \dots (3.21)$$

Note that use of Eq. 3.21 and of the proposed

TABLE 3.6 – MULTIRATE FLOW TEST DATA

t (hours)	p_{wf} (psia)
0	3,000
0.333	999
0.667	857
1.0	778.5
2.0	1,378.5
2.333	2,043
2.667	2,067.5
3.0	2,094

t (hours)	q_n (STB/D)
0	—
0.333	478.5
0.667	478.5
1.0	478.5
2.0	319.0
2.333	159.5
2.667	159.5
3.0	159.5

TABLE 3.7 – DATA FOR PLOTTING MULTIRATE FLOW TEST

$p_i - p_{wf}$ (psia)	$\frac{p_i - p_{wf}}{q_n}$ (psia/RB/D)	$\sum_{j=1}^n \frac{q_j - q_{j-1}}{q_n} \cdot \log(t_j - t_{j-1})$
—	—	—
2,001	4.18	-0.478
2,143	4.48	-0.176
2,221.5	4.64	0
1,621.5	5.08	0.452
957	6.00	1.459
923.5	5.79	1.232
906	5.68	1.130

plotting method implies that p_i is known from independent measurements.

Odeh and Jones⁷ discussed this analysis technique. They pointed out that it can be applied to the analysis of multirate flow tests commonly run on gas wells and oil wells. In these applications of the technique, it is essential to remember the assumption that the reservoir is infinite acting to the total elapsed time t for all flow rates combined. Further, note that the technique ignores any wellbore storage distortion created by any discrete rate changes.

Example 3.6 – Multirate Flow Test Analysis

Problem. Odeh and Jones⁷ present data from a 3-hour drawdown test on an oil well; in this test, the rate during the first hour averaged 478.5 STB/D; during the second hour, 319 STB/D; and during the third hour, 159.5 STB/D. Reservoir fluid viscosity is 0.6 cp; initial pressure is 3,000 psia; formation volume factor, B , is considered to be 1.0; and the reservoir is assumed to be infinite acting for the entire test. Assume that wellbore storage distortion is minimal at all times during the test. Pressures (p_{wf}) at various flow times are given in Table 3.6. From these data, determine the permeability/thickness product of the tested well.

Solution. We first prepare the data for plotting – i.e., at each time, we must determine $(p_i - p_{wf})/q_n$ and

$$\sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t_n - t_{j-1}).$$

$t = 0.333$ hour. Here,

$$q_n = q_1 = 478.5 \text{ STB/D},$$

$$t_n = 0.333 \text{ hour},$$

$$(p_i - p_{wf})/q_n = (3,000 - 999)/478.5 = 4.18, \text{ and}$$

$$\begin{aligned} \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t_n - t_{j-1}) \\ = \frac{(478.5 - 0)}{478.5} \log(0.333 - 0) = -0.478 \end{aligned}$$

$t = 1.0$ hour. Here,

$$q_n = 478.5 \text{ STB/D},$$

$$t_j = 1.0 \text{ hour},$$

$$(p_i - p_{wf})/q_n = (3,000 - 778.5)/478.5 = 4.64, \text{ and}$$

$$\begin{aligned} \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t_n - t_{j-1}) \\ = \frac{(478.5 - 0)}{478.5} \log(1.0 - 0) = 0. \end{aligned}$$

$t = 3.0$ hours. Here,

$$(p_i - p_{wf})/q_n = (3,000 - 2,094)/159.5 = 5.68, \text{ and}$$

$$\begin{aligned} \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t_n - t_{j-1}) \\ = \frac{1}{159.5} [478.5 \log(3.0 - 0) + (319 - 478.5) \\ \cdot \log(3.0 - 1.0) + (159.5 - 319) \\ \cdot \log(3.0 - 2.0)] \\ = 1.130. \end{aligned}$$

Calculations at these and other times are summarized in Table 3.7.

We next estimate the permeability/thickness product. From Fig. 3.13,

$$m' = \frac{(6.0 - 4.2)}{[1.459 - (-0.452)]} = 0.942.$$

Then,

$$\begin{aligned} kh = 162.6 \frac{\mu B}{m'} = \frac{(162.6)(0.6)(1.0)}{0.942} \\ = 104 \text{ md-ft}. \end{aligned}$$

Odeh and Jones do not state values for h , ϕ , c_t , and r_w used to construct these example test data. To illustrate skin-factor calculation, assume that $h = 10$ ft and, thus, that $k = 10.4$ md. Also assume that $k/\phi\mu c_t r_w^2 = 4.81 \times 10^7$. Then, since the graph indicated that $b' = 4.63$ [b' is the value of $(p_i - p_{wf})/q_n$ when the plotting function is zero],

$$\begin{aligned}
 s &= 1.151 \left[\frac{b'}{m'} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \\
 &= 1.151 \left[\frac{4.63}{0.942} - \log(4.8 \times 10^7) + 3.23 \right] \\
 &= 0.53.
 \end{aligned}$$

Exercises

3.1. A constant-rate drawdown test was run in a well with the following characteristics:

$$\begin{aligned}
 q &= 500 \text{ STB/D (constant),} \\
 \phi &= 0.2, \\
 \mu &= 0.8 \text{ cp,} \\
 c_t &= 10 \times 10^{-6} \text{ psi}^{-1}, \\
 r_w &= 0.3 \text{ ft,} \\
 h &= 56 \text{ ft,} \\
 B_o &= 1.2 \text{ RB/STB,} \\
 A_{wb} &= 0.022 \text{ sq ft,} \\
 \rho &= 50 \text{ lb/cu ft, and} \\
 &\text{liquid/gas interface is in well.}
 \end{aligned}$$

From the test data in Table 3.8, estimate formation permeability, skin factor, and area (in acres) drained by the well.

3.2. A drawdown test in which the rate decreased continuously throughout the test was run in a well with the following characteristics.

$$\begin{aligned}
 \phi &= 0.2, \\
 \mu &= 1.0 \text{ cp,} \\
 c_t &= 10 \times 10^{-6} \text{ psi}^{-1}, \\
 r_w &= 0.25 \text{ ft,} \\
 h &= 100 \text{ ft,} \\
 B_o &= 1.3 \text{ RB/STB,} \\
 A_{wb} &= 0.0218 \text{ sq ft,} \\
 \rho &= 55 \text{ lb/cu ft, and} \\
 &\text{liquid/gas interface is in well}
 \end{aligned}$$

From the test data in Table 3.9, estimate formation permeability and skin factor.

3.3. A constant-rate drawdown test was run on the well described in Problem 3.2. Rate was held constant at 600 STB/D. After 96.9 hours, the surface rate was changed abruptly to 300 STB/D. Data for the tests before and after the rate change are given in Table 3.10. From the data obtained with $q = 600$ STB/D, estimate k , s , and area of the reservoir. From the data obtained in the two-rate flow test after q changed to 300 STB/D, confirm the estimates of k and s , and calculate the current value of p^* .

3.4. For the multirate flow test described in Example 3.6, (a) calculate the value of the plotting function at $t = 0.5$, 1.5, and 2.5 hours, and (b) calculate the flowing bottomhole pressure at $t = 3.5$ hours, assuming that there is no change in rate for 3 hours $< t < 4$ hours.

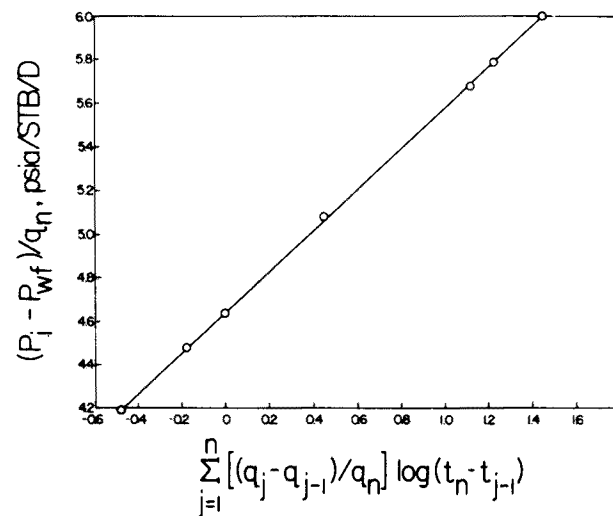


Fig. 3.13 – Example multirate flow test.

TABLE 3.8 – DATA FOR EXAMPLE CONSTANT-RATE DRAWDOWN TEST

t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)
0	3,000	0.491	2,302	32.8	1,543
0.0109	2,976	0.546	2,256	38.2	1,533
0.0164	2,964	1.09	1,952	43.7	1,525
0.0218	2,953	1.64	1,828	49.1	1,517
0.0273	2,942	2.18	1,768	54.6	1,511
0.0328	2,930	2.73	1,734	65.5	1,500
0.0382	2,919	3.28	1,712	87.4	1,482
0.0437	2,908	3.82	1,696	109.2	1,468
0.0491	2,897	4.37	1,684	163.8	1,440
0.0546	2,886	4.91	1,674	218.4	1,416
0.109	2,785	5.46	1,665	273.0	1,393
0.164	2,693	6.55	1,651	327.6	1,370
0.218	2,611	8.74	1,630		
0.273	2,536	10.9	1,587		
0.328	2,469	16.4	1,568		
0.437	2,352	27.3	1,554		

TABLE 3.9 – DATA FOR EXAMPLE VARIABLE-RATE DRAWDOWN TEST

t (hours)	p_{wf} (psi)	q (STB/D)	t (hours)	p_{wf} (psi)	q (STB/D)
0	5,000	200	3.03	4,797	122
0.114	4,927	145	3.64	4,797	121
0.136	4,917	143	4.37	4,798	119
0.164	4,905	142	5.24	4,798	118
0.197	4,893	141	6.29	4,798	117
0.236	4,881	140	7.54	4,799	116
0.283	4,868	138	9.05	4,799	114
0.340	4,856	137	10.9	4,800	113
0.408	4,844	136	13.0	4,801	112
0.490	4,833	135	15.6	4,801	110
0.587	4,823	133	18.8	4,802	109
0.705	4,815	132	22.5	4,803	108
0.846	4,809	131	27.0	4,803	107
1.02	4,804	129	32.4	4,804	105
1.22	4,801	128	38.9	4,805	104
1.46	4,799	127	46.7	4,806	103
1.75	4,798	126	56.1	4,807	102
2.11	4,797	124	67.3	4,807	100
2.53	4,797	123	80.7	4,808	99
			96.9	4,809	98

TABLE 3.10 – DATA FOR EXAMPLE TWO-RATE FLOW TEST

t (hours)	p_{wf} at 600 STB/D (psi)	p_{wf} at 300 STB/D (psi)	t (hours)	p_{wf} at 600 STB/D (psi)	p_{wf} at 300 STB/D (psi)
0	5,000	3,833	3.03	4,012	4,324
0.114	4,710	3,978	3.64	4,002	4,330
0.136	4,665	4,000	4.37	3,992	4,334
0.164	4,616	4,025	5.24	3,982	4,339
0.197	4,563	4,051	6.29	3,972	4,343
0.236	4,507	4,079	7.54	3,963	4,346
0.283	4,449	4,108	9.05	3,953	4,350
0.340	4,390	4,138	10.9	3,944	4,353
0.408	4,332	4,167	13.0	3,935	4,356
0.490	4,277	4,194	15.6	3,926	4,359
0.587	4,227	4,219	18.8	3,918	4,361
0.705	4,182	4,242	22.5	3,909	4,363
0.846	4,144	4,261	27.0	3,900	4,364
1.02	4,112	4,276	32.4	3,891	4,365
1.22	4,087	4,289	38.9	3,883	4,365
1.46	4,067	4,299	46.7	3,874	4,364
1.75	4,050	4,307	56.1	3,865	4,362
2.11	4,036	4,314	67.3	3,855	4,359
2.53	4,024	4,319	80.7	3,845	4,354
			96.9	3,833	4,349

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Chapter 4

Analysis of Well Tests Using Type Curves

4.1 Introduction

This chapter discusses the quantitative use of type curves in well test analysis. The objective of this chapter is limited basically to illustrating how a representative sample of type curves can be used as analysis aids. Other major type curves in use today are discussed in the SPE well testing monograph.¹ However, type curves for specialized situations are appearing frequently in the literature, and even that monograph is not completely current. We hope that the fundamentals of type-curve use presented in this chapter will allow the reader to understand and to apply newer type curves as they appear in the literature.

Specific type curves discussed include (1) Ramey *et al.*'s type curves²⁻⁴ for buildup and constant-rate drawdown tests; (2) McKinley's type curves^{5,6} for the same applications; and (3) Gringarten *et al.*'s⁷ type curves for vertically fractured wells with uniform flux.

4.2 Fundamentals of Type Curves

Many type curves commonly are used to determine formation permeability and to characterize damage and stimulation of the tested well. Further, some are used to determine the beginning of the MTR for a Horner analysis. Most of these curves were generated by simulating constant-rate pressure drawdown (or injection) tests; however, most also can be applied to buildup (or falloff) tests if an equivalent shut-in time⁸ is used as the time variable on the graph.

Conventional test analysis techniques (such as the Horner method for buildup tests) share these objectives. However, type curves are advantageous because they may allow test interpretation even when wellbore storage distorts most or all of the test data; in that case, conventional methods fail.

The use of type curves for fractured wells has a further advantage. In a single analytical technique, type curves combine the linear flow that occurs at early times in many fractured reservoirs, the radial flow that may occur later after the radius of investigation has moved beyond the region influenced by the fracture, and the effects of reservoir boundaries that may appear before a true MTR line is

established in a pressure transient test on a fractured well.

Fundamentally, a type curve is a preplotted family of pressure drawdown curves. The most fundamental of these curves (Ramey's²) is a plot of dimensionless pressure change, p_D , vs. dimensionless time change, t_D . This curve, reproduced in Fig. 4.1 (identical to Fig. 1.6), has two parameters that distinguish the curves from one another: the skin factor s and a dimensionless wellbore storage constant, C_{sD} . For an infinite-acting reservoir, specification of C_{sD} and s uniquely determines the value of p_D at a given value of t_D . Proof of this follows from application of the techniques discussed in Appendix B. If we put the differential equation describing a flow test in dimensionless form (along with its initial and boundary conditions), then the solution, p_D , is determined uniquely by specification of the independent variables (in this case, t_D and r_D), of all dimensionless parameters that appear in the equation, and of initial and boundary conditions (in this case, s and C_{sD}). Further, in most such solutions, we are interested in wellbore pressures of a tested well; here, dimensionless radius, $r_D = r/r_w$, has a fixed value of unity and thus does not appear as a parameter in the solution.

Thus, type curves are generated by obtaining solutions to the flow equations (e.g., the diffusivity equation) with specified initial and boundary conditions. Some of these solutions are analytical; others are based on finite-difference approximations generated by computer reservoir simulators. For example, Ramey's type curves were generated from analytical solutions to the diffusivity equation, with the initial condition that the reservoir be at uniform pressure before the drawdown test, and with boundary conditions of (1) infinitely large outer drainage radius and (2) constant *surface* withdrawal rate combined with wellbore storage, which results in variable *sandface* withdrawal rate. A skin factor, s , is used to characterize wellbore damage or stimulation; as we have seen, this causes an additional pressure drop, Δp_s , which is proportional to the instantaneous sandface flow rate (which changes with

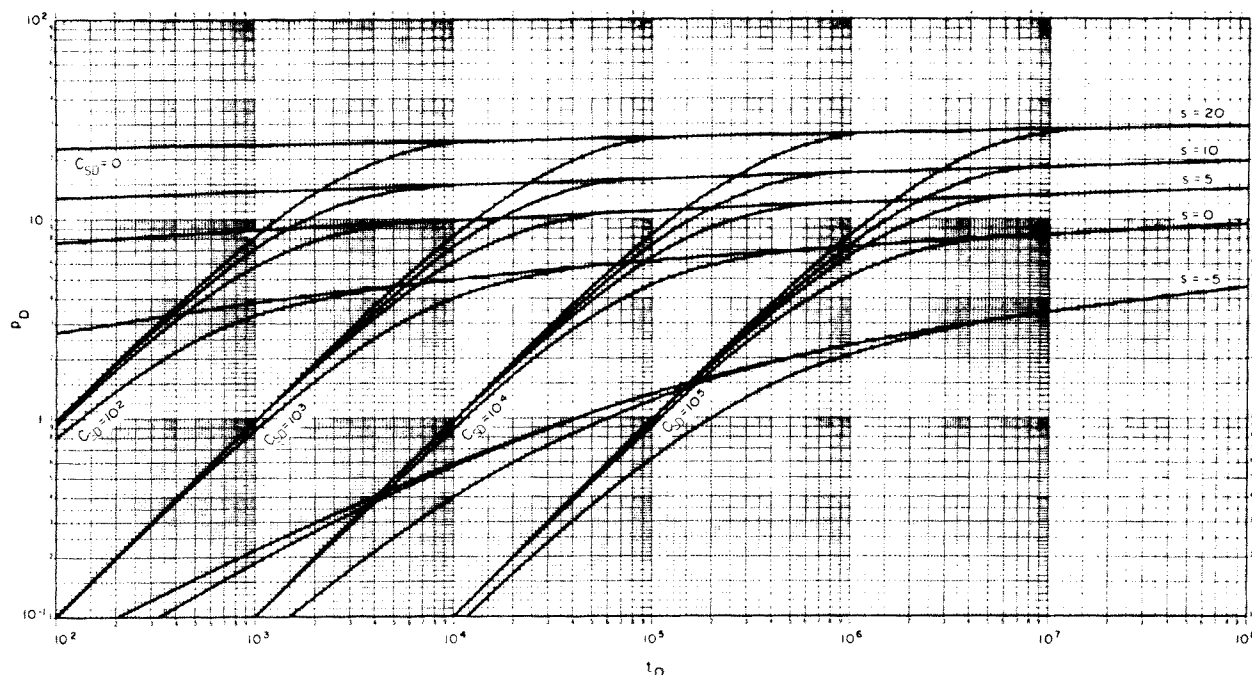


Fig. 4.1 – Type curves for constant production rate, infinite-acting reservoir (Ramey).

time while wellbore storage is a dominant influence). Dimensionless pressure drawdown at the wellbore, p_D , predicted by these solutions thus can be plotted as a function of elapsed time, t_D , for fixed values of C_{SD} and s . When curves are drawn for the range of s and C_{SD} of greatest practical importance, the type curve results (Fig. 4.1).

To use a type curve to analyze an actual drawdown test, the analyst plots pressure change, $p_i - p_{wf}$ vs. flow time, t , on the same size graph paper as the type curve. Then one finds the preplotted curve that most nearly has the same shape as the actual test data plot. When the match is found, s , C_{SD} , and corresponding values of $[p_D, (p_i - p_{wf})]$ and (t_D, t) will have been established, and k then can be determined. These sentences summarize the *principle* – but the *practice* differs in detail from the principle and is not necessarily as straightforward as this brief discussion implies.

4.3 Ramey's Type Curves

Ramey's² type curves were generated for the situation of a constant-rate pressure drawdown test in a reservoir with slightly compressible, single-phase liquid flowing; sufficient homogeneity such that the radial diffusivity equation adequately models flow in the reservoir; uniform pressure in the drainage area of the well before production; infinite-acting reservoir (no boundary effects during the flow period of interest for test analysis purposes); constant withdrawal rate at the surface; and wellbore storage and concentrated wellbore damage or stimulation characterized by a skin factor, s . This list of assumptions is tedious, but it is also important. When one or more of these assumptions is not valid in a specific case, there is no assurance that use of the type curves can lead to a valid test interpretation. (Some of these limitations can be removed, as we will

note later in this chapter. Of major importance is that the curves can be used for buildup tests and for gas well tests.) The result of Ramey's work is shown in Fig. 4.1.

Some important properties of these curves follow.

1. Examination of the analytical solution on which the type curves are based shows that, at earliest times when wellbore unloading is responsible for 100% of the flow in a drawdown test (or afterflow rate equals rate before shut-in in a buildup test), Δp is a linear function of Δt (Δp is pressure change since the test began and Δt is time elapsed since the test began). Thus, the $\log \Delta p$ - $\log \Delta t$ curve is also linear with a slope of unity (a 45° line) and the wellbore storage constant C_s can be determined from any point ($\Delta t, \Delta p$) on this line (Fig. 4.2) from the relation

$$C_s = \frac{qB}{24} \left(\frac{\Delta t}{\Delta p} \right)_{\text{unit-slope line}}$$

Note that, in a well with a liquid/gas interface in the wellbore,

$$C_s = \frac{25.65 A_{wb}}{\rho} \text{ bbl/psi, } \dots \dots \dots (4.1)$$

and for a wellbore filled with a single-phase liquid or with gas,

$$C_s = c_{wb} V_{wb} \text{ bbl/psi, } \dots \dots \dots (4.2)$$

and

$$C_{SD} = 0.894 C_s / \phi c_t h r_w^2 \dots \dots \dots (4.3)$$

Successful application of Ramey's type curves for quantitative analysis depends significantly on our ability to establish the correct value of C_{SD} to be used for curve matching – type curves for a given value of s and for different values of C_{SD} have very similar shapes, so it is difficult to find the best fit without prior knowledge of C_{SD} . Direct calculation of C_s ,

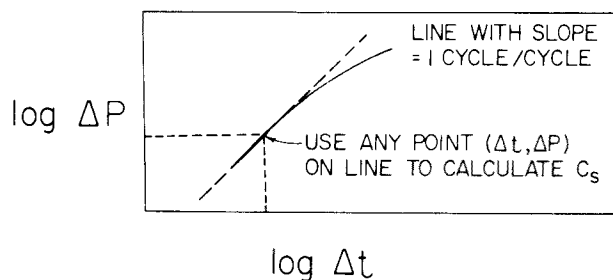


Fig. 4.2 – Use of unit slope line to calculate wellbore storage constant.

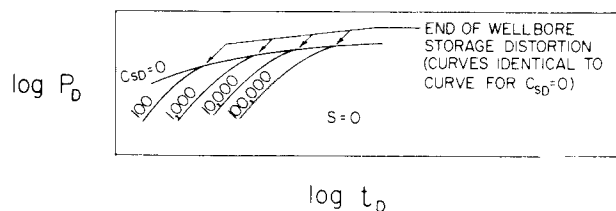


Fig. 4.3 – Use of type curves to determine end of wellbore storage distortion.

and thus C_{SD} , from known values of A_{wb} and ρ or c_{wb} and V_{wb} does not characterize test conditions as well as the value of C_s determined from actual test performance as reflected in the unit-slope lines.⁹

2. Wellbore storage has ceased distorting the pressure transient test data when the type curve for the value of C_{SD} characterizing the test becomes identical to the type curve for $C_{SD}=0$ (Fig. 4.3). (This usually occurs about one and a half to two cycles from the end of the unit-slope line.) Thus, these type curves can be used to determine how much data (if any) can be analyzed by conventional methods such as the Horner plot for buildup tests.

3. The type curves, which were developed for drawdown tests, also can be used for buildup test analysis if an equivalent shut-in time, $\Delta t_e = \Delta t / (1 + \Delta t / t_p)$, is used as the time variable. An intuitive proof of this assertion for small Δt (where $\Delta t \approx \Delta t_e$) follows.

The equation of the MTR in a drawdown test can be expressed as

$$p_i - p_{wf} = m \log t + C_1.$$

The equation of the MTR in a buildup test is

$$\begin{aligned} p_i - p_{ws} &= m \log[(t_p + \Delta t) / \Delta t] \\ &= m \log(t_p + \Delta t) - m \log \Delta t. \end{aligned}$$

If $\log(t_p + \Delta t) \approx \log t_p$ (an adequate assumption for $\Delta t_{\max} \leq 0.1 t_p$),

$$\begin{aligned} p_i - p_{ws} &\approx m \log t_p - m \log \Delta t \\ &\approx (p_i - p_{wf}) - m \log \Delta t - C_1, \end{aligned}$$

or

$$(p_{ws} - p_{wf}) \approx m \log \Delta t + C_1.$$

Thus, the equations for MTR's in drawdown and buildup test plots have similar form if we use the analogies

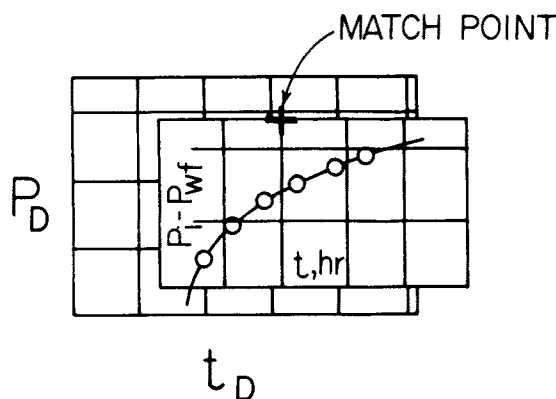


Fig. 4.4 – Horizontal and vertical shifting to find position of fit and match points.

$$(p_i - p_{wf}) \text{ drawdown} \sim (p_{ws} - p_{wf}) \text{ buildup},$$

and

$$t_{\text{drawdown}} \sim \Delta t_{\text{buildup}}.$$

If these analogies can be used for the larger values of Δt in the MTR, then we would expect intuitively that the approximation used to develop them would be even better for the smaller values of Δt in the ETR.

The practical implication of this analysis is this. For use with type curves, we plot actual drawdown test data as $(p_i - p_{wf})$ vs. t and buildup test data as $(p_{ws} - p_{wf})$ vs. Δt , but we must remember that Δt_e must be used instead of Δt whenever $\Delta t > 0.1 t_p$.

4. A log-log plot of p_D vs. t_D differs from a log-log plot of $(p_i - p_{wf})$ vs. t (for a drawdown test) only by a shift in the origin of the coordinate system—i.e., $\log t_D$ differs from $\log t$ by a constant and $\log p_D$ differs from $\log (p_i - p_{wf})$ by another constant. To show this, we note that

$$t_D = \frac{0.000264 kt}{\phi \mu c_t r_w^2},$$

and

$$p_D = \frac{kh(p_i - p_{wf})}{141.2 qB\mu}.$$

Thus,

$$\log t_D = \log t + \log \frac{0.000264 k}{\phi \mu c_t r_w^2},$$

and

$$\log p_D = \log (p_i - p_{wf}) + \log \frac{kh}{141.2 qB\mu}.$$

The significance of this result is that the plot of an actual drawdown test ($\log t$ vs. $\log \Delta p$) should have a shape identical to that of a plot of $\log t_D$ vs. $\log p_D$, but we have to displace both the horizontal and

vertical axes (i.e., shift the origin of the plot) to find the position of best fit (Fig. 4.4).

Once a fit is found by vertical and horizontal shifting, we choose a match point to determine the relationship between actual time and dimensionless time and between actual pressure drawdown and dimensionless pressure for the test being analyzed. Any point on the graph paper will suffice as a match point (i.e., the result is independent of the choice of match point). For the match point chosen, we determine the corresponding values of (t, t_D) and $[(p_i - p_{wf}), p_D]$. Then, from definition of p_D and t_D ,

$$k = 141.2 \frac{qB\mu}{h} \left(\frac{p_D}{p_i - p_{wf}} \right)_{MP}, \dots \dots \dots (4.4)$$

and

$$\phi C_{ti} = \frac{0.000264 k}{\mu_i r_w^2} \left(\frac{t}{t_D} \right)_{MP} \dots \dots \dots (4.5)$$

5. Although the type curves were developed from solutions to flow equations for slightly compressible liquids, they also can be used to analyze gas well tests. Transformation of the flow equations to model gas flow in terms of the pseudopressure¹⁰ $\psi(p)$ and comparison of these solutions expressed in terms of dimensionless pseudopressure, ψ_D , with solutions p_D for slightly compressible liquids,¹¹ show that, as a high-order-accuracy approximation for transient flow,

$$\psi_D(t_D, r_D, s', C_{sD}) = p_D(t_D, r_D, s, C_{sD}),$$

where, for gases,

$$t_D = \frac{0.000264 kt}{\phi \mu_i c_{gi} r_w^2}, \dots \dots \dots (4.6)$$

$$\psi_D = \frac{kh T_{sc} [\psi(p_i) - \psi(p_{wf})]}{50,300 p_{sc} q_g T}, \dots \dots \dots (4.7)$$

$$s' = s + D|q_g|, \dots \dots \dots (4.8)$$

and

$$\psi(p) = 2 \int_{p_{\beta}}^p \frac{p}{\mu(p) z(p)} dp \dots \dots \dots (4.9)$$

To use this result as stated would require that we (1) prepare a table or graph of values of $\psi(p)$ vs. p from Eq. 4.9 based on the properties of the specific gas in the well being tested; (2) plot $[\psi(p_i) - \psi(p_{wf})]$ vs. t on log-log paper; and (3) find the best fit just as for a slightly compressible liquid (with the values of s providing the best fit being interpreted for the gas well as $s + D|q_g|$).

Steps 1 and 2 can be simplified in some cases. When μz is directly proportional to pressure (for $p > 3,000$ psia for some gases), Eq. 4.7 can be replaced by the definition

$$p_D = \frac{kh(p_i - p_{wf})}{141.2 q_g \mu_i B_{gi}}, \dots \dots \dots (4.10)$$

where

$$B_{gi} = 5.04 \frac{z_i T}{p_i} \text{ RB/Mscf.} \dots \dots \dots (4.11)$$

Thus, when $\mu z/p = \text{constant}$, we can plot $(p_i - p_{wf})$ for type-curve use just as for a slightly compressible liquid. Match-point interpretation is

$$k = 141.2 \frac{q_g \mu_i B_{gi}}{h} \left(\frac{p_D}{p_i - p_{wf}} \right)_{MP}, \dots \dots \dots (4.12)$$

$$\phi C_{ti} = \frac{0.000264 k}{\mu_i r_w^2} \left(\frac{t}{t_D} \right)_{MP} \dots \dots \dots (4.13)$$

Note that all gas properties are to be evaluated at original reservoir pressure for a test in an infinite-acting reservoir (or, more generally, at the uniform reservoir pressure preceding the drawdown test or at the current average reservoir pressure for a buildup test).

In some other situations, μz is constant (e.g., in many cases for $p < 2,000$ psia); as a result, Eq. 4.7 can be replaced by the definition

$$p_D = \frac{kh(p_i^2 - p_{wf}^2)}{1,422 q_g \mu_i z_i T} \dots \dots \dots (4.14)$$

Thus, when $\mu z = \text{constant}$, we can plot $(p_i^2 - p_{wf}^2)$ for type-curve use. Match-point interpretation becomes

$$k = 1,422 \frac{q_g \mu_i z_i T}{h} \left(\frac{p_D}{p_i^2 - p_{wf}^2} \right)_{MP}, \dots \dots \dots (4.15)$$

$$\phi C_{ti} = \frac{0.000264 k}{\mu_i r_w^2} \left(\frac{t}{t_D} \right)_{MP} \dots \dots \dots (4.16)$$

Determination of whether either approximation ($\mu z/p = \text{constant}$ or $\mu z = \text{constant}$) is valid must be based on plots or tabulations using the properties of the specific gas in the reservoir being tested.

Use of Ramey's Type Curves

The theory of Ramey's type curves leads to the following procedure for using the curves for test analysis. The procedure is given for a slightly compressible liquid; Eqs. 4.6 through 4.16 show the changes necessary when a gas well test is analyzed.

1. Plot $(p_i - p_{wf})$ vs. t (drawdown test) or $(p_{ws} - p_{wf})$ vs. $\Delta t_e = \Delta t / (1 + \Delta t/t_p)$ (buildup test) on log-log paper the same size as Ramey's type curve. Caution: Unless a type curve undistorted in the reproduction process is used, it will not have the same dimensions as commercially available graph paper and finding a fit may be misleading or impossible. The best solution is to use an undistorted type curve; an acceptable alternative is to plot test data on tracing paper, using the grid on the distorted type curve as a plotting aid.

2. If the test has a uniform-slope region (45° line at earliest times), choose any point $[t, (p_i - p_{wf})]$ or $[\Delta t, (p_{ws} - p_{wf})]$ on the unit-slope line and calculate the wellbore storage constant C_s :

$$C_s = \frac{qB}{24} \left(\frac{t}{p_i - p_{wf}} \right)_{\text{unit-slope line}}$$

TABLE 4.1 – CONSTANT-RATE DRAWDOWN TEST DATA

t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)
0.0109	2,976	0.164	2,693	2.18	1,768
0.0164	2,964	0.218	2,611	2.73	1,734
0.0218	2,953	0.273	2,536	3.28	1,712
0.0273	2,942	0.328	2,469	3.82	1,696
0.0328	2,930	0.382	2,408	4.37	1,684
0.0382	2,919	0.437	2,352	4.91	1,674
0.0437	2,908	0.491	2,302	5.46	1,665
0.0491	2,897	0.546	2,256	6.55	1,651
0.0546	2,886	1.09	1,952	8.74	1,630
0.109	2,785	1.64	1,828	10.9	1,614
				16.4	1,587

Then calculate the dimensionless wellbore storage constant:

$$C_{sD} = \frac{0.894 C_s}{\phi c_t h r_w^2}$$

Note that estimates of ϕ and c_t are required at this point – with implications that are discussed later.

If a unit-slope line is not present, C_s and C_{sD} must be calculated from wellbore properties, and inaccuracies may result if these properties do not describe actual test behavior.⁹

3. Using type curves with C_{sD} as calculated in Step 2, find the curve that most nearly fits all the plotted data. This curve will be characterized by some skin factor, s ; record its value. Interpolation between curves should improve the precision of the analysis, but may prove difficult. Even for fixed C_{sD} from the unit-slope curve, the analyst may experience difficulty in determining that one value of s provides a better fit than another, particularly if all data are distorted by wellbore storage or if the “scatter” or “noise” that characterizes much actual field data is present. If C_{sD} is not known with certainty, the possible ambiguity in finding the best fit is even more pronounced.

4. With the actual test data plot placed in the position of best fit, record corresponding values of $(p_i - p_{wf}, p_D)$ and (t, t_D) from any convenient match point.

5. Calculate k and ϕc_t (Eqs. 4.4 and 4.5):

$$k = 141.2 \frac{qB\mu}{h} \left(\frac{p_D}{p_i - p_{wf}} \right)_{MP}$$

$$\phi c_t = \frac{0.000264 k}{\mu r_w^2} \left(\frac{t}{t_D} \right)_{MP}$$

Eq. 4.5 does not establish ϕc_t based on test performance unless it is possible to establish C_{sD} without assuming values for ϕc_t – it simply reproduces those values assumed in Step 2.

In summary, the procedure outlined in Steps 1 through 5 provides estimates of k , s , and C_s .

Example 4.1 – Drawdown Test Analysis Using Ramey's Type Curves

Problem. Determine k , s , and C_s from the data below and in Table 4.1, which were obtained in a pressure drawdown test on an oil well.

TABLE 4.2 – DRAWDOWN DATA TABULATED FOR PLOTTING

t (hours)	$p_i - p_{wf}$ (psi)	t (hours)	$p_i - p_{wf}$ (psi)	t (hours)	$p_i - p_{wf}$ (psi)
0.0109	24	0.164	307	2.18	1,232
0.0164	36	0.218	389	2.73	1,266
0.0218	47	0.273	464	3.28	1,288
0.0273	58	0.328	531	3.82	1,304
0.0328	70	0.382	592	4.37	1,316
0.0382	81	0.437	648	4.91	1,326
0.0437	92	0.491	698	5.46	1,335
0.0491	103	0.546	744	6.55	1,349
0.0546	114	1.09	1,048	8.74	1,370
0.109	215	1.64	1,172	10.9	1,386
				16.4	1,413

$$q = 500 \text{ STB/D,}$$

$$\phi = 0.2,$$

$$\mu = 0.8 \text{ cp,}$$

$$c_t = 10 \times 10^{-6} \text{ psi}^{-1},$$

$$r_w = 0.3 \text{ ft,}$$

$$h = 56 \text{ ft,}$$

$$B_o = 1.2 \text{ RB/STB, and}$$

$$p_i = 3,000 \text{ psia.}$$

Solution. We must first prepare the data for plotting (Table 4.2). The data are plotted in Fig. 4.5.

From the unit-slope line (on which the data lie for $t \leq 0.0218$ hour),

$$\begin{aligned} C_s &= \frac{qB}{24} \left[\frac{t}{(p_i - p_{wf})} \right]_{\text{point on unit-slope line}} \\ &= \frac{(500)(1.2)}{(24)} \times \frac{(0.0218)}{(47)} \\ &= 0.0116 \text{ RB/psi.} \end{aligned}$$

Then,

$$\begin{aligned} C_{sD} &= \frac{0.894 C_s}{\phi c_t h r_w^2} \\ &= \frac{(0.894)(0.0116)}{(0.2)(1 \times 10^{-5})(56)(0.3)^2} \\ &= 1.03 \times 10^3 \\ &\approx 1 \times 10^3. \end{aligned}$$

For $C_{sD} = 10^3$, the best-fitting type curve is for $s = 5$. A time match point is $t = 1$ hour when $t_D = 1.93 \times 10^4$. A pressure match point is $(p_i - p_{wf}) = 100$ psi, when $p_D = 0.85$.

From the match, we also note that wellbore storage distortion ends at $t \approx 5.0$ hours (i.e., the type curve for $C_{sD} = 10^3$ becomes identical to the type curve for $C_{sD} = 0$).

From the pressure match point,

$$\begin{aligned} k &= 141.2 \frac{qB\mu}{h} \left(\frac{p_D}{p_i - p_{wf}} \right)_{MP} \\ &= \frac{(141.2)(500)(1.2)(0.8)}{(56)} \left(\frac{0.85}{100} \right) \\ &= 10.3 \text{ md.} \end{aligned}$$

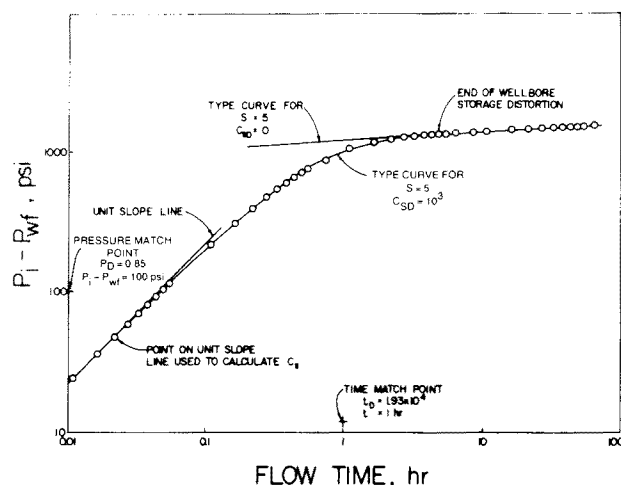


Fig. 4.5 – Drawdown test analysis with Ramey's type curve.

From the time match point,

$$\begin{aligned}\phi c_t &= \frac{0.000264 k}{\mu r_w^2} \left(\frac{t}{t_D} \right)_{MP} \\ &= \frac{(0.000264)(10.3)}{(0.8)(0.3)^2} \left(\frac{1}{1.93 \times 10^4} \right) \\ &= 1.96 \times 10^{-6} \text{ psi}^{-1}.\end{aligned}$$

Compare those with values used to determine C_{SD} from C_s :

$$\begin{aligned}\phi c_t &= (0.2)(1 \times 10^{-5}) \\ &= 2 \times 10^{-6} \dots \text{values in} \\ &= \text{values out.}\end{aligned}$$

4.4 McKinley's Type Curves

McKinley⁵ proposed type curves with the primary objective of characterizing damage or stimulation in a drawdown or buildup test in which wellbore storage distorts most or all of the data, thus making this characterization possible with relatively short-term tests.

In constructing his type curves, McKinley observed that the ratio of pressure change, Δp , to flow rate causing the change, qB , is a function of several dimensionless quantities:

$$\frac{\Delta p}{qB} = f\left(\frac{kh\Delta t}{\mu C_s}, \frac{k\Delta t}{\phi \mu c_t r_w^2}, \frac{r_e}{r_w}, \frac{\Delta t}{t_p}\right).$$

Type curves with this many parameters would be difficult, if not impossible, to use. Accordingly, McKinley simplified the problem in the following way.

1. He assumed that the well has produced sufficiently long (essentially to stabilization) that the last group, $\Delta t/t_p$, is not important.

2. He ignored boundary effects except approximately and, thus, ignored r_e/r_w in the basic log used to construct the type curves.

3. His analysis of simulated buildup and drawdown curves showed that, during the wellbore-storage-dominated portion of a test, the parameter $kh\Delta t/\mu C_s$ was much more important in determining $\Delta p/qB$ than was the parameter $k\Delta t/\phi \mu c_t r_w^2$. Accordingly, he let $k/\phi \mu c_t r_w^2 = 10 \times 10^6$ md-psi/cp-sq ft (an average value) for all his type curves. It is important to emphasize that even when $k/\phi \mu c_t r_w^2$ varies from this average value by one or two orders of magnitude, the shape of the type curves is not affected significantly. The reason for this approximation was McKinley's judgment that the loss of accuracy is more than offset by the gain in sensitivity in the type curves – i.e., that the shape of each curve is distinctly different at earliest times (Fig. 4.6).

4. To take into account the remaining parameters that *do* have a significant influence on test results, McKinley plotted his type curves as Δt (ordinate) vs. $5.615 C_s \Delta p/qB$ (abscissa), with the single parameter $kh/5.615 C_s \mu$. A small-scale version of McKinley's curves is shown in Fig. 4.6.

5. Note that the skin factor s does not appear as a parameter in the McKinley curves. Instead, McKinley's curves assess damage or stimulation by noting that the earliest wellbore-storage-distorted data are dominated by the effective near-well transmissibility $(kh/\mu)_{wb}$; thus, a type-curve match of the earliest data in a test should allow calculation of this quantity. Later, after wellbore storage distortion has diminished, the pressure/time behavior is governed by the transmissibility in the formation, kh/μ ; this quantity also can be estimated from a type-curve match – but for the later data only.

6. McKinley approximated boundary effects by plotting the simulator-generated type curves for about one-fifth log cycle beyond the end of wellbore storage distortion (where the curve has the same shape as for $C_s = 0$) and then making the curves vertical. This step roughly simulates drainage conditions of 40-acre spacing. Note that this gives the curves early-, middle-, and late-time regions – but remember that the curves were designed to be used primarily to analyze *early-time* data. When the curves are applied to drawdown tests, they *must* be applied to early-time data only; they do not properly simulate boundary effects in drawdown tests.

Use of McKinley's Type Curves

Before providing a step-by-step procedure for using McKinley's type curves, we note that he actually prepared three different curves: one for the time range 0.01 to 10 minutes; one for 1 to 1,000 minutes; and one for 10^3 to 10^6 minutes. The curve for the time range 1 to 1,000 minutes is by far the most useful; accordingly, it is the only one provided with this text. The complete set is provided with Ref. 1.

The steps for using McKinley's curves follow.

1. Plot Δt (minutes) as ordinate vs. $\Delta p = p_i - p_{wf}$ (or $p_{ws} - p_{wf}$) as abscissa on 3×5 cycle log-log paper the same size as McKinley's type curve if undistorted type curves are used. Otherwise, use tracing paper for actual test data plotting. The time range on the axis should correspond exactly to one of the type curves (e.g., it should span the time range of

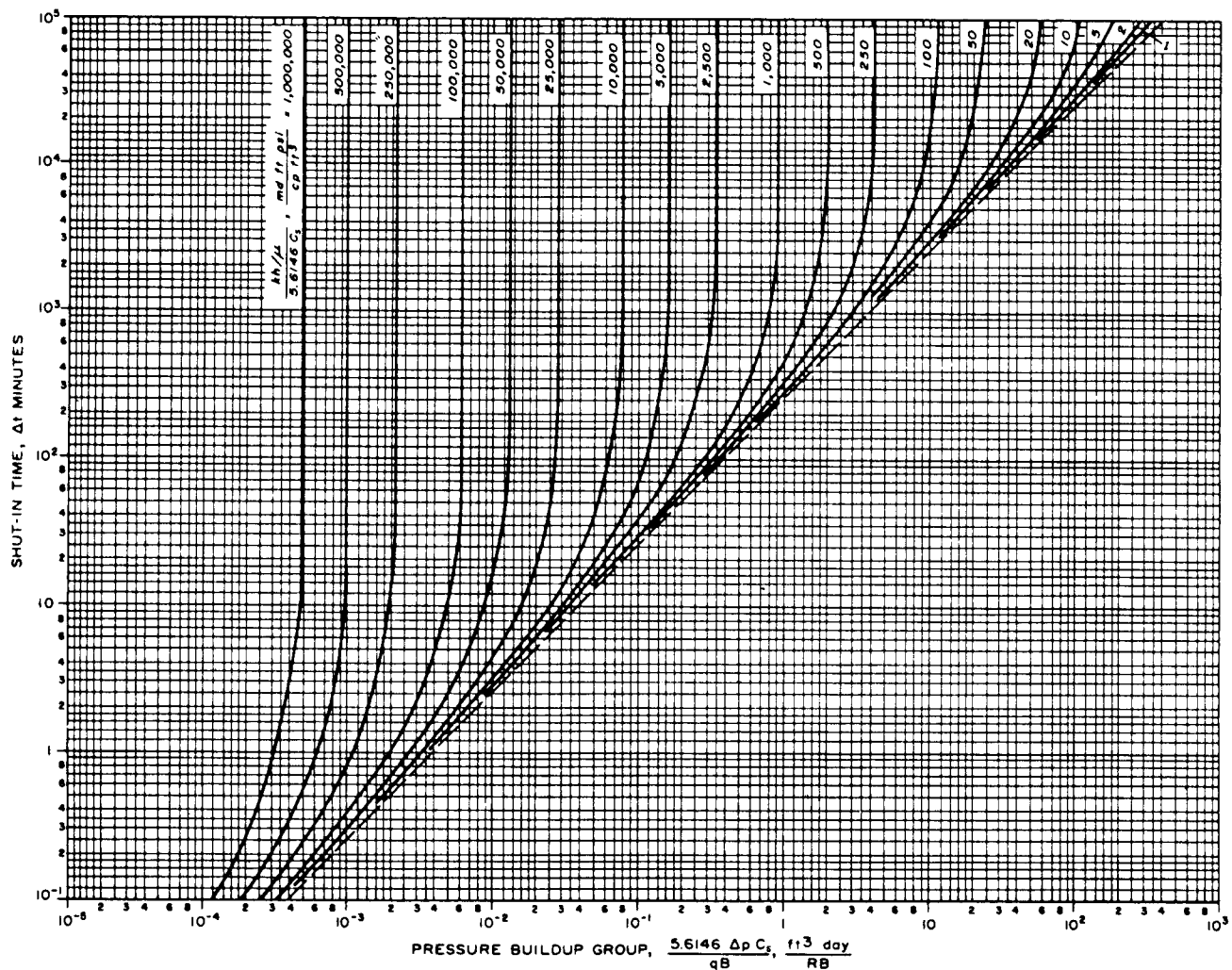


Fig. 4.6 – McKinley's type curve.

1 to 1,000 minutes *or* 0.01 to 10 minutes *or* 10^3 to 10^6 minutes).

2. Match the time axis of the test data plot with one of McKinley's. Move the data along the plot horizontally (*no vertical shifting allowed*) until the earliest data fall along one of the type curves.

3. Record the parameter value $(kh/\mu)/5.615 C_s$ for the correct type curve.

4. Choose a data match point (any Δp from the test graph paper and the corresponding value of $5.615 \Delta p C_s / qB$ from the type curve).

5. Determine the wellbore storage constant C_s from values of $\Delta p = \Delta p_{MP}$ and $5.615 \Delta p C_s / qB = (5.615 \Delta p C_s / qB)_{MP}$ at the match point:

$$C_s = \frac{(5.615 \Delta p C_s / qB)_{MP}}{\Delta p_{MP}} \times \frac{qB}{5.615}$$

6. Calculate near-well transmissibility, $(kh/\mu)_{wb}$, from the parameter value recorded in Step 3 and the wellbore storage constant determined in Step 5:

$$(kh/\mu)_{wb} = \left(\frac{kh/\mu}{5.615 C_s} \right)_{wb} \times 5.615 C_s$$

7. If the data trend away from the type curve providing the earliest fit (indicating that formation transmissibility is different from effective near-well

transmissibility), shift the data plot horizontally to find another type curve that better fits the later data. A shift to a higher value of $(kh/\mu)/5.615 C_s$ indicates damage; a shift to a lower value indicates stimulation.

8. Calculate formation transmissibility:

$$(kh/\mu)_f = \left(\frac{kh/\mu}{5.615 C_s} \right)_{\text{Step 7}} \times (5.615 C_s)_{\text{Step 5}}$$

Note that we *do not* find a new pressure match point to redetermine C_s ; C_s is found once and for all in Step 5. In fact, if only data reflecting formation transmissibility (after wellbore storage distortion has disappeared) are analyzed, error will result using the McKinley method. (However, conventional methods work here, so no problem arises.) The match point must be found with early, wellbore-storage-distorted data (Figs. 4.7 and 4.8).

Flow efficiency also can be estimated fairly directly from the data plotted for use with McKinley's type curves.⁶ The definition of flow efficiency, E , is

$$E \approx \frac{p^* - p_{wf} - \Delta p_s}{p^* - p_{wf}} = \frac{\Delta p^* - \Delta p_s}{\Delta p^*}$$

The quantities Δp^* and Δp_s can be estimated from

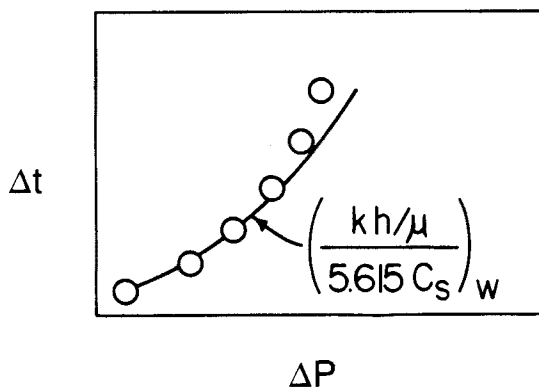


Fig. 4.7 – Early data fit on McKinley's type curve.

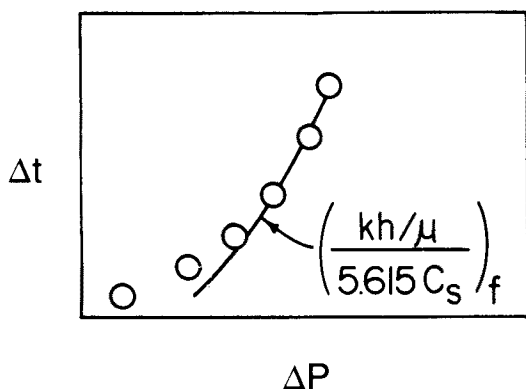


Fig. 4.8 – Later data fit on McKinley's type curve.

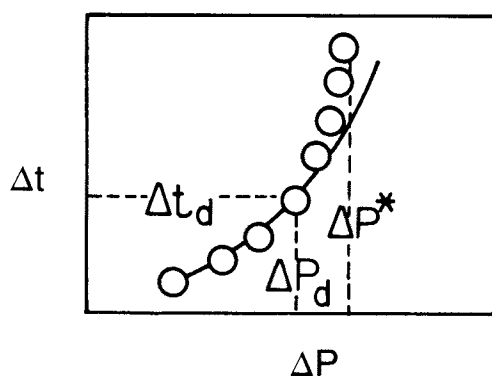


Fig. 4.9 – Data for flow efficiency calculation from McKinley's type curve.

the McKinley type curves in the following manner (Fig. 4.9):

1. Δp^* is the vertical asymptote approached by Δp in the McKinley plot.

2. Δp_s can be calculated from Δp_d , the time at which the actual test data depart from the earliest-fitting type curve. Picking a time of departure is subjective, so no great accuracy is assured for this reason alone.

McKinley⁶ states that Δp_s and Δp_d are related by

$$\Delta p_s = \left(1 - \frac{k_{wb}}{k_f}\right) \Delta p_d.$$

3. Thus, E can be calculated:

$$E \approx \frac{\Delta p^* - \Delta p_s}{\Delta p^*}.$$

Example 4.2 – Drawdown Test Analysis Using McKinley's Type Curves

Problem. Estimate near-well and formation permeability and flow efficiency using the data presented in Example 4.1 from a drawdown test on an oil well.

Solution. We first prepare those data, most of which lie in the time range 1 minute $< t < 1,000$ minutes for plotting as Δt (minutes) vs. $\Delta p = p_i - p_{wf}$ (Table 4.3). From the data plot (Fig. 4.10) and the match with the best-fitting McKinley curve for the early data,

$$\left(\frac{kh}{\mu}\right)_{wb} \times \frac{1}{5.615 C_s} = 5,000.$$

We also note that the data depart from the best-fitting curve (early time) at $\Delta t_d \approx 100$ minutes; here, $\Delta p_d \approx 1,180$ psi.

A match point for the early fit is $\Delta p_{MP} = 107$ psi when $5.615 \Delta p C_s / qB = 0.010$. The best fit of the later data is for $(kh/\mu)_f (1/5.615 C_s) \approx 10,000$. Thus, from the match-point data,

$$\begin{aligned} C_s &= \left(\frac{5.615 \Delta p C_s}{qB}\right)_{MP} \left(\frac{1}{\Delta p}\right)_{MP} \times \frac{qB}{5.615} \\ &= (0.010) \left(\frac{1}{107}\right) \left(\frac{500 \times 1.2}{5.615}\right) \\ &= 0.01 \text{ RB/psi} \end{aligned}$$

The near-well transmissibility and apparent permeability, k_{wb} , are then

$$\begin{aligned} \left(\frac{kh}{\mu}\right)_{wb} &= \left(\frac{kh/\mu}{5.615 C_s}\right)_{wb} (5.615 C_s) \\ &= (5,000)(5.615)(0.010) \\ &= 281 \text{ md-ft/cp.} \end{aligned}$$

Then,

$$k_{wb} = \frac{(281)(0.8)}{(56)} = 4.01 \text{ md.}$$

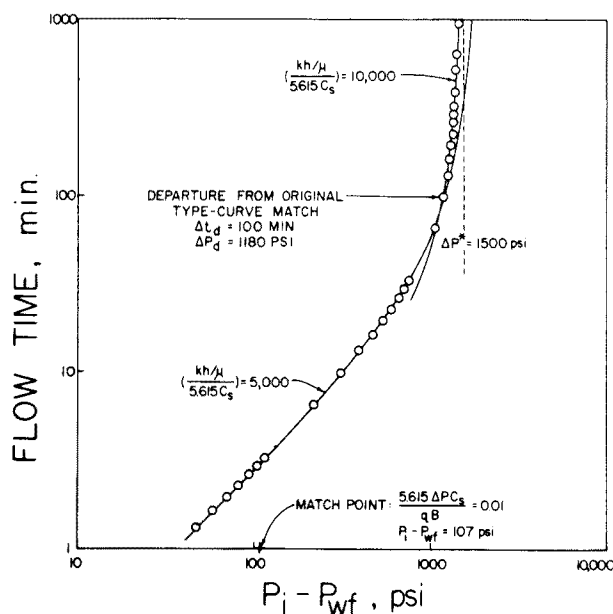


Fig. 4.10 – Drawdown test analysis with McKinley's type curve.

The formation transmissibility and permeability are

$$\begin{aligned} \left(\frac{kh}{\mu} \right)_f &= \frac{(kh/\mu \times 5.615 C_s)_f}{(kh/\mu \times 5.615 C_s)_{wb}} \times \left(\frac{kh}{\mu} \right)_{wb} \\ &= \frac{10,000}{5,000} \times 281 \\ &= 562 \text{ md-ft/cp,} \end{aligned}$$

and

$$\begin{aligned} k_f &= \left(\frac{kh}{\mu} \right)_f \times \frac{\mu}{h} \\ &= (562) \left(\frac{0.8}{56} \right) \\ &= 8.03 \text{ md.} \end{aligned}$$

Flow efficiency becomes

$$\Delta p^* \approx 1,500 \text{ psi (Fig. 4.10),}$$

$$\Delta p_s = \left(1 - \frac{k_{wb}}{k_f} \right) \Delta p_d$$

$$\Delta p_s = \left(1 - \frac{4.01}{8.03} \right) (1,180) = 590 \text{ psi,}$$

$$E \approx \frac{1,500 - 590}{1,500} = 0.607.$$

4.5 Gringarten *et al.*⁷ Type Curves for Fractured Wells

Gringarten *et al.*⁷ developed type curves for hydraulically fractured wells in which vertical

TABLE 4.3 – DRAWDOWN DATA FOR MCKINLEY CURVE ANALYSIS

Δt (minutes)	$p_i - p_{wf}$ (psi)	Δt (minutes)	$p_i - p_{wf}$ (psi)	Δt (minutes)	$p_i - p_{wf}$ (psi)
1.31	47	16.4	464	197	1,288
1.64	58	19.7	531	229	1,304
1.97	70	22.9	592	262	1,316
2.29	81	26.2	648	295	1,326
2.62	92	29.5	698	328	1,335
2.95	103	32.8	744	393	1,349
3.28	114	65.4	1,048	524	1,370
6.54	215	98.4	1,172	654	1,386
9.84	307	131	1,232	984	1,413
13.1	389	164	1,266		

fractures with two equal-length wings were created. The curves discussed in this section assume *uniform flux* into the fracture (same flow rate per unit cross-sectional area of fracture from wellbore to fracture tip). High fracture conductivity is required to achieve uniform flux, but this is not identical to an infinitely conductive fracture (no pressure drop from fracture tip to wellbore), as Gringarten *et al.* demonstrated.⁷

The study was made for finite reservoirs (i.e., boundary effects become important at later times in the test). The reservoir is assumed to be at uniform pressure, p_i , initially. The type curve (Fig. 4.11), developed for a constant-rate drawdown test for a slightly compressible liquid, also can be used for buildup tests (for $\Delta t_{\max} \leq 0.1 t_p$) and for gas wells* using the modifications discussed earlier. Wellbore storage effects are ignored.

All the dimensionless variables and parameters considered important are taken into account in Fig. 4.11, which is a log-log plot of p_D vs. $t_D r_w^2 / L_f^2$ with parameter x_e / L_f . In these parameters, L_f is the fracture half-length and x_e is the distance from the well to the side of the square drainage area in which it is assumed to be centered. Dimensionless pressure has the usual definition,

$$p_D = \frac{kh(p_i - p_{wf})}{141.2 q B \mu} \text{ (drawdown test),}$$

and

$$\frac{t_D r_w^2}{L_f^2} = \frac{0.000264 kt}{\phi \mu c_t L_f^2} = t_{DLf} \dots \dots \dots (4.17)$$

Several features of Fig. 4.11 are of interest:

1. The slope of the log-log plot is 1/2 up to $t_{DLf} \approx 0.16$ for $x_e / L_f > 1$. This is linear flow. We have shown that, in linear flow,

*This statement may be an oversimplification. Some gas wells exhibit time-dependent non-Darcy flow in the fracture, unlike liquids.¹²

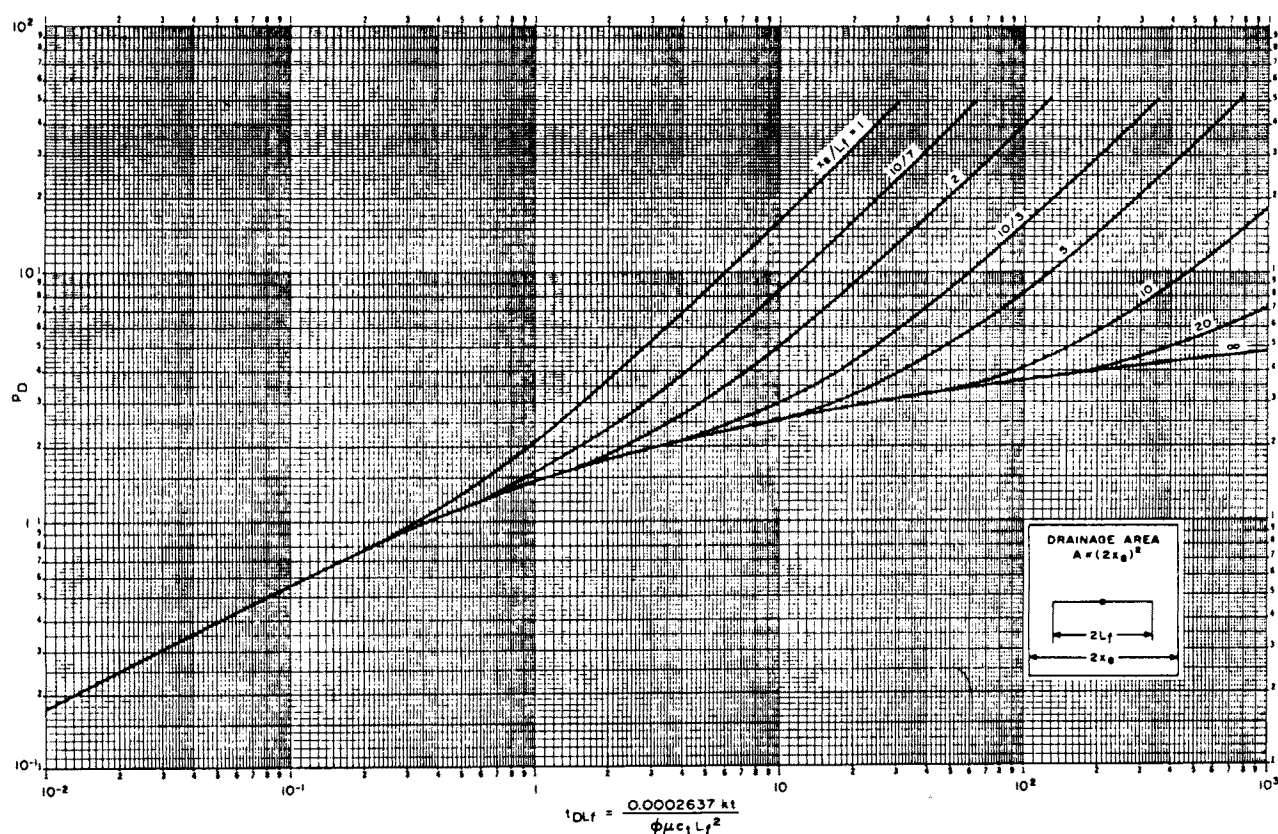


Fig. 4.11 – Gringarten *et al.* type curve for vertically-fractured well centered in closed square, no wellbore storage, uniform flux.

$$p_i - p_{wf} = ct^{1/2},$$

or

$$p_D = c' t_{DLf}^{1/2}.$$

Then,

$$\log p_D = \log c' + 1/2 \log t_{DLf}.$$

2. Although not apparent on a log-log plot, a semilog plot of the data in Fig. 4.11 (p_D vs. $\log t_{DLf}$) is a straight line, signifying radial flow when, for $x_e/L_f > 5$, $t_{DLf} \approx 2$. The straight-line terminates, of course, when boundary effects become important, but a match of actual test data with Fig. 4.11 can show the amount of data in the radial flow region (and, thus, can be analyzed for permeability by the conventional p_{wf} vs. $\log t$ or Horner plots). Fig. 4.11 thus combines, in a single graph, the linear flow and radial flow regions (and a region with neither), boundary effects, and the effect of various fracture lengths. If fracture conductivity is high and constant throughout the test and if wellbore storage has negligible effect on earliest data, then this figure allows a rather complete analysis of a hydraulically fractured well—specifically, estimation of fracture length, L_f , and formation permeability, k . The method is frequently superior to the nontype-curve methods discussed in Chap. 2.

Steps in use of Fig. 4.11 as a type curve for test analysis include the following.

1. Plot $(p_i - p_{wf})$ (drawdown test) or $(p_{ws} -$

$p_{wf})$ (buildup test) on the ordinate vs. t (drawdown test) or Δt_e (buildup test) on the abscissa on a 3×5 cycle log-log paper if an undistorted version of the type curve is available. Otherwise, use tracing paper.

2. Select the best match by sliding the actual test data plot both horizontally and vertically.

3. Note the value of the match points $[(p_D)_{MP}, (p_i - p_{wf})_{MP}]$ and $[(t_{DLf})_{MP}, t_{MP}]$.

4. Estimate formation permeability from the pressure match point:

$$k = 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(p_i - p_{wf})_{MP}}.$$

5. Estimate fracture length from the time match point:

$$L_f = \left[\frac{0.000264 kt_{MP}}{\phi \mu c_t (t_{DLf})_{MP}} \right]^{1/2} \dots \dots \dots (4.18)$$

Three useful checks are sometimes possible:

1. If a half-slope (linear flow) region appears on the test data plot, replot data from the region as p_{wf} (or p_{ws}) vs. $\sqrt{t}$ (or $\sqrt{\Delta t_e}$); from the slope m_L and linear flow theory,

$$L_f \sqrt{k} = \frac{4.064 qB}{hm_L} \sqrt{\frac{\mu}{\phi c_t}},$$

which should agree with the result from the type-curve analysis.

2. If a radial flow region appears (before boundary effects become important—that is, before the data deviate from the $x_e/L_f = \infty$ curve), a plot of p_{wf}

TABLE 4.4 – FRACTURED WELL BUILDUP TEST DATA

Δt (hours)	$p_{ws} - p_{wf}$ (psi)	Δt (hours)	$p_{ws} - p_{wf}$ (psi)
0	0	0.75	89
0.0833	31	0.833	100
0.167	43	0.917	100
0.250	54	1.00	100
0.330	66	1.25	114
0.417	66	2.00	136
0.500	72	2.50	159
0.583	78	4.00	181
0.667	83	4.75	206
		6.00	218

vs. $\log t$ [p_{ws} vs. $\log \Delta t$ or $\log (t_p + \Delta t) / \Delta t$] should show that $k = 162.6 qB\mu/mh$, in agreement with type-curve analysis.

3. If a well proves to be in a finite-acting reservoir, it may be possible to estimate x_e from a matching parameter, x_e/L_f , to compare with the known (or assumed) value of x_e to check the quality of the match.

Example 4.3 – Buildup Test Analysis for a Vertically Fractured Well

Problem. Gringarten *et al.*¹³ presented buildup test data for a well believed to be fractured vertically. From these data, presented below and in Table 4.4, estimate fracture length and formation permeability. Producing time, t_p , was significantly greater than maximum shut-in time, so that $\Delta t \approx \Delta t_e$.

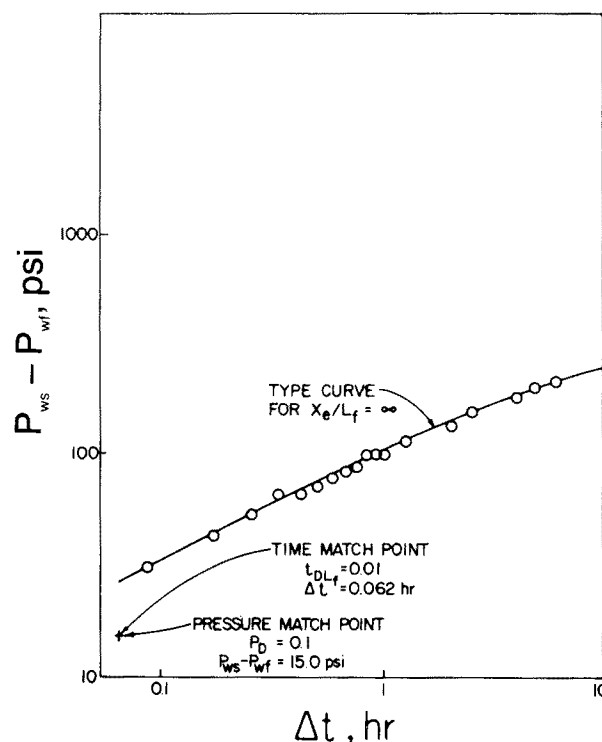


Fig. 4.12 – Buildup test analysis for vertically-fractured well with Gringarten type curve.

$$\begin{aligned}
 q &= 2,750 \text{ STB/D,} \\
 \mu &= 0.23 \text{ cp,} \\
 B &= 1.76 \text{ RB/STB,} \\
 h &= 230 \text{ ft,} \\
 \phi &= 0.3, \text{ and} \\
 c_f &= 30 \times 10^{-6} \text{ psi}^{-1}.
 \end{aligned}$$

Solution. Fig. 4.12 is a plot of $\Delta p = p_{ws} - p_{wf}$ vs. Δt . An adequate fit is characterized by the match points ($t = 0.062$ hour, $t_{DLf} = 0.01$) and ($\Delta p = 15.2$ psi, $p_D = 0.1$). From the pressure match point,

$$\begin{aligned}
 k &= 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}} \\
 &= \frac{(141.2)(2,750)(1.76)(0.23)(0.1)}{(230)(15.2)} \\
 &= 4.5 \text{ md.}
 \end{aligned}$$

TABLE 4.5 – CONSTANT-RATE DRAWDOWN TEST DATA

t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)
0.0109	2,976	0.218	2,611	3.28	1,712	32.8	1,543
0.0164	2,964	0.273	2,536	3.82	1,696	38.2	1,533
0.0218	2,953	0.328	2,469	4.37	1,684	43.7	1,525
0.0273	2,942	0.382	2,408	4.91	1,674	49.1	1,517
0.0328	2,930	0.437	2,352	5.46	1,665	54.6	1,511
0.0382	2,919	0.491	2,302	6.55	1,651	65.5	1,500
0.0437	2,908	0.546	2,256	8.74	1,630	87.4	1,482
0.0491	2,897	1.09	1,952	10.9	1,614	109.2	1,468
0.0546	2,886	1.64	1,828	16.4	1,587	163.8	1,440
0.109	2,785	2.18	1,768	21.8	1,568	218.4	1,416
0.164	2,693	2.73	1,734	27.3	1,554	273.0	1,393
						327.6	1,370

TABLE 4.6 – BUILDUP TEST DATA

Δt (hours)	p_{ws} (psi)	Δt (hours)	p_{ws} (psi)	Δt (hours)	p_{ws} (psi)	Δt (hours)	p_{ws} (psi)
0.0	1,370	0.546	2,114	5.46	2,703	43.7	2,828
0.109	1,586	1.09	2,418	6.55	2,717	49.1	2,833
0.164	1,677	1.64	2,542	8.74	2,737	54.6	2,837
0.218	1,760	2.18	2,602	10.9	2,752	65.5	2,844
0.273	1,834	2.73	2,635	16.4	2,777	87.4	2,853
0.328	1,901	3.28	2,657	21.8	2,793	109.2	2,858
0.382	1,963	3.82	2,673	27.3	2,805	163.8	2,863
0.437	2,018	4.37	2,685	32.8	2,814	218.4	2,864
0.491	2,068	4.91	2,695	38.2	2,822		

From the time match point,

$$L_f = \left(\frac{0.000264 k \Delta t_{MP}}{\phi \mu c_t (t_{DL_f})_{MP}} \right)^{1/2}$$

$$= \left[\frac{(0.000264)(4.5)(0.062)}{(0.3)(0.23)(3 \times 10^{-5})(0.01)} \right]^{1/2}$$

$$= 59.7 \text{ ft.}$$

Exercises

Examples 4.1 and 4.2 were based on a portion of the following data for a drawdown test followed by a pressure buildup test.

- $q = 500 \text{ STB/D (constant)}$,
- $\phi = 0.2$,
- $\mu = 0.8 \text{ cp}$,
- $c_t = 10 \times 10^{-6} \text{ psi}^{-1}$,
- $p_i = 3,000 \text{ psi}$,
- $r_w = 0.3 \text{ ft}$,
- $h = 56 \text{ ft}$,
- $B_o = 1.2 \text{ RB/STB}$,
- $A_{wb} = 0.022 \text{ sq ft}$,
- $\rho = 50 \text{ lbm/cu ft}$,
- single-phase liquid,
- liquid/gas interface in wellbore, and
- well centered in a cylindrical drainage area with
- $r_e = 1,000 \text{ ft}$.

The drawdown test data are presented in Table 4.5; buildup test data ($t_p = 327.6$ hours) are given in Table 4.6.

4.1 Using conventional analysis techniques (p_{wf} vs. $\log t$ plot in ETR and MTR, p_{wf} vs. t plot in LTR), estimate k , s , E , t_{wbs} , V_p , r_e (assume cylindrical reservoir), and r_i at the beginning and end

of the MTR for the drawdown test.

4.2 Analyze the drawdown test data as completely as possible using Ramey's type curves. Can data in the LTR be analyzed with these curves? Why?

4.3 Analyze the drawdown test data using McKinley's type curves. Estimate k , k_{wb} , and E . Can the data in the LTR be analyzed with these curves? Why?

4.4 Analyze the buildup test using the Horner plotting technique. Estimate (1) k , s , E , t_{wbs} , and r_i at the beginning and end of the MTR, (2) $\bar{p}$ from the MBH and modified Muskat techniques, and (3) reservoir pore volume (using $\bar{p}$ before and after the drawdown test).

4.5 Analyze the buildup test as completely as possible using Ramey's type curves. Is there a shut-in time, Δt_{max} , beyond which the type-curve technique should not be used? Why?

4.6 Analyze the buildup test using McKinley's type curves. Estimate k_{wb} , k , and E . Is there a shut-in time beyond which the type-curve technique should not be used? Why?

4.7 In the buildup test analyzed in Example 4.3, does a linear flow region appear? If so, analyze the data using the conventional equations for linear flow in a reservoir. Does a radial flow region appear? If so, analyze the data using conventional methods.

4.8 A drawdown test was run in a vertically fractured oil well; the results are given in Table 4.7. Using the Gringarten *et al.* type curve, estimate fracture length and formation permeability. Identify linear flow and radial flow regions and verify the type-curve analysis with conventional analysis of these regions. As part of the conventional analysis of the radial flow region, estimate r_i at the beginning and end of the MTR and estimate fracture length from skin-factor calculation. The test data were as follows.

TABLE 4.7 – FRACTURED WELL DRAWDOWN TEST DATA

t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)
0	4,000	1.5	3,932	20	3,823
0.15	3,982	2.0	3,922	30	3,803
0.2	3,978	3.0	3,907	40	3,789
0.3	3,975	4.0	3,896	50	3,778
0.4	3,969	5.0	3,886	60	3,768
0.5	3,965	6.0	3,879	80	3,755
0.6	3,960	8.0	3,866	100	3,744
0.8	3,957	10	3,856		
1.0	3,950	15	3,837		

$q = 200 \text{ STB/D (constant)}$,
 $B = 1.288 \text{ RB/STB}$,
 $h = 12 \text{ ft}$,
 $\phi = 0.1$,
 $\mu = 0.5 \text{ cp}$,
 $c_t = 20 \times 10^{-6} \text{ psi}^{-1}$, and
 wellbore unloading distortion negligible at all times.

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Chapter 5

Gas Well Testing

5.1 Introduction

This chapter discusses deliverability tests of gas wells. The discussion includes basic theory of transient and pseudosteady-state flow of gases, expressed in terms of the pseudopressure $\psi(p)$ and of approximations to the pseudopressure approach that are valid at high and low pressures. This is followed by an examination of flow-after-flow, isochronal, and modified isochronal deliverability tests. The chapter concludes with an introduction to the application of pseudopressure in gas well test analysis.

5.2 Basic Theory of Gas Flow in Reservoirs

Investigations^{1,2} have shown that gas flow in infinite-acting reservoirs can be expressed by an equation similar to that for flow of slightly compressible liquids if pseudopressure $\psi(p)$ is used instead of pressure:

$$\psi(p_{wf}) = \psi(p_i) + 50,300 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[1.151 \cdot \log\left(\frac{1,688 \phi \mu_i c_{ti} r_w^2}{kt}\right) - (s + D|q_g|) \right], \quad (5.1)$$

where the pseudopressure is defined by the integral

$$\psi(p) = 2 \int_{p_\beta}^p \frac{p}{\mu z} dp. \quad (5.2)$$

The term $D|q_g|$ reflects a non-Darcy flow pressure loss – i.e., it takes into account the fact that, at high velocities near the producing well (characteristic of large gas production rates), Darcy's law does not predict correctly the relationship between flow rate and pressure drop. As a first approximation, this additional pressure drop can be added to the Darcy's law pressure drop, just as pressure drop across the altered zone is, and D can be considered constant. The absolute value of q_g , $|q_g|$, is used so that the term $D|q_g|$ is positive for either production or injection.

For stabilized flow³ ($r_i \geq r_e$),

$$\psi(p_{wf}) = \psi(\bar{p}) - 50,300 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + D|q_g| \right]. \quad (5.3)$$

Eqs. 5.1 and 5.3 provide the basis for analysis of gas well tests. As noted in Sec. 2.11, for $p > 3,000$ psi, these equations assume a simpler form (in terms of pressure, p); for $p < 2,000$ psi, they assume another simple form (in terms of p^2). Thus, we can develop procedures for analyzing gas well tests with equations in terms of $\psi(p)$, p , and p^2 . In most of this chapter, our equations will be written in terms of p^2 – not because p^2 is more generally applicable or more accurate (the equations in ψ best fit this role), but because the p^2 equations illustrate the general method and permit easier comparison with older methods of gas well test analysis that still are used widely.

Before developing the equations, let us generalize Eq. 5.1 to model a drawdown test starting from *any* uniform drainage-area pressure ($\bar{p}$) that may be much lower than initial pressure (p_i) after years of production:

$$\psi(p_{wf}) = \psi(\bar{p}) + 50,300 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[1.151 \cdot \log\left(\frac{1,688 \phi \mu_{\bar{p}} c_{t\bar{p}} r_w^2}{kt}\right) - (s + D|q_g|) \right], \quad (5.4)$$

where $p = \bar{p}$ for all r at $t_p = 0$. For $p < 2,000$ psia, $\mu z_g \approx \text{constant} \approx \mu_{\bar{p}} z_{\bar{p}g}$ for most gases; in this case,

$$\psi(p) = \frac{2}{\mu_{\bar{p}} z_{\bar{p}g}} \left(\frac{p^2}{2} - \frac{p_\beta^2}{2} \right).$$

Substituting into Eq. 5.4,

$$p_{wf}^2 = \bar{p}^2 + 1,637 \frac{q_g \mu_{\bar{p}} z_{\bar{p}g} T}{kh} \left[\log\left(\frac{1,688 \phi \mu_{\bar{p}} c_{t\bar{p}}}{kt_p}\right) \right]$$

$$-\left(\frac{s+D|q_g|}{1.151}\right)]. \dots\dots\dots (5.5)$$

For stabilized flow,

$$p_{wf}^2 = \bar{p}^2 - 1,422 \frac{q_g \mu \bar{p} z \bar{p}_g T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + D|q_g| \right]. \dots\dots\dots (5.6)$$

Eq. 5.6 is a complete deliverability equation. Given a value of flowing BHP, p_{wf} , corresponding to a given pipeline or backpressure, we can estimate the rate q_g at which the well will deliver gas. However, certain parameters must be determined before the equation can be used in this way:

1. The well flowed at rate q_g until $r_i \geq r_e$ (stabilized flow). In this case, note that Eq. 5.6 has the form

$$\bar{p}^2 - p_{wf}^2 = a q_g + b q_g^2, \dots\dots\dots (5.7)$$

where

$$a = 1,422 \frac{\mu \bar{p} z \bar{p}_g T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s \right], \dots\dots (5.8)$$

and

$$b = 1,422 \frac{\mu \bar{p} z \bar{p}_g T}{kh} D. \dots\dots\dots (5.9)$$

The constants a and b can be determined from flow tests for at least two rates in which q_g and the corresponding value of p_{wf} are measured; $\bar{p}$ also must be known.

2. The well flowed for times such that $r_i \leq r_e$ (transient flow). In this case, we will need to estimate kh , s , and D from transient tests (drawdown or buildup) modeled by Eq. 5.5 (or some adaptation of it using superposition); these parameters then can be combined with known (or assumed) values of $\bar{p}$ and r_e in Eq. 5.6 to provide deliverability estimates.

The gas flow rate q_g , used in Eqs. 5.1 through 5.7, should include all substances that are flowing in the vapor phase in the reservoir, with their volumes expressed at standard conditions. These substances include the gas produced as such at the surface, and condensate and liquid water produced at the surface that existed in the vapor phase in the reservoir. Calculation of the vapor equivalent of condensate is discussed in Appendix A of Ref. 4. Craft and Hawkins⁵ summarize the calculation of the vapor equivalent of produced *fresh* (nonformation) water.

Most of the remainder of this chapter provides detailed information on testing procedures that lead to estimates of the parameters required to provide deliverability estimates. This discussion is based on recommendations in the ERCB gas well testing manual.⁴

5.3 Flow-After-Flow Tests

In this testing method, a well flows at a selected constant rate until pressure stabilizes—i.e., pseudosteady state is reached. The stabilized rate and pressure are recorded; rate is then changed and the

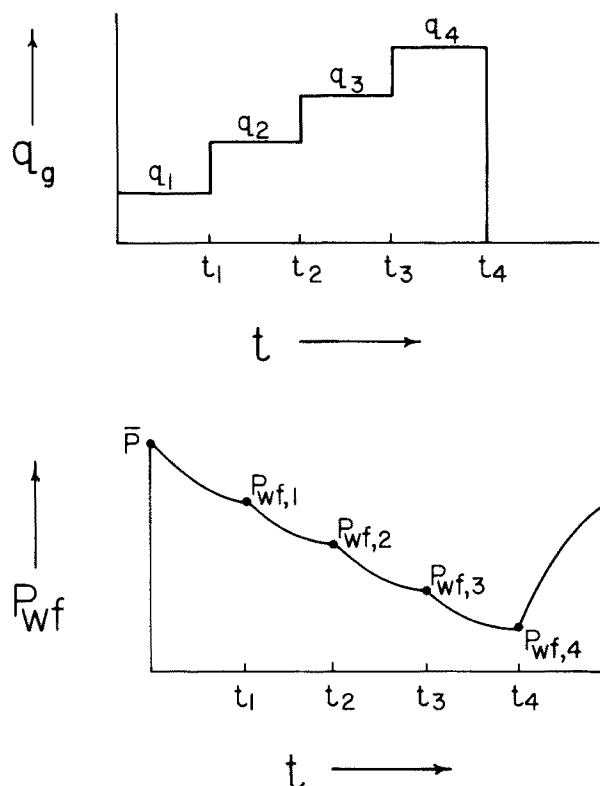


Fig. 5.1 – Rates and pressures in flow-after-flow test.

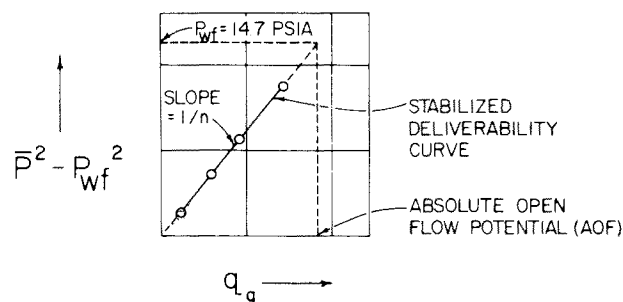


Fig. 5.2 – Empirical deliverability plot for flow-after-flow test.

TABLE 5.1 – STABILIZED FLOW TEST DATA

Test	p_{wf} (psia)	q_g (MMscf/D)
1	403.1	4.288
2	394.0	9.265
3	378.5	15.552
4	362.6	20.177

well flows until the pressure stabilizes again at the new rate. The process is repeated for a total of three or four rates.

Rates and pressures in a typical test follow the pattern indicated in Fig. 5.1. Two fundamentally different techniques can be used to analyze these test data.

Empirical Method

An empirical observation – with a rather tenuous theoretical basis – is that a plot of $\Delta p^2 = \bar{p}^2 - p_{wf}^2$ vs. q_g (Fig. 5.2) on log-log paper is approximately a straight line for many wells in which the pseudosteady state is reached at each rate in a flow-after-flow test sequence. The equation of the line in this plot is

$$q_g = C(\bar{p}^2 - p_{wf}^2)^n. \quad (5.10)$$

This plot is an empirical correlation of field data. As in any other empirical correlation, there is substantial risk of error in extrapolating the plot a large distance beyond the region in which data were obtained. Unfortunately, such an extrapolation is frequently required. To estimate the absolute open flow potential (AOF) – the theoretical rate at which the well would produce if the flowing pressure p_{wf} were atmospheric – it may be necessary to extrapolate the curve far beyond the range of test data. An AOF determined from such a lengthy extrapolation may be incorrect.

The constants C and n in Eq. 5.10 are not constants at all. They depend on fluid properties that are pressure (and, thus, time) dependent. Accordingly, if this type of deliverability curve is used, periodic retesting of the well will show changes in C and perhaps in n .

We must emphasize that deliverability estimates based on this plot assume that pressures were stabilized ($r_i \geq r_e$) during the testing period used to construct the plot. If this is not the case, stabilized deliverability estimates from the curve can be highly misleading.

Theoretical Method

Eq. 5.7 suggests that we plot $(\bar{p}^2 - p_{wf}^2)/q_g$ vs. q_g ; the result (for *pseudosteady-state flow*) should be a straight line with slope b and intercept a . Because this line has a sounder theoretical basis than the log $\Delta p^2 - \log q_g$ plot, it should be possible to extrapolate it to determine AOF with less error and to correct deliverability estimates for changes in $\mu_{\bar{p}}$, $z_{\bar{p}g}$, etc., more readily.

TABLE 5.2 – STABILIZED FLOW TEST ANALYSIS

p_{wf} (psia)	q_g (MMscf/D)	$\bar{p}^2 - p_{wf}^2$ (psia ²)	$(\bar{p}^2 - p_{wf}^2)/q_g$ (psia ² /MMscf/D)
408.2	0	—	—
403.1	4.288	4,138	964.9
394.0	9.265	11,391	1,229
378.5	15.552	23,365	1,502
362.6	20.177	35,148	1,742
14.7	AOF	166,411	—

Example 5.1 – Stabilized Flow Test Analysis

Problem. The data in Table 5.1 were reported for a flow-after-flow (or four-point) test in Ref. 6. At each rate, pseudosteady state was reached. Initial (i.e., before the test) shut-in BHP, $\bar{p}$, was determined to be 408.2 psia. Estimate the AOF of the tested well using (1) the empirical plot and (2) the theoretical flow equation. In addition, plot deliverabilities estimated using the theoretical equation on the empirical curve plot.

Solution. We prepare a table of data (Table 5.2) to be plotted for both empirical and theoretical analyses.

1. *Empirical Method.* From a plot of $(\bar{p}^2 - p_{wf}^2)$ vs. q_g on log-log paper, and extrapolation of this plot to $\bar{p}^2 - p_{wf}^2 = 166,411$ (where $p_{wf} = 0$ psig or 14.7 psia), AOF ≈ 60 MMscf/D.

The slope of the curve, $1/n$, is

$$\begin{aligned} 1/n &= \frac{\log(\bar{p}^2 - p_{wf}^2)_2 - \log(\bar{p}^2 - p_{wf}^2)_1}{\log q_{g,2} - \log q_{g,1}} \\ &= \frac{\log\left(\frac{10^5}{10^3}\right)}{\log\left(\frac{42.5}{1.77}\right)} = 1.449. \end{aligned}$$

Thus, $n = 0.690$. Then,

$$\begin{aligned} C &= \frac{q_g}{(\bar{p}^2 - p_{wf}^2)^n} \\ &= \frac{42.5}{(10^5)^{0.690}} = 0.01508. \end{aligned}$$

Thus, the empirical deliverability equation is

$$q_g = 0.01508(\bar{p}^2 - p_{wf}^2)^{0.690}.$$

These data are plotted in Fig. 5.3.

2. *Theoretical Method.* The theoretical deliverability equation is

$$(\bar{p}^2 - p_{wf}^2)/q_g = a + bq_g.$$

Fig. 5.4 is a plot of $(\bar{p}^2 - p_{wf}^2)/q_g$ vs. q_g for the test data. Two points on the best straight line through the data are (2.7; 900) and (23.9; 1,900). Thus,

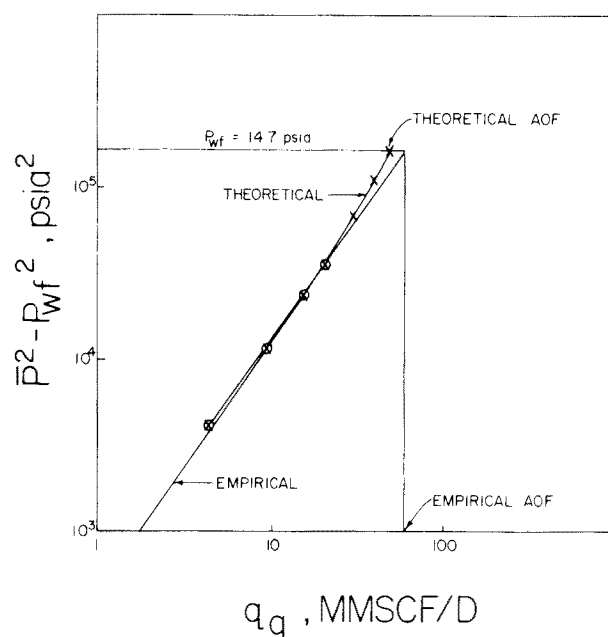


Fig. 5.3 – Stabilized gas well deliverability test.

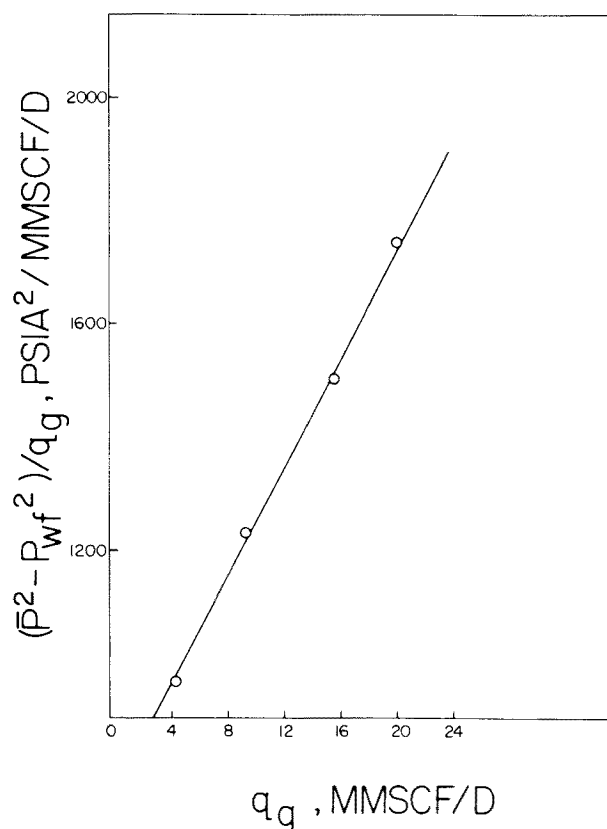


Fig. 5.4 – Stabilized deliverability test, theoretical flow equation, constant determination.

$$900 = a + 2.7 b,$$

$$1,900 = a + 23.9 b.$$

Solving for a and b , we find that $a=773$ and $b=47.17$. Thus, the theoretical deliverability equation is

$$47.17 q_g^2 + 773 q_g = (\bar{p}^2 - p_{wf}^2).$$

We can solve this quadratic equation for the AOF:

$$47.17 q_g^2 + 773 q_g - 166,411 = 0.$$

The solution is

$$q_g = \text{AOF} = 51.8 \text{ MMscf/D.}$$

We also can determine points on the deliverability curve as calculated from the theoretical equation: $\bar{p}^2 - p_{wf}^2 = 47.17 q_g^2 + 773 q_g$. See Table 5.3. These results are plotted in Fig. 5.3. The plot is *almost* linear, but there is sufficient curvature to cause a 15.8% error in calculated AOF.

5.4 Isochronal Tests

The objective of isochronal testing⁷ is to obtain data to establish a stabilized deliverability curve for a gas well without flowing the well for sufficiently long to achieve stabilized conditions ($r_i \geq r_e$) at *each* (or, in some cases, *any*) rate. This procedure is needed for

lower-permeability reservoirs, where it frequently is impractical to achieve $r_i = r_e$ during the test. An isochronal test is conducted by flowing a well at a fixed rate, then shutting it in until the pressure builds up to an unchanging (or almost unchanging) value, $\bar{p}$. The well then is flowed at a second rate for the same length of time, followed by another shut-in, etc. If possible, the final flow period should be long enough to achieve stabilized flow. If this is impossible or impractical, it is still possible to predict the stabilized deliverability characteristics (with increased potential for error, of course).

In obtaining data in this testing program, it is essential to record flowing BHP, p_{wf} , as a function of time at each flow rate.

Fig. 5.5 illustrates rate and pressures in an isochronal testing sequence. This figure illustrates the following important points about the isochronal testing sequence.

1. Flow periods, excepting the final one, are of equal length [i.e., $t_1 = (t_3 - t_2) = (t_5 - t_4) \leq (t_7 - t_6)$].
2. Shut-in periods have the objective of letting $p \approx \bar{p}$ rather than the objective of equal length. Thus, in general, $(t_2 - t_1) \neq (t_4 - t_3) \neq (t_6 - t_5)$.
3. A final flow period in which the well stabilizes (i.e., r_i reaches r_e at time t_7) is desirable but not essential.

The most general theory of isochronal tests is based on equations using pseudopressure. However, we will once again present the theory in terms of the low-pressure approximations to these equations (p^2

TABLE 5.3 – THEORETICAL DELIVERABILITIES

q_g (MMsc/D)	$\bar{p}^2 - p_{wf}^2$ (psia ²)
4.288	4,182
9.265	11,210
15.552	23,430
20.177	34,800
30	65,640
40	106,400
49.8 = AOF	166,600

equations) because (1) they are somewhat simpler and less abstract than equations in pseudopressure and (2) they allow direct comparison with more conventional analysis methods⁷ based on plots of $(\bar{p}^2 - p_{wf}^2)$ vs. q_g on log-log paper.

Eqs. 5.5 and 5.6 provide the basic method for interpreting isochronal tests.

For transient flow ($r_i < r_e$),

$$p_{wf}^2 = \bar{p}^2 + 1,637 \frac{\mu \bar{p} z \bar{p}_g}{kh} T q_g$$

$$\left[\log \left(\frac{1,688 \phi \mu \bar{p} c_{tp} r_w^2}{kt} \right) - \left(\frac{s+D|q_g|}{1.151} \right) \right] \dots \dots \dots (5.5)$$

For stabilized flow ($r_i \geq r_e$),

$$p_{wf}^2 = \bar{p}^2 - 1,422 \frac{\mu \bar{p} z \bar{p}_g}{kh} T q_g \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + (s+D|q_g|) \right] \dots \dots \dots (5.6)$$

In addition to the flow equations, an important theoretical consideration in understanding isochronal tests is the radius-of-investigation concept. We observed previously that the radius of investigation achieved at a given time in a flow test is *independent of flow rate* and, thus, should be the same at a given time for each flow rate in an isochronal test. Further, the radius of investigation at a given time can be considered to be proportional to a drainage radius at that time, because it is near (but slightly less than) the point beyond which there has been no appreciable drawdown in reservoir pressure and thus no fluid drainage. Accordingly, at a given time, the same portion of the reservoir is being drained at each rate and, as a good approximation, stabilized flow conditions exist to a point just beyond $r = r_i$. Thus, a deliverability curve can be drawn at each fixed time (hence, the name isochronal) through

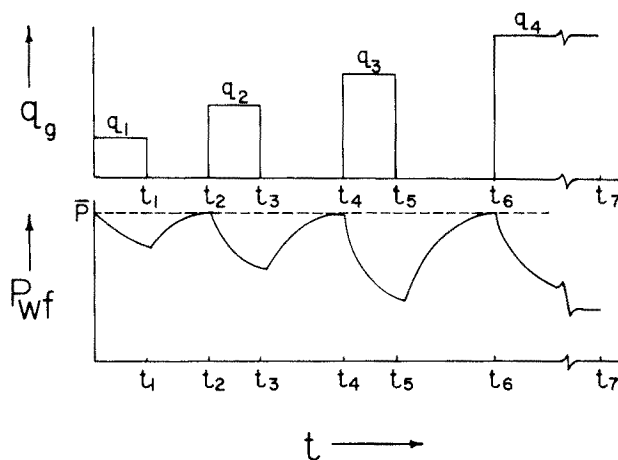


Fig. 5.5 – Rates and pressures in isochronal test.

points $(q_g, \bar{p}^2 - p_{wf}^2)$ obtained at that time at several different rates, and a truly stabilized deliverability curve can be drawn when $r_i \geq r_e$.

These assertions can be made more quantitative if we note that for flowing time t , at each rate, there corresponds a drainage radius, $r_d' = cr_i$, that is independent of rate. Admittedly anticipating a convenient result, we let $r_d' = 1.585 r_i$ (but the p vs. $\log r$ plot constructed in the solution to Problem 1.2 shows that such an r_d' approximates quite closely the point beyond which no appreciable fluid is being drained). At time t_1 ,

$$p_{wf}^2 = \bar{p}^2 + 1,422 \frac{\mu \bar{p} z \bar{p}_g}{kh} T q_g \left[\frac{1}{2} \ln \left(\frac{1,688 \phi \mu \bar{p} c_{tp} r_w^2}{kt_1} \right) - (s+D|q_g|) \right]$$

Because

$$\begin{aligned} r_d'^2 &= (1.585)^2 r_i^2 \\ &= \frac{(1.585)^2 kt_1}{948 \phi \mu \bar{p} c_{tp}} \\ &= \frac{kt_1}{377 \phi \mu \bar{p} c_{tp}} \end{aligned}$$

we may write the transient flow equation as

$$p_{wf}^2 = \bar{p}^2 - 1,422 \frac{\mu \bar{p} z \bar{p}_g}{kh} T q_g \left[\ln \left(\frac{r_d'}{r_w} \right) - 0.75 + (s+D|q_g|) \right]$$

Compare with the stabilized deliverability equation:

$$p_{wf}^2 = \bar{p}^2 - 1,422 \frac{\mu \bar{p} z \bar{p}_g}{kh} T q_g \left[\ln \left(\frac{r_e}{r_w} \right) \right]$$

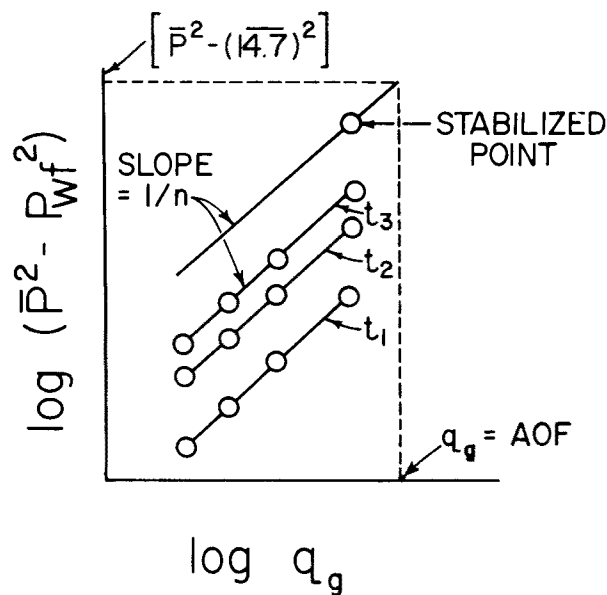


Fig. 5.6 – Empirical deliverability plot for Isochronal test.

$$-0.75 + (s + D|q_g|)].$$

The equations are identical in form because we have defined a time-dependent drainage radius, r_d' , as

$$r_d' = \left(\frac{kt}{377 \phi \mu_{\bar{p}} c_{t\bar{p}}} \right)^{1/2}.$$

Thus, we conclude that, at each fixed time t_1 , an analysis of different rates used in an isochronal test should be possible just as it is for a stabilized test – except, of course, the data will not yield a truly stabilized deliverability curve. This is possible only if stabilized data are available or if they can be estimated.

Analysis of Test Data:

One Rate Continued to Stabilization

Experience⁷ shows that reasonably satisfactory results can be obtained with the empirical method using the following procedure.

1. The best straight line is drawn through the points $(\bar{p}^2 - p_{wf}^2, q_g)$ obtained at a fixed value of time with the different rates used in the isochronal testing program. The data are plotted on log-log paper, just as when analyzing a stabilized deliverability curve.

2. Lines should be drawn for several values of time t , and the slope $1/n$ should be established for each isochronal deliverability curve.

3. A line with the slope $1/n$ determined from the nonstabilized, fixed-time curves then is drawn through the single stabilized point, $(q_g, \bar{p}^2 - p_{wf}^2)$. This establishes the stabilized deliverability curve. Once the stabilized deliverability curve is determined, AOF is established in the usual way, as indicated in Fig. 5.6.

The theoretical method for analyzing isochronal test data is based on the theoretical equations for

TABLE 5.4 – ISOCHRONAL TEST DATA

Test	Duration (hours)	p_{wf} or p_{ws} (psia)	q_g (MMscf/D)
Initial shut-in	48	1,952	–
First flow	12	1,761	2.6
First shut-in	15	1,952	–
Second flow	12	1,694	3.3
Second shut-in	17	1,952	–
Third flow	12	1,510	5.0
Third shut-in	18	1,952	–
Fourth flow	12	1,320	6.3
Extended flow (stabilized)	72	1,151	6.0
Final shut-in	100	1,952	–

TABLE 5.5 – ISOCHRONAL TEST ANALYSIS

q_g (MMscf/D)	$\bar{p}^2 - p_{wf}^2$ (psia ²)	$(\bar{p}^2 - p_{wf}^2) / q_g$ (psia ² /MMscf/D)
2.6	709,000	273,000
3.3	941,000	285,000
5.0	1,530,000	306,000
6.3	2,070,000	328,200

stabilized flow and transient flow written in the form below. For stabilized flow,

$$\bar{p}^2 - p_{wf}^2 = a q_g + b q_g^2, \quad \dots \dots \dots (5.7)$$

where

$$a = 1,422 \frac{\mu_{\bar{p}} z_{\bar{p}g} T}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s \right], \quad \dots \dots (5.8)$$

and

$$b = 1,422 \frac{\mu_{\bar{p}} z_{\bar{p}g} TD}{kh} \quad \dots \dots \dots (5.9)$$

For transient flow,

$$\bar{p}^2 - p_{wf}^2 = a_t q_g + b q_g^2, \quad \dots \dots \dots (5.11)$$

where b has the same meaning as for stabilized flow and where a_t , a function of time, is given by

$$a_t = 1,422 \frac{\mu_{\bar{p}} z_{\bar{p}g} T}{kh} \left[\frac{1}{2} \ln \left(\frac{kt}{1,688 \phi \mu_{\bar{p}} c_{t\bar{p}} r_w^2} \right) + s \right]. \quad \dots \dots \dots (5.12)$$

Thus,

$$\bar{p}^2 - p_{wf}^2 = a_t q_g + b q_g^2, \quad r_i < r_e, \quad \dots \dots \dots (5.11)$$

and

$$\bar{p}^2 - p_{wf}^2 = a q_g + b q_g^2, \quad r_i \geq r_e. \quad \dots \dots \dots (5.7)$$

An analysis method of isochronal tests consistent with the theoretical equations follows.

1. For a fixed value of t , determine b from a plot of $(\bar{p}^2 - p_{wf}^2) / q_g$ vs. q_g .

2. Using the stabilized data point $[q_{gs}, (\bar{p}^2 - p_{wf}^2)_s]$, determine a from

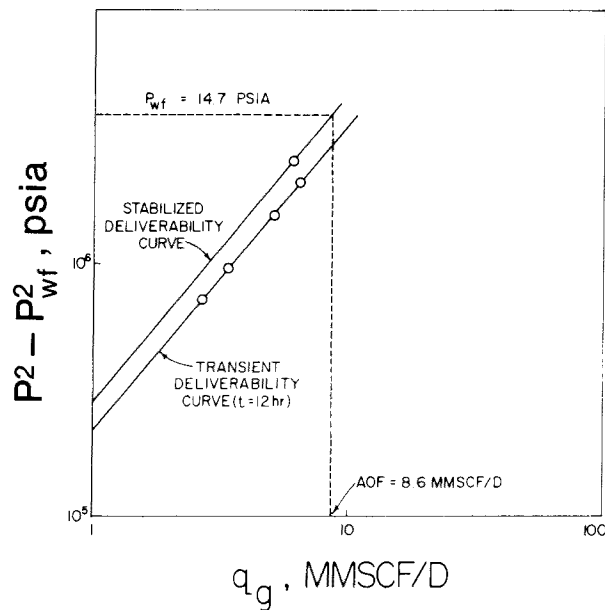


Fig. 5.7 – Isochronal deliverability test analysis.

$$a = [(\bar{p}^2 - p_{wf}^2)_s - bq_{gs}^2] / q_{gs}.$$

3. The stabilized deliverability curve uses the constants determined in Steps 1 and 2:

$$\bar{p}^2 - p_{wf}^2 = aq_g + bq_g^2.$$

This equation can be used to calculate the AOF, for example:

$$AOF = \frac{-a + \sqrt{a^2 + 4b(\bar{p}^2 - 14.7^2)}}{2b}.$$

Example 5.2 – Isochronal Gas Well Test Analysis

Problem. Determine the stabilized deliverability curve and AOF from the test data in Table 5.4 (adapted from data given in Ref. 4) using (1) the empirical method and (2) the theoretical method. For simplicity, data at only a single flow time are reported.

Solution. 1. *Empirical Method.* Data from the 12-hour flow tests are analyzed first to determine the slope of the deliverability curve (Table 5.5). We also note that at stabilized rate $q_g = 6.0$ MMscf/D, $\bar{p}^2 - p_{wf}^2 = 2,485,000$ psia², and that when $q_g = AOF$, $\bar{p}^2 - p_{wf}^2 = 3,810,000$ psia².

The data are plotted in Fig. 5.7; from the plot, $AOF \approx 8.6$ MMscf/D.

2. *Theoretical Method.* We first determine b , using a plot of $(\bar{p}^2 - p_{wf}^2)/q_g$ vs. q_g from the 12-hour test data. The line drawn (Fig. 5.8) passes through the points with greatest pressure drawdown and, thus, least potential error: [$q_g = 5.0$ MMscf/D, $(\bar{p}^2 - p_{wf}^2)/q_g = 306,000$ psia²/MMscf/D] and [$q_g = 6.3$ MMscf/D, $(\bar{p}^2 - p_{wf}^2)/q_g = 328,000$ psia²/MMscf/D]. Thus,

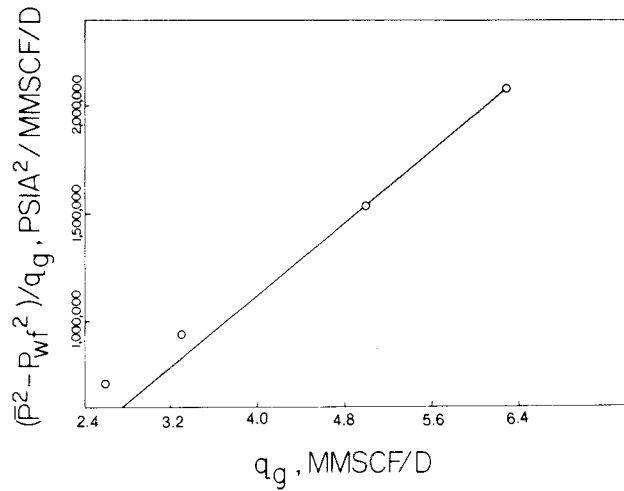


Fig. 5.8 – Isochronal deliverability test analysis, theoretical flow equation, constant determination.

$$306,000 = a_{12} + 5.0 b,$$

$$328,000 = a_{12} + 6.3 b.$$

Solving, we find that $b = 17,080$.

From the stabilized test, $q_g = 6.0$ MMscf/D and $\bar{p}^2 - p_{wf}^2 = 2,485,000$ psia². Thus,

$$a = \frac{(\bar{p}^2 - p_{wf}^2)_s - bq_{gs}^2}{q_{gs}} = \frac{2,485,000 - (17,080)(6.0)^2}{6.0}$$

$$= 311,700.$$

Thus, the stabilized deliverability curve is

$$\bar{p}^2 - p_{wf}^2 = 311,700 q_g + 17,080 q_g^2.$$

Solving for AOF, we find that it is equal to 8.38 MMscf/D. This value is quite close to the value established using the empirical method. This is at least partly because the extrapolation of the empirical curve is not far beyond the observed data.

Analysis of Test Data:

No Stabilized Flow Attained

In very low-permeability reservoirs, stabilized flow cannot be achieved in a reasonable length of time. In such cases, a satisfactory determination of the stabilized deliverability curve can be based on use of the theoretical equations for transient flow (Eq. 5.5) and stabilized flow (Eq. 5.6). We have noted that the stabilized equation can be expressed as

$$\bar{p}^2 - p_{wf}^2 = aq_g + bq_g^2, \quad \dots \dots \dots (5.7)$$

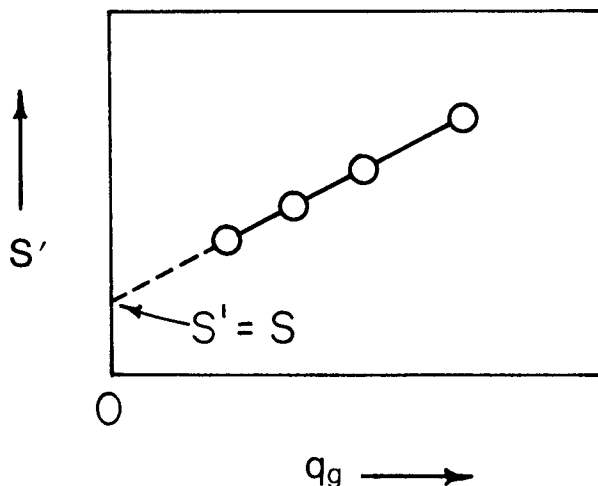


Fig. 5.9 – Skin factor determination.

where

$$a = 1,422 \frac{\mu \bar{p} z \bar{p}_g T}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s \right], \dots (5.8)$$

and

$$b = 1,422 \frac{\mu \bar{p} z \bar{p}_g TD}{kh} \dots (5.9)$$

We also noted that the transient equation has the form

$$\bar{p}^2 - p_{wf}^2 = a_i q_g + b q_g^2, \dots (5.11)$$

where

$$a_i = 1,422 \frac{\mu \bar{p} z \bar{p}_g T}{kh} \left[\frac{1}{2} \ln \left(\frac{kt}{1,688 \phi \mu \bar{p} c_{tp} r_w^2} \right) + s \right] \dots (5.8)$$

The objective of determining the stabilized flow equation can be achieved if the constants a and b can be determined. We note that the constant b can be determined from the isochronal test data as illustrated [by plotting $(\bar{p}^2 - p_{wf}^2)/q_g$ vs. q_g for fixed value of t and determining the slope, b , of the resulting best straight line]. Constant a , however, is more troublesome: The only satisfactory means of determining a is through knowledge of each term in the defining equation for this quantity. Thus, we need estimates of kh , s , and r_e . (Other quantities in Eq. 5.8 are usually available.)

Since an isochronal test consists of a series of drawdown and buildup tests, kh and s usually can be determined from them. Determination of kh is straightforward in principle; determination of s is less straightforward. Recall that a single test provides only an estimate of $s' = s + D|q_g|$. Accordingly, to determine s , we must analyze at least two tests: either

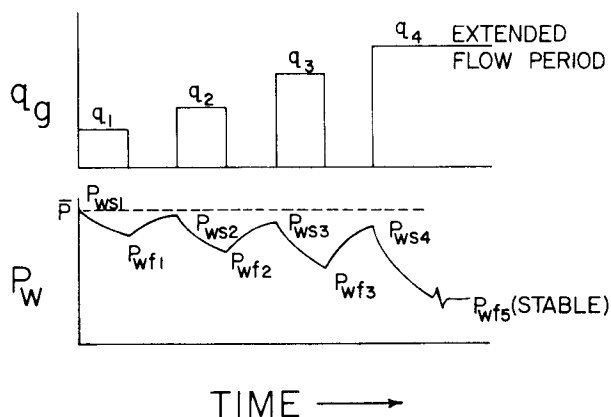


Fig. 5.10 – Rates and pressures in modified isochronal test.

drawdown tests run at different rates or buildup tests following drawdown tests at different rates. We can then plot s' vs. q_g ; extrapolation to $q_g = 0$ provides an estimate of true skin factor, s (see Fig. 5.9). The drainage radius r_e must be estimated from expected well spacing (or knowledge of reservoir geometry in a small or irregular reservoir).

The constants a and b determined in this way then can be substituted into the stabilized deliverability equation, Eq. 5.7. If a plot of $\log(\bar{p}^2 - p_{wf}^2)$ vs. $\log q_g$ also is desired, data points to be plotted can be determined from the equation.

5.5 Modified Isochronal Tests

The objective of modified isochronal tests is to obtain the same data as in an isochronal test without using the sometimes lengthy shut-in periods required for pressure to stabilize completely before each flow test is run.

In the modified isochronal test (Fig. 5.10), shut-in periods of the same duration as the flow periods are used, and the final shut-in BHP (p_{ws}) before the beginning of a new flow period is used as an approximation to $\bar{p}$ in the test analysis procedure. For example, for the first flow period, use $(\bar{p}^2 - p_{wf,1}^2) = (p_{ws,1}^2 - p_{wf,1}^2)$; for the second flow period, use $(\bar{p}^2 - p_{wf,2}^2) = (p_{ws,2}^2 - p_{wf,2}^2)$. Otherwise, the analysis procedure is the same as for the “true” isochronal test.

Note that the modified isochronal procedure uses *approximations*. Isochronal tests are modeled exactly by rigorous theory (if reservoir and fluid properties cooperate); modified isochronal tests are not. However, modified isochronal tests are used widely because they conserve time and money and because they have proved to be excellent approximations to true isochronal tests.

TABLE 5.6 – MODIFIED ISOCHRONAL TEST DATA

Test	Duration (hours)	p_{wf} or p_{ws} (psia)	q_g (MMscf/D)
Pretest shut-in	20	1,948	—
First flow	12	1,784	4.50
First shut-in	12	1,927	—
Second flow	12	1,680	5.60
Second shut-in	12	1,911	—
Third flow	12	1,546	6.85
Third shut-in	12	1,887	—
Fourth flow	12	1,355	8.25
Extended flow (stabilized)	81	1,233	8.00
Final shut-in	120	1,948	—

TABLE 5.7 – MODIFIED ISOCHRONAL TEST ANALYSIS

q_g (MMscf/D)	p_{ws} (psia)	p_{wf} (psia)	$p_{ws}^2 - p_{wf}^2$ (psia ²)	$(p_{ws}^2 - p_{wf}^2) / q_g$
4.50	1,948	1,784	612,048	136,011
5.60	1,927	1,680	890,929	159,094
6.85	1,911	1,546	1,261,805	184,205
8.25	1,887	1,355	1,724,744	209,060
8.00	1,948*	1,233	2,274,415	—

*Note that $\bar{p}$, the true current reservoir pressure, is used for the stabilized test analysis.

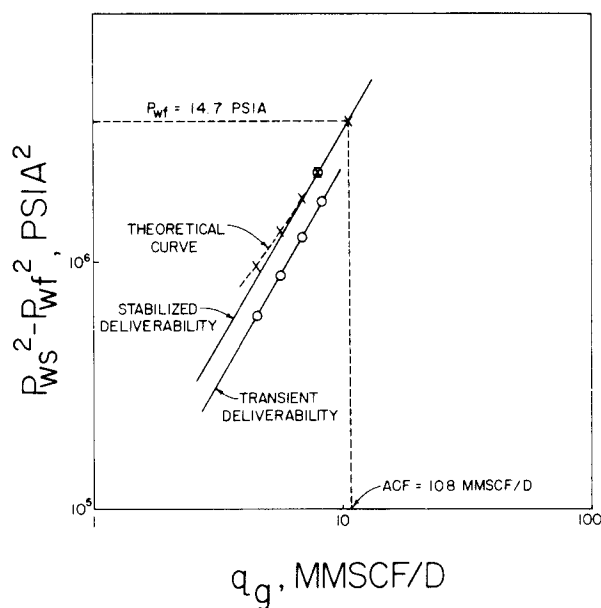


Fig. 5.11 – Modified isochronal test analysis.

Example 5.3 – Modified Isochronal Test Analysis

Problem. Estimate the AOF from the data in Table 5.6 obtained in a modified isochronal test,⁴ using both empirical and theoretical methods.

Solution. We first prepare the data for plotting (Table 5.7). Fig. 5.11 shows the data plot for the empirical method. This is a plot of $(p_{ws}^2 - p_{wf}^2)$ vs. q_g on log-log paper. The transient points are used to establish the slope of the curve, and a line with the same slope is drawn through the single stabilized point. The AOF is the value of q_g when $p_{ws}^2 - p_{wf}^2 = \bar{p}^2 - p_{wf}^2 = 1,948^2 - 14.7^2 = 3,790,000$ psia²;

TABLE 5.8 – THEORETICAL STABILIZED DELIVERABILITIES

q_g (MMscf/D)	$\bar{p}^2 - p_{wf}^2$ (psia ²)
4.5	972,000
5.6	1,330,000
6.85	1,794,000
8.0	2,274,000
10.8	3,660,000

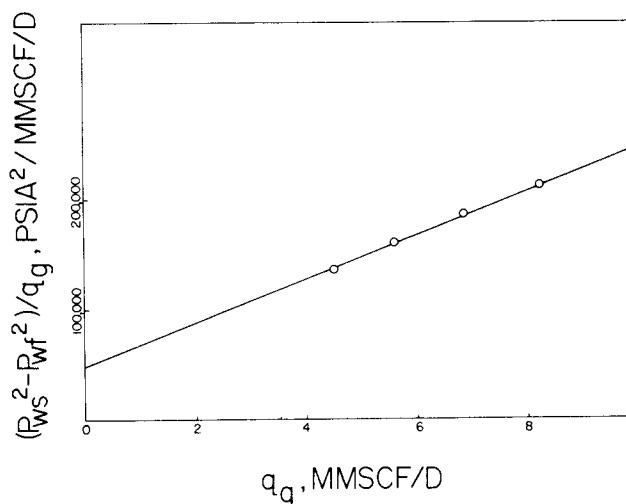


Fig. 5.12 – Modified isochronal test analysis, theoretical flow equation, constant determination.

this value is AOF = 10.8 MMscf/D.

For the theoretical method, we establish the constant b from the slope of a plot of $(p_{ws}^2 - p_{wf}^2) / q_g$ vs. q_g ; in this case,

$$b = \frac{243,000 - 48,000}{10} = 19,500,$$

using data from Fig. 5.12. Then,

$$a = \frac{(\bar{p}^2 - p_{wf}^2)_s - b q_{gs}^2}{q_{gs}} = \frac{2,274,415 - (19,500)(8.0)^2}{8.0} = 128,300.$$

Thus, the equation of the stabilized deliverability curve is

$$\bar{p}^2 - p_{wf}^2 = 128,300 q_g + 19,500 q_g^2.$$

Solving this equation for the AOF (q_g when $\bar{p}^2 - p_{wf}^2 = 3,790,000$), we find that it is equal to 11.0 MMscf/D.

It is also of interest to calculate points on the stabilized deliverability curve and to plot them on the

**TABLE 5.9 – GAS PROPERTIES
FOR EXAMPLE 5.4**

p (psia)	μ_g (cp)	z	$p/\mu z$ (psia/cp)
150	0.01238	0.9856	12,290
300	0.01254	0.9717	24,620
450	0.01274	0.9582	36,860
600	0.01303	0.9453	48,710
750	0.01329	0.9332	60,470
900	0.01360	0.9218	71,790
1,050	0.01387	0.9112	83,080
1,200	0.01428	0.9016	93,205
1,350	0.01451	0.8931	104,200
1,500	0.01485	0.8857	114,000
1,650	0.01520	0.8795	123,400
1,800	0.01554	0.8745	132,500
1,950	0.01589	0.8708	140,900
2,100	0.01630	0.8684	148,400
2,250	0.01676	0.8671	154,800
2,400	0.01721	0.8671	160,800
2,550	0.01767	0.8683	166,200
2,700	0.01813	0.8705	171,100
2,850	0.01862	0.8738	175,200
3,000	0.01911	0.8780	178,800
3,150	0.01961	0.8830	181,900

**TABLE 5.10 – PSEUDOPRESSURE
FOR EXAMPLE 5.4**

p (psia)	$\psi(p)$ (psia ² /cp)
150	1.844×10^6
300	7.381×10^6
450	1.660×10^7
600	2.944×10^7
750	4.582×10^7
900	6.566×10^7
1,050	8.888×10^7
1,200	1.154×10^8
1,350	1.451×10^8
1,500	1.779×10^8
1,650	2.135×10^8
1,800	2.518×10^8
1,950	2.929×10^8
2,100	3.363×10^8
2,250	3.817×10^8
2,400	4.291×10^8
2,550	4.781×10^8
2,700	5.287×10^8
2,850	5.807×10^8
3,000	6.338×10^8
3,150	6.879×10^8

empirical data plot. The values are given in Table 5.8. These data are plotted in Fig. 5.11.

5.6 Use of Pseudopressure in Gas Well Test Analysis

Accuracy of gas well test analysis can be improved in some cases if the pseudopressure $\psi(p)$ is used instead of approximations written in terms of pressure or pressure squared. In this section, we discuss the calculation of pseudopressure and provide an introduction to direct use of this quantity in gas-well drawdown test analysis. Detailed discussion, including systematic development of working equations and application to drawdown, buildup, and deliverability tests, is provided in Ref. 4.

Calculation of Pseudopressure

Gas pseudopressure, $\psi(p)$, is defined by the integral

$$\psi(p) = 2 \int_{p_\beta}^p \frac{p}{\mu z} dp, \dots \dots \dots (5.2)$$

where p_β is some arbitrary low base pressure. To evaluate $\psi(p)$ at some value of p , we can evaluate the integral in Eq. 5.2 numerically, using values for μ and z for the *specific* gas under consideration, evaluated at reservoir temperature. An example will illustrate this calculation.

Example 5.4 – Calculation of Gas Pseudopressure

Problem. Calculate the gas pseudopressure $\psi(p)$ for a reservoir containing 0.7 gravity gas at 200°F as a function of pressure in the range 150 to 3,150 psia. Gas properties as functions of pressure are given in Table 5.9.

Solution. We will select $p_\beta = 0$ and use the fact that, as $p_\beta \rightarrow 0$, $p/\mu z \rightarrow 0$. We will use the trapezoidal rule for our numerical integration.

For $p = 150$ psia,

$$\begin{aligned} \psi(150) &= 2 \int_{p_\beta}^p \frac{p}{\mu z} dp \\ &\approx 2 \frac{\left[\left(\frac{p}{\mu z} \right)_0 + \left(\frac{p}{\mu z} \right)_{150} \right]}{2} (150 - 0) \\ &= \frac{2[0 + 12,290]}{2} (150) \\ &= 1.844 \times 10^6 \text{ psia}^2/\text{cp}. \end{aligned}$$

For $p = 300$ psia,

$$\begin{aligned} \psi(300) &\approx 1.844 \times 10^6 \\ &+ 2 \left(\frac{12,290 + 24,620}{2} \right) (300 - 150) \\ &= 7.381 \times 10^6 \text{ psia}^2/\text{cp}. \end{aligned}$$

Proceeding in a similar way, we can construct Table 5.10. These results are plotted in Fig. 5.13.

Drawdown Test Analysis Using Pseudopressure

Transient flow at a constant rate from an infinite-acting gas reservoir is modeled by Eq. 5.1.

$$\begin{aligned} \psi(p_{wf}) &= \psi(p_i) + 50,300 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \\ &\quad \cdot \left\{ 1.151 \log \left[1,688 \frac{\phi \mu_i c_{ti} r_w^2}{kt} \right] \right\} \end{aligned}$$

TABLE 5.11 – DRAWDOWN TEST DATA

t (hours)	p_{wf} (psia)	$\psi(p_{wf})$ (psia ² /cp)
0	3,000	6.338×10^8
0.024	2,964	6.210×10^8
0.096	2,920	6.055×10^8
0.244	2,890	5.947×10^8
0.686	2,866	5.864×10^8
2.015	2,848	5.801×10^8
6.00	2,833	5.747×10^8
17.96	2,817	5.693×10^8
53.82	2,802	5.640×10^8
161.	2,786	5.585×10^8
281	2,777	5.553×10^8
401	2,771	5.532×10^8
521	2,766	5.517×10^8
641	2,763	5.505×10^8
761	2,760	5.494×10^8
881	2,757	5.485×10^8

TABLE 5.12 – DRAWDOWN TEST DATA
FOR CURVE MATCHING

t (hours)	$\psi(p_i) - (p_{wf})$ (psia ² /cp)
0.024	0.128×10^8
0.096	0.283×10^8
0.244	0.391×10^8
0.686	0.474×10^8
2.015	0.537×10^8
6.00	0.591×10^8
17.96	0.645×10^8
53.82	0.698×10^8
161	0.753×10^8
281	0.785×10^8
401	0.806×10^8
521	0.821×10^8
641	0.833×10^8
761	0.844×10^8
881	0.853×10^8

$$- (s + D|q_g|) \Big] \Big\} \dots \dots \dots (5.1)$$

This equation describes the MTR in a gas well test, just as Eq. 3.1 describes flow of a slightly compressible liquid. In general, of course, drawdown tests in gas wells also have ETR's (usually dominated by wellbore storage distortion) and LTR's (in which boundary effects become important). Analysis of a gas well test using pseudopressure as the dependent variable is illustrated in Example 5.5.

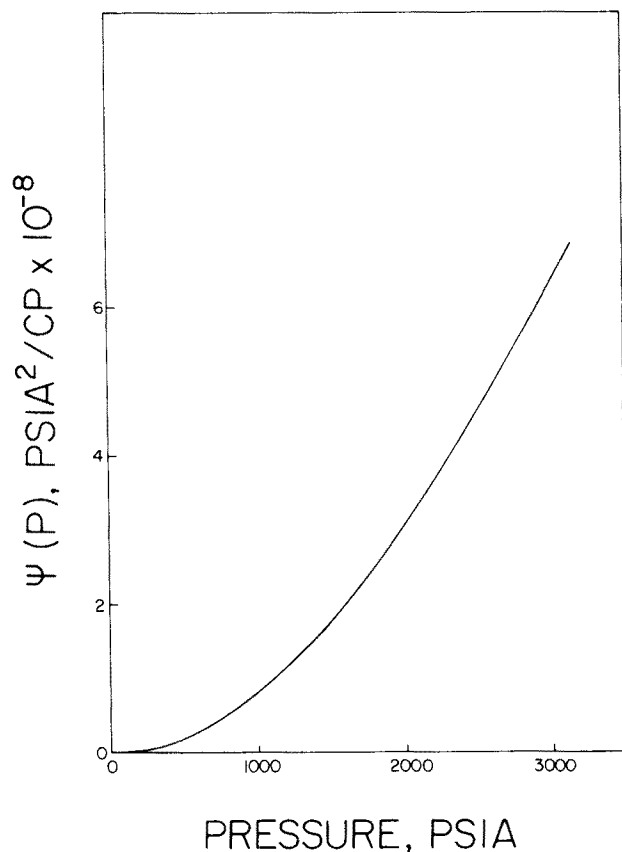


Fig. 5.13 – Pseudopressure vs. pressure.

Example 5.5 – Analysis of Gas Well Drawdown Test Using Pseudopressures

Problem. A constant-rate drawdown test was run on a gas well with the properties given below; results are shown in Table 5.11.

$p_i = 3,000$ psia,
 $\phi = 0.19$,
 $S_{wi} = 0.211$,
 $V_w = 286$ cu ft,
 $h = 10$ ft,
 $T = 200^\circ\text{F}$,
 $r_w = 0.365$ ft,
 $\mu_i = 0.01911$ cp,
 $q_g = 1,000$ Mscf/D,
 $\gamma_g = 0.7$,
 $c_{ti} = 0.235 \times 10^{-3}$ psi⁻¹,
 drainage area = 640 acres (square), and
 well centered in drainage area.

This gas is the same as that analyzed in Example 5.4;

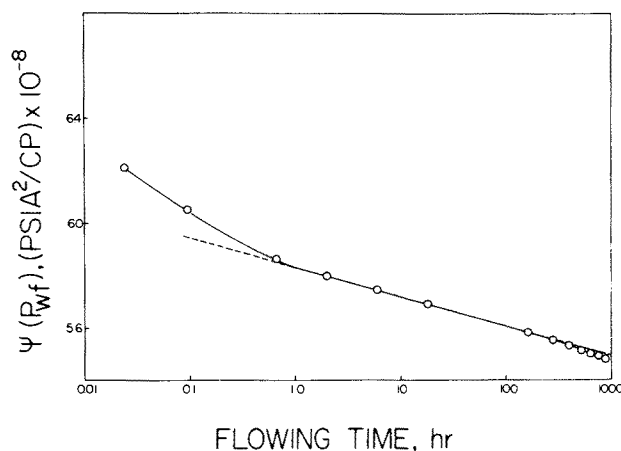


Fig. 5.14 – Drawdown test analysis using pseudo-pressures.

TABLE 5.13 – STABILIZED
DELIVERABILITY TEST DATA

Test	Rate (MMscf/D)	p_{wf} (psia)
initial buildup	–	3,127
1	3.710	3,087
2	5.980	3,059
3	8.191	3,035
4	14.290	2,942

TABLE 5.14 – ISOCHRONAL DELIVERABILITY
TEST DATA

q (Mscf/D)	$\bar{p}$ (psia)	p_{wf} (psia)	t (hours)
983	352.4	344.7	0.5
977		342.4	1.0
970		339.5	2.0
965		337.6	3.0
2,631		329.5	0.5
2,588	352.3	322.9	1.0
2,533		315.4	2.0
2,500		310.5	3.0
3,654		318.7	0.5
3,565		309.5	1.0
3,453	351.0	298.6	2.0
3,390		291.9	3.0
4,782		305.5	0.5
4,625		293.6	1.0
4,438		279.6	2.0
4,318	349.5	270.5	3.0

accordingly, we can determine $\psi(p_{wf})$ for each p_{wf} . These values are included in Table 5.11. From these data, determine formation permeability and apparent skin factor.

Solution. The first step in the analysis procedure is to plot $\psi(p_{wf})$ vs. $\log t$. This plot is shown in Fig. 5.14. The curve shape suggests wellbore storage distortion up to $t \approx 1$ hour, and boundary effects starting at about 200 hours. A log-log plot of $\Delta\psi = [\psi(p_i) - \psi(p_{wf})]$ is useful to confirm this suspicion. Thus, we tabulate (Table 5.12) and plot $\Delta\psi$ vs. t .

Qualitative curve matching of $\log \Delta\psi$ vs. $\log t$ with Ramey's type curve (not shown) indicates an end to wellbore storage distortion at about 1 hour for skin factors in the range 0 to 5 regardless of C_{SD} , confirming the indication on the $\psi(p_{wf})$ vs. $\log t$ plot.

The absolute value of the slope of the MTR line is

$$m = [(5.944 - 5.497) \times 10^8] / 4$$

$$= 11.18 \times 10^6 \text{ psia}^2 / \text{cp-cycle}.$$

Eq. 5.1 shows that the proper interpretation of this slope is

$$m = 50,300 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} (1.151).$$

Thus, for $p_{sc} = 14.7$ psia and $T_{sc} = 520^\circ \text{R}$,

$$k = 1,637 \frac{q_g T}{mh}$$

TABLE 5.15 – GAS WELL
BUILDUP TEST DATA

Δt (days)	p_{ws} (psia)
0	2,430
0.001883	2,542
0.003392	2,600
0.005738	2,650
0.009959	2,692
0.01903	2,726
0.04287	2,756
0.1144	2,785
0.3289	2,814
0.9724	2,843
2.903	2,872
7.903	2,896
12.90	2,907
17.90	2,913
22.90	2,917
27.90	2,920
32.90	2,921
37.90	2,922

$$= \frac{(1,637)(1,000)(660)}{(11.18 \times 10^6)(10)} = 9.66 \text{ md}.$$

From Eq. 5.1, we see that the apparent skin factor, $s' = s + D|q_g|$, is

$$s' = 1.151 \left[\frac{\psi(p_i) - \psi(p_{1 \text{ hr}})}{m} - \log \left(\frac{k}{\phi \mu_i c_{ti} r_w^2} \right) + 3.23 \right].$$

Thus,

$$s' = 1.151 \left\{ \frac{6.338 \times 10^8 - 5.833 \times 10^8}{11.18 \times 10^6} - \log \left[\frac{9.66}{(0.19)(0.01911)(2.35 \times 10^{-4})(0.365)^2} \right] + 3.23 \right\} = -0.21$$

The well is apparently slightly stimulated.

Radius of investigation at the beginning and end of the MTR is found from Eq. 1.25:

$$r_i = \left(\frac{kt}{948 \phi \mu_i c_{ti}} \right)^{1/2}$$

$$= \left[\frac{(9.66)(1)}{(948)(0.19)(0.01911)(2.35 \times 10^{-4})} \right]^{1/2}$$

= 109 ft at start of MTR ($t \approx 1$ hour),

and

$$r_i = (109) \left(\frac{200}{1} \right)^{1/2}$$

= 1,550 ft at end of MTR.

The distance x_e from the well to the edge of the 640-acre square in which it is centered is 2,640 ft; thus, the time at which the observed deviation from the MTR occurs agrees qualitatively with the time at which boundary effects should begin to appear.

Exercises

5.1. The data in Table 5.13 (from Ref. 6) were obtained on a well believed to be stabilized at each rate. Using equations in p^2 (strictly speaking, not applicable in this pressure range), estimate the AOF using (a) the empirical method and (b) the theoretical method.

Also, do the following: (c) plot the theoretical deliverability curve on the same graph paper as the empirical curve; (d) since p^2 equations are not accurate at this pressure level, develop and outline a theoretical method based on equations in p ; and (e) apply equations in p to these data; in particular, calculate the AOF.

5.2. Cullender⁷ presented data from an isochronal test and from an earlier, longer test that led to approximate stabilization in 214 hours test time.

In the 214-hour test, the rate was 1,156 Mscf/D, the shut-in pressure was 441.6 psia, and the flowing BHP was 401.4 psia. Using the data in Table 5.14, (a) determine the AOF with both empirical and theoretical methods, and (b) establish plots (on the same graph paper) of the empirical and theoretical stabilized deliverability curves.

5.3. Confirm $\psi(p)$ results stated in Example 5.4 for pressures in the range $450 \leq p \leq 3,150$ psia.

5.4. The well discussed in Example 5.5 was produced at 2,000 Mscf/D for 90 days and then shut in for a pressure buildup test. Data obtained in the buildup test are given in Table 5.15. Determine formation permeability and apparent skin factor using an analysis procedure based on equations written in terms of pseudopressure, $\psi(p)$.

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Chapter 6

Other Well Tests

6.1 Introduction

This concluding chapter surveys four well-testing techniques not yet discussed in the text: interference tests; pulse tests; drillstem tests; and wireline formation tests. These tests and others covered in previous chapters by no means exhaust the subject; however, the comprehensive treatment needed by the practitioner is provided by SPE monographs^{1,2} and the Canadian gas well testing manual.³

6.2 Interference Testing

Interference tests have two major objectives. They are used (1) to determine whether two or more wells are in pressure communication (i.e., in the same reservoir) and (2) when communication exists, to provide estimates of permeability k and porosity/compressibility product, ϕc_t , in the vicinity of the tested wells.

An interference test is conducted by producing from or injecting into at least one well (the active well) and by observing the pressure response in at least one other well (the observation well). Fig. 6.1 indicates the typical test program with one active well and one observation well.

As the figure indicates, an active well starts producing from a reservoir at uniform pressure at Time 0. Pressure in an observation well, a distance r away, begins to respond after some time lag (related to the time for the radius of investigation corresponding to the rate change at the active well to reach the observation well). The pressure in the active well begins to decline immediately, of course. The magnitude and timing of the deviation in pressure response at the observation well depends on reservoir rock and fluid properties in the vicinity of the active and observation wells.

Vela and McKinley⁴ showed that these properties are values from the area investigated in the test – a rectangle with sides of length $2r_i$ and $2r_i + r$ (see Fig. 6.2). In Fig. 6.2, r_i is the radius of investigation achieved by the active well during the test and r is the distance between active and observation wells. The essential point is that the region investigated is much greater than some small area between wells, as intuition might suggest.

In an infinite-acting, homogeneous, isotropic reservoir, the simple Ei -function solution to the diffusivity equation describes the pressure change at the observation well as a function of time:

$$p_i - p_r = -70.6 \frac{qB\mu}{kh} Ei\left(-948 \frac{\phi\mu c_t r^2}{kt}\right). \quad \dots (6.1)$$

This is simply a restatement of a familiar result. The pressure drawdown at radius r (i.e., the observation well) resulting from production from the active well at rate q , starting from a reservoir initially at uniform pressure p_i , is given by the Ei -function solution. Eq. 6.1 assumes that the skin factor of the active well does not affect the drawdown at the observation well. Wellbore storage effects also are assumed negligible at both the active and observation wells when Eq. 6.1 is used to model an interference test. Jargon⁵ shows that both these assumptions can lead to error in test analysis in some cases.

A convenient analysis technique for interference tests is the use of type curves. Fig. 6.3 is a type curve presented by Earlougher;¹ it is simply the Ei function expressed as a function of its usual argument in flow problems, $948 \phi\mu c_t r^2 / kt$. Note that Eq. 6.1 can be expressed completely in terms of dimensionless variables:

$$\frac{p_i - p_r}{(141.2 \frac{qB\mu}{kh})} = -\frac{1}{2} Ei\left[\left(-\frac{1}{4}\right) \cdot \left(\frac{\phi\mu c_t r_w^2}{0.000264 kt}\right) \left(\frac{r}{r_w}\right)^2\right],$$

or

$$p_D = -\frac{1}{2} Ei\left(\frac{-r_D^2}{4t_D}\right), \quad \dots \dots \dots (6.2)$$

where

$$p_D = \frac{(p_i - p_r) kh}{141.2 qB\mu},$$

$$r_D = r/r_w,$$

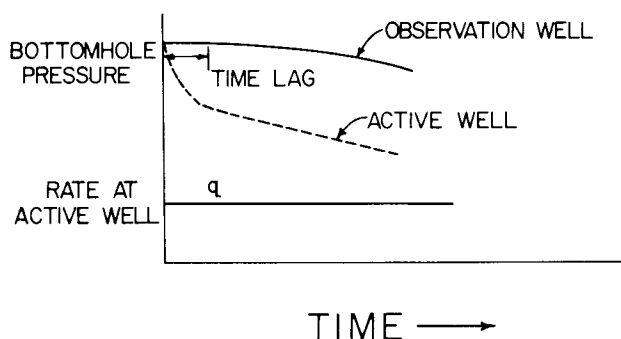


Fig. 6.1 – Pressure response in interference test.

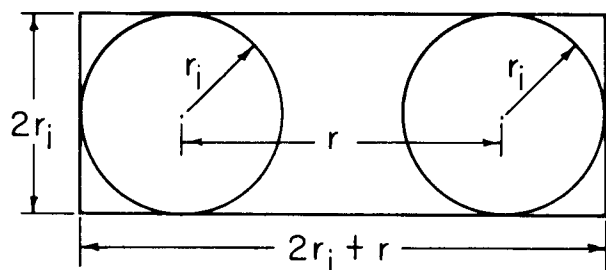


Fig. 6.2 – Region investigated in interference test.

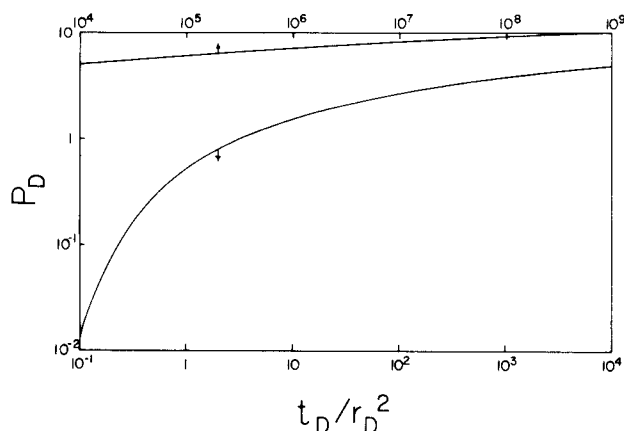


Fig. 6.3 – Exponential integral solution.

and

$$t_D = \frac{0.000264 kt}{\phi \mu c_t r_w^2}$$

Fig. 6.3 can be used in the following way to analyze interference tests.

1. Plot pressure drawdown in an observation well, $\Delta p = p_i - p_r$, vs. elapsed time t on the same size log-log paper as the full-scale, type-curve version of Fig. 6.3 using an undistorted curve (the reader can prepare such a curve easily).

2. Slide the plotted test data over the type curve until a match is found. (Horizontal and vertical sliding both are required.)

3. Record pressure and time match points, $(p_D)_{MP}$, Δp_{MP} and $[(t_D/r_D^2)_{MP}, t_{MP}]$.

4. Calculate permeability k in the test region from the pressure match point:

$$k = 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}}$$

5. Calculate ϕc_t from the time match point:

$$\phi c_t = \left(\frac{0.000264 k}{\mu r^2} \right) \left| \frac{t_{MP}}{(t_D/r_D^2)_{MP}} \right|$$

Example 6.1 – Interference Test in Water Sand

Problem. An interference test was run in a shallow-water sand. The active well, Well 13, produced 466 STB/D water. Pressure response in shut-in Well 14, which was 99 ft from Well 13, was measured as a function of time elapsed since the drawdown in Well 13 began. Estimated rock and fluid properties include $\mu_w = 1.0$ cp, $B_w = 1.0$ RB/STB, $h = 9$ ft, $r_w = 3$ in., and $\phi = 0.3$. Total compressibility is unknown. Pressure readings in Well 14 were as given in Table 6.1. Estimate formation permeability and total compressibility.

Solution. We assume that the aquifer is homogeneous, isotropic, and infinite-acting; we use the Ei -function type curves to estimate k and c_t . Data to be plotted are presented in Table 6.2. The data fit the Ei -function type curve well. A pair of match points are $(\Delta t = 128$ minutes, $t_D/r_D^2 = 10)$ and $(\Delta p = 5.1$ psi, $p_D = 1.0)$. (See Fig. 6.4.) Thus,

$$\begin{aligned} k &= 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}} \\ &= \frac{(141.2)(466)(1.0)(1.0)}{(9.0)} \frac{(1.0)}{(5.1)} \\ &= 1,433 \text{ md,} \end{aligned}$$

and

$$c_t = \frac{0.000264}{\phi r^2} \frac{k}{\mu} \frac{(t_{MP}/60)}{(t_D/r_D^2)_{MP}}$$

$$= \frac{(0.000264)(1,433)(128/60)}{(0.3)(99)^2(1.0)(10)}$$

$$= 2.74 \times 10^{-5} \text{ psi}^{-1}.$$

6.3 Pulse Testing

Pulse tests⁶ have the same objectives as conventional interference tests – to determine whether well pairs are in pressure communication and to estimate k and ϕc_i in the area of the tested wells. The tests are conducted by sending a coded signal or pulse sequence from an active well (producer or injector) to a shut-in observation well. The pulse sequence is created by producing from (or injecting into) the active well, then shutting it in, and repeating that sequence in a regular pattern. An example is indicated in Fig. 6.5.

The reason for the sequence of pressure pulses is that we readily can determine the effect of an active well on an observation well amid the established trend in reservoir pressure and random perturbations (noise) to that trend. Highly sensitive pressure gauges usually are required to detect these small coded pulses, which may have magnitudes of less than 0.1 psi.

Pulse testing has several advantages over conventional interference tests:

1. It disrupts normal operations much less than interference testing does. It lasts a minimum time, which may range from a few hours to a few days.

2. There are fewer interpretation problems caused by random noise and by trends in reservoir pressure as they affect pressure response at observation wells.

3. Pulse test analysis usually can be based on simple solutions to the flow equations – specifically, superposition of Ei -function solutions, which assume infinite-acting, homogeneous reservoirs. In many cases, longer interference tests require that boundaries be taken into account.

Analysis techniques for pulse tests usually are based on simulating the pressure response in an observation well with the familiar Ei -function solution to the diffusivity equation, using superposition to model the rate changes in the pulsing sequence. From the simulations of pulse tests, charts relating key characteristics of the tests to reservoir properties have been developed.⁷ Before we discuss these charts (Figs. 6.7 through 6.14) and their application, it will be useful to introduce nomenclature used in pulse test analysis, using the system of Earlougher¹ (and his schematic pulse-test rate and pressure-response history).

Fig. 6.6 illustrates the time lag t_L which is the time elapsed between the end of a pulse and the pressure peak caused by the pulse. The radius-of-investigation concept prepares us to expect a time lag. A finite period of time is required for a pulse caused by producing an active well to move to a responding well, and the subsequent transient created by a shut-in period also requires a finite time period to affect pressure response.

The amplitude Δp of a pulse can be represented

TABLE 6.1 – PRESSURE/TIME DATA FROM INTERFERENCE TEST

Δt (minutes)	p_{ws} (psia)
0	148.92
5	148.92
25	144.91
40	143.72
50	143.18
100	141.47
200	139.72
300	138.70
400	137.99
580	137.12

TABLE 6.2 – INTERFERENCE TEST DATA FOR LOG-LOG PLOT

Δt (minutes)	$\Delta p = p_i - p_{wf}$ (psia)
0	0
5	0
25	4.01
40	5.20
50	5.74
100	7.45
200	9.20
300	10.22
400	10.93
580	11.80

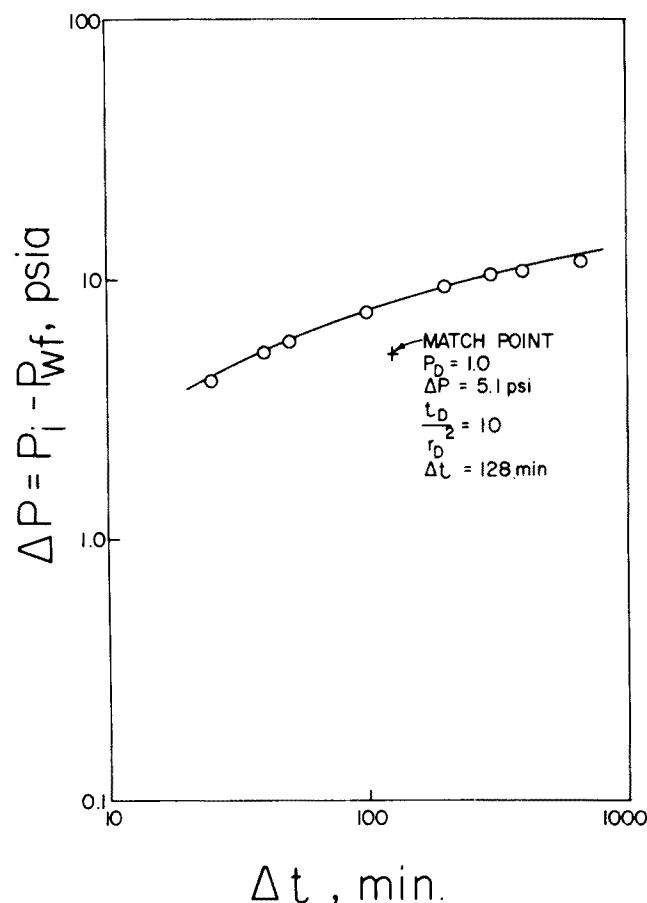


Fig. 6.4 – Interference test data from water reservoir.

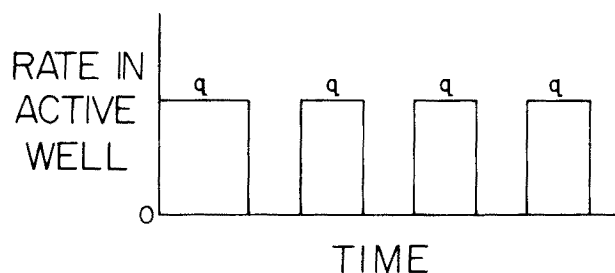


Fig. 6.5 – Typical rate schedule in pulse test.

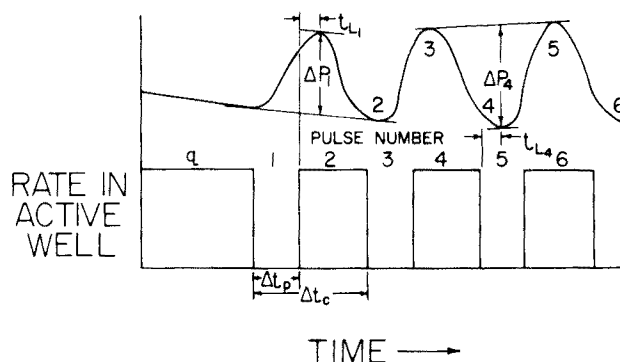


Fig. 6.6 – Pressure response in pulse test.

conveniently as the vertical distance between two adjacent peaks (or valleys) and a line parallel to this through the valley (or peak), as illustrated in Fig. 6.6.

The length of the pulse period and total cycle length (including both shut-in and flow periods) are represented by Δt_p and Δt_c , respectively.

Analysis of simulated pulse tests shows that Pulse 1 (the first odd pulse) and Pulse 2 (the first even pulse) have characteristics that differ from all subsequent pulses; beyond these initial responses, all odd pulses have similar characteristics and all even pulses also have similar characteristics.

We now define dimensionless variables that are used in Figs. 6.7 through 6.14, which were designed for quantitative analysis of pulse tests: dimensionless time lag, $(t_L)_D = 0.000264 kt_L / \phi \mu c_f r_w^2$; dimensionless distance between active and observation wells, $r_D = r / r_w$; and dimensionless pressure-response amplitude, $\Delta p_D = kh \Delta p / 141.2 q B \mu$ (with sign convention that $\Delta p / q$ is always positive).

Figs. 6.7 through 6.14 provide the correlations to be used in pulse test analysis. Figs. 6.7 and 6.8 are to be used for the first odd (i.e., first) pulse; Figs. 6.9 and 6.10 for the first even (i.e., second) pulse; Figs. 6.11 and 6.12 for all other odd-numbered pulses (third, fifth, etc.); and Figs. 6.13 and 6.14 for all other even-numbered pulses (fourth, sixth, etc.). The figures use the ratios (1) $F' = \text{pulse length } (\Delta t_p) \text{ to cycle length } (\Delta t_c)$ and (2) time lag (t_L) to cycle length (Δt_c) . The figures appropriate for a given pulse number are used to obtain values of $\Delta p_D (t_L / \Delta t_c)^2$ and $[(t_L)_D / r_D^2]$, which are then used to provide estimates of k and ϕc_f .

$$k = 141.2 \frac{q B \mu \Delta p_D (t_L / \Delta t_c)^2}{h \Delta p (t_L / \Delta t_c)^2},$$

$$\phi c_f = \frac{0.000264 k t_L}{\mu r^2 [(t_L)_D / r_D^2]}.$$

Example 6.2 illustrates how these figures are applied.

Example 6.2 – Pulse Test Analysis

Problem. A pulse test was run in a reservoir in which the distance between wells, r , was 933 ft. Formation fluid viscosity, μ , was 0.8 cp; formation thickness, h ,

was 26 ft; and porosity, ϕ , was 0.08. In the test following rate stabilization, the active well was shut in for 2 hours, then produced for 2 hours, shut in for 2 hours, etc. Production rate, q , was 425 STB/D and formation volume factor, B , was 1.26 RB/STB. The amplitude Δp of the fourth pulse (Fig. 6.15) was 0.629 psi, and the time lag was 0.4 hour. From these data, estimate k and ϕc_f .

Solution. To analyze the fourth pulse, we use Figs. 6.13 and 6.14. Using Fig. 6.13 first to determine $\Delta p_D (t_L / \Delta t_c)^2$, and thus k , we note that

$$F' = \Delta t_p / \Delta t_c = 2 / (2 + 2) = 0.5,$$

$$t_L / \Delta t_c = 0.4 / 4 = 0.1.$$

Then, from Fig. 6.13,

$$\Delta p_D (t_L / \Delta t_c)^2 = 0.00221,$$

and

$$\begin{aligned} k &= 141.2 \frac{q B \mu \Delta p_D (t_L / \Delta t_c)^2}{h \Delta p (t_L / \Delta t_c)^2} \\ &= \frac{(141.2)(425)(1.26)(0.8)(0.00221)}{(26)(0.629)(0.1)^2} \\ &= 817 \text{ md.} \end{aligned}$$

From Fig. 6.14,

$$(t_L)_D / r_D^2 = 0.091.$$

Thus,

$$\begin{aligned} \phi c_f &= \frac{0.000264 k t_L}{\mu r^2 [(t_L)_D / r_D^2]} \\ &= \frac{(0.000264)(817)(0.4)}{(0.8)(933)^2 (0.091)} \\ &= 1.36 \times 10^{-6} \end{aligned}$$

Then,

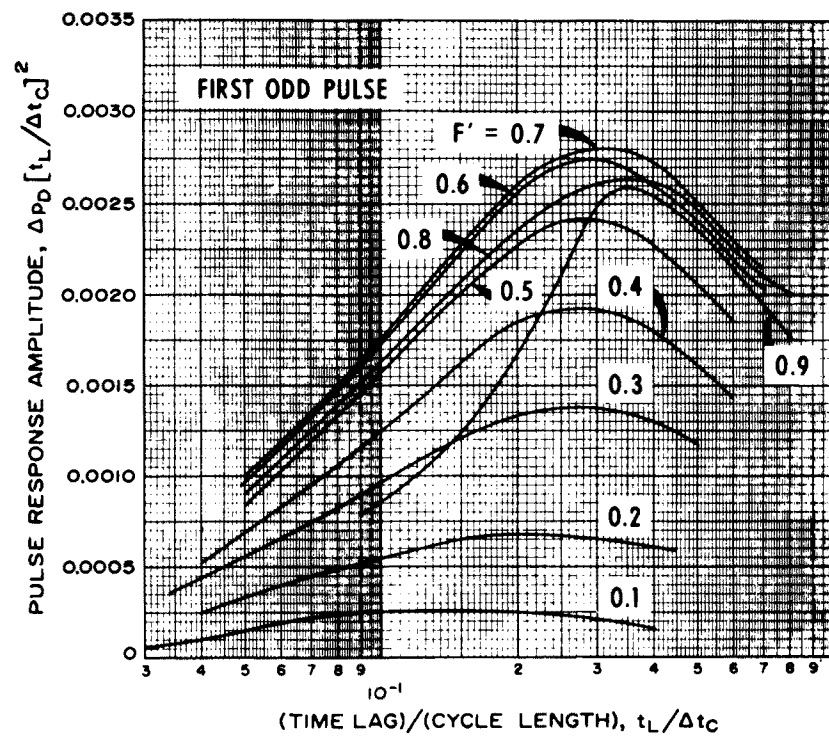


Fig. 6.7 – Relation between time lag and response amplitude for first odd pulse.¹

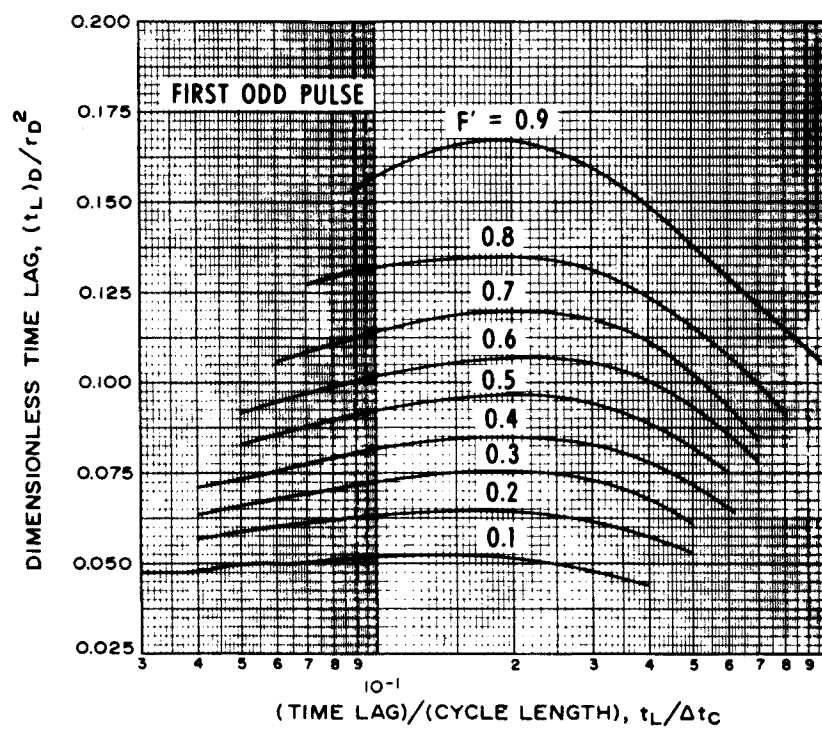


Fig. 6.8 – Relation between time lag and cycle length for first odd pulse.¹

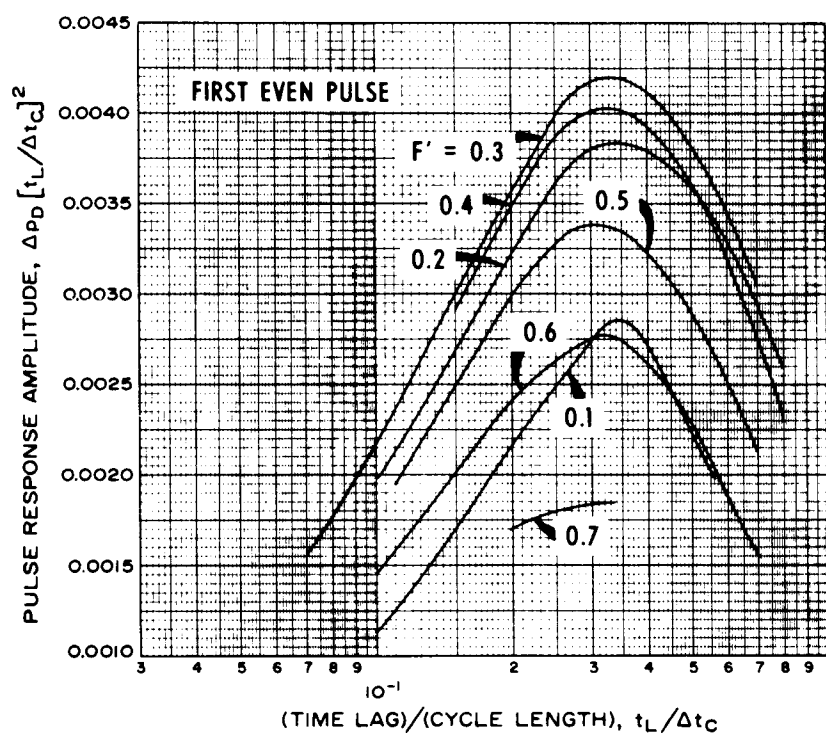


Fig. 6.9 – Relation between time lag and response amplitudes for first even pulse.¹

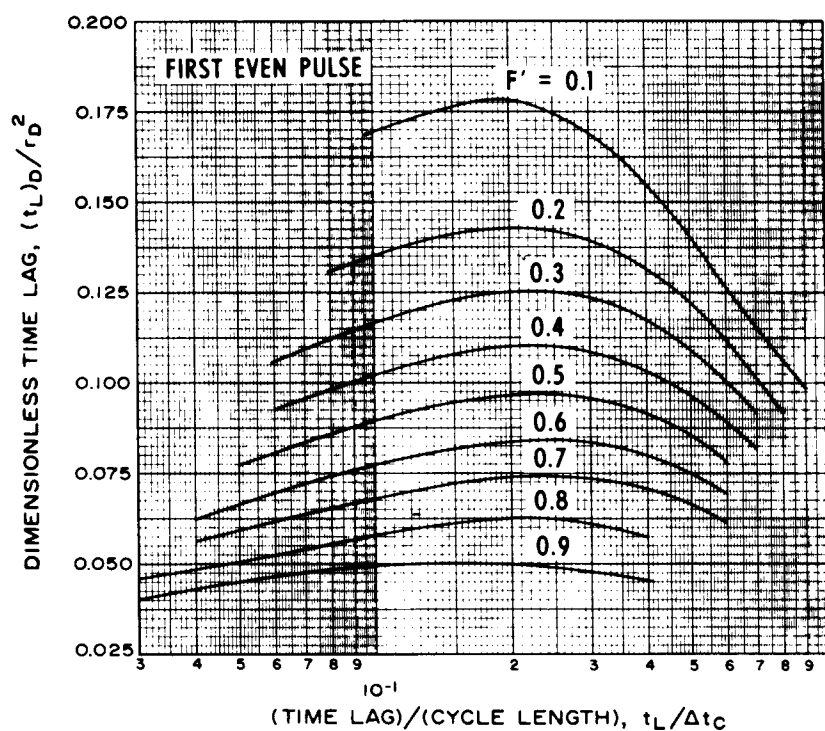


Fig. 6.10 – Relation between time lag and cycle length for first even pulse.¹

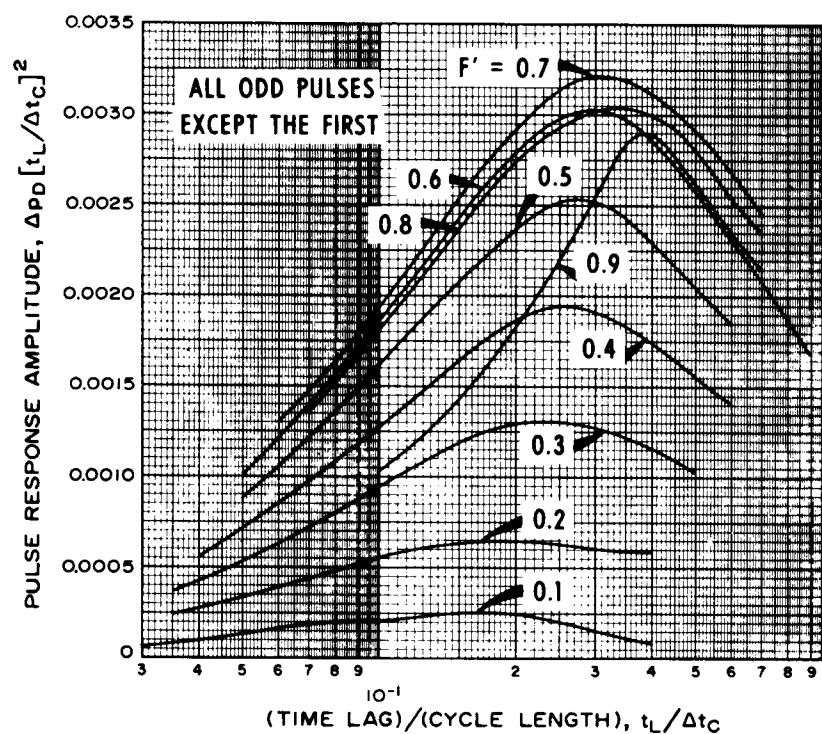


Fig. 6.11 – Relation between time lag and response amplitude for all odd pulses after the first.¹

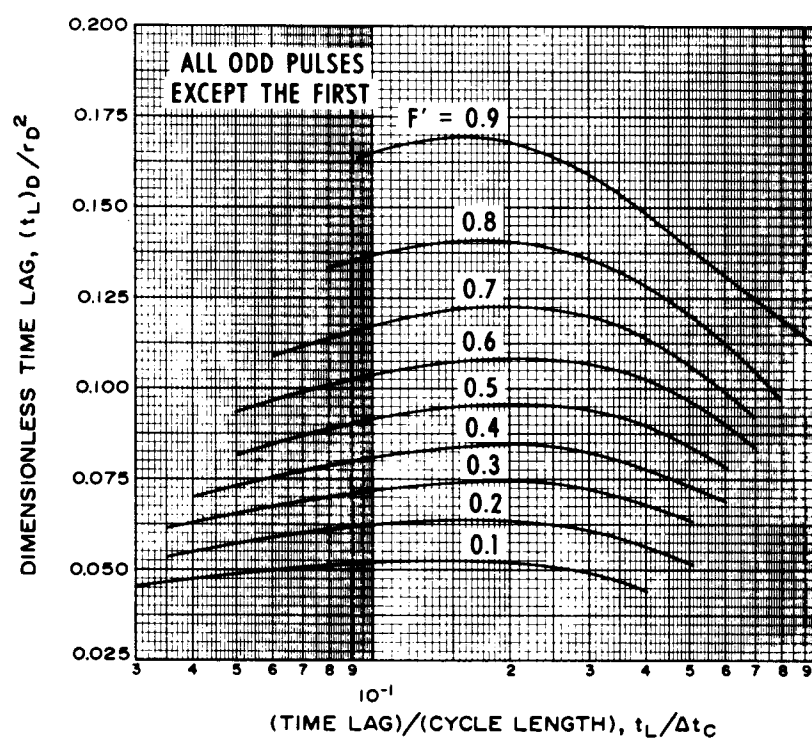


Fig. 6.12 – Relation between time lag and cycle length for all odd pulses after the first.¹

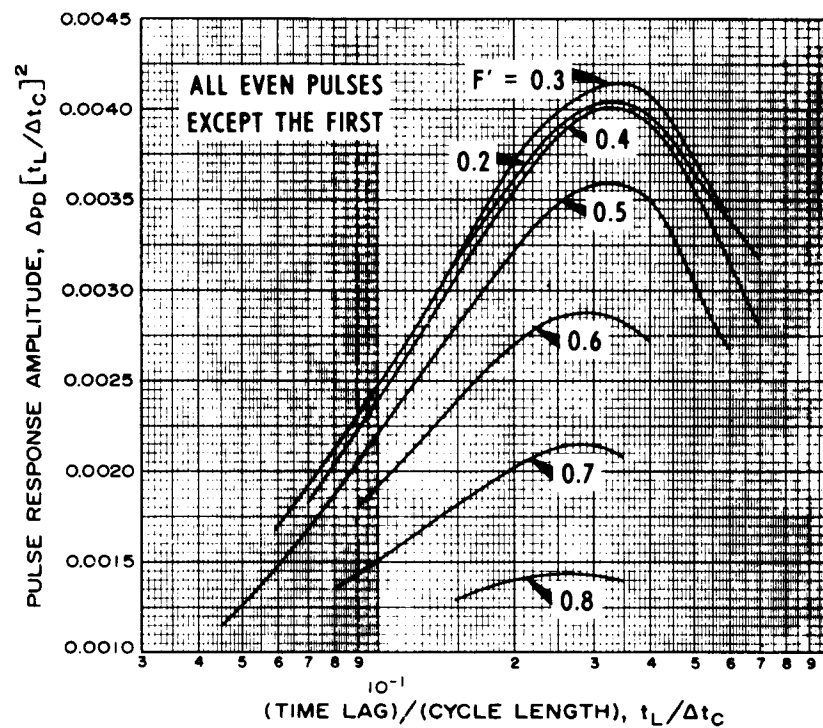


Fig. 6.13 – Relation between time lag and response amplitude for all even pulses after the first.¹

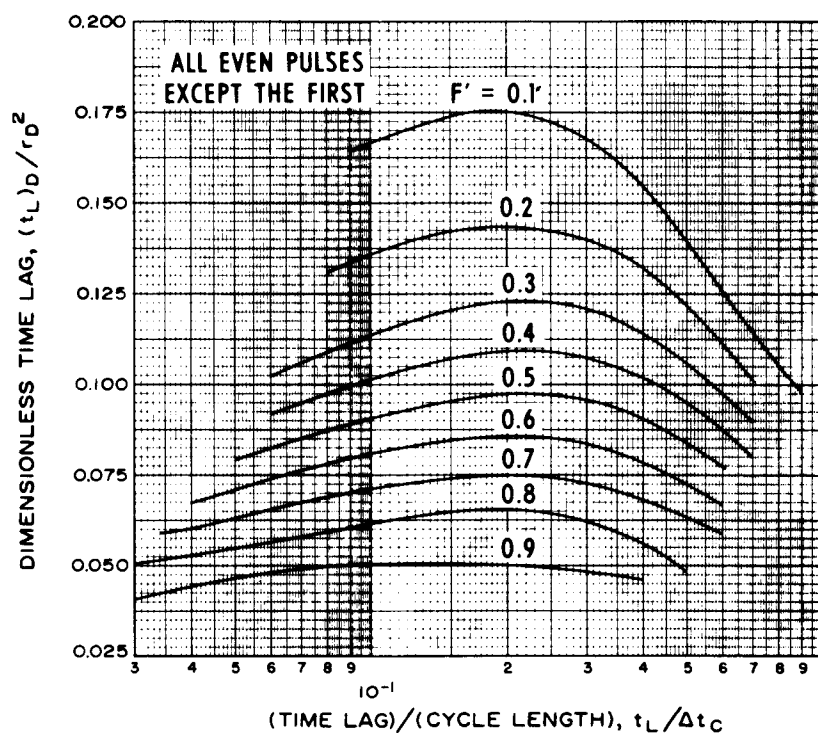


Fig. 6.14 – Relation between time lag and cycle length for all even pulses after the first.¹

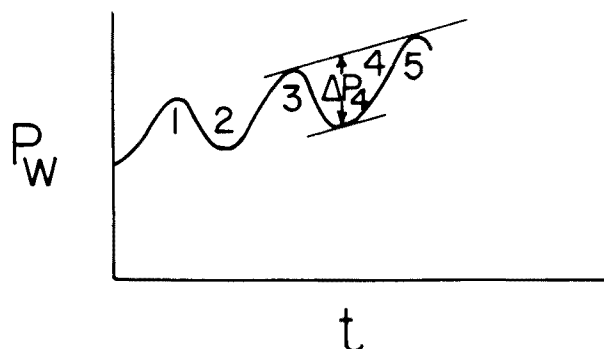


Fig. 6.15 – Schematic of pressure response in pulse test.

$$c_t = \frac{1.36 \times 10^{-6}}{0.08} = 17 \times 10^{-6} \text{ psi}^{-1}.$$

6.4 Drillstem Tests

A drillstem test (DST)^{8,9} provides a means of estimating formation and fluid properties before completion of a well. Basically, a DST is a temporary completion of a well. The DST tool is an arrangement of packers and valves placed on the end of the drillpipe. This arrangement can be used to isolate a zone of interest and to let it produce into the drillpipe or drillstem. A fluid sample is obtained in the test; thus, the test can tell us the types of fluids the well will produce if it is completed in the tested formation.

With the surface-actuated valves on a DST device, it is possible to have a sequence of flow periods followed by shut-in periods. A pressure recorder on the DST device can record pressures during the flow and shut-in periods. The pressures recorded during the shut-in periods can be particularly valuable for estimating formation characteristics such as permeability/thickness product and skin factor. These data also can be used to determine possible pressure depletion during the test.

To illustrate how a typical DST is performed, we will examine a schematic chart (Fig. 6.16) of pressure vs. time from a test with two flow periods and two shut-in periods.

At Point A, the tool is lowered into the hole. Between Points A and B, the ever-increasing mud-column pressure is recorded; at Point B, the tool is on bottom. When the packers are set, the mud column is compressed and a still higher pressure is recorded at Point C. The tool is opened for an initial flow period, and the pressure drops to Point D as shown. As fluid accumulates in the drillstem above the pressure gauge, the pressure rises. Finally, at Point E, the well is shut in for an initial pressure buildup test. After a suitable shut-in period, the well is reopened for a second final flow period, from Point G to Point H. This final flow period is followed by a final shut-in period (from Point H to Point I). The packers are then released, and the hydrostatic pressure of the mud column is again

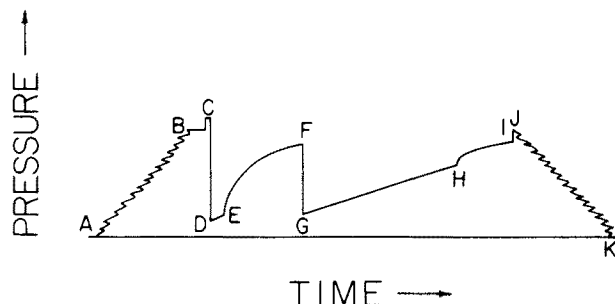


Fig. 6.16 – Schematic of drillstem test pressure chart.

imposed on the pressure gauge. The testing device is then removed from the hole (Point J to Point K).

The initial flow period is usually brief (5 to 10 minutes); its purpose is to draw down the pressure slightly near the wellbore (perhaps letting any mud-filtrate-invaded zone bleed back to or below static reservoir pressure). The initial shut-in period, often 30 to 60 minutes, is designed to let the pressure build back to true static formation pressure. This initial shut-in pressure on a DST may be the best measurement made of static reservoir pressure.

The second flow period is designed to capture a large sample of formation fluid and to draw down the pressure in the formation to the maximum distance and extent possible within the time that is possible to allow for the DST – frequently 30 minutes to several hours. The second shut-in period is designed to obtain good pressure buildup data so that formation properties can be estimated. In addition, comparison of the final (or extrapolated) pressure from the second shut-in period to the initial shut-in pressure can indicate that pressure depletion has occurred during the DST and that the well thus has been tested in a small, noncommercial reservoir. The desired length of the second shut-in period varies from equal to the second flow period (for high-permeability formations) to twice the length of the second flow period (for low-permeability formations).

Theory much like that used for an ordinary pressure buildup test following production at constant rate is used for analyzing the shut-in periods on a DST. This is true even though the flow rate preceding a shut-in period in a DST usually decreases continuously. Usually, the *average* production rate can be used as a good approximation in buildup test analyses; this average rate of production is determined by dividing the fluid recovery by the length of the flow period.

To analyze the buildup test, we plot p_{ws} vs. $\log(t_p + \Delta t)/\Delta t$, where t_p is now the *actual* flowing time at the average rate q . The permeability/thickness product is found from the relationship $kh = 162.6 qB\mu/m$. Usually, a fluid sample will not yet have been analyzed in the laboratory; accordingly, correlations (Appendix D) relating μ and B to produced fluid properties must be used.

TABLE 6.3 – DRILLSTEM TEST DATA

Δt (minutes)	p_{ws} (psi)
0	350
5	965
10	1,215
15	1,405
20	1,590
25	1,685
30	1,725
35	1,740
40	1,753
45	1,765

Static reservoir pressure is found by extrapolating the buildup tests to infinite shut-in time. In a dual shut-in test, we have the opportunity to extrapolate both buildup tests to infinite shut-in time and to compare the estimated static reservoir pressure. If the static reservoir pressure from the final shut-in period is significantly lower than that from the initial shut-in period, it is possible that the reservoir was partially pressure depleted even during the relatively short DST, implying that the formation tested is probably noncommercial.

Skin factor is calculated from the conventional skin-factor equation:

$$s = 1.151 \left[\frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \dots \dots \dots (2.4)$$

There is a further complication in DST analysis. At the time of the test, reservoir rock and fluid properties that appear in Eq. 2.4 may not yet be known accurately. This is particularly true of porosity, ϕ , and total compressibility, c_t . Accordingly, one may be forced to use the best available estimates and to recognize that skin-factor and radius-of-investigation calculations, which also depend on these properties, may be subject to considerable uncertainty. Use of Eq. 2.3 instead of Eq. 2.4 [Eq. 2.3 includes the sometimes important term $\log (t_p + \Delta t) / t_p$] to calculate s may be justified in some cases if other data in the equation are known with unusual accuracy.

6.5 Wireline Formation Tests

In many areas, hole conditions prohibit use of DST's as temporary wellbore completions. In these areas, and in others where the costs of the required numbers of DST's for complete evaluation are prohibitive, wireline formation tests^{10,11} frequently are used in formation evaluation work.

A wireline formation tester is, in effect, a sample chamber of up to several gallons capacity combined with pressure gauges. The test chambers are forced against the borehole wall in a sealing pad, and the formation is perforated by firing a shaped charge. The signal to fire the charge is transmitted on logging cable. Fluid is collected during sampling, and

pressure is recorded. Following sample collection, shut-in pressures are recorded as they build up with time.

Flow into the sample chamber is probably approximately spherical (i.e., into a point rather than spread uniformly across an entire productive interval). For this reason, the shut-in test cannot be analyzed as in a DST, although theory based on steady-state spherical flow may explain wireline test buildup pressure satisfactorily in some cases.¹¹ The device is useful for obtaining samples of formation fluid and estimating initial formation pressure; extensive use of the device for this latter application has been reported in the literature.¹¹

Exercises

6.1 Determine the duration of an interference test required to achieve a pressure drawdown of 25 psig at the observation well for the reservoir described in Example 6.1 if the active well produces 500 STB/D throughout the test. What will be the radius of investigation at this time? If the skin factor in the active well is 2.0, what will be the drawdown in the active well?

6.2 For the pulse test described in Example 6.2, given the results of the test analysis of the fourth pulse, determine the time lag and pressure response for (a) the first odd pulse; (b) the first even pulse; (c) the third pulse; (d) the fifth pulse; and (e) the sixth pulse.

6.3 Johnston-Schlumberger⁹ reports data below from a DST:

initial flow period = 5 minutes,
initial shut-in period = 30 minutes,
final flow period = 60 minutes, and
final shut-in period = 45 minutes.

Data obtained in the final shut-in period were as given in Table 6.3. In the initial shut-in period, the pressure reached and remained stable at 1,910 psi. Total fluid recovered in both initial and final flow periods filled 300 ft of 2½-in.-ID drill collars (0.0061 bbl/ft) and 300 ft of 4½-in. drillpipe (0.0142 bbl/ft). The produced fluid was 35° API oil with a measured gas rate of 47 Mscf/D at the surface (assumed solution gas). Formation temperature was measured at 120°F. Porosity is estimated to be 10%; total compressibility is 8.4×10^{-6} psi⁻¹; wellbore radius is 4.5 in; and formation thickness is 10 ft.

Estimate formation permeability, skin factor, flow efficiency, and radius of investigation achieved in the test.

6.4 Smolen and Litsey¹¹ propose that “borehole-corrected,” steady-state, spherical flow into the wireline formation tester can be modeled by

$$k = 3,300 q \mu / \Delta p,$$

where

k = permeability, md,
 q = flow rate, cm³/s (reservoir conditions),
 μ = fluid viscosity (usually mud filtrate), cp,
and
 Δp = drawdown from formation pressure, psi.

A test showed that formation pressure was 3,850 psi. Pressure was drawn down in the sample chamber to an approximately constant 1,850 psi by withdrawing 10 cm^3 of filtrate ($\mu = 0.5 \text{ cp}$) from the formation in 16 seconds.

Estimate formation permeability from the test data.

Derive an equation for steady-state spherical flow and show that it has the form $k = \text{constant} \times qB\mu/[r(p_i - p_{wf})]$. State the assumptions required for the equation to model a wireline formation tester flow test.

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Appendix A

Development of Differential Equations for Flow in Porous Media

Introduction

In this appendix, we develop some of the basic differential equations that describe the flow of fluids in porous media. Results presented include equations for three-dimensional flow of slightly compressible liquids, and for radial flow of slightly compressible liquids, gases, and simultaneous flow of oil, water, and gas. In developing these equations, we start with continuity equations (mass balances); then we introduce flow laws (such as Darcy's law) and appropriate equations of state for the fluid considered.

Continuity Equation for Three-Dimensional Flow

To develop continuity equations, we use a mass balance on a small element of porous material. The balance has the following form.

$$(\text{rate of mass flow into element}) - (\text{rate of mass flow out of element}) = (\text{rate of accumulation of mass within element}).$$

Our element is shown in Fig. A-1. It has dimensions Δx , Δy , and Δz , in the x , y , and z coordinate system; for convenience, the coordinate system is oriented such that gravitational forces are in the $(-z)$ direction. We denote the components of the volumetric rate of flow per unit cross-sectional area (cubic feet per hour-square feet or feet per hour are typical engineering units) by u_x , u_y , and u_z .

The rate at which mass enters the element in the x direction is $\rho u_x \Delta y \Delta z$ (lbm/cu ft $\times$ ft/hr $\times$ sq ft = lbm/hr); the rate at which the mass leaves the element in the x direction is $[\rho u_x + \Delta(\rho u_x)] \Delta y \Delta z$. Similar expressions describe rates of mass entering and leaving in the y and z directions. The result of adding these expressions is

$$(\text{rate of mass flow into element}) - (\text{rate of mass flow out of element})$$

$$\begin{aligned} &= \rho u_x \Delta y \Delta z + \rho u_y \Delta x \Delta z + \rho u_z \Delta x \Delta y - [\rho u_x \\ &\quad + \Delta(\rho u_x)] \Delta y \Delta z - [\rho u_y + \Delta(\rho u_y)] \Delta x \Delta z \\ &\quad - [\rho u_z + \Delta(\rho u_z)] \Delta x \Delta y \\ &= -\Delta(\rho u_x) \Delta y \Delta z - \Delta(\rho u_y) \Delta x \Delta z - \Delta(\rho u_z) \\ &\quad \cdot \Delta x \Delta y. \end{aligned}$$

To determine the rate at which fluid accumulates within the element, we first note that the mass within the element (of porosity ϕ) at a given time is $\rho \phi \Delta x \Delta y \Delta z$ (lbm/cu ft $\times$ cu ft = lbm). Thus, the rate at which this mass changes over a time interval Δt is

$$\frac{(\rho \phi|_{t+\Delta t} - \rho \phi|_t)}{\Delta t} \Delta x \Delta y \Delta z,$$

where time, t , is in hours. Accordingly, the mass balance becomes

$$\begin{aligned} &-\Delta(\rho u_x) \Delta y \Delta z - \Delta(\rho u_y) \Delta x \Delta z - \Delta(\rho u_z) \Delta x \Delta y \\ &= \frac{\rho \phi|_{t+\Delta t} - \rho \phi|_t}{\Delta t} \Delta x \Delta y \Delta z. \end{aligned}$$

If we divide each term by $\Delta x \Delta y \Delta z$ and take the limit as $\Delta t, \Delta x, \Delta y$, and $\Delta z \rightarrow 0$, the result is

$$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = -\frac{\partial}{\partial t}(\rho \phi). \quad \dots\dots\dots (\text{A.1})$$

Continuity Equation for Radial Flow

From a mass balance similar to that used to develop Eq. A.1, the continuity equation for one-dimensional, radial flow can be shown to be

$$\frac{1}{r} \frac{\partial}{\partial r}(r \rho u_r) = -\frac{\partial}{\partial t}(\phi \rho), \quad \dots\dots\dots (\text{A.2})$$

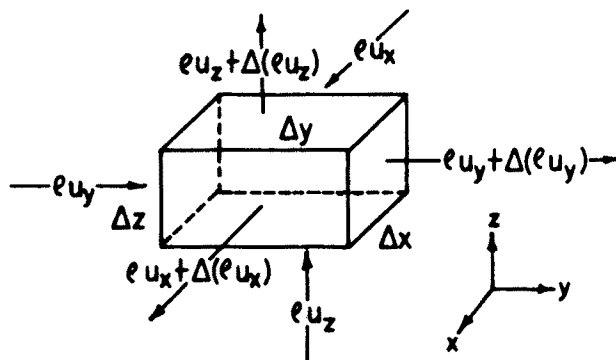


Fig. A-1 – Element of porous medium used for mass balance.

where u_r is the volumetric flow rate per unit cross-section area in the radial direction. Eq. A.2 is less general than Eq. A.1; in particular, radial flow *only* is assumed [i.e., there is no flow in the z or θ “direction” in a cylindrical (r, z, θ) coordinate system].

Flow Laws

Liquid flow usually is described by Darcy’s law. This law, when applied in the coordinate system orientation we have chosen to describe three-dimensional flow, becomes, in field units,

$$\begin{aligned} u_x &= -0.001127 \frac{k_x}{\mu} \frac{\partial p}{\partial x}, \\ u_y &= -0.001127 \frac{k_y}{\mu} \frac{\partial p}{\partial y}, \\ u_z &= -0.001127 \frac{k_z}{\mu} \left(\frac{\partial p}{\partial z} + 0.00694\rho \right), \dots (A.3) \end{aligned}$$

The k_i are the permeabilities in direction i and p denotes pressure (psi). In Eq. A.3, we have assumed that the x and y directions are horizontal, so that gravity acts only in the z direction.

After substituting these equations into the continuity equation, the result is

$$\begin{aligned} &\frac{\partial}{\partial x} \left(\frac{k_x \rho}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k_y \rho}{\mu} \frac{\partial p}{\partial y} \right) \\ &+ \frac{\partial}{\partial z} \left[\frac{k_z \rho}{\mu} \left(\frac{\partial p}{\partial z} + 0.00694\rho \right) \right] \\ &= \frac{1}{0.000264} \frac{\partial}{\partial t} (\phi \rho). \dots (A.4) \end{aligned}$$

For radial flow, the result is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r \rho k_r}{\mu} \frac{\partial p}{\partial r} \right) = \frac{1}{0.000264} \frac{\partial}{\partial t} (\rho \phi). \dots (A.5)$$

Single-Phase Flow of Slightly Compressible Fluids

To solve Eqs. A.4 and A.5 analytically, we must

introduce additional assumptions. First, we restrict our analysis to slightly compressible liquids – those with constant compressibility, c , where c is defined by the equation

$$c = \frac{-1}{V} \frac{dV}{dp} = \frac{1}{\rho} \frac{d\rho}{dp}. \dots (A.6)$$

For constant compressibility c , integration of Eq. A.6 gives

$$\rho = \rho_o e^{c(p-p_o)}, \dots (A.7)$$

where ρ_o is the value of ρ at some reference pressure p_o . Eq. A.7 describes most single-phase liquids adequately.

If we now introduce Eq. A.7 into Eq. A.4 and assume that (1) $k_x = k_y = k_z = \text{constant}$, (2) gravitational forces are negligible, (3) $\phi = \text{constant}$, and (4) $\mu = \text{constant}$, Eq. A.4 becomes

$$\begin{aligned} &\frac{\partial}{\partial x} \left[e^{c(p-p_o)} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[e^{c(p-p_o)} \frac{\partial p}{\partial y} \right] \\ &+ \frac{\partial}{\partial z} \left[e^{c(p-p_o)} \frac{\partial p}{\partial z} \right] \\ &= \frac{\phi \mu}{0.000264 k} \frac{\partial}{\partial t} \left[e^{c(p-p_o)} \right]. \end{aligned}$$

Simplifying,

$$\begin{aligned} &\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \\ &+ c \left[\left(\frac{\partial p}{\partial x} \right)^2 + \left(\frac{\partial p}{\partial y} \right)^2 + \left(\frac{\partial p}{\partial z} \right)^2 \right] \\ &= \frac{\phi \mu c}{0.000264 k} \frac{\partial p}{\partial t}. \end{aligned}$$

If we further assume that

$$c \left[\left(\frac{\partial p}{\partial x} \right)^2 + \left(\frac{\partial p}{\partial y} \right)^2 + \left(\frac{\partial p}{\partial z} \right)^2 \right]$$

is negligible compared with other terms in the equation (which requires either small c or small pressure gradients, or both), then

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\phi \mu c}{0.000264 k} \frac{\partial p}{\partial t}. \dots (A.8)$$

For radial flow, the corresponding equation is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{0.000264 k} \frac{\partial p}{\partial t}. \dots (A.9)$$

Eqs. A.8 and A.9 are diffusivity equations. Analytical solutions to Eq. A.9 are known for several simple boundary conditions; these solutions are used for most well test analysis. We must remember that Eqs. A.8 and A.9 are *not* general; instead, they are based on several important assumptions, including (1) the single-phase liquid flowing has small and constant compressibility; (2) k is constant and the same in all directions (isotropic); (3) ϕ is constant; and (4) pressure gradients are small.

Single-Phase Gas Flow

For gas flow characterized by Darcy's law and for a gas described by the equation of state,

$$\rho = \frac{M}{RT} \frac{p}{z},$$

Eq. A.4 becomes, for constant ϕ and k and negligible gravitational forces,

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{p}{\mu z_g} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{p}{\mu z_g} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{p}{\mu z_g} \frac{\partial p}{\partial z} \right) \\ = \frac{\phi}{0.000264 k} \frac{\partial}{\partial t} \left(\frac{p}{z_g} \right). \end{aligned} \quad \text{..... (A.10)}$$

In this section only, we use z_g to denote the real gas law compressibility factor so that we can continue to use the symbol z for one of the space variables. For gases, μ and z_g are functions of pressure and cannot be assumed constant except in special cases. To reduce Eq. A.10 to a form similar to Eq. A.8, we define a pseudopressure,¹ $\psi(p)$, as follows:

$$\psi(p) = 2 \int_{p_\beta}^p \frac{p}{\mu z_g} dp, \quad \text{..... (A.11)}$$

where p_β is a low base pressure. Now,

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{p}{z_g} \right) &= \frac{d \left(\frac{p}{z_g} \right)}{dp} \frac{\partial p}{\partial t} \\ &= \frac{c_g p}{z_g} \frac{\partial p}{\partial t}, \end{aligned}$$

since

$$c_g = \frac{1}{p} \frac{dp}{dp} = \frac{z_g}{p} \frac{d \left(\frac{p}{z_g} \right)}{dp}.$$

Also note that

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial p} \frac{\partial p}{\partial t} = \frac{2p}{\mu z_g} \frac{\partial p}{\partial t},$$

and

$$\frac{\partial \psi}{\partial x} = \frac{2p}{\mu z_g} \frac{\partial p}{\partial x}.$$

Similar expressions apply for $\partial \psi / \partial y$ and $\partial \psi / \partial z$. Thus, Eq. A.10 becomes

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\partial \psi}{\partial z} \right) \\ = \frac{\phi \mu c_g}{0.000264 k} \frac{\partial \psi}{\partial t}. \end{aligned} \quad \text{..... (A.12)}$$

For radial flow, the equivalent of Eq. A.12 is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) = \frac{\phi \mu c_g}{0.000264 k} \frac{\partial \psi}{\partial t}. \quad \text{..... (A.13)}$$

Eqs. A.12 and A.13 are similar in form to Eqs. A.8 and A.9, but there is one important difference. The coefficient $\phi \mu c / 0.000264 k$ is constant in Eqs. A.8 and A.9; in Eqs. A.12 and A.13, it is a function of pressure. Despite this complication, many solutions

to Eqs. A.8 and A.9 prove to be accurate approximate solutions of Eqs. A.12 and A.13.

Simultaneous Flow of Oil, Water, and Gas

In this section, we outline a detailed derivation of an equation describing radial, simultaneous flow of oil, gas, and water. More complete discussions and more general equations are given by Matthews and Russell² and Martin.³

We assume that a porous medium contains oil, water, and gas, and that each phase has saturation-dependent effective permeability (k_o , k_w , and k_g); time-dependent saturation (S_o , S_w , and S_g); pressure-dependent formation volume factor (B_o , B_w , and B_g); and pressure-dependent viscosity (μ_o , μ_w , and μ_g). When gravitational forces and capillary pressures are negligible, the differential equation describing this type of flow is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi c_t}{0.000264 \lambda_t} \frac{\partial p}{\partial t}, \quad \text{..... (A.14)}$$

where

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w}, \quad \text{..... (A.15)}$$

$$c_t = S_o c_o + S_w c_w + S_g c_g + c_f, \quad \text{..... (A.16)}$$

and

$$c_o = - \frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_s}{dp}. \quad \text{..... (A.17)}$$

Note in Eq. A.17 that effective oil compressibility depends not only on the usual change in liquid volume with pressure but also on the change with pressure of the dissolved GOR, R_s .

A similar expression can be written for water compressibility:

$$c_w = - \frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp}. \quad \text{..... (A.18)}$$

Exercises

A.1 Derive Eq. A.2; state the assumptions required in this derivation.

A.2 Derive Eq. A.5; state the assumptions required in this derivation.

A.3 Derive Eq. A.9; state the assumptions required in this derivation.

A.4 Derive Eq. A.13; state the assumptions required in this derivation. Compare the assumptions required to derive Eq. A.9 with those required to derive Eq. A.13.

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Appendix B

Dimensionless Variables

Introduction

It is convenient and customary to present graphical or tabulated solutions to flow equations, such as Eq. A.9, in terms of dimensionless variables. In this way, it is possible to present compactly solutions for a wide range of parameters ϕ , μ , c , and k , and variables r , p , and t .

In this appendix, we show how many of the dimensionless variables that appear in the well-testing literature arise logically and directly in the differential equations (and in their initial and boundary conditions) that describe flow in porous media.

Radial Flow of a Slightly Compressible Fluid

In this section, we identify the dimensionless variables and parameters required to characterize the solutions to the equations describing radial flow of a slightly compressible liquid in a reservoir. We assume that Eq. A.9 adequately models this flow. Specifically, we analyze the situation in which (1) pressure throughout the reservoir is uniform before production; (2) fluid is produced at a constant rate from a single well of radius r_w centered in the reservoir; and (3) there is no flow across the outer boundary (with radius r_e) of the reservoir. Stated mathematically, the differential equation, and initial and boundary conditions are

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{0.000264 k} \frac{\partial p}{\partial t}, \quad \dots \dots \dots (B.1)$$

at $t=0$, $p=p_i$ for all r ,

at $r=r_e$, $q=0$ for $t>0$, $\dots \dots \dots (B.2)$

or

$$\frac{\partial p}{\partial r} \Big|_{r_e} = 0, \quad \dots \dots \dots (B.3)$$

at $r=r_w$, $q = \frac{-0.001127 (2\pi r_w h)}{B} \frac{k}{\mu} \frac{\partial p}{\partial r} \Big|_{r_w}$ for $t>0$

or

$$\frac{\partial p}{\partial r} \Big|_{r_w} = - \frac{qB\mu}{0.00708 khr_w} \dots \dots \dots (B.4)$$

This boundary condition arises from Darcy's law in the form similar to that used in Eq. A.3:

$$u_r = -0.001127 \frac{k_r}{\mu} \frac{\partial p}{\partial r} = \frac{qB}{2\pi hr}.$$

Our objective in this analysis is to restate the differential equation, and initial and boundary conditions in dimensionless form so we can determine the dimensionless variables and parameters that characterize this flow situation and that can be used to characterize solutions. These dimensionless parameters and variables are not unique (i.e., more than one choice can be made for each). Further, we want to emphasize that these dimensionless variables are *defined* rather than *derived* quantities. These ideas will become clearer as we proceed.

We *define* a dimensionless radius, $r_D = r/r_w$ (any other convenient reference length, such as r_e , could have been used). From the form of the differential equation, we also note that a convenient definition of dimensionless time is $t_D = 0.000264 kt / \phi \mu c r_w^2$.

The initial and boundary conditions suggest that a convenient definition of dimensionless pressure is

$$p_D = \frac{0.00708 kh (p_i - p)}{qB\mu}.$$

With this definition, the boundary condition (Eq. B.4) becomes

$$\frac{-qB\mu}{0.00708 khr_w} \frac{\partial p_D}{\partial r_D} \Big|_{r_D=1} = \frac{-qB\mu}{0.00708 khr_w},$$

or simply

$$\frac{\partial p_D}{\partial r_D} \Big|_{r_D=1} = 1.$$

Expressed in terms of dimensionless variables, the differential equation and its initial and boundary conditions become

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial p_D}{\partial r_D} \right) = \frac{\partial p_D}{\partial t_D}, \quad \dots \dots \dots (B.5)$$

$$p_D = 0 \text{ for all } r_D \text{ at } t_D = 0, \dots\dots\dots (\text{B.6})$$

$$\left. \frac{\partial p_D}{\partial r_D} \right|_{r_D=r_e/r_w=r_{De}} = 0 \text{ for } t_D > 0, \dots\dots\dots (\text{B.7})$$

$$\left. \frac{\partial p_D}{\partial r_D} \right|_{r_D=1} = 1 \text{ for } t_D > 0. \dots\dots\dots (\text{B.8})$$

The implication is that any solution, p_D , of Eq. B.5 is a unique function of r_D and t_D for fixed r_{De} ; no other dimensionless variables appear in either the dimensionless differential equation or in the dimensionless initial or boundary conditions describing this particular problem. Thus, if we wish to present solutions to Eq. B.5, we could do so compactly either by tabulating or by plotting p_D at $r_D = 1$ ($r = r_w$) as a function of the variable t_D with the parameter r_{De} . In fact, such tables and graphs have been prepared and presented; this problem is precisely the well known van Everdingen and Hurst constant terminal-rate problem* for which they present solutions as functions of t_D and r_{De} .

We have implied that the groups of parameters and variables that arise from the differential equation and its initial and boundary conditions are dimensionless. We will now provide the proof for the case just considered.

Dimensionless variables are

$$r_D = r/r_w,$$

$$t_D = \frac{0.000264 \, k t}{\phi \mu c r_w^2},$$

and

$$p_D = \frac{0.00708 \, k h (p_i - p)}{q B \mu}.$$

Obviously, r_D is dimensionless. To show that t_D and p_D are dimensionless, we introduce the symbol [], which denotes "has units of." Let m denote mass, L , length, and t , time. The quantities that appear in t_D and p_D have the following basic units:

$$\begin{aligned} k &\sim [L^2], \\ t &\sim [t], \\ \phi &\sim [1] \text{ (i.e., dimensionless)}, \\ \mu &\sim [m/Lt], \\ c &\sim [Lt^2/m], \\ r_w &\sim [L], \\ h &\sim [L], \\ p &\sim [m/Lt^2], \\ q &\sim [L^3/t], \text{ and} \\ B &\sim [1]. \end{aligned}$$

A comment on the units of pressure, p , and compressibility, c (which has the units psi^{-1}) may be helpful: Pressure is defined as force per unit area. From Newton's second law, force is mass times acceleration; thus, force has the fundamental units

$$\text{Force} \sim [mL/t^2].$$

Thus, pressure has the fundamental units

$$\text{Pressure} \sim [mL/t^2 L^2] \text{ or } [m/Lt^2],$$

*See Appendix C.

and compressibility has the fundamental units

$$\text{Compressibility} \sim [Lt^2/m].$$

Then,

$$t_D = \frac{0.000264 \, k t}{\phi \mu c r_w^2} \sim \frac{[L^2][t]}{[1] \left| \frac{m}{Lt} \right| \left| \frac{Lt^2}{m} \right| [L^2]} \sim [1].$$

Thus, t_D has units of unity, or, more plainly, is dimensionless. Similarly,

$$\begin{aligned} p_D &= \frac{0.00708 \, k h (p_i - p)}{q B \mu} \\ &\sim \frac{[L^2][L] \left| \frac{m}{Lt^2} \right|}{\left| \frac{L^3}{t} \right| [1] \left| \frac{m}{Lt} \right|} \sim [1]. \end{aligned}$$

Thus, p_D also is dimensionless.

Again, we stress the reason for introduction of somewhat unnatural dimensionless quantities. They allow solutions for wide ranges of k , h , c , t , r_e , r_w , q , μ , and B to be presented compactly (tables or graphs) as functions of a minimum number of variables and parameters. Such tabulations and graphs are in widespread use in well test analysis.

Radial Flow With Constant BHP

To illustrate further how the choice of appropriate dimensionless variables depends on the specific differential equation, and initial and boundary conditions, we now determine appropriate dimensionless variables for radial flow of a slightly compressible liquid through a wellbore of radius r_w from a reservoir of radius r_e . There is no flow across the outer boundary (at $r = r_e$); initial pressure, p_i , is uniform before production; and flowing BHP in the wellbore, p_{wf} , is held constant once production begins. The mathematical problem is then

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{0.000264 \, k} \frac{\partial p}{\partial t}, \dots\dots\dots (\text{B.9})$$

$$p = p_i \text{ at } t = 0 \text{ for all } r, \dots\dots\dots (\text{B.10})$$

$$p = p_{wf} \text{ at } r = r_w \text{ for all } t > 0, \dots\dots\dots (\text{B.11})$$

$$\left. \frac{\partial p}{\partial r} \right|_{r=r_e} = 0 \text{ for all } t > 0. \dots\dots\dots (\text{B.12})$$

As before, our approach will be to eliminate all parameters and variables with dimensions from the differential equation, and initial and boundary conditions. The appropriate definition of r_D is again $r_D = r/r_w$; for t_D , it is

$$t_D = \frac{0.000264 \, k t}{\phi \mu c r_w^2}.$$

These definitions will eliminate all parameters from Eq. B.9. The appropriate definition of p_D for this situation is

$$p_D = \frac{p_i - p}{p_i - p_{wf}}.$$

With these definitions, the mathematical statement of the problem becomes.

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial p_D}{\partial r_D} \right) = \frac{\partial p_D}{\partial t_D}, \dots \dots \dots (B.13)$$

$$p_D = 0 \text{ at } t_D = 0 \text{ for all } r_D, \dots \dots \dots (B.14)$$

$$p_D = 1 \text{ at } r_D = 1 \text{ for all } t_D > 0, \dots \dots \dots (B.15)$$

$$\left. \frac{\partial p_D}{\partial r_D} \right|_{r_D=r_e/r_w=r_{De}} = 0 \text{ for all } t_D > 0. \dots \dots \dots (B.16)$$

Thus, dimensionless pressure p_D is a function of dimensionless time, t_D , and dimensionless radius r_D for a given value of the dimensionless parameter r_{eD} since no other variables or parameters appear in the differential equation, or initial or boundary conditions. However, the appropriate definition of dimensionless pressure is different in this constant-pressure case from the constant-rate case—with the appropriate definition dictated by the boundary conditions.

Because this constant-pressure case is of considerable practical importance, we proceed further with our analysis. It is of interest to determine instantaneous production rate and cumulative produced volume for this case. Instantaneous production rate q is

$$q = - \frac{0.001127 (2\pi r_w h)}{B} \frac{k}{\mu} \frac{\partial p}{\partial r} \bigg|_{r_w}, \dots \dots \dots (B.17)$$

and cumulative production Q_p is

$$Q_p = \frac{1}{24} \int_0^t q dt. \dots \dots \dots (B.18)$$

It is convenient to define a dimensionless production rate, q_D :

$$q_D = \frac{q B \mu}{0.00708 kh (p_i - p_{wf})}.$$

Then,

$$q_D = \left. \frac{\partial p_D}{\partial r_D} \right|_{r_D=1} \dots \dots \dots (B.19)$$

We also define a dimensionless cumulative produced volume, Q_{pD} .

$$Q_{pD} = \int_0^{t_D} q_D dt_D = \left[\frac{B \mu}{0.00708 kh (p_i - p_{wf})} \right]$$

$$\cdot \left(\frac{0.000264 k}{\phi \mu c r_w^2} \right) \int_0^t q dt.$$

Therefore,

$$Q_p = 1.119 \frac{\phi c h r_w^2 (p_i - p_{wf})}{B} Q_{pD}. \dots \dots \dots (B.20)$$

The implication of our analysis is that cumulative production, Q_p (stock-tank barrels), can be determined from dimensionless cumulative production, Q_{pD} , which is based on solutions to Eqs. B.13 through B.16. Since Q_{pD} is based on these solutions, it is a function of t_D and r_{De} only (for $r_D = 1$); thus, it appears possible that Q_{pD} could be tabulated (or plotted) as a function of t_D for selected values of r_{De} . In fact, this has been done by van Everdingen and Hurst; this problem is their well known constant-pressure case.

The Q_{pD} values also can be interpreted as dimensionless cumulative water influx from a radial aquifer into a reservoir if r_w is interpreted as the reservoir radius and r_e as the aquifer radius.

Exercise

B.1 Consider the radial flow of a gas expressed in terms of the pseudopressure, ψ , by Eq. A.13. Define dimensionless pseudopressure as

$$\psi_D = \frac{kh T_{sc}}{50,300 q_g p_{sc} T} (\psi_i - \psi),$$

dimensionless time as

$$t_D = \frac{0.000264 kt}{\phi \mu_i c_{gi} r_w^2},$$

and

$$r_D = r/r_w,$$

- T = reservoir temperature, °R,
- T_{sc} = standard-condition temperature, °R,
- p_{sc} = standard-condition pressure, psia,
- q_g = gas flow rate, Mscf/D,
- μ_i = gas viscosity evaluated at original reservoir pressure, cp, and
- c_{gi} = gas compressibility evaluated at original reservoir pressure, psi⁻¹.

The cylindrical reservoir is initially at uniform pressure, p_i ; there is no flow across the outer boundary of radius r_e ; and flow rate into the wellbore of radius r_w is constant (expressed at standard conditions).

Write the differential equation, and initial and boundary conditions in dimensional form; then rewrite them in dimensionless form.

Appendix C

Van Everdingen and Hurst Solutions to Diffusivity Equations

Introduction

In Appendix B, we showed that solutions to differential equations describing flow in a petroleum reservoir for given initial and boundary conditions can be expressed compactly using dimensionless variables and parameters. In this appendix, we examine four of these solutions that are important in reservoir engineering applications.

Constant Rate at Inner Boundary, No Flow Across Outer Boundary

This solution of the diffusivity equation models radial flow of a slightly compressible liquid in a homogeneous reservoir of uniform thickness; reservoir at uniform pressure p_i before production; no flow across the outer boundary (at $r=r_e$); and production at constant rate q from the single well (centered in the reservoir) with wellbore radius r_w . The solution—pressure as a function of time and radius for fixed values of r_e , r_w , and rock and fluid properties—is expressed most conveniently in terms of dimensionless variables and parameters:

$$p_D = f(t_D, r_D, r_{eD}), \quad \dots \dots \dots (C.1)$$

where

$$p_D = \frac{0.00708 \, kh(p_i - p)}{qB\mu},$$

$$t_D = \frac{0.000264kt}{\phi\mu c_l r_w^2},$$

$$r_D = r/r_w,$$

and

$$r_{eD} = r_e/r_w.$$

Eq. C.1 states that p_D is a function of the variables t_D and r_D for a fixed value of the parameter r_{eD} . The most important solution is that for pressure at the wellbore radius ($r=r_w$ or $r_D=1$):

$$p_D|_{r_D=1} = f(t_D, r_{eD}).$$

When expressed in terms of dimensionless pressure

evaluated at $r_D=1$, Eq. 1.6 shows the functional form of $f(t_D, r_{eD})$ —an infinite series of exponentials and Bessel functions. This series has been evaluated¹ for several values of r_{eD} over a wide range of values of t_D . Chatas² tabulated these solutions; a modification of Chatas' tabulation is presented in Tables C.1 and C.2.

Some important characteristics of this tabulation include the following.

1. Table C.1 presents values of p_D in the range $t_D < 1,000$ for an infinite-acting reservoir. For $t_D < 0.01$, p_D can be approximated by the relation

$$p_D \approx 2\sqrt{t_D/\pi}. \quad \dots \dots \dots (C.2)$$

2. Table C.1 is valid for finite reservoirs with $t_D < 0.25 r_{eD}^2$.

3. For $100 < t_D < 0.25 r_{eD}^2$, Table C.1 can be extended by use of the equation

$$p_D \approx 0.5(\ln t_D + 0.80907). \quad \dots \dots \dots (C.3)$$

(This equation is identical to Eq. 1.10 for $s=0$. It begins to become slightly less accurate at $t_D \approx 0.0625 r_{eD}^2$, but there is no simple approximation between this value and $t_D \approx 0.25 r_{eD}^2$. Fortunately, this slight loss in accuracy is not of sufficient magnitude to cause problems in most practical applications.)

4. Table C.2 presents p_D as a function of t_D for $1.5 < r_{eD} < 10$.

(a) For values of t_D smaller than the value listed for a given r_{eD} , the reservoir is infinite acting, and Table C.1 should be used to determine p_D .

(b) For values of t_D larger than the largest value listed for a given r_{eD} (or, more correctly, for $25 < t_D$ and $0.25 r_{eD}^2 < t_D$), p_D can be calculated from²

$$p_D \approx \frac{2(t_D + 0.25)}{r_{eD}^2 - 1} - \frac{(3r_{eD}^4 - 4r_{eD}^4 \ln r_{eD} - 2r_{eD}^2 - 1)}{4(r_{eD}^2 - 1)^2}. \quad \dots \dots (C.4)$$

(c) A special case of Eq. C.4 arises when $r_{eD}^2 \gg 1$; then,

TABLE C.1 – p_D vs. t_D – INFINITE RADIAL SYSTEM, CONSTANT RATE AT INNER BOUNDARY

t_D	p_D	t_D	p_D	t_D	p_D
0	0	0.15	0.3750	60.0	2.4758
0.0005	0.0250	0.2	0.4241	70.0	2.5501
0.001	0.0352	0.3	0.5024	80.0	2.6147
0.002	0.0495	0.4	0.5645	90.0	2.6718
0.003	0.0603	0.5	0.6167	100.0	2.7233
0.004	0.0694	0.6	0.6622	150.0	2.9212
0.005	0.0774	0.7	0.7024	200.0	3.0636
0.006	0.0845	0.8	0.7387	250.0	3.1726
0.007	0.0911	0.9	0.7716	300.0	3.2630
0.008	0.0971	1.0	0.8019	350.0	3.3394
0.009	0.1028	1.2	0.8672	400.0	3.4057
0.01	0.1081	1.4	0.9160	450.0	3.4641
0.015	0.1312	2.0	1.0195	500.0	3.5164
0.02	0.1503	3.0	1.1665	550.0	3.5643
0.025	0.1669	4.0	1.2750	600.0	3.6076
0.03	0.1818	5.0	1.3625	650.0	3.6476
0.04	0.2077	6.0	1.4362	700.0	3.6842
0.05	0.2301	7.0	1.4997	750.0	3.7184
0.06	0.2500	8.0	1.5557	800.0	3.7505
0.07	0.2680	9.0	1.6057	850.0	3.7805
0.08	0.2845	10.0	1.6509	900.0	3.8088
0.09	0.2999	15.0	1.8294	950.0	3.8355
0.1	0.3144	20.0	1.9601	1,000.0	3.8584
		30.0	2.1470		
		40.0	2.2824		
		50.0	2.3884		

Notes: For $t_D < 0.01$, $p_D \approx 2\sqrt{t_D/\pi}$.For $100 < t_D < 0.25 r_{eD}^2$, $p_D \approx 0.5 (\ln t_D + 0.80907)$.

$$p_D \approx \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - 3/4. \quad \text{..... (C.5)}$$

5. The p_D solutions in Tables C.1 and C.2 also apply to a *reservoir* of radius r_w surrounded by an *aquifer* of radius r_e when there is a constant rate of water influx from the aquifer into the reservoir. For this case, values of r_{eD} in the range 1.5 to 10.0, as in Table C.2, are of practical importance. For most well problems, r_{eD} is larger than 10.0, and the approximations given by Eq. C.3 for $100 < t_D < 0.25 r_{eD}^2$ and Eq. C.5 for $0.25 r_{eD}^2 < t_D$ are the really useful results of the van Everdingen and Hurst analysis.

6. When analyzing a variable-rate problem with the p_D solution, we use superposition just as we did with the *Ei*-function solution in Sec. 1.5.

Example C.1 – Use of p_D Solutions for No-Flow Boundary

Problem. In a large laboratory flow experiment, fluid was produced into a 1-ft-radius perforated cylinder from a sand-packed model with a radius of 10 ft. No fluid flowed across the external radius of the model. Properties of the sandpack and produced fluid include the following.

$$\begin{aligned} k &= 1 \text{ darcy,} \\ h &= 0.5 \text{ ft,} \\ p_i &= 15 \text{ psia,} \\ \mu &= 2 \text{ cp,} \\ q &= 1.0 \text{ STB/D,} \\ B &= 1.0 \text{ RB/STB,} \\ \phi &= 0.3, \text{ and} \\ c_t &= 0.11 \times 10^{-3} \text{ psi}^{-1}. \end{aligned}$$

Estimate the pressure on the inner boundary of the sandpack at times of 0.001, 0.01, and 0.1 hour.

Solution. We first calculate t_D and r_{eD} :

$$\begin{aligned} r_{eD} &= 10/1 = 10, \\ t_D &= \frac{0.000264 kt}{\phi \mu c_t r_w^2} \\ &= \frac{(0.000264)(1,000)}{(0.3)(2)(0.11 \times 10^{-3})(1)} t = 4 \times 10^3 t. \end{aligned}$$

Then, the following data result.

t (hour)	t_D	p_D	Source of p_D
0.001	4	1.275	Table C.1 (reservoir infinite acting)
0.01	40	2.401	Table C.2 ($r_{eD} = 10$)
0.1	400	9.6751	Eq. C.4

Note that for $t_D = 400$, $t_D > 0.25 r_{eD}^2 = (0.25)(10)^2 = 25$; thus, Eq. C.4 is used to calculate p_D .

$$\begin{aligned} p_D &= \frac{2(t_D + 0.25)}{r_{eD}^2 - 1} \\ &\quad - \frac{(3r_{eD}^4 - 4r_{eD}^4 \ln r_{eD} - 2r_{eD}^2 - 1)}{4(r_{eD}^2 - 1)^2} \\ &= 9.6751. \end{aligned}$$

A rearrangement of the definition of p_D results in

$$\begin{aligned} p &= p_i - 141.2 \frac{qB\mu}{kh} p_D \\ &= 15 - \frac{(141.2)(1.0)(1.0)(2)}{(1000)(0.5)} p_D \\ &= 15 - 0.565 p_D. \end{aligned}$$

Thus, we have these values:

t (hour)	p (psia)
0.001	14.28
0.01	13.64
0.1	9.53

Constant Rate at Inner Boundary, Constant Pressure at Outer Boundary

This solution of the diffusivity equation models radial flow of a slightly compressible liquid in a homogeneous reservoir of uniform thickness; reservoir at uniform pressure p_i before production; unchanging pressure, also p_i , at the outer boundary (at $r = r_e$); and production at constant rate q from the single well (centered in the reservoir) with wellbore radius r_w . The solutions, p_D (evaluated at $r_D = 1$), as a function of t_D for fixed values of r_{eD} in the range $1.5 < r_{eD} < 3,000$, are given in Table C.3. The dimensionless variables p_D , t_D , r_D , and r_{eD} have the same definitions as in the previous section.

TABLE C.2 – p_D vs. t_D – FINITE RADIAL SYSTEM WITH CLOSED EXTERIOR BOUNDARY, CONSTANT RATE AT INNER BOUNDARY

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD} = 3.0$		$r_{eD} = 3.5$		$r_{eD} = 4.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
0.06	0.251	0.22	0.443	0.40	0.565	0.52	0.627	1.0	0.802	1.5	0.927
0.08	0.288	0.24	0.459	0.42	0.576	0.54	0.636	1.1	0.830	1.6	0.948
0.10	0.322	0.26	0.476	0.44	0.587	0.56	0.645	1.2	0.857	1.7	0.968
0.12	0.355	0.28	0.492	0.46	0.598	0.60	0.662	1.3	0.882	1.8	0.988
0.14	0.387	0.30	0.507	0.48	0.608	0.65	0.683	1.4	0.906	1.9	1.007
0.16	0.420	0.32	0.522	0.50	0.618	0.70	0.703	1.5	0.929	2.0	1.025
0.18	0.452	0.34	0.536	0.52	0.628	0.75	0.721	1.6	0.951	2.2	1.059
0.20	0.484	0.36	0.551	0.54	0.638	0.80	0.740	1.7	0.973	2.4	1.092
0.22	0.516	0.38	0.565	0.56	0.647	0.85	0.758	1.8	0.994	2.6	1.123
0.24	0.548	0.40	0.579	0.58	0.657	0.90	0.776	1.9	1.014	2.8	1.154
0.26	0.580	0.42	0.593	0.60	0.666	0.95	0.791	2.0	1.034	3.0	1.184
0.28	0.612	0.44	0.607	0.65	0.688	1.0	0.806	2.25	1.083	3.5	1.255
0.30	0.644	0.46	0.621	0.70	0.710	1.2	0.865	2.50	1.130	4.0	1.324
0.35	0.724	0.48	0.634	0.75	0.731	1.4	0.920	2.75	1.176	4.5	1.392
0.40	0.804	0.50	0.648	0.80	0.752	1.6	0.973	3.0	1.221	5.0	1.460
0.45	0.884	0.60	0.715	0.85	0.772	2.0	1.076	4.0	1.401	5.5	1.527
0.50	0.964	0.70	0.782	0.90	0.792	3.0	1.328	5.0	1.579	6.0	1.594
0.55	1.044	0.80	0.849	0.95	0.812	4.0	1.578	6.0	1.757	6.5	1.660
0.60	1.124	0.90	0.915	1.00	0.832	5.0	1.828			7.0	1.727
0.65	1.204	1.0	0.982	2.0	1.215					8.0	1.861
0.70	1.284	2.0	1.649	3.0	1.506					9.0	1.994
0.75	1.364	3.0	2.316	4.0	1.977					10.0	2.127
0.80	1.444	5.0	3.649	5.0	2.398						

$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6.0$		$r_{eD} = 7.0$		$r_{eD} = 8.0$		$r_{eD} = 9.0$		$r_{eD} = 10.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
2.0	1.023	3.0	1.167	4.0	1.275	6.0	1.436	8.0	1.556	10.0	1.651	12.0	1.732
2.1	1.040	3.1	1.180	4.5	1.322	6.5	1.470	8.5	1.582	10.5	1.673	12.5	1.750
2.2	1.056	3.2	1.192	5.0	1.364	7.0	1.501	9.0	1.607	11.0	1.693	13.0	1.768
2.3	1.072	3.3	1.204	5.5	1.404	7.5	1.531	9.5	1.631	11.5	1.713	13.5	1.784
2.4	1.087	3.4	1.215	6.0	1.441	8.0	1.559	10.0	1.653	12.0	1.732	14.0	1.801
2.5	1.102	3.5	1.227	6.5	1.477	8.5	1.586	10.5	1.675	12.5	1.750	14.5	1.817
2.6	1.116	3.6	1.238	7.0	1.511	9.0	1.613	11.0	1.697	13.0	1.768	15.0	1.832
2.7	1.130	3.7	1.249	7.5	1.544	9.5	1.638	11.5	1.717	13.5	1.786	15.5	1.847
2.8	1.144	3.8	1.259	8.0	1.576	10.0	1.663	12.0	1.737	14.0	1.803	16.0	1.862
2.9	1.158	3.9	1.270	8.5	1.607	11.0	1.711	12.5	1.757	14.5	1.819	17.0	1.890
3.0	1.171	4.0	1.281	9.0	1.638	12.0	1.757	13.0	1.776	15.0	1.835	18.0	1.917
3.2	1.197	4.2	1.301	9.5	1.668	13.0	1.810	13.5	1.795	15.5	1.851	19.0	1.943
3.4	1.222	4.4	1.321	10.0	1.698	14.0	1.845	14.0	1.813	16.0	1.867	20.0	1.968
3.6	1.246	4.6	1.340	11.0	1.757	15.0	1.888	14.5	1.831	17.0	1.897	22.0	2.017
3.8	1.269	4.8	1.360	12.0	1.815	16.0	1.931	15.0	1.849	18.0	1.926	24.0	2.063
4.0	1.292	5.0	1.378	13.0	1.873	17.0	1.974	17.0	1.919	19.0	1.955	26.0	2.108
4.5	1.349	5.5	1.424	14.0	1.931	18.0	2.016	19.0	1.986	20.0	1.983	28.0	2.151
5.0	1.403	6.0	1.469	15.0	1.988	19.0	2.058	21.0	2.051	22.0	2.037	30.0	2.194
5.5	1.457	6.5	1.513	16.0	2.045	20.0	2.100	23.0	2.116	24.0	2.096	32.0	2.236
6.0	1.510	7.0	1.556	17.0	2.103	22.0	2.184	25.0	2.180	26.0	2.142	34.0	2.278
7.0	1.615	7.5	1.598	18.0	2.160	24.0	2.267	30.0	2.340	28.0	2.193	36.0	2.319
8.0	1.719	8.0	1.641	19.0	2.217	26.0	2.351	35.0	2.499	30.0	2.244	38.0	2.360
9.0	1.823	9.0	1.725	20.0	2.274	28.0	2.434	40.0	2.658	34.0	2.345	40.0	2.401
10.0	1.927	10.0	1.808	25.0	2.560	30.0	2.517	45.0	2.817	38.0	2.446	50.0	2.604
11.0	2.031	11.0	1.892	30.0	2.846					40.0	2.496	60.0	2.806
12.0	2.135	12.0	1.975							45.0	2.621	70.0	3.008
13.0	2.239	13.0	2.059							50.0	2.746	80.0	3.210
14.0	2.343	14.0	2.142							60.0	2.996	90.0	3.412
15.0	2.447	15.0	2.225							70.0	3.246	100.0	3.614

Notes: For t_D smaller than values listed in this table for a given r_{eD} , reservoir is infinite acting. Find p_D in Table C.1.

For $25 < t_D$ and t_D larger than values in table,

$$p_D \approx \frac{(\frac{1}{2} + 2t_D)}{(r_{eD}^2 - 1)} - \frac{3r_{eD}^4 - 4r_{eD}^2 \ln r_{eD} - 2r_{eD}^2 - 1}{4(r_{eD}^2 - 1)^2}$$

For Wells in bounded reservoirs with

$$r_{eD}^2 \gg 1,$$

$$p_D \approx \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4}.$$

TABLE C.3 – p_D vs. t_D – FINITE RADIAL SYSTEM WITH FIXED CONSTANT PRESSURE
AT EXTERIOR BOUNDARY, CONSTANT RATE AT INNER BOUNDARY

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD} = 3.0$		$r_{eD} = 3.5$		$r_{eD} = 4.0$		$r_{eD} = 6.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
0.050	0.230	0.20	0.424	0.30	0.502	0.50	0.617	0.50	0.620	1.0	0.802	4.0	1.275
0.055	0.240	0.22	0.441	0.35	0.535	0.55	0.640	0.60	0.665	1.2	0.857	4.5	1.320
0.060	0.249	0.24	0.457	0.40	0.564	0.60	0.662	0.70	0.705	1.4	0.905	5.0	1.361
0.070	0.266	0.26	0.472	0.45	0.591	0.70	0.702	0.80	0.741	1.6	0.947	5.5	1.398
0.080	0.282	0.28	0.485	0.50	0.616	0.80	0.738	0.90	0.774	1.8	0.986	6.0	1.432
0.090	0.292	0.30	0.498	0.55	0.638	0.90	0.770	1.0	0.804	2.0	1.020	6.5	1.462
0.10	0.307	0.35	0.527	0.60	0.659	1.0	0.799	1.2	0.858	2.2	1.052	7.0	1.490
0.12	0.328	0.40	0.552	0.70	0.696	1.2	0.850	1.4	0.904	2.4	1.080	7.5	1.516
0.14	0.344	0.45	0.573	0.80	0.728	1.4	0.892	1.6	0.945	2.6	1.106	8.0	1.539
0.16	0.356	0.50	0.591	0.90	0.755	1.6	0.927	1.8	0.981	2.8	1.130	8.5	1.561
0.18	0.367	0.55	0.606	1.0	0.778	1.8	0.955	2.0	1.013	3.0	1.152	9.0	1.580
0.20	0.375	0.60	0.619	1.2	0.815	2.0	0.980	2.2	1.041	3.4	1.190	10.0	1.615
0.22	0.381	0.65	0.630	1.4	0.842	2.2	1.000	2.4	1.065	3.8	1.232	12.0	1.667
0.24	0.386	0.70	0.639	1.6	0.861	2.4	1.016	2.6	1.087	4.5	1.266	14.0	1.704
0.26	0.390	0.75	0.647	1.8	0.876	2.6	1.030	2.8	1.106	5.0	1.290	16.0	1.730
0.28	0.393	0.80	0.654	2.0	0.887	2.8	1.042	3.0	1.123	5.5	1.309	18.0	1.749
0.30	0.396	0.85	0.660	2.2	0.895	3.0	1.051	3.5	1.153	6.0	1.325	20.0	1.762
0.35	0.400	0.90	0.665	2.4	0.900	3.5	1.069	4.0	1.183	7.0	1.347	22.0	1.771
0.40	0.402	0.95	0.669	2.6	0.905	4.0	1.080	5.0	1.225	8.0	1.361	24.0	1.777
0.45	0.404	1.0	0.673	2.8	0.908	4.5	1.087	6.0	1.232	9.0	1.370	26.0	1.781
0.50	0.405	1.2	0.682	3.0	0.910	5.0	1.091	7.0	1.242	10.0	1.376	28.0	1.784
0.60	0.405	1.4	0.688	3.5	0.913	5.5	1.094	8.0	1.247	12.0	1.382	30.0	1.787
0.70	0.405	1.6	0.690	4.0	0.915	6.0	1.096	9.0	1.250	14.0	1.385	35.0	1.789
0.80	0.405	1.8	0.692	4.5	0.916	6.5	1.097	10.0	1.251	16.0	1.386	40.0	1.791
		2.0	0.692	5.0	0.916	7.0	1.097	12.0	1.252	18.0	1.386	50.0	1.792
		2.5	0.693	5.5	0.916	8.0	1.098	14.0	1.253				
		3.0	0.693	6.0	0.916	10.0	1.099	16.0	1.253				

$r_{eD} = 8.0$		$r_{eD} = 10.0$		$r_{eD} = 15.0$		$r_{eD} = 20.0$		$r_{eD} = 25.0$		$r_{eD} = 30.0$		$r_{eD} = 40.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
7.0	1.499	10.0	1.651	20.0	1.960	30.0	2.148	50.0	2.389	70.0	2.551	120	2.813
7.5	1.527	12.0	1.730	22.0	2.003	35.0	2.219	55.0	2.434	80.0	2.615	140	2.888
8.0	1.554	14.0	1.798	24.0	2.043	40.0	2.282	60.0	2.476	90.0	2.672	160	2.953
8.5	1.580	16.0	1.856	26.0	2.080	45.0	2.338	65.0	2.514	100.0	2.723	180	3.011
9.0	1.604	18.0	1.907	28.0	2.114	50.0	2.388	70.0	2.550	120.0	2.812	200	3.063
9.5	1.627	20.0	1.952	30.0	2.146	60.0	2.475	75.0	2.583	140.0	2.886	220	3.109
10.0	1.648	25.0	2.043	35.0	2.218	70.0	2.547	80.0	2.614	160.0	2.950	240	3.152
12.0	1.724	30.0	2.111	40.0	2.279	80.0	2.609	85.0	2.643	165.0	2.965	260	3.191
14.0	1.786	35.0	2.160	45.0	2.332	90.0	2.658	90.0	2.671	170.0	2.979	280	3.226
16.0	1.837	40.0	2.197	50.0	2.379	100.0	2.707	95.0	2.697	175.0	2.992	300	3.259
18.0	1.879	45.0	2.224	60.0	2.455	105.0	2.728	100.0	2.721	180.0	3.006	350	3.331
20.0	1.914	50.0	2.245	70.0	2.513	110.0	2.747	120.0	2.807	200.0	3.054	400	3.391
22.0	1.943	55.0	2.260	80.0	2.558	115.0	2.764	140.0	2.878	250.0	3.150	450	3.440
24.0	1.967	60.0	2.271	90.0	2.592	120.0	2.781	160.0	2.936	300.0	3.219	500	3.482
26.0	1.986	65.0	2.279	100.0	2.619	125.0	2.796	180.0	2.984	350.0	3.269	550	3.516
28.0	2.002	70.0	2.285	120.0	2.655	130.0	2.810	200.0	3.024	400.0	3.306	600	3.545
30.0	2.016	75.0	2.290	140.0	2.677	135.0	2.823	220.0	3.057	450.0	3.332	650	3.568
35.0	2.040	80.0	2.293	160.0	2.689	140.0	2.835	240.0	3.085	500.0	3.351	700	3.588
40.0	2.055	90.0	2.297	180.0	2.697	145.0	2.846	260.0	3.107	600.0	3.375	800	3.619
45.0	2.064	100.0	2.300	200.0	2.701	150.0	2.857	280.0	3.126	700.0	3.387	900	3.640
50.0	2.070	110.0	2.301	220.0	2.704	160.0	2.876	300.0	3.142	800.0	3.394	1,000	3.655
60.0	2.076	120.0	2.302	240.0	2.706	180.0	2.906	350.0	3.171	900.0	3.397	1,200	3.672
70.0	2.078	130.0	2.302	260.0	2.707	200.0	2.929	400.0	3.189	1,000	3.399	1,400	3.681
80.0	2.079	140.0	2.302	280.0	2.707	240.0	2.958	450.0	3.200	1,200	3.401	1,600	3.685
		160.0	2.303	300.0	2.708	280.0	2.975	500.0	3.207	1,400	3.401	1,800	3.687
						300.0	2.980	600.0	3.214			2,000	3.688
						400.0	2.992	700.0	3.217			2,500	3.689
						500.0	2.995	800.0	3.218				
								900.0	3.219				

Notes: For t_D smaller than values listed in this table for a given r_{eD} , reservoir is infinite acting. Find p_D in Table C.1.

For t_D larger than values listed in this table, $p_D \approx \ln r_{eD}$.

TABLE C.3 – CONTINUED

$r_{eD} = 50.0$		$r_{eD} = 60.0$		$r_{eD} = 70.0$		$r_{eD} = 80.0$		$r_{eD} = 90.0$		$r_{eD} = 100.0$		$r_{eD} = 200.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
200	3.064	300	3.257	500	3.512	600	3.603	800	3.747	1,000	3.859	1,500	4.061
220	3.111	400	3.401	600	3.603	700	3.680	900	3.806	1,200	3.949	2,000	4.205
240	3.154	500	3.512	700	3.680	800	3.747	1,000	3.858	1,400	4.026	2,500	4.317
260	3.193	600	3.602	800	3.746	900	3.805	1,200	3.949	1,600	4.092	3,000	4.498
280	3.229	700	3.676	900	3.803	1,000	3.857	1,300	3.988	1,800	4.150	3,500	4.485
300	3.263	800	3.739	1,000	3.854	1,200	3.946	1,400	4.025	2,000	4.200	4,000	4.552
350	3.339	900	3.792	1,200	3.937	1,400	4.019	1,500	4.058	2,500	4.303	5,000	4.663
400	3.405	1,000	3.832	1,400	4.003	1,500	4.051	1,800	4.144	3,000	4.379	6,000	4.754
450	3.461	1,200	3.908	1,600	4.054	1,600	4.080	2,000	4.192	3,500	4.434	7,000	4.829
500	3.512	1,400	3.959	1,800	4.095	1,800	4.130	2,500	4.285	4,000	4.478	8,000	4.834
550	3.556	1,600	3.996	2,000	4.127	2,000	4.171	3,000	4.349	4,500	4.510	9,000	4.949
600	3.595	1,800	4.023	2,500	4.181	2,500	4.248	3,500	4.394	5,000	4.534	10,000	4.996
650	3.630	2,000	4.043	3,000	4.211	3,000	4.297	4,000	4.426	5,500	4.552	12,000	5.072
700	3.661	2,500	4.071	3,500	4.228	3,500	4.328	4,500	4.448	6,000	4.565	14,000	5.129
750	3.688	3,000	4.084	4,000	4.237	4,000	4.347	5,000	4.464	6,500	4.579	16,000	5.171
800	3.713	3,500	4.090	4,500	4.242	4,500	4.360	6,000	4.482	7,000	4.583	18,000	5.203
850	3.735	4,000	4.092	5,000	4.245	5,000	4.368	7,000	4.491	7,500	4.588	20,000	5.227
900	3.754	4,500	4.093	5,500	4.247	6,000	4.376	8,000	4.496	8,000	4.593	25,000	5.264
950	3.771	5,000	4.094	6,000	4.247	7,000	4.380	9,000	4.498	9,000	4.598	30,000	5.282
1,000	3.787	5,500	4.094	6,500	4.248	8,000	4.381	10,000	4.499	10,000	4.601	35,000	5.290
1,200	3.833			7,000	4.248	9,000	4.382	11,000	4.499	12,500	4.604	40,000	5.294
1,400	3.862			7,500	4.248	10,000	4.382	12,000	4.500	15,000	4.605		
1,600	3.881			8,000	4.248	11,000	4.382	14,000	4.500				
1,800	3.892												
2,000	3.900												
2,200	3.904												
2,400	3.907												
2,600	3.909												
2,800	3.910												

$r_{eD} = 300.0$		$r_{eD} = 400.0$		$r_{eD} = 500.0$		$r_{eD} = 600.0$		$r_{eD} = 700.0$		$r_{eD} = 800.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
6,000	4.754	15,000	5.212	20,000	5.365	40,000	5.703	50,000	5.814	70,000	5.983
8,000	4.898	20,000	5.356	25,000	5.468	45,000	5.762	60,000	5.905	80,000	6.049
10,000	5.010	30,000	5.556	30,000	5.559	50,000	5.814	70,000	5.982	90,000	6.108
12,000	5.101	40,000	5.689	35,000	5.636	60,000	5.904	80,000	6.048	100,000	6.160
14,000	5.177	50,000	5.781	40,000	5.702	70,000	5.979	90,000	6.105	120,000	6.249
16,000	5.242	60,000	5.845	45,000	5.759	80,000	6.041	100,000	6.156	140,000	6.322
18,000	5.299	70,000	5.889	50,000	5.810	90,000	6.094	120,000	6.239	160,000	6.382
20,000	5.348	80,000	5.920	60,000	5.894	100,000	6.139	140,000	6.305	180,000	6.432
24,000	5.429	90,000	5.942	70,000	5.960	120,000	6.210	160,000	6.357	200,000	6.474
28,000	5.491	100,000	5.957	80,000	6.013	140,000	6.262	180,000	6.398	250,000	6.551
30,000	5.517	110,000	5.967	90,000	6.055	160,000	6.299	200,000	6.430	300,000	6.599
40,000	5.606	120,000	5.975	100,000	6.088	180,000	6.326	250,000	6.484	350,000	6.630
50,000	5.652	125,000	5.977	120,000	6.135	200,000	6.345	300,000	6.514	400,000	6.650
60,000	5.676	130,000	5.980	140,000	6.164	250,000	6.374	350,000	6.530	450,000	6.663
70,000	5.690	140,000	5.983	160,000	6.183	300,000	6.387	400,000	6.540	500,000	6.671
80,000	5.696	160,000	5.988	180,000	6.195	350,000	6.392	450,000	6.545	550,000	6.676
90,000	5.700	180,000	5.990	200,000	6.202	400,000	6.395	500,000	6.548	600,000	6.679
100,000	5.702	200,000	5.991	250,000	6.211	500,000	6.397	600,000	6.550	700,000	6.682
120,000	5.703	240,000	5.991	300,000	6.213	600,000	6.397	700,000	6.551	800,000	6.684
140,000	5.704	260,000	5.991	350,000	6.214			800,000	6.551	1,000,000	6.684
150,000	5.704			400,000	6.214						

TABLE C.3 – CONTINUED

$r_{eD} = 900.0$		$r_{eD} = 1,000.0$		$r_{eD} = 1,200.0$		$r_{eD} = 1,400.0$		$r_{eD} = 1,600.0$		$r_{eD} = 1,800.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
8.0(10 ⁴)	6.049	1.0(10 ⁵)	6.161	2.0(10 ⁵)	6.507	2.0(10 ⁵)	6.507	2.5(10 ⁵)	6.619	3.0(10 ⁵)	6.710
9.0(10 ⁴)	6.108	1.2(10 ⁵)	6.252	3.0(10 ⁵)	6.704	2.5(10 ⁵)	6.619	3.0(10 ⁵)	6.710	4.0(10 ⁵)	6.854
1.0(10 ⁵)	6.161	1.4(10 ⁵)	6.329	4.0(10 ⁵)	6.833	3.0(10 ⁵)	6.709	3.5(10 ⁵)	6.787	5.0(10 ⁵)	6.965
1.2(10 ⁵)	6.251	1.6(10 ⁵)	6.395	5.0(10 ⁵)	6.918	3.5(10 ⁵)	6.785	4.0(10 ⁵)	6.853	6.0(10 ⁵)	7.054
1.4(10 ⁵)	6.327	1.8(10 ⁵)	6.452	6.0(10 ⁵)	6.975	4.0(10 ⁵)	6.849	5.0(10 ⁵)	6.962	7.0(10 ⁵)	7.120
1.6(10 ⁵)	6.392	2.0(10 ⁵)	6.503	7.0(10 ⁵)	7.013	5.0(10 ⁵)	6.950	6.0(10 ⁵)	7.046	8.0(10 ⁵)	7.183
1.8(10 ⁵)	6.447	2.5(10 ⁵)	6.605	8.0(10 ⁵)	7.038	6.0(10 ⁵)	7.026	7.0(10 ⁵)	7.114	9.0(10 ⁵)	7.238
2.0(10 ⁵)	6.494	3.0(10 ⁵)	6.681	9.0(10 ⁵)	7.056	7.0(10 ⁵)	7.082	8.0(10 ⁵)	7.167	1.0(10 ⁶)	7.280
2.5(10 ⁵)	6.587	3.5(10 ⁵)	6.738	1.0(10 ⁶)	7.067	8.0(10 ⁵)	7.123	9.0(10 ⁵)	7.210	1.5(10 ⁶)	7.407
3.0(10 ⁵)	6.652	4.0(10 ⁵)	6.781	1.2(10 ⁶)	7.080	9.0(10 ⁵)	7.154	1.0(10 ⁶)	7.244	2.0(10 ⁶)	7.459
4.0(10 ⁵)	6.729	4.5(10 ⁵)	6.813	1.4(10 ⁶)	7.085	1.0(10 ⁶)	7.177	1.5(10 ⁶)	7.334	3.0(10 ⁶)	7.489
4.5(10 ⁵)	6.751	5.0(10 ⁵)	6.837	1.6(10 ⁶)	7.088	1.5(10 ⁶)	7.229	2.0(10 ⁶)	7.364	4.0(10 ⁶)	7.495
5.0(10 ⁵)	6.766	5.5(10 ⁵)	6.854	1.8(10 ⁶)	7.089	2.0(10 ⁶)	7.241	2.5(10 ⁶)	7.373	5.0(10 ⁶)	7.495
5.5(10 ⁵)	6.777	6.0(10 ⁵)	6.868	1.9(10 ⁶)	7.089	2.5(10 ⁶)	7.243	3.0(10 ⁶)	7.376	5.1(10 ⁶)	7.495
6.0(10 ⁵)	6.785	7.0(10 ⁵)	6.885	2.0(10 ⁶)	7.090	3.0(10 ⁶)	7.244	3.5(10 ⁶)	7.377	5.2(10 ⁶)	7.495
7.0(10 ⁵)	6.794	8.0(10 ⁵)	6.895	2.1(10 ⁶)	7.090	3.1(10 ⁶)	7.244	4.0(10 ⁶)	7.378	5.3(10 ⁶)	7.495
8.0(10 ⁵)	6.798	9.0(10 ⁵)	6.901	2.2(10 ⁶)	7.090	3.2(10 ⁶)	7.244	4.2(10 ⁶)	7.378	5.4(10 ⁶)	7.495
9.0(10 ⁵)	6.800	1.0(10 ⁶)	6.904	2.3(10 ⁶)	7.090	3.3(10 ⁶)	7.244	4.4(10 ⁶)	7.378	5.6(10 ⁶)	7.495
1.0(10 ⁶)	6.801	1.2(10 ⁶)	6.907	2.4(10 ⁶)	7.090						

$r_{eD} = 2,000.0$		$r_{eD} = 2,200.0$		$r_{eD} = 2,400.0$		$r_{eD} = 2,600.0$		$r_{eD} = 2,800.0$		$r_{eD} = 3,000.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
4.0(10 ⁵)	6.854	5.0(10 ⁵)	6.966	6.0(10 ⁵)	7.057	7.0(10 ⁵)	7.134	8.0(10 ⁵)	7.201	1.0(10 ⁶)	7.312
5.0(10 ⁵)	6.966	5.5(10 ⁵)	7.013	7.0(10 ⁵)	7.134	8.0(10 ⁵)	7.201	9.0(10 ⁵)	7.260	1.2(10 ⁶)	7.403
6.0(10 ⁵)	7.056	6.0(10 ⁵)	7.057	8.0(10 ⁵)	7.200	9.0(10 ⁵)	7.259	1.0(10 ⁶)	7.312	1.4(10 ⁶)	7.480
7.0(10 ⁵)	7.132	6.5(10 ⁵)	7.097	9.0(10 ⁵)	7.259	1.0(10 ⁶)	7.312	1.2(10 ⁶)	7.403	1.6(10 ⁶)	7.545
8.0(10 ⁵)	7.196	7.0(10 ⁵)	7.133	1.0(10 ⁶)	7.310	1.2(10 ⁶)	7.401	1.6(10 ⁶)	7.542	1.8(10 ⁶)	7.602
9.0(10 ⁵)	7.251	7.5(10 ⁵)	7.167	1.2(10 ⁶)	7.398	1.4(10 ⁶)	7.475	2.0(10 ⁶)	7.644	2.0(10 ⁶)	7.651
1.0(10 ⁶)	7.298	8.0(10 ⁵)	7.199	1.6(10 ⁶)	7.526	1.6(10 ⁶)	7.536	2.4(10 ⁶)	7.719	2.4(10 ⁶)	7.732
1.2(10 ⁶)	7.374	8.5(10 ⁵)	7.229	2.0(10 ⁶)	7.611	1.8(10 ⁶)	7.588	2.8(10 ⁶)	7.775	2.8(10 ⁶)	7.794
1.4(10 ⁶)	7.431	9.0(10 ⁵)	7.256	2.4(10 ⁶)	7.668	2.0(10 ⁶)	7.631	3.0(10 ⁶)	7.797	3.0(10 ⁶)	7.820
1.6(10 ⁶)	7.474	1.0(10 ⁶)	7.307	2.8(10 ⁶)	7.706	2.4(10 ⁶)	7.699	3.5(10 ⁶)	7.840	3.5(10 ⁶)	7.871
1.8(10 ⁶)	7.506	1.2(10 ⁶)	7.390	3.0(10 ⁶)	7.720	2.8(10 ⁶)	7.746	4.0(10 ⁶)	7.870	4.0(10 ⁶)	7.908
2.0(10 ⁶)	7.530	1.6(10 ⁶)	7.507	3.5(10 ⁶)	7.745	3.0(10 ⁶)	7.765	5.0(10 ⁶)	7.905	4.5(10 ⁶)	7.935
2.5(10 ⁶)	7.566	2.0(10 ⁶)	7.579	4.0(10 ⁶)	7.760	3.5(10 ⁶)	7.799	6.0(10 ⁶)	7.922	5.0(10 ⁶)	7.955
3.0(10 ⁶)	7.584	2.5(10 ⁶)	7.631	5.0(10 ⁶)	7.775	4.0(10 ⁶)	7.821	7.0(10 ⁶)	7.930	6.0(10 ⁶)	7.979
3.5(10 ⁶)	7.593	3.0(10 ⁶)	7.661	6.0(10 ⁶)	7.780	5.0(10 ⁶)	7.845	8.0(10 ⁶)	7.934	7.0(10 ⁶)	7.992
4.0(10 ⁶)	7.597	3.5(10 ⁶)	7.677	7.0(10 ⁶)	7.782	6.0(10 ⁶)	7.856	9.0(10 ⁶)	7.936	8.0(10 ⁶)	7.999
5.0(10 ⁶)	7.600	4.0(10 ⁶)	7.686	8.0(10 ⁶)	7.783	7.0(10 ⁶)	7.860	1.0(10 ⁷)	7.937	9.0(10 ⁶)	8.002
6.0(10 ⁶)	7.601	5.0(10 ⁶)	7.693	9.0(10 ⁶)	7.783	8.0(10 ⁶)	7.862	1.2(10 ⁷)	7.937	1.0(10 ⁷)	8.004
6.4(10 ⁶)	7.601	6.0(10 ⁶)	7.695	9.5(10 ⁶)	7.783	9.0(10 ⁶)	7.863	1.3(10 ⁷)	7.937	1.2(10 ⁷)	8.006
		7.0(10 ⁶)	7.696			1.0(10 ⁷)	7.863			1.5(10 ⁷)	8.006
		8.0(10 ⁶)	7.696								

Some important properties of these tabulated solutions include the following:

1. For values of t_D smaller than the smallest value listed for a given r_{eD} , the reservoir is infinite acting, and Table C.1 should be used to determine p_D .
2. For values of t_D larger than the largest value listed for a given r_{eD} (or for $t_D > r_{eD}^2$),

$$p_D \approx \ln r_{eD} \quad \text{..... (C.6)}$$

Example C.2 – Use of p_D Solutions for Constant-Pressure Boundary

Problem. An oil well is producing 300 STB/D from a water-drive reservoir that maintains pressure at the original oil/water contact at a constant 3,000 psia. Distance from the well to the oil/water contact is 900 ft. Well and reservoir properties include the following.

$$\phi = 0.21,$$

$$k = 80 \text{ md},$$

$$c_l = 20 \times 10^{-6} \text{ psi}^{-1},$$

$$r_w = 0.45 \text{ ft},$$

$$\mu = 0.8 \text{ cp},$$

$$h = 12 \text{ ft, and}$$

$$B = 1.25 \text{ RB/STB}.$$

Calculate pressure in the wellbore for production after 0.1, 1.0, and 100.0 days.

Solution. Since $t_D = 0.000264 \text{ kt}/\phi\mu c_l r_w^2$ (t in hours), then

$$t_D = \frac{(0.000264)(80)(24) t \text{ days}}{(0.21)(0.8)(20 \times 10^{-6})(0.45)^2}$$

$$= 7.45 \times 10^5 t.$$

Also,

$$r_{eD} = \frac{r_e}{r_w} = \frac{900}{0.45} = 2,000.$$

Thus, we obtain the following.

TABLE C.4 – Q_{pD} vs. t_D – INFINITE RADIAL SYSTEM, CONSTANT PRESSURE AT INNER BOUNDARY

Dimensionless Time t_D	Dimensionless Cumulative Production Q_{pD}	Dimensionless Time t_D	Dimensionless Cumulative Production Q_{pD}	Dimensionless Time t_D	Dimensionless Cumulative Production Q_{pD}	Dimensionless Time t_D	Dimensionless Cumulative Production Q_{pD}	Dimensionless Time t_D	Dimensionless Cumulative Production Q_{pD}
0.00	0.000	98	42.433	740	226.904	1,975	528.337	8,900	1,986.796
0.01	0.112	99	42.781	750	229.514	2,000	534.145	9,000	2,006.628
0.05	0.278	100	43.129	760	232.120	2,025	539.945	9,100	2,026.438
0.10	0.404	105	44.858	770	234.721	2,050	545.737	9,200	2,046.227
0.15	0.520	110	46.574	775	236.020	2,075	551.522	9,300	2,065.996
0.20	0.606	115	48.277	780	237.318	2,100	557.299	9,400	2,085.744
0.25	0.689	120	49.968	790	239.912	2,125	563.068	9,500	2,105.473
0.30	0.758	125	51.648	800	242.501	2,150	568.830	9,600	2,125.184
0.40	0.898	130	53.317	810	245.086	2,175	574.585	9,700	2,144.878
0.50	1.020	135	54.976	820	247.668	2,200	580.332	9,800	2,164.555
0.60	1.140	140	56.625	825	248.957	2,225	586.072	9,900	2,184.216
0.70	1.251	145	58.265	830	250.245	2,250	591.086	10,000	2,203.861
0.80	1.359	150	59.895	840	252.819	2,275	597.532	12,500	2,688.967
0.90	1.469	155	61.517	850	255.388	2,300	603.252	15,000	3,164.780
1	1.569	160	63.131	860	257.953	2,325	608.965	17,500	3,633.368
2	2.447	165	64.737	870	260.515	2,350	614.672	20,000	4,095.800
3	3.202	170	66.336	875	261.795	2,375	620.372	25,000	5,005.726
4	3.893	175	67.928	880	263.073	2,400	626.066	30,000	5,899.508
5	4.539	180	69.512	890	265.629	2,425	631.755	35,000	6,780.247
6	5.153	185	71.090	900	268.181	2,450	637.437	40,000	7,650.096
7	5.743	190	72.661	910	270.729	2,475	643.113	50,000	9,363.099
8	6.314	195	74.226	920	273.274	2,500	648.781	60,000	11,047.299
9	6.869	200	75.785	925	274.545	2,550	660.093	70,000	12,708.358
10	7.411	205	77.338	930	275.815	2,600	671.379	75,000	13,531.457
11	7.940	210	78.886	940	278.353	2,650	682.640	80,000	14,350.121
12	8.457	215	80.428	950	280.888	2,700	693.877	90,000	15,975.389
13	8.964	220	81.965	960	283.420	2,750	705.090	100,000	17,586.284
14	9.461	225	83.497	970	285.948	2,800	716.280	125,000	21,560.732
15	9.949	230	85.023	975	287.211	2,850	727.449	1.5(10 ⁵)	2.538(10 ⁴)
16	10.434	235	86.545	980	288.473	2,900	738.598	2.0(10 ⁵)	3.308(10 ⁴)
17	10.913	240	88.062	990	290.995	2,950	749.725	2.5(10 ⁵)	4.066(10 ⁴)
18	11.386	245	89.575	1,000	293.514	3,000	760.833	3.0(10 ⁵)	4.817(10 ⁴)
19	11.855	250	91.084	1,010	296.030	3,050	771.922	4.0(10 ⁵)	6.267(10 ⁴)
20	12.319	255	92.589	1,020	298.543	3,100	782.992	5.0(10 ⁵)	7.699(10 ⁴)
21	12.778	260	94.090	1,025	299.799	3,150	794.042	6.0(10 ⁵)	9.113(10 ⁴)
22	13.233	265	95.588	1,030	301.053	3,200	805.075	7.0(10 ⁵)	1.051(10 ⁵)
23	13.684	270	97.081	1,040	303.560	3,250	816.090	8.0(10 ⁵)	1.189(10 ⁵)
24	14.131	275	98.571	1,050	306.065	3,300	827.088	9.0(10 ⁵)	1.326(10 ⁵)
25	14.573	280	100.057	1,060	308.567	3,350	838.067	1.0(10 ⁶)	1.462(10 ⁵)
26	15.013	285	101.540	1,070	311.066	3,400	849.028	1.5(10 ⁶)	2.126(10 ⁵)
27	15.450	290	103.019	1,075	312.314	3,450	859.974	2.0(10 ⁶)	2.781(10 ⁵)
28	15.883	295	104.495	1,080	313.562	3,500	870.903	2.5(10 ⁶)	3.427(10 ⁵)
29	16.313	300	105.968	1,090	316.055	3,550	881.816	3.0(10 ⁶)	4.064(10 ⁵)
30	16.742	305	107.437	1,100	318.545	3,600	892.712	4.0(10 ⁶)	5.313(10 ⁵)
31	17.167	310	108.904	1,110	321.032	3,650	903.594	5.0(10 ⁶)	6.544(10 ⁵)
32	17.590	315	110.367	1,120	323.517	3,700	914.459	6.0(10 ⁶)	7.761(10 ⁵)
33	18.011	320	111.827	1,125	324.760	3,750	926.309	7.0(10 ⁶)	8.965(10 ⁵)
34	18.429	325	113.284	1,130	326.000	3,800	936.144	8.0(10 ⁶)	1.016(10 ⁶)
35	18.845	330	114.738	1,140	328.480	3,850	946.966	9.0(10 ⁶)	1.134(10 ⁶)
36	19.259	335	116.189	1,150	330.958	3,900	957.773	1.0(10 ⁷)	1.252(10 ⁶)
37	19.671	340	117.638	1,160	333.433	3,950	968.566	1.5(10 ⁷)	1.828(10 ⁶)
38	20.080	345	119.083	1,170	335.906	4,000	979.344	2.0(10 ⁷)	2.398(10 ⁶)
39	20.488	350	120.526	1,175	337.142	4,050	990.108	2.5(10 ⁷)	2.961(10 ⁶)
40	20.894	355	121.966	1,180	338.376	4,100	1,000.858	3.0(10 ⁷)	3.517(10 ⁶)
41	21.298	360	123.403	1,190	340.843	4,150	1,011.595	4.0(10 ⁷)	4.610(10 ⁶)
42	21.701	365	124.838	1,200	343.308	4,200	1,022.318	5.0(10 ⁷)	5.689(10 ⁶)
43	22.101	370	126.270	1,210	345.770	4,250	1,033.028	6.0(10 ⁷)	6.758(10 ⁶)
44	22.500	375	127.699	1,220	348.230	4,300	1,043.724	7.0(10 ⁷)	7.816(10 ⁶)
45	22.897	380	129.126	1,225	349.460	4,350	1,054.409	8.0(10 ⁷)	8.866(10 ⁶)
46	23.291	385	130.550	1,230	350.688	4,400	1,065.082	9.0(10 ⁷)	9.911(10 ⁶)
47	23.684	390	131.972	1,240	353.144	4,450	1,075.743	1.0(10 ⁸)	1.095(10 ⁷)
48	24.076	395	133.391	1,250	355.597	4,500	1,086.390	1.5(10 ⁸)	1.604(10 ⁷)
49	24.466	400	134.808	1,260	358.048	4,550	1,097.024	2.0(10 ⁸)	2.108(10 ⁷)
50	24.855	405	136.223	1,270	360.496	4,600	1,107.646	2.5(10 ⁸)	2.607(10 ⁷)
51	25.244	410	137.635	1,275	361.720	4,650	1,118.257	3.0(10 ⁸)	3.100(10 ⁷)
52	25.633	415	139.045	1,280	362.942	4,700	1,128.854	4.0(10 ⁸)	4.071(10 ⁷)
53	26.020	420	140.453	1,290	365.386	4,750	1,139.439	5.0(10 ⁸)	5.03 (10 ⁷)
54	26.406	425	141.859	1,300	367.828	4,800	1,150.012	6.0(10 ⁸)	5.98 (10 ⁷)
55	26.791	430	143.262	1,310	370.267	4,850	1,160.574	7.0(10 ⁸)	6.928(10 ⁷)
56	27.174	435	144.664	1,320	372.704	4,900	1,171.125	8.0(10 ⁸)	7.865(10 ⁷)
57	27.555	440	146.064	1,325	373.922	4,950	1,181.666	9.0(10 ⁸)	8.797(10 ⁷)
58	27.935	445	147.461	1,330	375.139	5,000	1,192.198	1.0(10 ⁹)	9.725(10 ⁷)
59	28.314	450	148.856	1,340	377.572	5,100	1,213.222	1.5(10 ⁹)	1.429(10 ⁸)
60	28.691	455	150.249	1,350	380.003	5,200	1,234.203	2.0(10 ⁹)	1.880(10 ⁸)
61	29.068	460	151.460	1,360	382.432	5,300	1,255.141	2.5(10 ⁹)	2.328(10 ⁸)
62	29.443	465	153.029	1,370	384.859	5,400	1,276.037	3.0(10 ⁹)	2.771(10 ⁸)

Notes: For $t_D < 0.01$, $Q_{pD} = 2\sqrt{t_D/\pi}$.

$$\text{For } t_D \geq 200, Q_{pD} \approx \frac{-4.29881 + 2.02566 t_D}{\ln t_D}$$

TABLE C.4 – CONTINUED

Dimension- less Time t_D	Dimension- less Cumulative Production Q_{pD}	Dimension- less Time t_D	Dimension- less Cumulative Production Q_{pD}	Dimension- less Time t_D	Dimension- less Cumulative Production Q_{pD}	Dimension- less Time t_D	Dimension- less Cumulative Production Q_{pD}	Dimension- less Time t_D	Dimension- less Cumulative Production Q_{pD}
63	29.818	470	154.416	1,375	386.070	5,500	1,296.893	4.0(10 ⁹)	3.645(10 ⁸)
64	30.192	475	155.801	1,380	387.283	5,600	1,317.709	5.0(10 ⁹)	4.510(10 ⁸)
65	30.565	480	157.184	1,390	389.705	5,700	1,338.486	6.0(10 ⁹)	5.368(10 ⁸)
66	30.937	485	158.565	1,400	392.125	5,800	1,359.225	7.0(10 ⁹)	6.220(10 ⁸)
67	31.308	490	159.945	1,410	394.543	5,900	1,379.927	8.0(10 ⁹)	7.066(10 ⁸)
68	31.679	495	161.322	1,420	396.959	6,000	1,400.593	9.0(10 ⁹)	7.909(10 ⁸)
69	32.048	500	162.698	1,425	398.167	6,100	1,421.224	1.0(10 ¹⁰)	8.747(10 ⁸)
70	32.417	510	165.444	1,430	399.373	6,200	1,441.820	1.5(10 ¹⁰)	1.288(10 ⁹)
71	32.785	520	168.183	1,440	401.786	6,300	1,462.383	2.0(10 ¹⁰)	1.697(10 ⁹)
72	33.151	525	169.549	1,450	404.197	6,400	1,482.912	2.5(10 ¹⁰)	2.103(10 ⁹)
73	33.517	530	170.914	1,460	406.606	6,500	1,503.408	3.0(10 ¹⁰)	2.505(10 ⁹)
74	33.883	540	173.639	1,470	409.013	6,600	1,523.872	4.0(10 ¹⁰)	3.299(10 ⁹)
75	34.247	550	176.357	1,475	410.214	6,700	1,544.305	5.0(10 ¹⁰)	4.087(10 ⁹)
76	34.611	560	179.069	1,480	411.418	6,800	1,564.706	6.0(10 ¹⁰)	4.868(10 ⁹)
77	34.974	570	181.774	1,490	413.820	6,900	1,585.077	7.0(10 ¹⁰)	5.643(10 ⁹)
78	35.336	580	184.473	1,500	416.220	7,000	1,605.418	8.0(10 ¹⁰)	6.414(10 ⁹)
79	35.697	590	187.166	1,525	422.214	7,100	1,625.729	9.0(10 ¹⁰)	7.183(10 ⁹)
80	36.058	600	189.852	1,550	428.196	7,200	1,646.011	1.0(10 ¹¹)	7.948(10 ⁹)
81	36.418	610	192.533	1,575	434.168	7,300	1,666.265	1.5(10 ¹¹)	1.17(10 ¹⁰)
82	36.777	620	195.208	1,600	440.128	7,400	1,686.490	2.0(10 ¹¹)	1.55(10 ¹⁰)
83	37.136	625	196.544	1,625	446.077	7,500	1,706.688	2.5(10 ¹¹)	1.92(10 ¹⁰)
84	37.494	630	197.878	1,650	452.016	7,600	1,726.859	3.0(10 ¹¹)	2.29(10 ¹⁰)
85	37.851	640	200.542	1,675	457.945	7,700	1,747.002	4.0(10 ¹¹)	3.02(10 ¹⁰)
86	38.207	650	203.201	1,700	463.863	7,800	1,767.120	5.0(10 ¹¹)	3.75(10 ¹⁰)
87	38.563	660	205.854	1,725	469.771	7,900	1,787.212	6.0(10 ¹¹)	4.47(10 ¹⁰)
88	38.919	670	208.502	1,750	475.669	8,000	1,807.278	7.0(10 ¹¹)	5.19(10 ¹⁰)
89	39.272	675	209.825	1,775	481.558	8,100	1,827.319	8.0(10 ¹¹)	5.89(10 ¹⁰)
90	39.626	680	211.145	1,800	487.437	8,200	1,847.336	9.0(10 ¹¹)	6.58(10 ¹⁰)
91	39.979	690	213.784	1,825	493.307	8,300	1,867.329	1.0(10 ¹²)	7.28(10 ¹⁰)
92	40.331	700	216.417	1,850	499.167	8,400	1,887.298	1.5(10 ¹²)	1.08(10 ¹¹)
93	40.684	710	219.046	1,875	505.019	8,500	1,907.243	2.0(10 ¹²)	1.42(10 ¹¹)
94	41.034	720	221.670	1,900	510.861	8,600	1,927.166		
95	41.385	725	222.980	1,925	516.695	8,700	1,947.065		
96	41.735	730	224.289	1,950	522.520	8,800	1,966.942		
97	42.084								

t (days)	t_D	p_D	Source	p_{wf} (psia)
0.1	7.45×10^4	6.015	0.5 (ln t_D + 0.80907)	2,735
1.0	7.45×10^5	7.161	Table C.3	2,684
100.0	7.45×10^7	7.601	ln r_{eD}	2,665

Note that for $r_{eD}=2,000$, the reservoir is infinite acting at $t_D=7.45 \times 10^4$. This means that Table C.1 rather than Table C.3 is to be used to determine p_D ; however, for this t_D , Eq. C.3, which “extends” Table C.1, is appropriate. For $t_D=7.45 \times 10^5$, p_D is found in Table C.3; for $t_D=7.45 \times 10^7$, p_D is calculated from Eq. C.6. Note also that

$$\begin{aligned}
 p_{wf} &= p_i - \frac{qB\mu}{0.00708 kh} p_D \\
 &= 3,000 - \frac{(300)(1.25)(0.8)}{(0.00708)(80)(12)} p_D \\
 &= 3,000 - 44.14 p_D.
 \end{aligned}$$

Constant Pressure at Inner Boundary, No Flow Across Outer Boundary

This solution of the diffusivity equation models radial flow of a slightly compressible liquid in a homogeneous reservoir of uniform thickness; reservoir at uniform pressure p_i before production;

no flow across the outer boundary (at $r=r_e$); and constant BHP p_{wf} at the single producing well (centered in the reservoir) with wellbore radius r_w . The solution – pressure as a function of time and radius for fixed values of r_e , r_w , and rock and fluid properties – is expressed most conveniently in terms of dimensionless variables and parameters:

$$p_D = f(t_D, r_D, r_{eD}),$$

where

$$\begin{aligned}
 p_D &= \frac{p_i - p}{p_i - p_{wf}}, \\
 t_D &= \frac{0.000264 kt}{\phi \mu c_t r_w^2}, \\
 r_D &= r/r_w, \\
 r_{eD} &= r_e/r_w.
 \end{aligned}$$

For this problem, instantaneous rate q and cumulative production Q_p are of more practical importance than p_D , and these quantities can be derived from the fundamental solutions, p_D . In Appendix B, we showed that a dimensionless production rate, q_D , and dimensionless cumulative production, Q_{pD} , can be defined as

$$q_D = \frac{qB\mu}{0.00708 kh (p_i - p_{wf})},$$

and

$$Q_{pD} = \int_0^{t_D} q_D dt_D$$

$$= \frac{B}{1.119 \phi c_t h r_w^2 (p_i - p_{wf})} Q_p.$$

Dimensionless cumulative production, Q_{pD} , is presented in Tables C.4 and C.5. Table C.4 is for infinite-acting reservoirs, and Table C.5 is for finite reservoirs with $1.5 \leq r_{eD} \leq 1 \times 10^6$. For $r_{eD} \geq 20$, values for both q_D and Q_{pD} are given. In Table C.5, for values of t_D smaller than the smallest value for a given r_{eD} , the reservoir is infinite-acting and Table C.4 should be used. For values of t_D larger than those in the table for a given value of r_{eD} , the reservoir has reached steady state, and

$$Q_{pD} = (r_{eD}^2 - 1)/2.$$

For $r_{eD} = \infty$ and $t_D \geq 200$, q_D can be approximated as³

$$Q_{pD} = \frac{(-4.29881 + 2.02566 t_D)}{\ln t_D}.$$

Since both q_D and Q_{pD} are based on solutions (p_D) that, for $r_D = 1$ ($r = r_w$), depend only on t_D and r_{eD} , q_D and Q_{pD} also should depend only on t_D and r_{eD} . Table C.5 confirms this expectation. For given r_{eD} and t_D , Q_{pD} is determined uniquely.

Although the Q_{pD} solutions can be used to model individual well problems, they more often are used to model water influx from an aquifer of radius r_e into a petroleum reservoir of radius r_w for a fixed reservoir pressure, p_{wf} . Superposition is used to model a variable pressure history, as illustrated in Example C.4.

Example C.3 – Use of Q_{pD} Solutions

Problem. An oil well is produced with a constant

TABLE C.5 – Q_{pD} vs. t_D – FINITE RADIAL SYSTEM WITH CLOSED EXTERIOR BOUNDARY, CONSTANT PRESSURE AT INNER BOUNDARY

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD} = 3.0$		$r_{eD} = 3.5$		$r_{eD} = 4.0$	
t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}
0.05	0.276	0.05	0.278	0.10	0.408	0.30	0.755	1.00	1.571	2.00	2.442
0.06	0.304	0.075	0.345	0.15	0.509	0.40	0.895	1.20	1.761	2.20	2.598
0.07	0.330	0.10	0.404	0.20	0.599	0.50	1.023	1.40	1.940	2.40	2.748
0.08	0.354	0.125	0.458	0.25	0.681	0.60	1.143	1.60	2.111	2.60	2.893
0.09	0.375	0.150	0.507	0.30	0.758	0.70	1.256	1.80	2.273	2.80	3.034
0.10	0.395	0.175	0.553	0.35	0.829	0.80	1.363	2.00	2.427	3.00	3.170
0.11	0.414	0.200	0.597	0.40	0.897	0.90	1.465	2.20	2.574	3.25	3.334
0.12	0.431	0.225	0.638	0.45	0.962	1.00	1.563	2.40	2.715	3.50	3.493
0.13	0.446	0.250	0.678	0.50	1.024	1.25	1.791	2.60	2.849	3.75	3.645
0.14	0.461	0.275	0.715	0.55	1.083	1.50	1.997	2.80	2.976	4.00	3.792
0.15	0.474	0.300	0.751	0.60	1.140	1.75	2.184	3.00	3.098	4.25	3.932
0.16	0.486	0.325	0.785	0.65	1.195	2.00	2.353	3.25	3.242	4.50	4.068
0.17	0.497	0.350	0.817	0.70	1.248	2.25	2.507	3.50	3.379	4.75	4.198
0.18	0.507	0.375	0.848	0.75	1.299	2.50	2.646	3.75	3.507	5.00	4.323
0.19	0.517	0.400	0.877	0.80	1.348	2.75	2.772	4.00	3.628	5.50	4.560
0.20	0.525	0.425	0.905	0.85	1.395	3.00	2.886	4.25	3.742	6.00	4.779
0.21	0.533	0.450	0.932	0.90	1.440	3.25	2.990	4.50	3.850	6.50	4.982
0.22	0.541	0.475	0.958	0.95	1.484	3.50	3.084	4.75	3.951	7.00	5.169
0.23	0.548	0.500	0.983	1.0	1.526	3.75	3.170	5.00	4.047	7.50	5.343
0.24	0.554	0.550	1.028	1.1	1.605	4.00	3.247	5.50	4.222	8.00	5.504
0.25	0.559	0.600	1.070	1.2	1.679	4.25	3.317	6.00	4.378	8.50	5.653
0.26	0.565	0.650	1.108	1.3	1.747	4.50	3.381	6.50	4.516	9.00	5.790
0.28	0.574	0.700	1.143	1.4	1.811	4.75	3.439	7.00	4.639	9.50	5.917
0.30	0.582	0.750	1.174	1.5	1.870	5.00	3.491	7.50	4.749	10	6.035
0.32	0.588	0.800	1.203	1.6	1.924	5.50	3.581	8.00	4.846	11	6.246
0.34	0.594	0.900	1.253	1.7	1.975	6.00	3.656	8.50	4.932	12	6.425
0.36	0.599	1.000	1.295	1.8	2.022	6.50	3.717	9.00	5.009	13	6.580
0.38	0.603	1.1	1.330	2.0	2.106	7.00	3.767	9.50	5.078	14	6.712
0.40	0.606	1.2	1.358	2.2	2.178	7.50	3.809	10.00	5.138	15	6.825
0.45	0.613	1.3	1.382	2.4	2.241	8.00	3.843	11	5.241	16	6.922
0.50	0.617	1.4	1.402	2.6	2.294	9.00	3.894	12	5.321	17	7.004
0.60	0.621	1.6	1.432	2.8	2.340	10.00	3.928	13	5.385	18	7.076
0.70	0.623	1.7	1.444	3.0	2.380	11.00	3.951	14	5.435	20	7.189
0.80	0.624	1.8	1.453	3.4	2.444	12.00	3.967	15	5.476	22	7.272
		2.0	1.468	3.8	2.491	14.00	3.985	16	5.506	24	7.332
		2.5	1.487	4.2	2.525	16.00	3.993	17	5.531	26	7.377
		3.0	1.495	4.6	2.551	18.00	3.997	18	5.551	30	7.434
		4.0	1.499	5.0	2.570	20.00	3.999	20	5.579	34	7.464
		5.0	1.500	6.0	2.599	22.00	3.999	25	5.611	38	7.481
				7.0	2.613	24.00	4.000	30	5.621	42	7.490
				8.0	2.619			35	5.624	46	7.494
				9.0	2.622			40	5.625	50	7.497
				10.0	2.624						

BHP of 2,000 psia for 1.0 hour from a reservoir initially at 2,500 psia. The reservoir is finite; there is no flow across the outer boundary. Other rock, fluid, and well properties include the following.

$$\begin{aligned} B &= 1.2 \text{ RB/STB}, \\ \mu &= 1 \text{ cp}, \\ k &= 0.294 \text{ md}, \\ r_w &= 0.5 \text{ ft}, \\ h &= 15 \text{ ft}, \\ \phi &= 0.15, \\ c_t &= 20 \times 10^{-6} \text{ psi}^{-1}, \text{ and} \\ r_e &= 1,000 \text{ ft}. \end{aligned}$$

Calculate the cumulative production, Q_p , in barrels.

Solution. We will calculate t_D and r_{eD} and read Q_{pD} from either Table C.4 or Table C.5.

$$\begin{aligned} t_D &= \frac{0.000264 kt}{\phi \mu c_t r_w^2} \\ &= \frac{(0.000264)(0.294)(1)}{(0.15)(1)(20 \times 10^{-6})(0.5)^2} = 103, \end{aligned}$$

$$r_{eD} = \frac{1,000}{0.5} = 2,000.$$

There is no entry in Table C.5 at this t_D . Thus, the reservoir is infinite acting, and from Table C.4, $Q_{pD} = 44.3$. Then,

$$\begin{aligned} Q_p &= 1.119 \phi c_t h r_w^2 (p_i - p_{wf}) Q_{pD} / B \\ &= (1.119)(0.15)(20 \times 10^{-6})(15)(0.5)^2 \\ &\quad \cdot (2,500 - 2,000)(44.3) / 1.2 \\ &= 0.233 \text{ STB}. \end{aligned}$$

Example C.4 – Analysis of Variable Pressure History With Q_{pD} Solution

Problem. A well is completed in a reservoir with an initial pressure of 6,000 psi. The well can be considered centered in the cylindrical reservoir; there is no flow across the outer boundary. Reservoir, fluid, and well properties include the following.

TABLE C.5 – CONTINUED

$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6.0$		$r_{eD} = 7.0$		$r_{eD} = 8.0$		$r_{eD} = 9.0$		$r_{eD} = 10.0$	
t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}	t_D	Q_{pD}
2.5	2.835	3.0	3.195	6.0	5.148	9.00	6.861	9	6.861	10	7.417	15	9.965
3.0	3.196	3.5	3.542	6.5	5.440	9.50	7.127	10	7.398	15	9.945	20	12.32
3.5	3.537	4.0	3.875	7.0	5.724	10	7.389	11	7.920	20	12.26	22	13.22
4.0	3.859	4.5	4.193	7.5	6.002	11	7.902	12	8.431	22	13.13	24	14.09
4.5	4.165	5.0	4.499	8.0	6.273	12	8.397	13	8.930	24	13.98	26	14.95
5.0	4.454	5.5	4.792	8.5	6.537	13	8.876	14	9.418	26	14.79	28	15.78
5.5	4.727	6.0	5.074	9.0	6.795	14	9.341	15	9.895	28	15.59	30	16.59
6.0	4.986	6.5	5.345	9.5	7.047	15	9.791	16	10.361	30	16.35	32	17.38
6.5	5.231	7.0	5.605	10.0	7.293	16	10.23	17	10.82	32	17.10	34	18.16
7.0	5.464	7.5	5.854	10.5	7.533	17	10.65	18	11.26	34	17.82	36	18.91
7.5	5.684	8.0	6.094	11	7.767	18	11.06	19	11.70	36	18.52	38	19.65
8.0	5.892	8.5	6.325	12	8.220	19	11.46	20	12.13	38	19.19	40	20.37
8.5	6.089	9.0	6.547	13	8.651	20	11.85	22	12.95	40	19.85	42	21.07
9.0	6.276	9.5	6.760	14	9.063	22	12.58	24	13.74	42	20.48	44	21.76
9.5	6.453	10	6.965	15	9.456	24	13.27	26	14.50	44	21.09	46	22.42
10	6.621	11	7.350	16	9.829	26	13.92	28	15.23	46	21.69	48	23.07
11	6.930	12	7.706	17	10.19	28	14.53	30	15.92	48	22.26	50	23.71
12	7.208	13	8.035	18	10.53	30	15.11	34	17.22	50	22.82	52	24.33
13	7.457	14	8.339	19	10.85	35	16.39	38	18.41	52	23.36	54	24.94
14	7.680	15	8.620	20	11.16	40	17.49	40	18.97	54	23.89	56	25.53
15	7.880	16	8.879	22	11.74	45	18.43	45	20.26	56	24.39	58	26.11
16	8.060	18	9.338	24	12.26	50	19.24	50	21.42	58	24.88	60	26.67
18	8.365	20	9.731	25	12.50	60	20.51	55	22.46	60	25.36	65	28.02
20	8.611	22	10.07	31	13.74	70	21.45	60	23.40	65	26.48	70	29.29
22	8.809	24	10.35	35	14.40	80	22.13	70	24.98	70	27.52	75	30.49
24	8.968	26	10.59	39	14.93	90	22.63	80	26.26	75	28.48	80	31.61
26	9.097	28	10.80	51	16.05	100	23.00	90	27.28	80	29.36	85	32.67
28	9.200	30	10.98	60	16.56	120	23.47	100	28.11	85	30.18	90	33.66
30	9.283	34	11.26	70	16.91	140	23.71	120	29.31	90	30.93	95	34.60
34	9.404	38	11.46	80	17.41	160	23.85	140	30.08	95	31.63	100	35.48
38	9.481	42	11.61	90	17.27	180	23.92	160	30.58	100	32.27	120	38.51
42	9.532	46	11.71	100	17.36	200	23.96	180	30.91	120	34.39	140	40.89
46	9.565	50	11.79	110	17.41	500	24.00	200	31.12	140	35.92	160	42.75
50	9.586	60	11.91	120	17.45			240	31.34	160	37.04	180	44.21
60	9.612	70	11.96	130	17.46			280	31.43	180	37.85	200	45.36
70	9.621	80	11.98	140	17.48			320	31.47	200	38.44	240	46.95
80	9.623	90	11.99	150	17.49			360	31.49	240	39.17	280	47.94
90	9.624	100	12.00	160	17.49			400	31.50	280	39.56	320	48.54
100	9.625	120	12.00	180	17.50			500	31.50	320	39.77	360	48.91
				200	17.50					360	39.88	400	49.14
				220	17.50					400	39.94	440	49.28
										440	39.97	480	49.36
										480	39.98		

$$\begin{aligned}
 k &= 31.6 \text{ md}, \\
 \phi &= 0.21, \\
 r_w &= 0.33 \text{ ft}, \\
 r_e &= 3,300 \text{ ft}, \\
 B_o &= 1.25 \text{ RB/STB}, \\
 h &= 20 \text{ ft}, \\
 \mu_o &= 0.8 \text{ cp, and} \\
 c_t &= 20 \times 10^{-6} \text{ psi}^{-1}.
 \end{aligned}$$

The well produced for 6 months with flowing BHP p_{wf} of 5,500 psi, for 6 more months with $p_{wf} = 4,500$ psi, and for 6 more months with $p_{wf} = 5,000$ psi. Calculate cumulative production after 18 months of production from this well.

Solution. Superposition is required to solve this problem. We can calculate the cumulative production by adding the cumulative production

from three wells, each beginning to produce when p_{wf} is changed and each producing with pressure drawdown equal to the difference in pressures before and after the change:

Well 1 produces for $(18 - 0) = 18$ months
with $(p_i - p_{wf1}) = 6,000 - 5,500 = 500$ psi.

Well 2 produces for $(18 - 6) = 12$ months
with $(p_{wf1} - p_{wf2}) = 5,500 - 4,500 = 1,000$ psi.

Well 3 produces for $(18 - 12) = 6$ months
with $(p_{wf2} - p_{wf3}) = 4,500 - 5,000 = -500$ psi.

For this well, $r_{eD} = 3,300/0.33 = 10,000$.

$$t_D = \frac{0.000264 kt}{\phi \mu c_t r_w^2}$$

TABLE C-5 – (CONTINUED)
 q_D and Q_{pD} vs. t_D – FINITE RADIAL SYSTEM WITH CLOSED EXTERIOR BOUNDARY –
CONSTANT PRESSURE AT INNER BOUNDARY

$r_{eD} = 20$			$r_{eD} = 50$			$r_{eD} = 100$		
t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}
100	0.3394	42.91	600	0.2652	189.0	2,000	0.2304	532
130	0.3174	52.76	800	0.2915	241	3,000	0.2179	757
160	0.2975	61.98	1,000	0.2393	290	4,000	0.2070	969
200	0.2728	73.38	1,300	0.2220	359	5,000	0.1967	1,171
240	0.2502	83.83	1,600	0.2060	473	6,000	0.1869	1,363
300	0.2197	97.91	2,000	0.1865	502	8,000	0.1686	1,718
400	0.1770	117.7	2,400	0.1682	573	1×10^4	0.1536	2,000
500	0.1426	133.6	3,000	0.1543	667	1.3×10^4	0.1304	2.46×10^3
600	0.1148	146.4	4,000	0.1133	795	1.6×10^4	0.1118	2.82×10^3
700	0.0925	156.7	5,000	0.0833	895	2×10^4	0.0910	3.23×10^3
800	0.0745	165.1	6,000	0.0682	974	2.4×10^4	0.0741	3.56×10^3
1,000	0.0483	177.1	8,000	0.0418	1,082	3×10^4	0.0645	3.94×10^3
1,300	0.0483	187.8	1×10^4	0.0254	1,148	4×10^4	0.0326	4.37×10^3
1,600	0.0132	193.4	1.3×10^4	0.0120	1,201	5×10^4	0.0195	4.62×10^3
2,000	0.0056	196.9	1.6×10^4	0.0056	1,227	6×10^4	0.0117	4.77×10^3
3,000	0.0006	199.2	2×10^4	0.0021	1,241	8×10^4	0.0042	4.92×10^3
			2.4×10^4	0.0006	1,246	1×10^5	0.0015	4.97×10^3
			3×10^4	0.0002	1,249	1.1×10^5	0.0009	4.98×10^3

$r_{eD} = 200$			$r_{eD} = 500$			$r_{eD} = 1,000$		
t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}
1×10^4	0.1943	2.19×10^3	1×10^5	0.1566	1.75×10^4	3×10^4	0.1773	5.89×10^3
1.3×10^4	0.1860	2.77×10^3	1.3×10^5	0.1498	2.21×10^4	4×10^4	0.1729	7.64×10^3
1.6×10^4	0.1820	3.33×10^3	1.6×10^5	0.1435	2.65×10^4	5×10^4	0.1697	9.35×10^3
2×10^4	0.1742	4.04×10^3	2×10^5	0.1354	3.21×10^4	1×10^5	0.1604	1.76×10^4
2.4×10^4	0.1668	4.72×10^3	2.4×10^5	0.1277	3.73×10^4	2×10^5	0.1518	3.32×10^4
3×10^4	0.1562	5.69×10^3	3×10^5	0.1170	4.47×10^4	3×10^5	0.1464	4.80×10^4
4×10^4	0.1401	7.17×10^3	4×10^5	0.1012	5.56×10^4	4×10^5	0.1416	6.24×10^4
5×10^4	0.1236	8.50×10^3	5×10^5	0.0875	6.50×10^4	5×10^5	0.1371	7.64×10^4
6×10^4	0.1126	9.68×10^3	6×10^5	0.0756	7.31×10^4	6×10^5	0.1327	8.98×10^4
8×10^4	0.0905	1.17×10^4	8×10^5	0.0565	8.62×10^4	7×10^5	0.1285	1.03×10^5
1×10^5	0.0728	1.33×10^4	1×10^6	0.0422	9.60×10^4	8×10^5	0.1244	1.16×10^5
1.3×10^5	0.0524	1.52×10^4	1.3×10^6	0.0273	1.06×10^5	9×10^5	0.1204	1.23×10^5
1.6×10^5	0.0378	1.65×10^4	1.6×10^6	0.0176	1.13×10^5	1×10^6	0.1166	1.40×10^5
2×10^5	0.0244	1.78×10^4	2×10^6	0.0098	1.18×10^5	1.4×10^6	0.1024	1.83×10^5
2.4×10^5	0.0138	1.86×10^4	2.4×10^6	0.0055	1.21×10^5	2×10^6	0.0844	2.39×10^5
3×10^5	0.0082	1.92×10^4	3×10^6	0.0023	1.23×10^5	2.4×10^6	0.0741	2.71×10^5
4×10^5	0.0028	1.97×10^4	4×10^6	0.0005	1.25×10^5	3×10^6	0.0610	3.11×10^5
5×10^5	0.0009	1.99×10^4	5×10^6	0.0001	1.25×10^5	4×10^6	0.0442	3.63×10^5
						5×10^6	0.0320	4.01×10^5
						7×10^6	0.0167	4.48×10^5
						8.4×10^6	0.0106	4.67×10^5
						1×10^7	0.0063	4.80×10^5
						1.4×10^7	0.0017	4.95×10^5
						2×10^7	0.0004	4.99×10^5
						3×10^7	0.0000	5.00×10^5

Notes: 1. For t_D smaller than values listed in this table for a given r_{eD} , reservoir is infinite acting. Find Q_{pD} in Table C.4.

2. For t_D larger than values listed in this table, $Q_{pD} \approx (r_{eD})^2 - 1/2$.

3. For t_D larger than values listed in this table, $q_D \approx 0.0$.

$$= \frac{(0.000264)(31.6)(730 \text{ hr/month})(t \text{ months})}{(0.21)(0.8)(2 \times 10^{-5})(0.33)^2}$$

$$= 1.664 \times 10^7 (t \text{ months}).$$

For Well 1, $t_D = (1.664 \times 10^7)(18) = 3.0 \times 10^8$.

From Table C.5,

$$Q_{pD} = 2.54 \times 10^7,$$

$$Q_p = 1.119 \phi c_t h r_w^2 (p_i - p_{wf}) Q_{pD} / B$$

$$= (1.119)(0.21)(2 \times 10^{-5})(20)(0.33)^2$$

$$\cdot (p_i - p_{wf}) Q_{pD} / 1.25$$

$$= 8.19 \times 10^{-6} (p_i - p_{wf}) Q_{pD}$$

$$= (8.19 \times 10^{-6})(500)(2.54 \times 10^7)$$

$$= 1.04 \times 10^5 \text{ STB.}$$

For Well 2, $t_D = (1.664 \times 10^7)(12) = 2.0 \times 10^8$.

$$Q_{pD} = 1.89 \times 10^7,$$

$$Q_p = (8.19 \times 10^{-6})(1,000)(1.89 \times 10^7)$$

$$= 1.55 \times 10^5 \text{ STB.}$$

For Well 3, $t_D = (1.664 \times 10^7)(6) = 1.0 \times 10^8$.

$$Q_{pD} = 1.06 \times 10^7,$$

$$Q_p = (8.19 \times 10^{-6})(-500)(1.06 \times 10^7)$$

$$= -0.434 \times 10^5.$$

Then

$$Q_p = Q_{p1} + Q_{p2} + Q_{p3}$$

$$= 1.04 \times 10^5 + 1.55 \times 10^5 - 0.434 \times 10^5$$

$$= 2.16 \times 10^5 \text{ STB.}$$

Exercises

C.1 An oil well is producing from a reservoir ($r_e = 900$ ft) with no fluid crossing the outer boundary of the reservoir. Initial reservoir pressure is 3,000 psia. Pressure in the wellbore is maintained at 2,000 psia. Well and reservoir properties include the following:

$$\phi = 0.21,$$

$$k = 80 \text{ md},$$

$$c_t = 20 \times 10^{-6} \text{ psi}^{-1},$$

$$r_w = 0.45 \text{ ft},$$

$$\mu = 0.8 \text{ cp},$$

$$h = 12 \text{ ft, and}$$

$$B = 1.2 \text{ RB/STB.}$$

Calculate instantaneous rate and cumulative production after 1.0 day of production.

TABLE C.5 – (CONTINUED)

$r_{eD} = 2,000$			$r_{eD} = 4,000$			$r_{eD} = 1.0 (10^4)$		
t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}
1×10^5	0.1604		9×10^5	0.1366		3×10^6	0.1263	4.06×10^5
2×10^5	0.1520		1×10^6	0.1356		4×10^6	0.1240	5.31×10^5
3×10^5	0.1475		1.3×10^6	0.1333		5×10^6	0.1222	6.76×10^5
4×10^5	0.1445	6.27×10^4	1.6×10^6	0.1315	2.26×10^5	6×10^6	0.1210	7.76×10^5
5×10^5	0.1422	7.70×10^4	2×10^6	0.1296	2.78×10^5	8×10^6	0.1188	1.02×10^6
6×10^5	0.1404	9.11×10^4	2.4×10^6	0.1280	3.30×10^5	1×10^7	0.1174	1.25×10^6
7×10^5	0.1389	1.05×10^5	3×10^6	0.1262	4.06×10^5	1.2×10^7	0.1162	1.49×10^6
8×10^5	0.1375	1.19×10^5	4×10^6	0.1237	5.31×10^5	1.4×10^7	0.1152	1.72×10^6
9×10^5	0.1363	1.33×10^5	5×10^6	0.1215	6.54×10^5	1.6×10^7	0.1143	1.95×10^6
1×10^6	0.1352	1.46×10^5	6×10^6	0.1194	7.74×10^5	1.8×10^7	0.1135	2.17×10^6
1.3×10^6	0.1320	1.86×10^5	8×10^6	0.1155	1.01×10^6	2×10^7	0.1128	2.40×10^6
1.6×10^6	0.1291	2.25×10^5	1×10^7	0.1118	1.24×10^6	2.4×10^7	0.1115	2.85×10^6
2×10^6	0.1254	2.76×10^5	1.2×10^7	0.1081	1.46×10^6	3×10^7	0.1098	3.51×10^6
2.4×10^6	0.1216	3.26×10^5	1.4×10^7	0.1046	1.67×10^6	4×10^7	0.1071	4.60×10^6
3×10^6	0.1166	3.97×10^5	1.6×10^7	0.1012	1.87×10^6	5×10^7	0.1050	5.66×10^6
4×10^6	0.1084	5.10×10^5	1.8×10^7	0.0979	2.07×10^6	7×10^7	0.0998	7.70×10^6
5×10^6	0.1008	6.14×10^5	2×10^7	0.0948	2.27×10^6	8×10^7	0.0975	8.69×10^6
7×10^6	0.0872	8.02×10^5	2.3×10^7	0.0902	2.54×10^6	9×10^7	0.0952	9.65×10^6
1×10^7	0.0701	1.04×10^6	3×10^7	0.0803	3.14×10^6	1×10^8	0.0930	1.06×10^7
1.3×10^7	0.0563	1.23×10^6	4×10^7	0.0681	3.88×10^6	1.2×10^8	0.0887	1.24×10^7
1.7×10^7	0.0421	1.42×10^6	5×10^7	0.0577	4.51×10^6	1.4×10^8	0.0846	1.41×10^7
2×10^7	0.0339	1.53×10^6	7×10^7	0.0415	5.49×10^6	1.7×10^8	0.0788	1.66×10^7
2.4×10^7	0.0253	1.65×10^6	8×10^7	0.0352	5.87×10^6	2×10^8	0.0734	1.89×10^7
3×10^7	0.0164	1.78×10^6	9×10^7	0.0298	6.20×10^6	2.4×10^8	0.0668	2.17×10^7
4×10^7	0.0079	1.89×10^6	1×10^8	0.0252	6.47×10^6	3×10^8	0.0580	2.54×10^7
5×10^7	0.0038	1.95×10^6	1.2×10^8	0.0181	6.90×10^6	4×10^8	0.0458	3.06×10^7
7×10^7	0.0009	1.99×10^6	1.4×10^8	0.0130	7.21×10^6	5×10^8	0.0362	3.47×10^7
1×10^8	0.0001	2.00×10^6	1.7×10^8	0.0079	7.52×10^6	6×10^8	0.0286	3.79×10^7
1.3×10^8	0.0000	2.00×10^6	2×10^8	0.0048	7.71×10^6	7×10^8	0.0226	4.04×10^7
			2.3×10^8	0.0029	7.82×10^6	8×10^8	0.0178	4.24×10^7
			2.6×10^8	0.0018	7.89×10^6	1×10^9	0.0111	4.53×10^7
			3×10^8	0.0009	7.94×10^6	1.4×10^9	0.0043	4.82×10^7
			4×10^8	0.0002	7.99×10^6	2×10^9	0.0011	4.96×10^7
			5×10^8	0.0000	8.00×10^6	3×10^9	0.0001	5.00×10^7

Notes: 1. For t_D smaller than values listed in this table for a given r_{eD} , reservoir is infinite acting. Find Q_{pD} in Table C.4.

2. For t_D larger than values listed in this table, $Q_{pD} \approx (r_{eD})^2 - 1/2$.

3. For t_D larger than values listed in this table, $q_D \approx 0$.

C.2 A single oil well is producing in the center of a circular, full water-drive reservoir. The pressure at the oil/water contact is constant at 3,340 psia and the radial distance to the oil/water contact is 1,500 ft. The well produced for the first 15 days at a rate of 500 STB/D, the next 14 days at 300 STB/D, and the last day at 200 STB/D. Calculate the cumulative production and wellbore pressure at the end of 30 days.

$$\begin{aligned}\phi &= 0.20, \\ k &= 75 \text{ md}, \\ c_t &= 17.5 \times 10^{-6} \text{ psi}^{-1}, \\ r_w &= 0.5 \text{ ft}, \\ \mu &= 0.75 \text{ cp}, \\ h &= 15 \text{ ft, and} \\ B &= 1.2 \text{ RB/STB}.\end{aligned}$$

C.3 An oil reservoir is initially at a pressure, p_i , of 2,734 psia. If the boundary pressure is suddenly lowered to 2,724 psia and held there, calculate the cumulative water influx into the reservoir after 100, 200, 400, and 800 days. Reservoir and aquifer properties include the following.

$$\begin{aligned}\phi &= 0.2, \\ k &= 83 \text{ md}, \\ c_t &= 8 \times 10^{-6} \text{ psi}^{-1}, \\ r_w &= 3000 \text{ ft (reservoir radius)}, \\ r_e &= 30,000 \text{ ft (aquifer radius)}, \\ \mu &= 0.62 \text{ cp, and} \\ h &= 40 \text{ ft}.\end{aligned}$$

C.4 If, in Exercise C.3, the reservoir boundary pressure suddenly dropped to 2,704 psia at the end of 100 days, calculate the total water influx at 400 days total elapsed time.

References

1. van Everdingen, A.F. and Hurst, W.: "The Application of the Laplace Transformation to Flow Problems in Reservoirs," *Trans., AIME* (1949) **186**, 305-324.
2. Chatas, A.T.: "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems," *Pet. Eng.* (Aug. 1953) B-44 through B-56.
3. Edwardson, M.J. *et al.*: "Calculation of Formation Temperature Disturbances Caused by Mud Circulation," *J. Pet. Tech.* (April 1962) 416-426; *Trans., AIME*, **225**.

TABLE C.5 – (CONTINUED)

$r_{eD} = 2.5(10^4)$			$r_{eD} = 1.0(10^5)$			$r_{eD} = 2.5(10^5)$			$r_{eD} = 1.0(10^6)$		
t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}	t_D	q_D	Q_{pD}
3×10^7	0.1103		1.4×10^8	0.1017		2×10^9	0.0897		2×10^{10}	0.0813	
4×10^7	0.1086		2×10^8	0.1000		3×10^9	0.0881		3×10^{10}	0.0800	
6×10^7	0.1064	6.77×10^6	2.4×10^8	0.0990		4×10^9	0.0870		4×10^{10}	0.0791	
7×10^7	0.1054	7.83×10^6	3×10^8	0.0980	3.10×10^7	5×10^9	0.0861	4.51×10^8	6×10^{10}	0.0778	
8×10^7	0.1047	8.86×10^6	3.5×10^8	0.0971	3.59×10^7	6×10^9	0.0854	5.37×10^8	8×10^{10}	0.0770	
9×10^7	0.1041	9.93×10^6	4×10^8	0.0966	4.07×10^7	7×10^9	0.0849	6.22×10^8	1×10^{11}	0.0763	7.95×10^{10}
1×10^8	0.1035	1.10×10^7	5×10^8	0.0956	5.03×10^7	8×10^9	0.0844	7.07×10^8	1.3×10^{11}	0.0756	1.02×10^{10}
1.4×10^8	0.1016	1.51×10^7	6×10^8	0.0948	5.98×10^7	1×10^{10}	0.0836	8.75×10^8	1.6×10^{11}	0.0750	1.25×10^{10}
2×10^8	0.0993	2.11×10^7	7×10^8	0.0941	6.93×10^7	1.4×10^{10}	0.0824	1.21×10^9	2×10^{11}	0.0743	1.55×10^{10}
2.6×10^8	0.0973	2.70×10^7	8×10^8	0.0935	7.87×10^7	2×10^{10}	0.0809	1.70×10^9	2.4×10^{11}	0.0737	1.84×10^{10}
3×10^8	0.0960	3.09×10^7	8.4×10^8	0.0933	8.24×10^7	3×10^{10}	0.0787	2.49×10^9	3×10^{11}	0.0730	2.28×10^{10}
3.3×10^8	0.0950	3.37×10^7	9×10^8	0.0930	8.80×10^7	4×10^{10}	0.0766	3.27×10^9	4×10^{11}	0.0719	3.21×10^{10}
3.6×10^8	0.0940	3.66×10^7	1×10^9	0.0925	9.73×10^7	4.4×10^{10}	0.0757	3.58×10^9	5×10^{11}	0.0709	3.72×10^{10}
4×10^8	0.0927	4.03×10^7	1.4×10^9	0.0911	1.34×10^8	4.7×10^{10}	0.0751	3.80×10^9	6×10^{11}	0.0697	4.42×10^{10}
4.4×10^8	0.0915	4.40×10^7	2×10^9	0.0896	1.80×10^8	5×10^{10}	0.0745	4.03×10^9	7×10^{11}	0.0686	5.11×10^{10}
5×10^8	0.0896	4.94×10^7	3×10^9	0.0877	2.77×10^8	5.4×10^{10}	0.0737	4.32×10^9	8×10^{11}	0.0676	6.32×10^{10}
5.4×10^8	0.0804	5.30×10^7	4×10^9	0.0861	3.64×10^8	6×10^{10}	0.0725	4.76×10^9	1×10^{12}	0.0656	7.13×10^{10}
6×10^8	0.0866	5.82×10^7	5×10^9	0.0845	4.49×10^8	7×10^{10}	0.0705	5.48×10^9	1.2×10^{12}	0.0636	8.42×10^{10}
6.4×10^8	0.0855	6.17×10^7	6×10^9	0.0829	5.33×10^8	7.4×10^{10}	0.0698	5.76×10^9	1.4×10^{12}	0.0617	9.67×10^{10}
7×10^8	0.0837	6.67×10^7	7×10^9	0.0814	6.15×10^8	8×10^{10}	0.0686	6.17×10^9	1.5×10^{12}	0.0607	1.03×10^{12}
7.4×10^8	0.0826	7.01×10^7	8×10^9	0.0799	6.95×10^8	8.4×10^{10}	0.0679	6.42×10^9	> 1.5×10^{12} not determined.		
8×10^8	0.0809	7.50×10^7	9×10^9	0.0784	7.75×10^8	9×10^{10}	0.0668	6.85×10^9			
8.4×10^8	0.0798	7.82×10^7	1×10^{10}	0.0770	8.52×10^8	1×10^{11}	0.0658	7.51×10^9	> 1.5×10^{12} not determined.		
9×10^8	0.0782	8.29×10^7	1.3×10^{10}	0.0728	1.08×10^9	1.1×10^{11}	0.0632	8.15×10^9			
1×10^9	0.0756	9.06×10^7	1.6×10^{10}	0.0689	1.29×10^9	1.3×10^{11}	0.0593	9.38×10^9	> 1.5×10^{12} not determined.		
1.3×10^9	0.0683	1.12×10^8	2×10^{10}	0.0639	1.56×10^9	1.6×10^{11}	0.0551	1.11×10^{10}			
1.6×10^9	0.0616	1.32×10^8	2.4×10^{10}	0.0594	1.80×10^9	2×10^{11}	0.0494	1.32×10^{10}	> 1.5×10^{12} not determined.		
2×10^9	0.0538	1.55×10^8	3×10^{10}	0.0531	2.14×10^9	2.4×10^{11}	0.0443	1.51×10^{10}			
2.4×10^9	0.0469	1.75×10^8	4×10^{10}	0.0441	2.62×10^9	3×10^{11}	0.0376	1.75×10^{10}	> 1.5×10^{12} not determined.		
3×10^9	0.0382	2.00×10^8	5×10^{10}	0.0366	3.03×10^9	4×10^{11}	0.0286	2.08×10^{10}			
4×10^9	0.0272	2.33×10^8	6×10^{10}	0.0304	3.36×10^9	5×10^{11}	0.0217	2.33×10^{10}	> 1.5×10^{12} not determined.		
5×10^9	0.0193	2.56×10^8	7×10^{10}	0.0253	3.64×10^9	7×10^{11}	0.0126	2.67×10^{10}			
6×10^9	0.0138	2.72×10^8	8×10^{10}	0.0210	3.87×10^9	1×10^{12}	0.0055	2.92×10^{10}	> 1.5×10^{12} not determined.		
8×10^9	0.0070	2.92×10^8	9×10^{10}	0.0174	4.06×10^9	1.3×10^{12}	0.0024	3.04×10^{10}			
1×10^{10}	0.0035	3.02×10^8	1×10^{11}	0.0145	4.22×10^9	1.6×10^{12}	0.0011	3.09×10^{10}	> 1.5×10^{12} not determined.		
1.4×10^{10}	0.0009	3.10×10^8	1.3×10^{11}	0.0083	4.55×10^9	2×10^{12}	0.0004	3.11×10^{10}			
2×10^{10}	0.0001	3.12×10^8	1.6×10^{11}	0.0048	4.74×10^9				> 1.5×10^{12} not determined.		
3×10^{10}	0.0000	3.12×10^8	2×10^{11}	0.0023	4.88×10^9						
			2.4×10^{11}	0.0011	4.94×10^9				> 1.5×10^{12} not determined.		
			3×10^{11}	0.0004	4.98×10^9						
			3.4×10^{11}	0.0002	4.99×10^9				> 1.5×10^{12} not determined.		
			4×10^{11}	0.0001	5.00×10^9						

Notes: 1. For t_D smaller than values listed in this table for a given r_{eD} , reservoir is infinite acting. Find Q_{pD} in Table C.4.

2. For t_D larger than values listed in this table, $Q_{pD} \approx (r_{eD})^2 - 1/2$.

3. For t_D larger than values listed in this table, $q_D \approx 0.0$.

Appendix D

Rock and Fluid Property Correlations

Introduction

Pressure transient test analysis requires knowledge of reservoir fluid properties such as viscosities, compressibilities, and formation volume factors. In addition, formation compressibility is a rock property frequently required for test analysis. For most of these properties, laboratory analysis provides the most accurate answer; however, in many cases, laboratory results are not available, and the test analyst must use empirical correlations of experimental data.

This appendix provides a summary of correlations that have proved useful for test analysis. These correlations are selected from those presented by Earlougher;¹ his collection of correlations is probably the best and the most complete in print at the time of this writing. We assume that the reader of this text has completed a study of the fundamentals of reservoir fluid properties. Such studies provide detail on the meaning, origin, and applicability of these correlations. Readers not familiar with this basis are referred to texts by Amyx *et al.*² and McCain.³ In this appendix, we simply reproduce figures from Earlougher's collection and illustrate the use of each with an example.

Pseudocritical Temperature and Pressure of Liquid Hydrocarbons

Pseudocritical temperature (T_{pc}) and pressure (p_{pc}) of undersaturated crude oils can be estimated from an approximate correlation developed by Trube⁴ and presented in Fig. D-1. These pseudocritical properties are required to estimate compressibility of an undersaturated crude oil, as illustrated by Example D.5.

To use Trube's correlation, one must know the specific gravity of the undersaturated reservoir liquid corrected to 60°F. Example D.1 illustrates use of the correlation.

Example D.1 – Estimation of Pseudocritical Temperature and Pressure for Undersaturated Crude Oil

Problem: Estimate the pseudocritical temperature,

T_{pc} , and pressure, p_{pc} , of an undersaturated crude oil with gravity of 30°API (specific gravity = 0.876 at 60°F).

Solution: From Fig. D-1, $T_{pc} = 1160^{\circ}\text{R}$ and $p_{pc} = 285$ psia.

Bubble-Point Pressure of Crude Oil

The test analyst may need to estimate the saturation or bubble-point pressure of a crude oil in some circumstances – e.g., to determine whether a reservoir is saturated or undersaturated at a certain pressure. Standing's correlation⁵ is useful for this estimate; Fig. D-2 shows this correlation.

To use Standing's correlation, one must know solution GOR, gas gravity, stock-tank oil gravity, and reservoir temperature. Example D.2 illustrates use of this correlation.

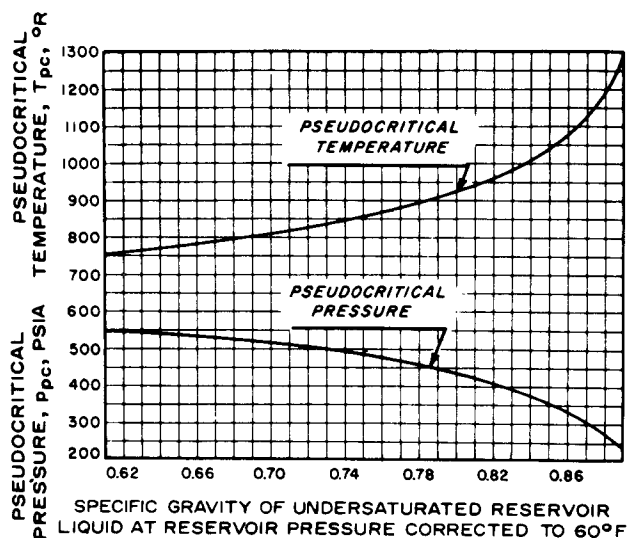


Fig. D-1 – Approximate correlation of liquid pseudocritical pressure and temperature with specific gravity.¹

Example D.2 – Estimation of Bubble-Point of Crude Oil

Problem. Estimate the bubble-point pressure of a crude oil from a reservoir producing at a GOR of 350 scf/STB (believed to be solution gas only) with gas gravity 0.75 and oil gravity 30° API from a reservoir with temperature 200°F.

Solution. On Fig. D-2, we start on the left by extending a horizontal line from the assumed GOR of 350 scf/STB to the line for a gas gravity of 0.75; from this point, we draw a vertical line that terminates at the line for a stock-tank oil gravity of 30° API; we then draw a horizontal line from that point that terminates at the intersection with the 200°F reservoir temperature line. Finally, we draw a vertical line from this point that intersects the bubble-point pressure scale at 1,930 psia. Thus, the estimated bubble-point or saturation pressure of the crude oil is 1,930 psia.

Solution GOR

Frequently, the saturation pressure of a reservoir oil is known, but solution GOR at saturation pressure (required for some test analyses) is unknown. Fig. D-2 also can be used for this estimate.

To determine solution GOR, one must know bubble-point pressure, reservoir temperature, stock-tank oil gravity, and gas gravity. Example D.3 illustrates use of Standing's⁵ correlation for estimating solution GOR.

Example D.3 – Estimation of Solution GOR

Problem. Estimate the solution GOR of a crude oil from a reservoir with bubble-point pressure 1,930 psia, temperature 200°F, oil gravity 30° API, and gas gravity 0.75.

Solution. In Fig. D-2, we start at the lower right by extending a vertical line from the bubble-point pressure to the 200°F reservoir-temperature line. We next draw a horizontal line to the left that terminates at the intersection with the line for tank-oil gravity of 30° API. We then draw a vertical line from this point to intersect the 0.75 gas-gravity line. Finally, we draw a horizontal line from that point to the GOR scale on the left and read the GOR to be 350 scf/STB.

Oil Formation Volume Factor

Fig. D-3 can be used to estimate the formation

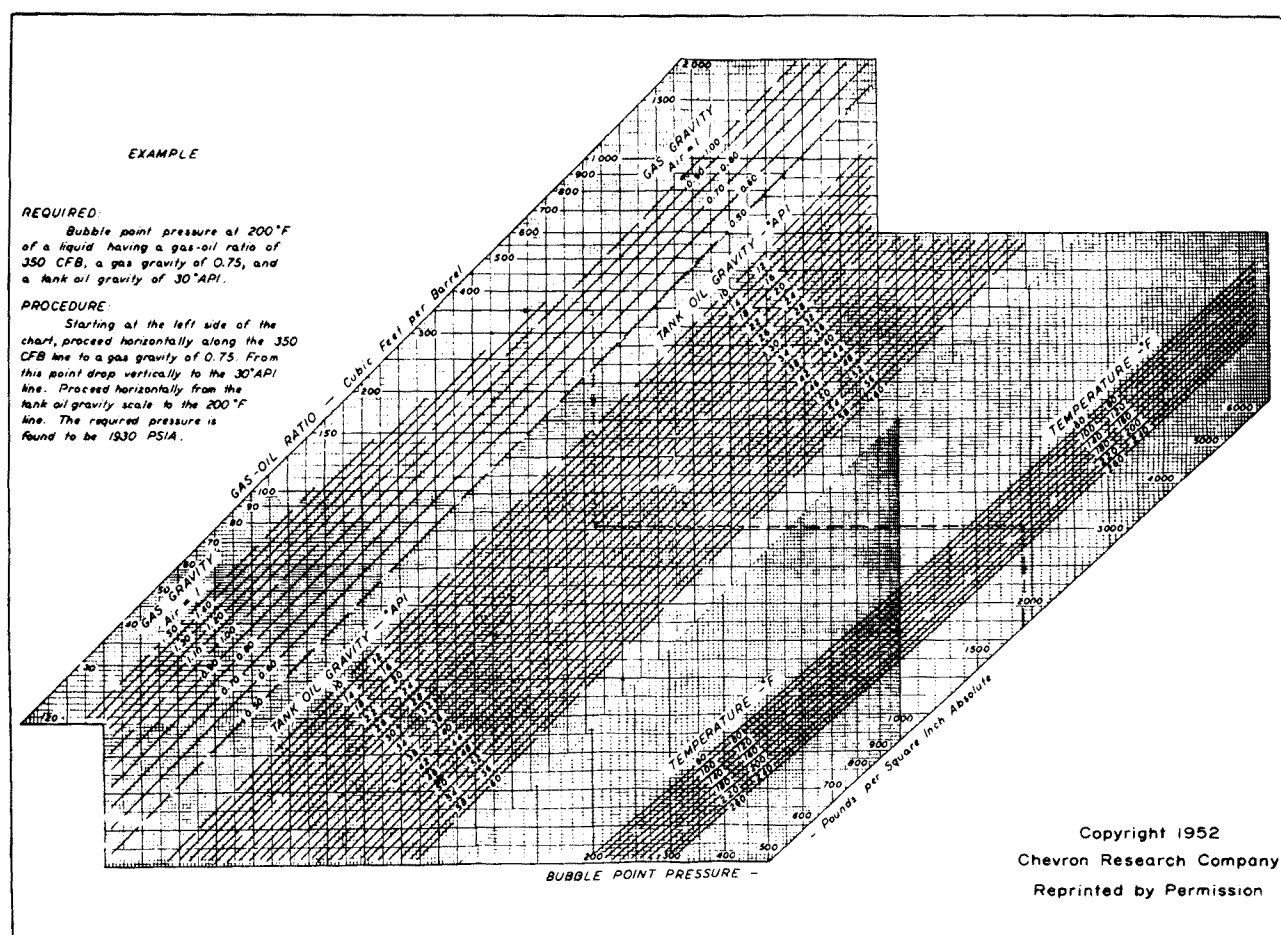
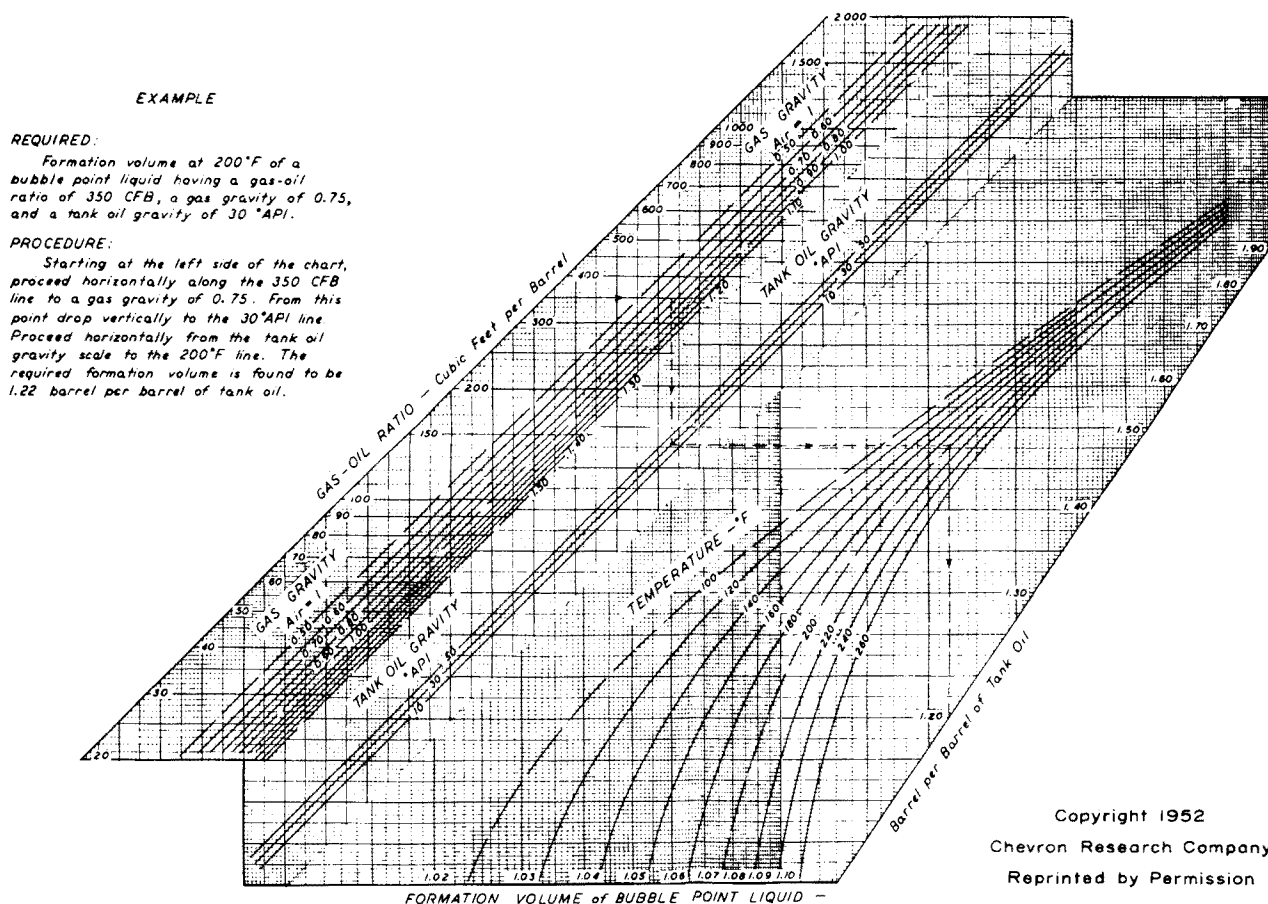
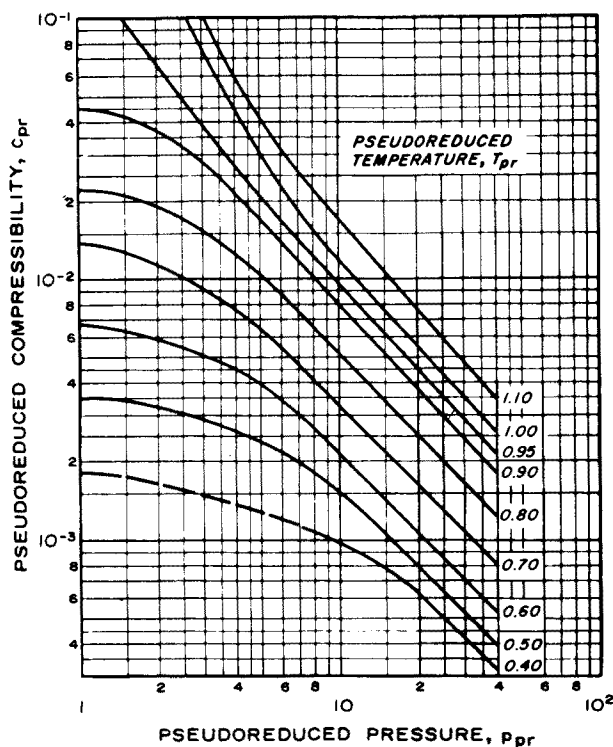


Fig. D-2 – Bubble-point pressure or dissolved GOR of oil.¹

Fig. D-3 – Oil formation volume factor.¹Fig. D-4 – Correlation of pseudoreduced compressibility for an undersaturated oil.¹

volume factor B_o of saturated crude oil. For this estimate, one must know solution GOR, gas gravity, stock-tank oil gravity, and reservoir temperature. Example D.4 illustrates use of this correlation, which also was developed by Standing.

Example D.4 – Estimation of Oil Formation Volume Factor

Problem. Estimate the formation volume factor of a saturated crude oil from a reservoir with solution GOR of 350 scf/STB, gas gravity of 0.75, tank oil gravity of 30°API, and reservoir temperature of 200°F.

Solution. In Fig. D-3, we start at the upper left and extend a horizontal line from a solution GOR of 350 scf/STB to intersect the line for gas gravity of 0.75. We then extend a vertical line from this point to intersect the 30°API tank-oil-gravity line. From this point of intersection, we draw a horizontal line to intersect the line for a reservoir temperature of 200°F. Finally, we draw a vertical line from that point to the formation-volume-factor scale at the lower right and read $B_o = 1.22$ RB/STB.

Compressibility of Undersaturated Oil

The correlation in Fig. D-4, developed by Trube,⁴ can be used to estimate the compressibility, c_o , of an

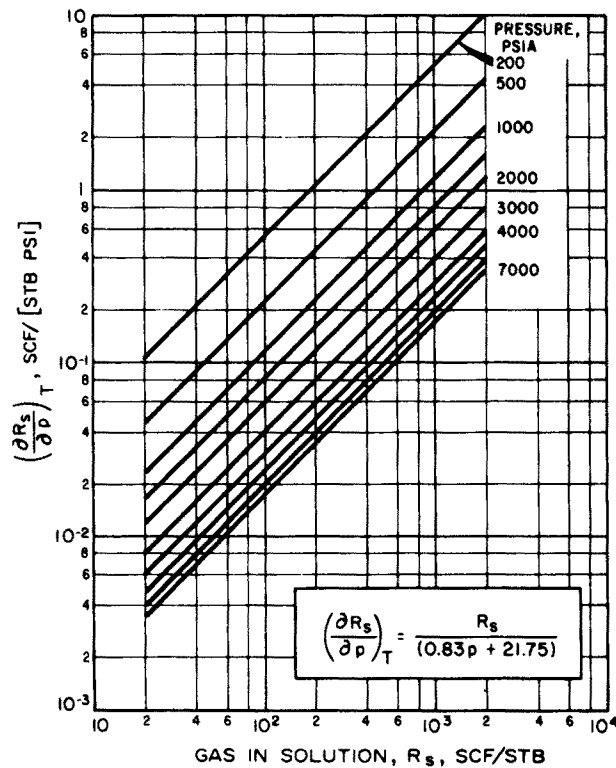


Fig. D-5 – Change of gas in solution in oil with pressure vs. gas in solution.¹

undersaturated crude oil.

The basic information required to use this figure is specific gravity of the undersaturated crude oil, reservoir temperature, and reservoir pressure. Given these values, we can estimate pseudocritical temperature (T_{pc}) and pressure (p_{pc}), pseudoreduced temperature ($T_{pr} = T/T_{pc}$) and pressure ($p_{pr} = p/p_{pc}$) and, thus, pseudoreduced compressibility (c_{pr}), which leads to oil compressibility (c_o) from the definition

$$c_o = c_{pr}/p_{pc}.$$

Example D.5 illustrates this sequence of calculations.

Example D.5 – Estimation of Undersaturated Oil Compressibility

Problem. Estimate the compressibility, c_o , of an undersaturated crude oil with 30°API gravity (0.876 specific gravity) at a reservoir temperature of 200°F and pressure of 5,000 psia.

Solution. In Example D.1, we found that the pseudocritical temperature of a 30°API oil was $T_{pc} = 1180^\circ\text{R}$ and that the pseudocritical pressure was $p_{pc} = 275$ psia. Thus,

$$T_{pr} = T/T_{pc} = (200 + 460)/1,160 = 0.569,$$

and

$$p_{pr} = p/p_{pc} = 5,000/285 = 17.5.$$

From Fig. D-4, then,

$$c_{pr} = 0.001.$$

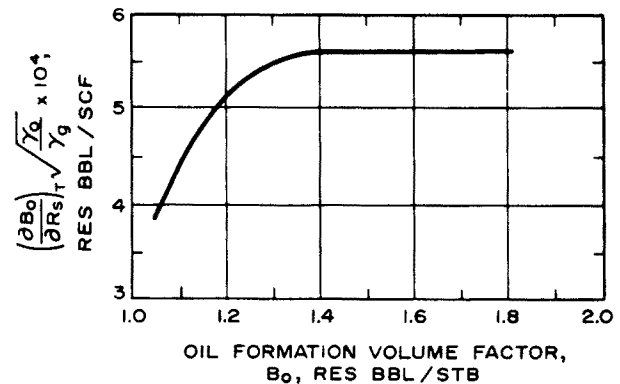


Fig. D-6 – Change of oil formation volume factor with gas in solution vs. oil formation volume factor.¹

By definition, $c_o = c_{pr}/p_{pc}$, so

$$c_o = 0.001/285 = 3.51 \times 10^{-6} \text{ psi}^{-1}.$$

Compressibility of Saturated Crude Oil

The apparent compressibility of saturated crude oil is significantly higher than that of undersaturated oil. The reason is that a pressure drop results in evolution of gas from the oil; the total volume of oil remaining actually *decreases* with pressure decline (although the density of the remaining liquid oil actually decreases slightly). The net result is that the *total* volume of oil and evolved gas becomes greater as pressure drops, leading to an apparent compressibility of the system that is appreciably higher than that of liquid oil alone. In equation form,

$$c_o = -\frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_s}{dp} \dots \dots \dots (D.1)$$

The first term accounts for the volume change in the liquid caused by (1) vaporization of some of the liquid and (2) increase in density of the remainder of the liquid. The derivative dB_o/dp is a positive number, so vaporization dominates. The second term accounts for the volume occupied by gas evolved as pressure decreases (or dissolved as pressure increases). The derivative dR_s/dp is positive, so this term is positive. Further, its numerical value is greater than that of the term $-(1/B_o)(dB_o/dp)$. Ramey⁶ proposed correlations that lead to an estimate of c_o for a saturated oil; these correlations are given in Figs. D-5 and D-6. To use these correlations, we must note that Eq. D.1 can be written as

$$c_o = -\frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_s}{dp}$$

$$= \frac{1}{B_o} \frac{dR_s}{dp} \left(B_g - \frac{dB_o}{dR_s} \right) \dots \dots \dots (D.2)$$

Fig. D-5 provides an estimate of dR_s/dp ; Fig. D-6 estimates dB_o/dR_s .

To estimate c_o , one needs values of reservoir pressure, p , solution GOR, R_s (which, in turn, can be estimated from Fig. D-2), tank-oil specific gravity (γ_o) and gas gravity (γ_g), oil formation volume factor, B_o (which can be estimated from Fig. D-3), and gas formation volume factor, B_g . (B_g can be calculated from reservoir temperature, pressure, and gas gravity, which leads to the real-gas-law deviation factor z : $B_g = 0.00504 Tz/p$ RB/scf.) Example D.6 illustrates use of these correlations.

Example D.6 – Estimation of Saturated Oil Compressibility

Problem. Pressure in an oil reservoir has dropped below the initial bubble point to 2,500 psia. Reservoir fluid characteristics include the following:

$$\gamma_o = 0.825 \text{ (40° API),}$$

$$\gamma_g = 0.7,$$

$$T = 200^\circ\text{F} = 660^\circ\text{R, and}$$

$$z = 0.851.$$

Estimate the apparent compressibility of this oil at its current saturation pressure of 2,500 psia.

Solution. To evaluate c_o , we must determine each term in Eq. D.2.

$$c_o = \frac{1}{B_o} \frac{dR_s}{dp} \left(B_g - \frac{dB_o}{dR_s} \right).$$

From knowledge of p , γ_g , γ_o , and T , we can estimate R_s from Fig. D-2, as in Example D.3; the result is $R_s = 640$ scf/STB. From knowledge of R_s , γ_g , γ_o , and T , we can estimate B_o from Fig. D-3, as in Example D.4; the result is $B_o = 1.36$ RB/STB.

We can calculate B_g since T , p , and z are known:

$$B_g = 0.00504 Tz/p$$

$$= (0.00504)(660)(0.851)/2,500$$

$$= 0.001132 \text{ RB/scf.}$$

From the inset in Fig. D-5,

$$\frac{dR_s}{dp} = \frac{R_s}{(0.83p + 21.75)}$$

$$= \frac{640}{[(0.83)(2,500) + 21.75]}$$

$$= 0.3052 \text{ scf/STB-psi.}$$

(This result also could be read from the curves plotted in Fig. D-5.)

From Fig. D-6,

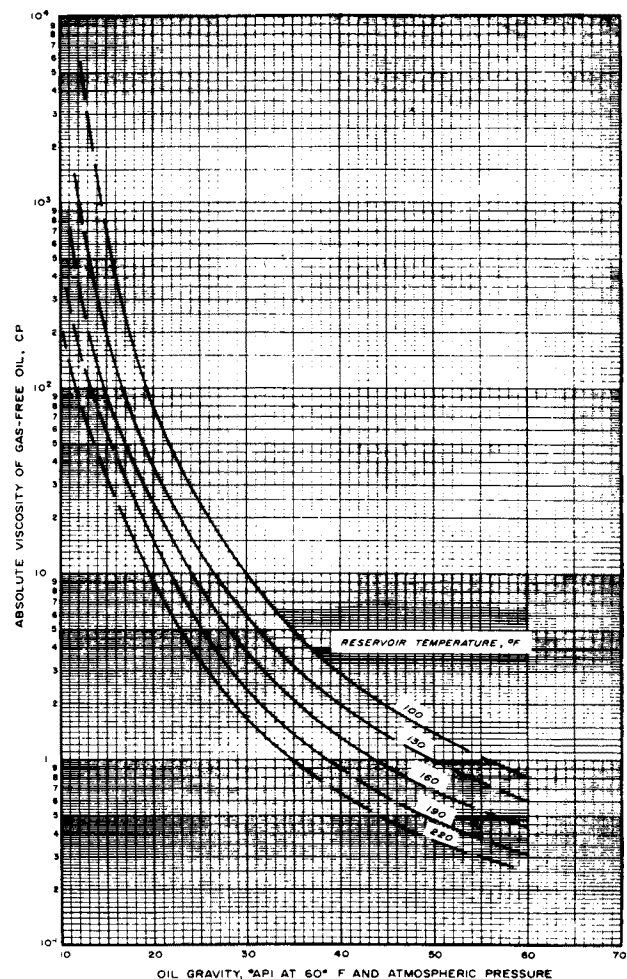


Fig. D-7 – Dead oil viscosity at reservoir temperature and atmospheric pressure.¹

$$\frac{dB_o}{dR_s} \sqrt{\frac{\gamma_o}{\gamma_g}} \times 10^4 = 5.6.$$

Thus,

$$\frac{dB_o}{dR_s} = 5.6 \times 10^{-4} \sqrt{\frac{\gamma_g}{\gamma_o}}$$

$$= (5.6 \times 10^{-4}) \sqrt{\frac{0.7}{0.825}}$$

$$= 0.516 \times 10^{-3} \text{ RB/scf.}$$

We now can calculate c_o :

$$c_o = \frac{1}{B_o} \frac{dR_s}{dp} \left[B_g - \frac{dB_o}{dR_s} \right]$$

$$= \left(\frac{1}{1.36} \right) (0.3052) (0.001132 - 0.000516)$$

$$= 0.138 \times 10^{-3} \text{ psi}^{-1}$$

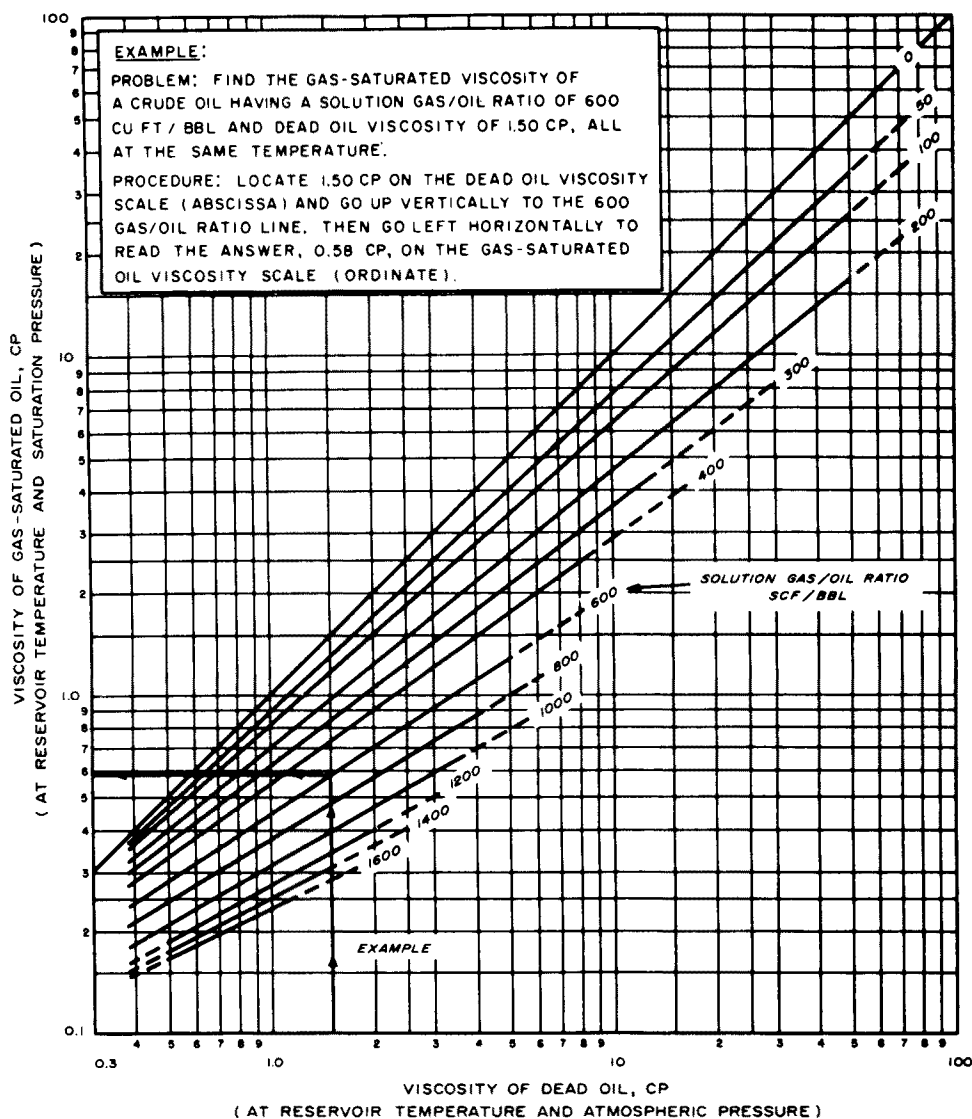


Fig. D-8 – Viscosity of gas-saturated crude oil at reservoir temperature and pressure. Dead oil viscosity from laboratory data or from Fig. D-7.¹

Oil Viscosity

Viscosity of saturated crude oil can be estimated from Beal's correlation⁷ (Fig. D-7) combined with Chew and Connally's data⁸ (Fig. D-8). Viscosity estimates for undersaturated oil require the same figures plus an additional correlation of Beal's, Fig. D-9. Fig. D-7 provides an estimate of dead (gas-free) oil viscosity, μ_{od} ; Fig. D-8 provides an estimate of gas-saturated oil viscosity, μ_{ob} , at saturation pressure; and Fig. D-9 provides an estimate of viscosity increase above bubble-point pressure.

These viscosity estimates require knowledge of reservoir temperature, oil gravity, solution GOR, and, in the case of an undersaturated oil, bubble-point and reservoir pressures. Example D.7 illustrates this estimation procedure.

Example D.7 – Estimation of Oil Viscosity

Problem. Estimate the viscosity of an undersaturated oil at a reservoir pressure of 5,000 psia and at the oil's saturation pressure of 1,930 psia. Solution GOR

is 350 scf/STB, oil gravity is 30°API, and reservoir temperature is 200°F.

Solution. From Fig. D-7, the dead oil viscosity, μ_{od} , is 2.15 cp. From Fig. D-8, the viscosity of gas-saturated oil (μ_{ob}) at the bubble point is 1.0 cp. Finally, using data from Fig. D-9, the viscosity of the undersaturated oil (μ_o) at a pressure of 5,000 psi is

$$\mu_o = 1 + (0.067) \left(\frac{5,000 - 1,930}{1,000} \right) = 1.21 \text{ cp.}$$

Solubility of Gas in Water

Solubility of natural gas in water can be estimated from correlations of Dodson and Standing,⁹ presented in Figs. D-10 and D-11. Fig. D-10 gives the solubility of natural gas in pure (nonsaline) water; Fig. D-11 provides a means of correcting solubility in pure water for brine salinity.

To estimate solubility of gas in water, one must know reservoir temperature and pressure and total

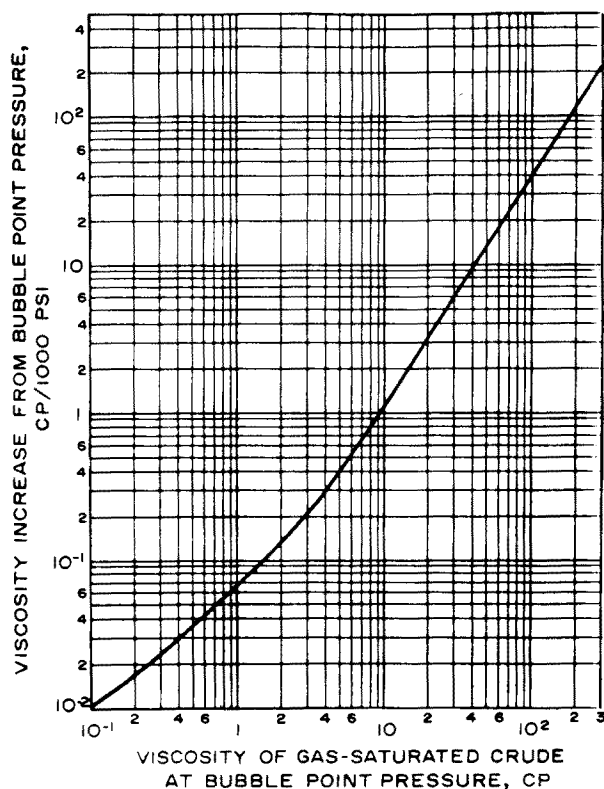


Fig. D-9 – Rate of increase in oil viscosity above bubble-point pressure.¹

solids content of the water. Example D.8 illustrates this estimation procedure.

Example D.8 – Estimation of Gas Solubility in Water

Problem. Estimate the solubility of natural gas in a formation water with 20,000 ppm dissolved solids, reservoir temperature of 200°F, and reservoir pressure of 5,000 psia.

Solution. From Fig. D-10, the solubility (R_{swp}) of natural gas in *pure* water at 200°F and 5,000 psia is 20.2 scf/STB. From Fig. D-11, the correction factor (R_{sw}/R_{swp}) for 20,000-ppm salinity at 200°F is 0.92. Thus,

$$\begin{aligned} R_{sw} &= (R_{swp}) (R_{sw}/R_{swp}) \\ &= (20.2)(0.92) \\ &= 18.6 \text{ scf/STB.} \end{aligned}$$

Water Formation Volume Factor

The formation volume factor B_w can be estimated by using Fig. D-12, an additional correlation developed by Dodson and Standing,⁹ along with Figs. D-10 and

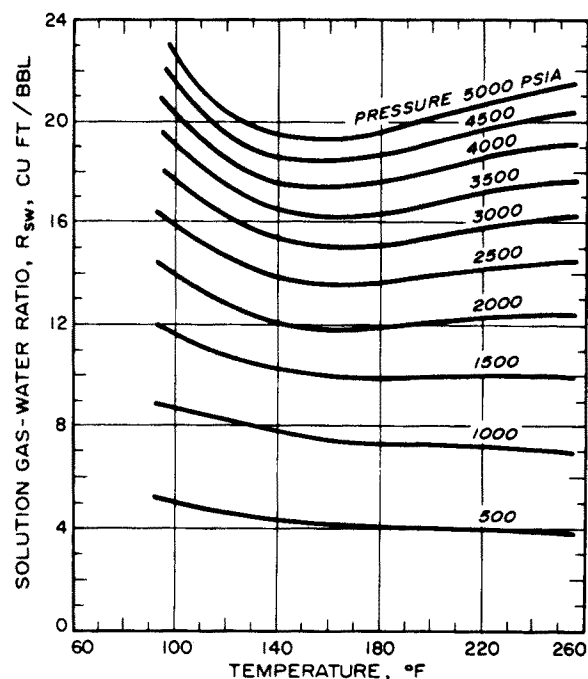


Fig. D-10 – Solubility of natural gas in pure water.¹

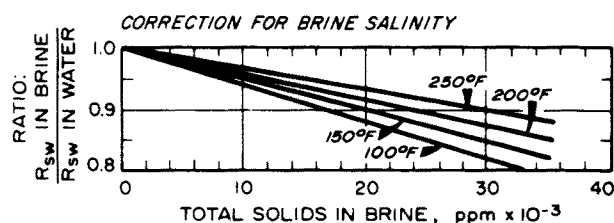


Fig. D-11 – Correction of natural gas solubility for dissolved solids.¹

D-11. This estimate requires the same information used to estimate the solubility of gas in water (reservoir temperature and pressure and total solids content of the water). Example D.9 illustrates the estimation procedure.

Example D.9 – Estimation of Water Formation Volume Factor

Problem. Estimate the formation volume factor of brine with 20,000 ppm dissolved solids at a temperature of 200°F and pressure of 5,000 psia.

Solution. From Fig. D-12, the formation volume factor of pure water is 1.021 RB/STB; for gas-saturated pure water, it is 1.030 RB/STB. Brine with 20,000 ppm dissolved solids contains less dissolved gas than pure water; from Fig. D-11, the ratio of solubility in brine to that in pure water is 0.92. Using this value to interpolate between volume factors for pure and gas-saturated water, then,

$$\begin{aligned} B_w &= 1.021 + (1.030 - 1.021)(0.92) \\ &= 1.029 \text{ RB/STB.} \end{aligned}$$

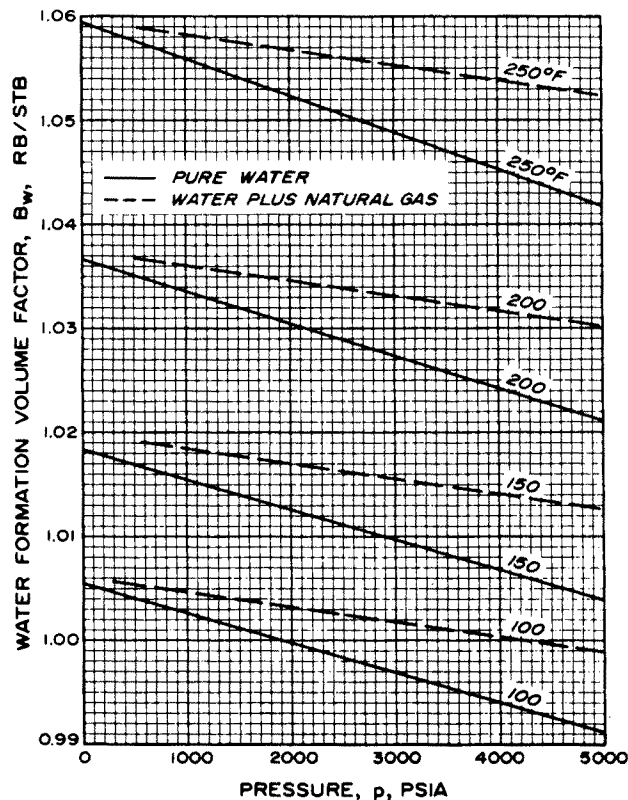


Fig. D-12 – Formation volume factor for pure water, gas-free and gas-saturated.¹

Compressibility of Water in Undersaturated Reservoirs

The formation water that occurs in an undersaturated oil reservoir will not release gas as pressure is decreased; this would lead to formation of or increase in a gas saturation in the reservoir. In such a case, we will assume that the formation water is saturated with gas at reservoir pressure. One implication of this is that, as gas is released from solution in the water, it is assumed to be redissolved in the undersaturated oil.

For this assumed system behavior, Dodson and Standing's⁹ correlations (given in Figs. D-13 and D-14) can be used to estimate the compressibility of water in an undersaturated oil reservoir.

These water-compressibility estimates require knowledge of reservoir temperature and pressure and formation-water salinity. Example D.10 illustrates the calculation procedure.

Example D.10 – Estimation of Water Compressibility in an Undersaturated Reservoir

Problem. Estimate the compressibility of a formation water containing 20,000 ppm dissolved solids in a reservoir with temperature 200°F and pressure

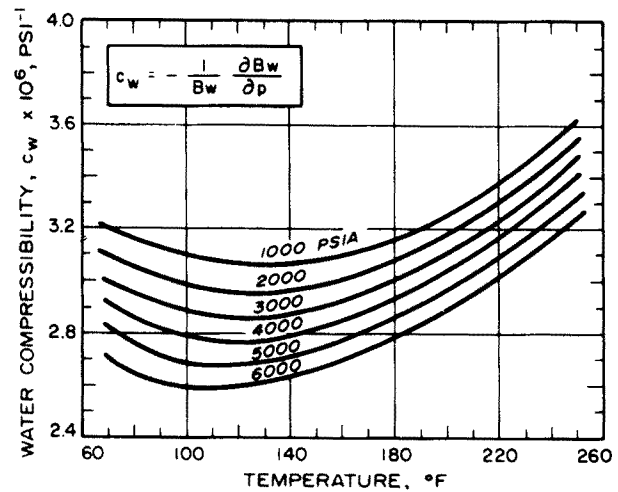


Fig. D-13 – Compressibility of gas-free water.¹

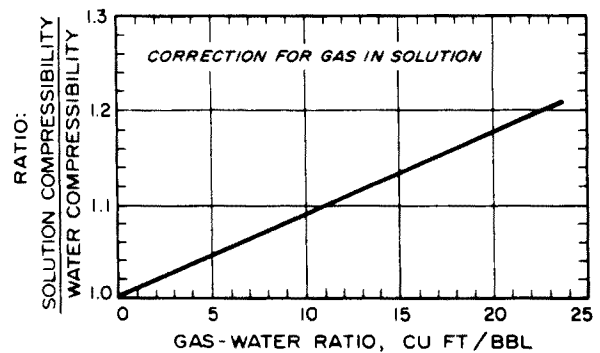


Fig. D-14 – Effect of dissolved gas on water compressibility.¹

5,000 psia. Oil in the reservoir has a saturation pressure of 1,930 psia.

Solution. We found in Example D.8 that solubility of gas in water at the stated conditions is 18.6 scf/STB. From Fig. D-13, the compressibility (c_{wp}) of pure (gas-free) water is $2.96 \times 10^{-6} \text{ psi}^{-1}$. At a gas/water ratio of 18.6 scf/STB, the correction factor for gas in solution is 1.16 (Fig. D-14). Thus,

$$\begin{aligned} c_w &= c_{wp} (c_w / c_{wp}) \\ &= (2.96 \times 10^{-6})(1.16) \\ &= 3.43 \times 10^{-6} \text{ psi}^{-1}. \end{aligned}$$

Compressibility of Water in a Saturated Reservoir

In a saturated reservoir, gas released from solution in the formation water either will begin to form or will increase a gas saturation as reservoir pressure is lowered; as Ramey⁶ pointed out, this dramatically increases the apparent water compressibility. In this case, the water compressibility is calculated from

$$c_w = -\frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp} \quad \text{..... (D.3)}$$

The term $-(1/B_w)(dB_w/dp)$ is still determined

using Fig. D-13 (for gas-free water) and Fig. D-14 (to correct for the effect of gas in solution). Ramey's correlation,⁶ presented in Fig. D-15, is used to estimate dR_{sw}/dp for fresh water, and Fig. D-11 is used to correct for the effect of salinity on dR_{sw}/dp .

This compressibility estimate requires knowledge of formation-water salinity, reservoir temperature and pressure, and formation volume factor of the gas dissolved in the water. Example D.11 illustrates this estimation procedure.

Example D.11 – Estimation of Water Compressibility in a Saturated Reservoir

Problem. Estimate the apparent compressibility of formation water containing 30,000 ppm dissolved solids in an undersaturated oil reservoir at 200°F and

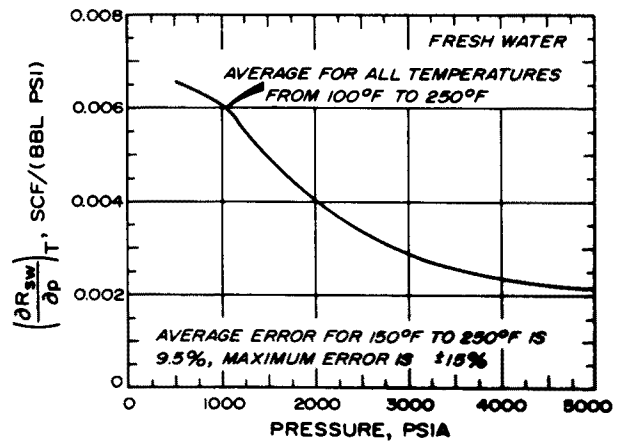


Fig. D-15 – Change of natural gas in solution in formation water with pressure vs. pressure.¹

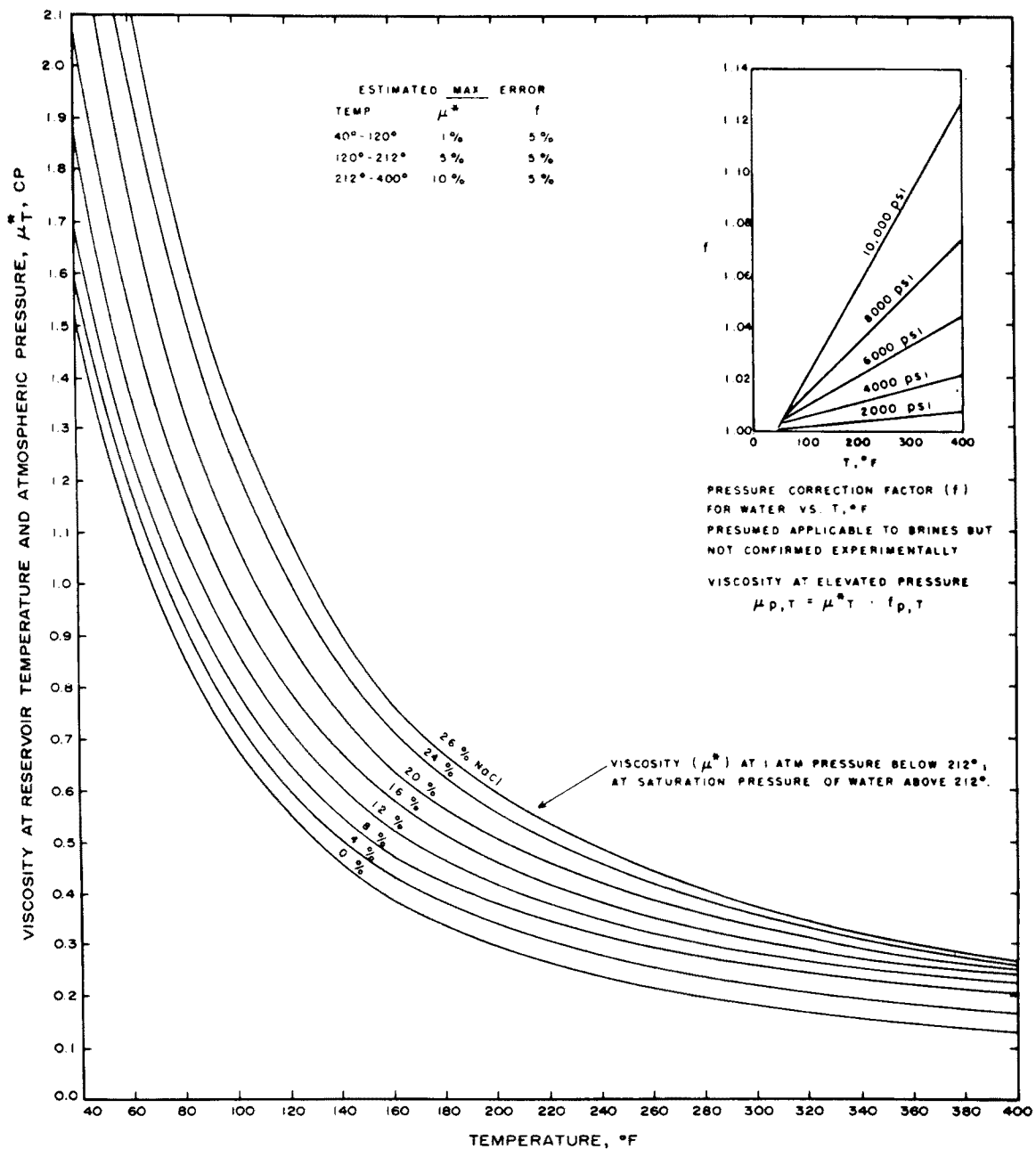


Fig. D-16 – Water viscosity at various salinities and temperatures.¹

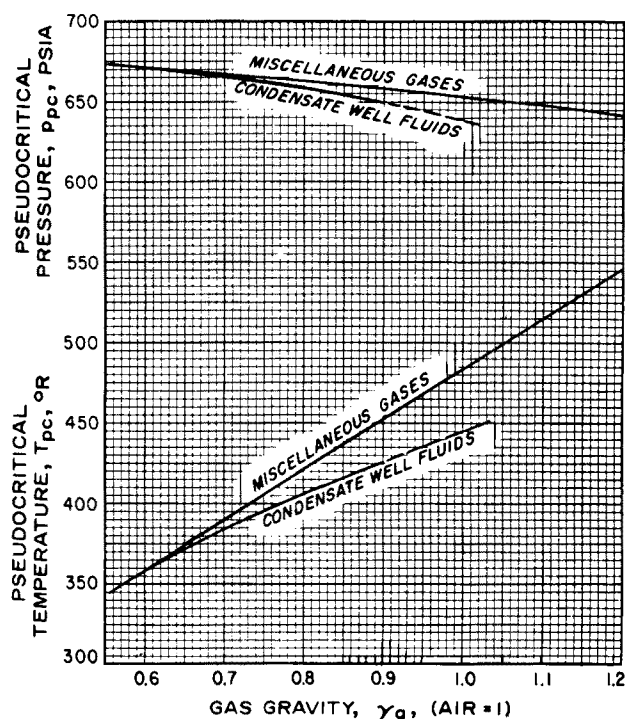


Fig. D-17 – Correlation of pseudocritical properties of condensate well fluids and miscellaneous natural gases with fluid gravity.¹

2,500 psia. Formation volume factor of the 0.7-gravity gas is 0.001132 RB/scf.

Solution. From Fig. D-15, for fresh water, $dR_{sw}/dp = 0.0033$. The correction factor for the effect of salinity (Fig. D-11) is 0.875; thus, for brine, $dR_{sw}/dp = (0.0033)(0.875) = 0.0029$ scf/STB-psi. From Fig. D-12, using procedures outlined in Example D.9, $B_w = 1.033$ RB/STB. From Figs. D-13 and D-14, using procedures of Example D.10,

$$\begin{aligned} -\frac{1}{B_w} \frac{dB_w}{dp} &= (3.13 \times 10^{-6})(1.11) \\ &= 3.47 \times 10^{-6} \text{ psi}^{-1}. \end{aligned}$$

Thus,

$$\begin{aligned} c_w &= -\frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp} \\ &= 3.47 \times 10^{-6} + \frac{(0.001132)(0.0029)}{(1.033)} \\ &= 6.65 \times 10^{-6} \text{ psi}^{-1}. \end{aligned}$$

Water Viscosity

Fig. D-16, first presented in the literature by Matthews and Russell,¹⁰ allows estimates of water viscosity as a function of reservoir temperature, salinity, and reservoir pressure. Example D.12 illustrates use of this correlation.

Example D.12 – Estimation of Water Viscosity

Problem. Estimate the viscosity of water containing 20,000 ppm (2%) dissolved solids at 200°F and 5,000 psia.

Solution. From Fig. D-16, the viscosity μ_T^* of water with 2% NaCl at 200°F and atmospheric pressure is 0.32 cp. The correction factor f for 5,000-psia pressure is 1.016 (inset at upper right of figure). Thus,

$$\mu_w = \mu_T^* f = (0.32)(1.016) = 0.33 \text{ cp.}$$

Pseudocritical Properties of Gas

Pseudocritical temperature, T_{pc} , and pressure, p_{pc} , are useful quantities that allow application of generalized correlations of gas properties needed in applications. The most accurate values of these quantities are calculated from compositions of natural gas mixtures.^{2,3} Approximate values can be determined from a correlation developed by Brown *et al.*¹¹ This correlation, presented in Fig. D-17, is based on gas gravity, γ_g ; values of critical properties depend on whether the gas is from a gas condensate reservoir (the “condensate well fluids” curve) or from a relatively dry gas reservoir (the “miscellaneous gases” curve). Example D.13 illustrates use of this correlation.

Example D.13 – Estimation of Pseudocritical Gas Properties

Problem. Estimate T_{pc} and p_{pc} for a dry gas of gravity (γ_g) 0.7.

Solution. From Fig. D-17, $T_{pc} = 390^\circ\text{R}$ and $p_{pc} = 665$ psia.

Gas-Law Deviation Factor (z-Factor) and Gas Formation Volume Factor

Application of the real gas law,

$$pV = znRT, \dots\dots\dots (D.4)$$

to relate pressure, volume, and temperature for gases requires values of the deviation factor z . One application of particular importance is in calculating gas formation volume factor, B_g , from

$$B_g = 0.00504 Tz/p \text{ (RB/scf)}. \dots\dots\dots (D.5)$$

Fig. D-18, developed by Standing and Katz,¹² can be used to estimate z . One must know pseudoreduced pressure, p_{pr} , and pseudoreduced temperature, T_{pr} , to determine z . By definition,

$$p_{pr} = p/p_{pc} \dots\dots\dots (D.6)$$

and

$$T_{pr} = T/T_{pc} \dots\dots\dots (D.7)$$

In these definitions, pressures must be expressed in psia (psig + 14.7) and temperatures in degrees Rankine ($^\circ\text{F} + 460$).

To use this correlation, one must know reservoir

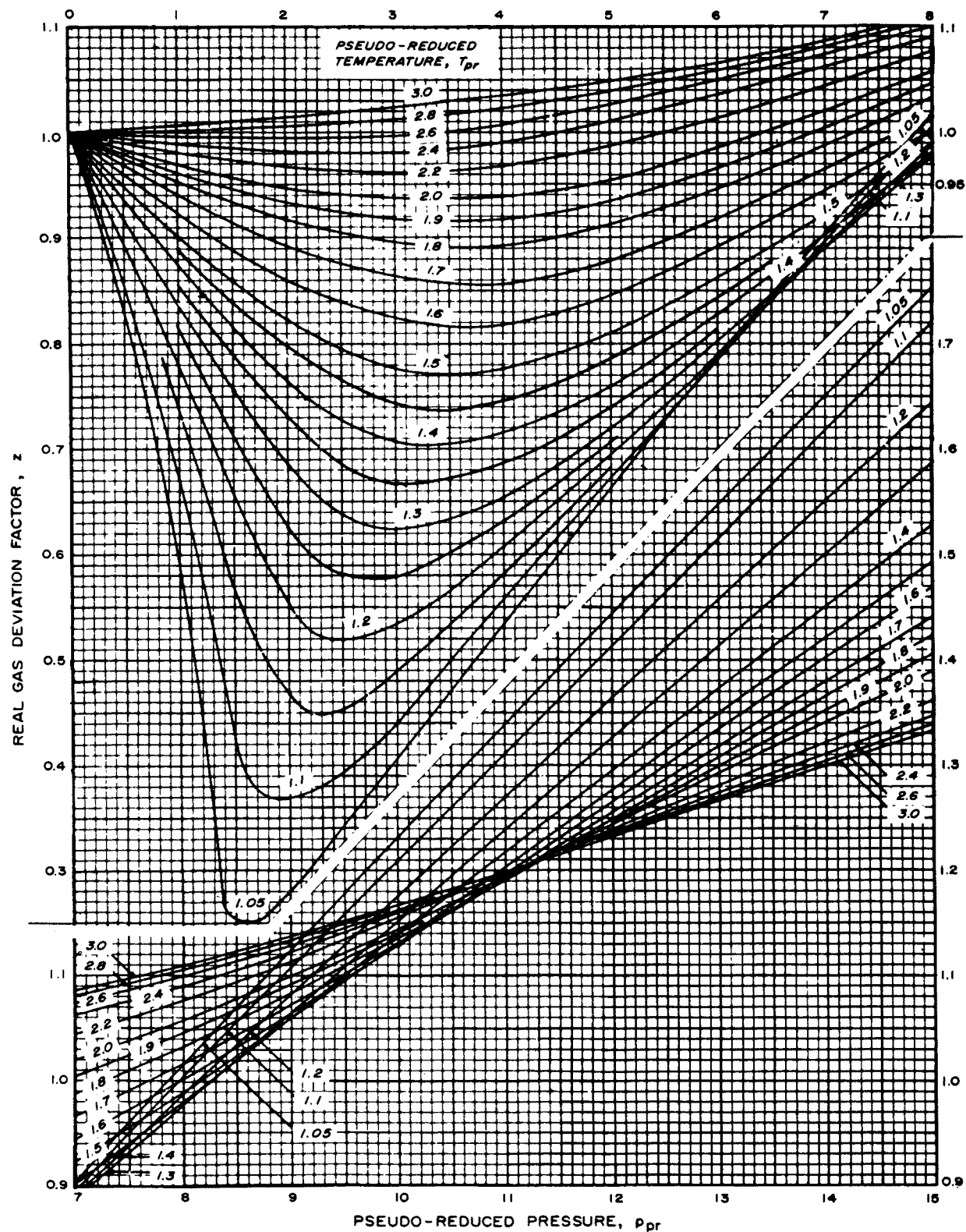


Fig. D-18 – Gas-law deviation factor for natural gases as a function of pseudoreduced pressure and temperature.¹

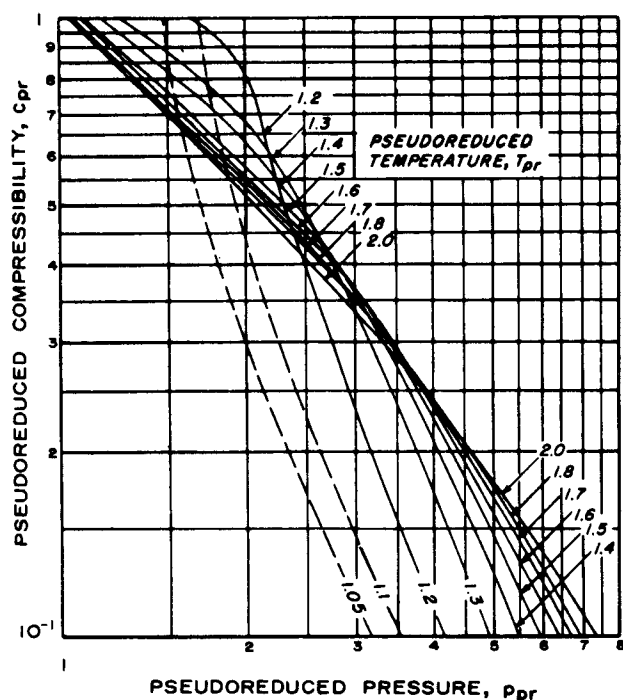


Fig. D-19A – Correlation of pseudoreduced compressibility for natural gases.¹

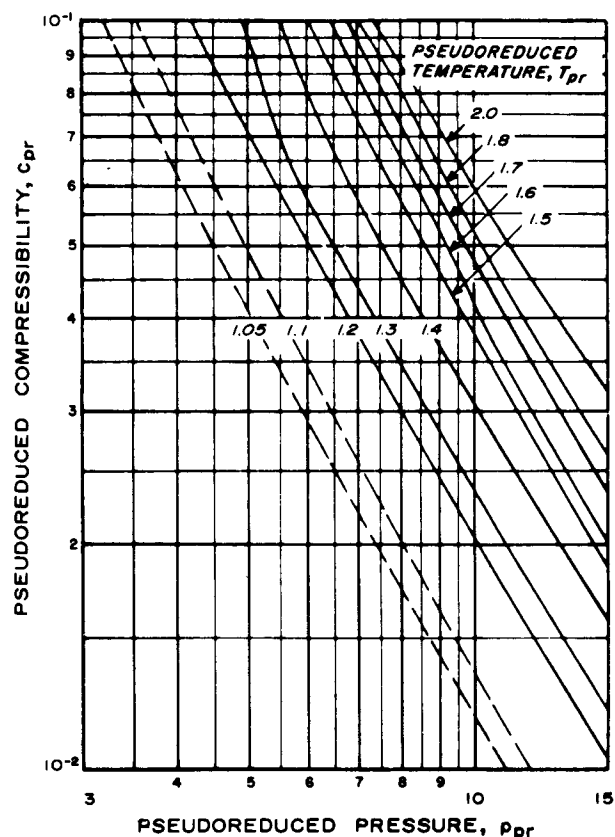


Fig. D-19B – Correlation of pseudoreduced compressibility for natural gases.¹

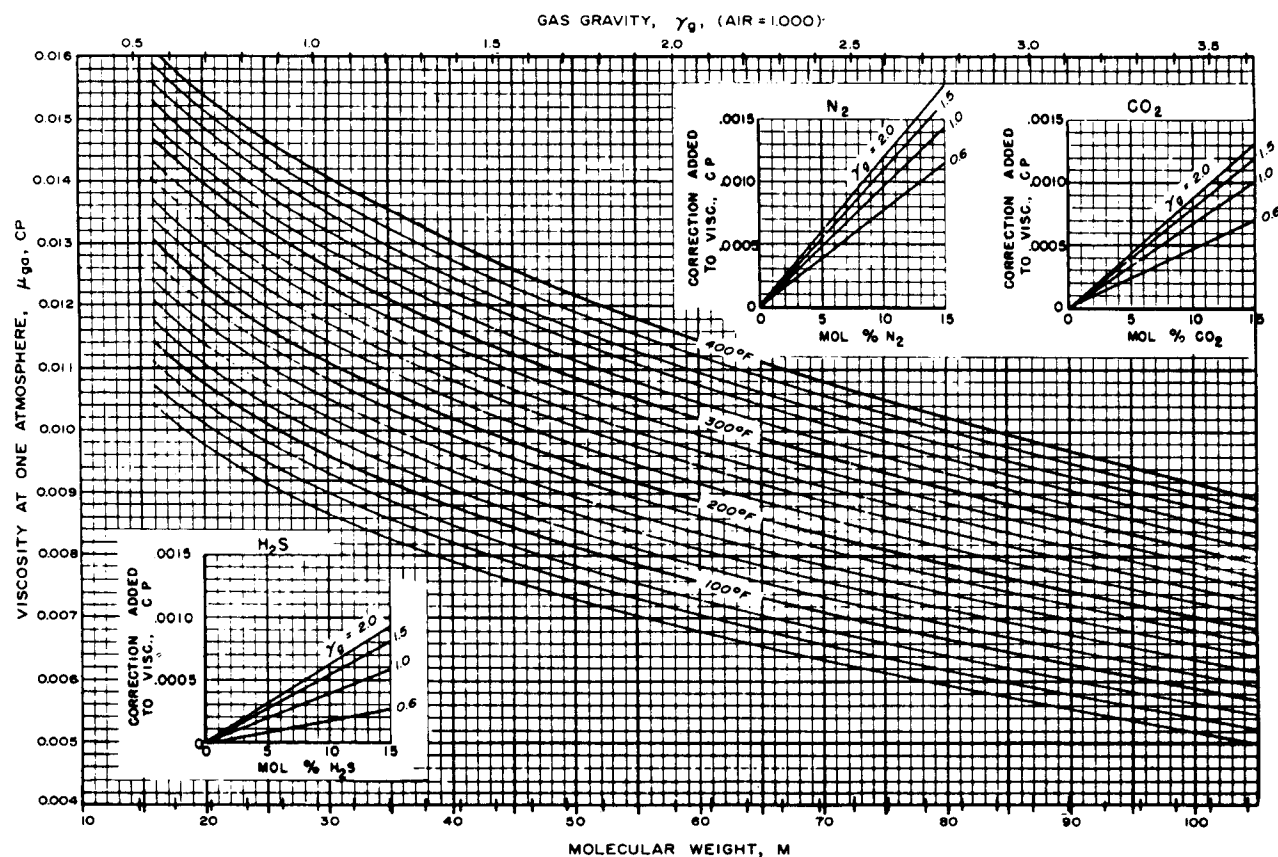


Fig. D-20 – Viscosity of natural gases at 1 atm.¹

temperature and pressure and pseudocritical temperature and pressure (from either composition or gas gravity). Example D.14 illustrates use of this correlation.

Example D.14 – Estimation of Gas-Law Deviation Factor and Gas Formation Volume Factor

Problem. Estimate z for 0.7-gravity gas in a reservoir with temperature 200°F and pressure 2,500 psia. Use this value of z to determine the gas formation volume factor B_g .

Solution. In Example D.13, we found that $T_{pc} = 390^\circ\text{R}$ and $p_{pc} = 665$ psia. Then,

$$T_{pr} = T/T_{pc} = (200 + 460)/390 = 1.69,$$

and

$$p_{pr} = p/p_{pc} = 2,500/665 = 3.76.$$

From Fig. D-18, $z = 0.851$. Then,

$$\begin{aligned} B_g &= 0.00504 Tz/p \\ &= (0.00504)(660)(0.851)/2,500 \\ &= 0.001132 \text{ RB/scf.} \end{aligned}$$

Gas Compressibility

Figs. D-19A and D-19B (developed by Trube¹³) lead to estimates of gas compressibility, c_g . In these figures (which cover different ranges of the independent variables), pseudoreduced compressibility, c_{pr} , is plotted as a function of pseudoreduced pressure, p_{pr} , with the parameter pseudoreduced temperature, T_{pr} . Pseudoreduced compressibility is defined as $c_{pr} = c_g p_{pc}$; thus, gas compressibility is found from the relation

$$c_g = c_{pr}/p_{pc} \quad \text{..... (D.8)}$$

Use of Figs. D-19A and D-19B requires knowledge of reservoir temperature and pressure and pseudocritical temperature and pressure of the gas (from either composition or gravity). Example D.15 illustrates use of this correlation.

Example D.15 – Estimation of Gas Compressibility

Problem. Estimate the compressibility of a 0.7-gravity gas at a reservoir temperature of 200°F and pressure of 2,500 psia.

Solution. In Example D.14, we found that $T_{pr} = 1.69$, $p_{pr} = 3.76$, and $p_{pc} = 665$ psia for these conditions. From Fig. D-19A, $c_{pr} = 0.26$. Thus,

$$\begin{aligned} c_g &= c_{pr}/p_{pc} = 0.26/665 \\ &= 0.00039 \text{ psi}^{-1}. \end{aligned}$$

Gas Viscosity

Figs. D-20, D-21A, and D-21B (from the work of Carr *et al.*¹⁴) can be used to estimate gas viscosity at

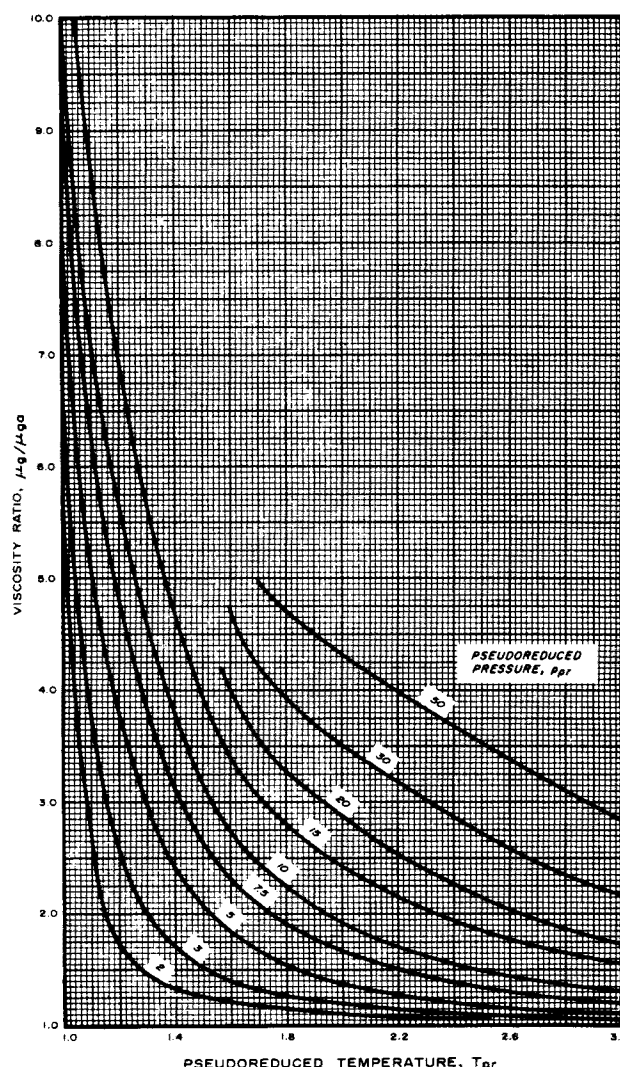


Fig. D-21A – Effect of temperature and pressure on gas viscosity.¹

reservoir conditions. From knowledge of reservoir temperature and gas gravity (or its equivalent, molecular weight), we can estimate the viscosity of a hydrocarbon gas, μ_{g1} , at atmospheric pressure. Insets in Fig. D-20 allow corrections to this viscosity for nonhydrocarbon components of the gas. Figs. D-21A and D-21B (two different ways of plotting the same data) permit calculation of gas viscosity at reservoir temperature and pressure, given viscosity at atmospheric pressure, pseudoreduced temperature, T_{pr} , and pseudoreduced pressure, p_{pr} . Example D.16 illustrates application of these figures.

Example D.16 – Estimation of Gas Viscosity

Problem. Estimate the viscosity of a 0.7-gravity hydrocarbon gas (no H_2S , N_2 , or CO_2) at 200°F and 2,500 psia.

Solution. From Fig. D-20, the viscosity μ_{g1} of 0.7-gravity gas [molecular weight = $0.7 \times 28.97 = 20.3$ lbm/(lbm-mole)] at 200°F and atmospheric pressure

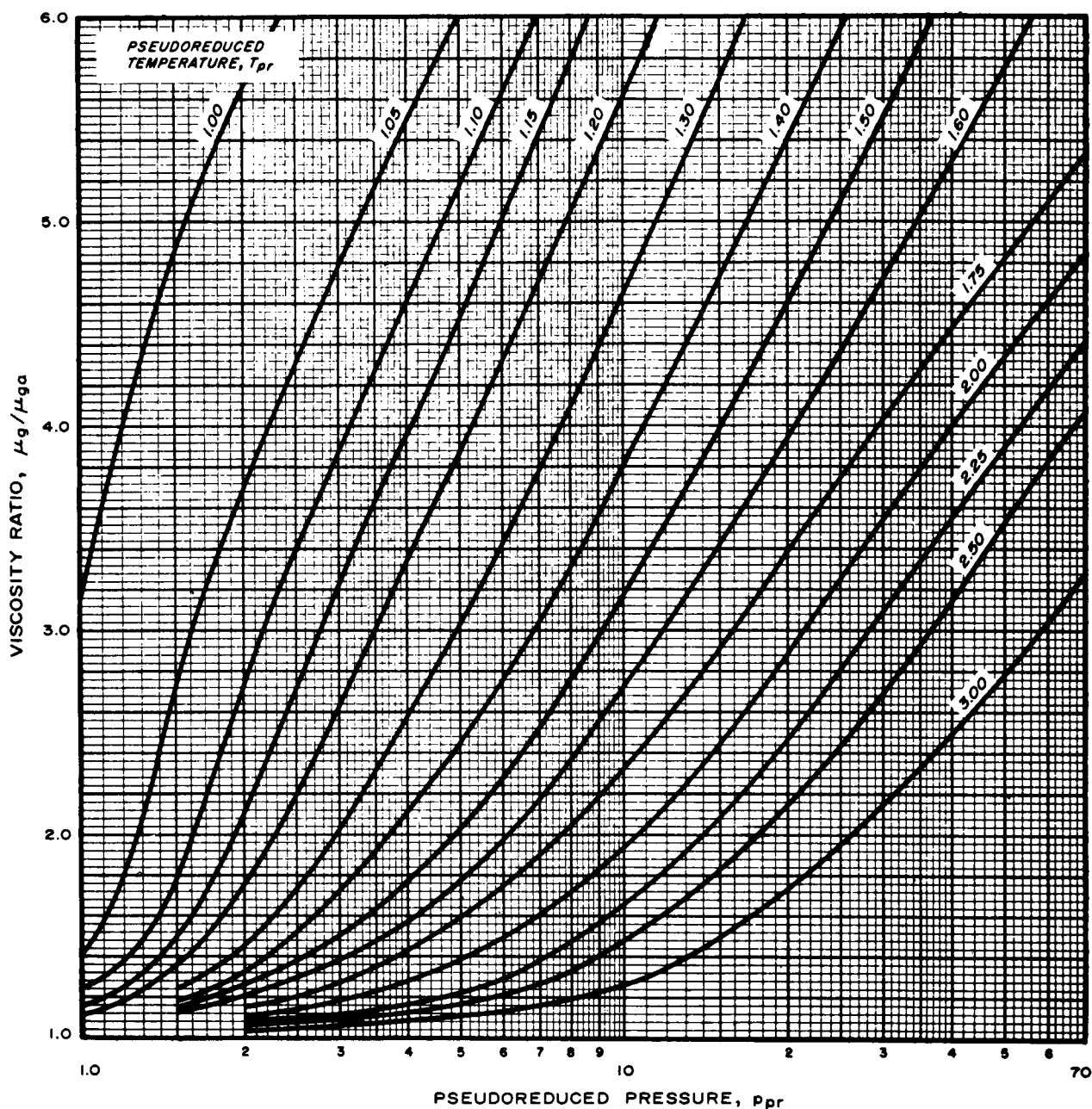


Fig. D-21B – Effect of pressure and temperature on gas viscosity.¹

is 0.01225 cp. In Example D.14, we found that, at these conditions, $T_{pr} = 1.69$ and $p_{pr} = 3.76$. Thus, from Fig. D-21A or D-21B, $\mu_g/\mu_{gl} = 1.45$. At 200°F and 2,500 psia, gas viscosity is

$$\begin{aligned}\mu_g &= (\mu_g/\mu_{gl})\mu_{gl} \\ &= (0.01225)(1.45) \\ &= 0.0178 \text{ cp.}\end{aligned}$$

Formation Compressibility

Formation compressibility, c_f , is defined as

$$c_f = -\frac{1}{V_p} \left(\frac{\partial V_p}{\partial p} \right)_T, \dots\dots\dots (D.9)$$

where V_p = pore volume of porous medium.

Formation compressibility is a complex function of rock type, porosity, pore pressure, overburden pressure, and, in general, the stresses in different directions in the formation. No reliable correlation of this quantity with the controlling variables has been presented in the literature; indeed, laboratory determinations of c_f are difficult, and many reported values of this quantity are doubtless erroneous because conditions in the field were not duplicated in the laboratory. A much-used correlation, developed by Hall,¹⁵ is presented in Fig. D-22. This correlation relates c_f to a single variable – porosity. As reported by Earlougher,¹ this correlation is known to be incorrect by an order of magnitude or more in specific situations. Thus, while the correlation is easy to use, the result may be seriously in error for any given application.

Use of this correlation is illustrated in Example D.17. The result may be of no greater accuracy than simply assuming $c_f \approx 4 \times 10^{-6} \text{ psi}^{-1}$, since only one of the many variables affecting c_f has been taken into account.

Example D.17 – Estimation of Formation Compressibility

Problem. Estimate the formation compressibility c_f for a reservoir with 20% porosity.

Solution. From Fig. D-22, $c_f = 3.6 \times 10^{-6} \text{ psi}^{-1}$.

Exercises

Results of pressure transient test analysis sometimes are combined with rock and fluid properties to calculate the following quantities:

Total reservoir flow rate,

$$q_{Rt} = q_o B_o + q_w B_w + (q_g - R_s q_o / 1,000) B_g.$$

Total mobility,

$$\lambda_t = k_o / \mu_o + k_w / \mu_w + k_g / \mu_g.$$

Total compressibility,

$$c_t = c_o S_o + c_w S_w + c_g S_g + c_f.$$

The following exercises require calculation of q_{Rt} , λ_t , and c_t for two cases.

D.1. Calculate q_{Rt} , λ_t , and c_t for an undersaturated oil reservoir with the following properties.

$$q_o = 100 \text{ STB/D},$$

$$q_w = 20 \text{ STB/D},$$

$$q_g = q_o R_s \text{ (reservoir produces dissolved gas only),}$$

$$\text{Reservoir pressure} = 4,000 \text{ psia},$$

$$\text{Reservoir temperature} = 220^\circ \text{F},$$

$$R_s = 400 \text{ scf/STB},$$

$$\gamma_g = 0.7,$$

$$\gamma_o = 0.85,$$

$$\text{Water salinity} = 25,000 \text{ ppm (2.5\% NaCl)},$$

$$k_o = 20 \text{ md},$$

$$k_w = 0.93 \text{ md},$$

$$k_g = 0 \text{ (no free-gas saturation),}$$

$$\phi = 0.18,$$

$$S_o = 0.65,$$

$$S_w = 0.35, \text{ and}$$

$$S_g = 0.$$

D.2. Calculate q_{Rt} , λ_t , and c_t for a saturated oil reservoir with the following properties:

$$q_o = 100 \text{ STB/D},$$

$$q_w = 5 \text{ STB/D},$$

$$q_g = 250 \text{ Mscf/D},$$

$$\text{Oil gravity} = 38^\circ \text{API},$$

$$\text{Gas gravity} = 0.8,$$

$$\text{Reservoir pressure} = 2,000 \text{ psia},$$

$$\text{Reservoir temperature} = 200^\circ \text{F},$$

$$k_o = 100 \text{ md},$$

$$k_w = 3.3 \text{ md},$$

$$k_g = 7.25 \text{ md},$$

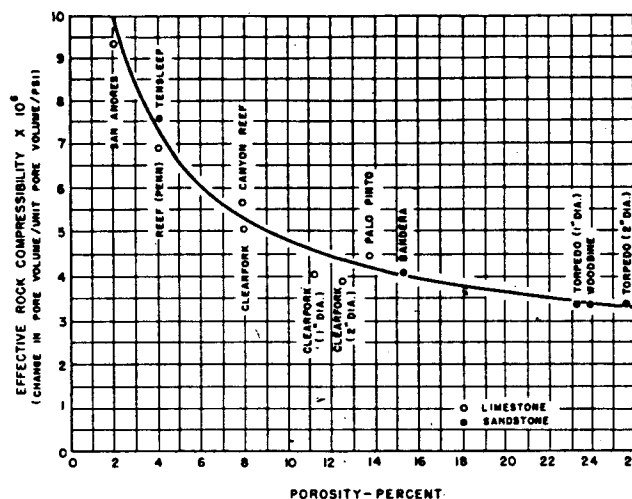


Fig. D-22 – Formation compressibility.¹⁵

Water salinity = 27,500 ppm,

$$S_w = 0.25,$$

$$S_g = 0.05,$$

$$S_o = 0.70, \text{ and}$$

$$\phi = 0.18.$$

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Appendix E

A General Theory of Well Testing

The purpose of this appendix is to summarize an approach to well test analysis that is not limited either to wells in infinite-acting reservoirs or to wells centered in cylindrical reservoirs. This theory is stated more clearly in terms of dimensionless variables, which we have largely avoided in the text because they are abstract and thus are less easily understood than other approaches. For some applications, though, use of dimensionless variables is essential – and general theory is such an area.

We have seen that for $t \leq 948 \phi \mu c_i r_e^2 / k$ (infinite-acting reservoir) and for constant-rate flow for a well centered in a cylindrical reservoir,

$$p_i - p_{wf} = -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_i r_w^2}{kt} \right) - 2s \right], \quad \dots \dots \dots (1.11)$$

and for $t > 948 \phi \mu c_i r_e^2 / k$ (pseudosteady state),

$$p_i - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[\frac{0.000527 kt}{\phi \mu c_i r_e^2} + \ln \left(\frac{r_e}{r_w} \right) - 0.75 + s \right]. \quad (1.12 \text{ modified})$$

We can generalize by writing these equations in terms of dimensionless variables:

$$\frac{0.00708 kh (p_i - p_{wf})}{qB\mu} = \frac{1}{2} \left(\ln \frac{0.000264 kt}{\phi \mu c_i r_w^2} + 0.809 \right) + s,$$

or

$$\frac{0.00708 kh (p_i - p_{wf})}{qB\mu} = \frac{1}{2} (\ln t_D + 0.809) + s \\ = p_D + s, \quad t_D \leq 0.25 r_{De}^2, \quad \dots \dots \dots (E.1)$$

and

$$\frac{0.00708 kh (p_i - p_{wf})}{qB\mu} = \left[\frac{2}{(r_e/r_w)^2} \cdot \left(\frac{0.000264 kt}{\phi \mu c_i r_w^2} \right) + \ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} + s \right],$$

or

$$\frac{0.00708 kh (p_i - p_{wf})}{qB\mu} = \left(\frac{2t_D}{r_{De}^2} + \ln r_{De} - 0.75 \right) + s \\ = p_D + s, \quad t_D > 0.25 r_{De}^2. \quad \dots \dots \dots (E.2)$$

Even more generally, we can write

$$\frac{0.00708 kh (p_i - p_{wf})}{qB\mu} = p_D(t_D) + s. \quad \dots \dots (E.3)$$

Thus, given a value t_D , there exists a rule for determining p_D for a well centered in a cylindrical reservoir – either Eq. E.1 or Eq. E.2. Thus, this general expression includes transient flow and pseudosteady-state flow as special cases.

The value of this generalization becomes clearer if now we reconsider a subject we introduced in Chap. 3 – n changes in rate in the producing history of a well (Fig. E-1).

The total pressure drawdown at the well, expressed in terms of dimensionless variables, is found by superposition, as we showed in Chap. 3 for the special case of transient flow. Here, note that rate q_1 acts for time, t ; $(q_2 - q_1)$ for time $(t - t_1)$, ... ; and $(q_n - q_{n-1})$ for time $(t - t_{n-1})$. Thus, we write, in general,

$$\frac{0.00708 kh [p_i - p_{wf}(t_D)]}{B\mu} = (q_1 - 0)[p_D(t_D - 0) + s] + (q_2 - q_1)[p_D(t_D - t_{D1}) + s] + \dots \\ + (q_n - q_{n-1})[p_D(t_D - t_{D,n-1}) + s],$$

or, more compactly,

$$\frac{0.00708 kh [p_i - p_{wf}(t_D)]}{B\mu} = \left[\sum_{j=1}^n \Delta q_j p_D(t_D - t_{Dj}) \right] + q_n s, \quad \dots \dots \dots (E.4)$$

where

$$\Delta q_j = q_j - q_{j-1} \text{ (and } q_0 \equiv 0),$$

$$t_{D0} \equiv 0.$$

Eq. E.4 is general – i.e., it applies to a reservoir in which, for some values of $t_D - t_{Dj}$, p_D can be the pseudosteady-state solution, and, for other values of $t_D - t_{Dj}$, p_D can be the transient solution.

As an example, consider a pressure buildup test in a cylindrical reservoir. Let $q_1 = q$; $q_2 = 0$; $t_{D1} = t_{pD} + \Delta t_D$; and $t_{D2} - t_{D1} = \Delta t_D$. Then,

$$\frac{0.00708 kh (p_i - p_{ws})}{B\mu} = qp_D(t_{pD} + \Delta t_D)$$

$$- qp_D(\Delta t_D).$$

If and only if flow is transient for total time $t_{pD} + \Delta t_D$, then

$$\frac{0.00708 kh (p_i - p_{ws})}{B\mu} = \frac{1}{2} q \left[\ln(t_{pD} + \Delta t_D) \right.$$

$$\left. + 0.809 \right] - \frac{1}{2} q (\ln \Delta t_D + 0.809)$$

or

$$p_i - p_{ws} = 70.6 \frac{qB\mu}{kh} \ln \left(\frac{t_{pD} + \Delta t_D}{\Delta t_D} \right)$$

$$= 162.6 \frac{qB\mu}{kh} \log \left(\frac{t_p + \Delta t}{\Delta t} \right).$$

In fact, the arguments leading to Eq. E.4 understate its generality. For constant-rate flow in cylindrical reservoirs, p_D can be calculated for all t_D from simple equations – but the method is not restricted to cylindrical reservoirs. It applies to *any* drainage configuration for which p_D can be calculated as a function of t_D (using finite-difference simulation or any other convenient means).

We now examine a useful method¹⁻³ for determining p_D as a function of t_D for more general reservoir shapes; this method uses the Matthews-Brons-Hazebroek functions⁴ developed for use in determining average reservoir pressure. We start by noting that for a pressure buildup test in a reservoir of general shape,

$$\frac{0.00708 kh (p_i - p_{ws})}{qB\mu} = p_D(t_D + \Delta t_D) -$$

$$p_D(\Delta t_D). \quad \dots \dots \dots (E.5)$$

For Δt_D sufficiently small (e.g., $\Delta t_D \leq 0.25 r_{De}^2$ in a cylindrical reservoir), flow will be transient regardless of drainage-area configuration, and $p_D(\Delta t_D) = 1/2 (\ln \Delta t_D + 0.809)$. If we now add and subtract $1/2 \ln(t_D + \Delta t_D)$ on the right side of Eq. E.5 written at these sufficiently small values of Δt_D , the result is

$$\frac{0.00708 kh (p_i - p_{ws})}{qB\mu} = \frac{1}{2} \ln \left(\frac{t + \Delta t}{\Delta t} \right)$$

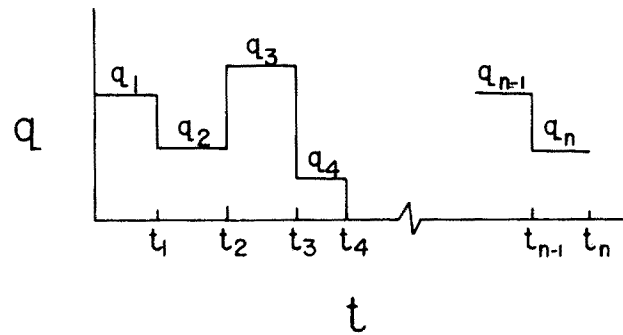


Fig. E-1 – n rate changes in well's producing history.

$$- \frac{1}{2} \left[\ln(t_D + \Delta t_D) + 0.809 \right] + p_D(t_D + \Delta t_D). \quad \dots \dots \dots (E.6)$$

Now, for some small values of $\Delta t_D \leq \Delta t_{Ds}$, $\ln(t_D + \Delta t_D) \approx \ln(t_D)$ and $p_D(t_D + \Delta t_D) \approx p_D(t_D)$.

For $\Delta t_D \leq \Delta t_{Ds}$, then,

$$\frac{0.00708 kh (p_i - p_{ws})}{qB\mu} = \frac{1}{2} \ln \left(\frac{t + \Delta t}{\Delta t} \right) + p_D(t_D)$$

$$- \frac{1}{2} \left[\ln(2.246 t_D) \right]$$

$$= \frac{1}{2} \ln \left(\frac{t + \Delta t}{\Delta t} \right) + \text{constant}. \quad \dots \dots \dots (E.7)$$

The implication of Eq. E.7 is simply that for sufficiently small Δt , a plot of p_{ws} vs. $\ln[(t + \Delta t)/\Delta t]$ or $\log[(t + \Delta t)/\Delta t]$ will be linear and will have a slope, m , related to permeability. This linear relationship exists, of course, only for sufficiently small values of Δt . Once we have established the linear relationship, though, we can *extrapolate* it to larger times. In particular, we can extrapolate p_{ws} to $(t + \Delta t)/\Delta t = 1$. Matthews *et al.*⁴ did this and chose to call the extrapolated pressure p^* . Eq. E.7 shows that

$$\frac{0.00708 kh}{qB\mu} (p_i - p^*) = p_D(t_D) - \frac{1}{2} \ln(2.246 t_D).$$

$$\dots \dots \dots (E.8)$$

A material balance shows that

$$c_t V_p (p_i - \bar{p}) = 5.615 qBt/24$$

$$= c_t A h \phi (p_i - \bar{p}), \quad \dots \dots \dots (E.9)$$

where A is the area drained by a well (square feet). (Eq. E.9 is valid regardless of drainage-area shape.)

Now define t_{DA} as $0.000264 kt/\phi\mu c_t A$. Substituting for t from Eq. E.9 gives

$$t_{DA} = \frac{0.000264 kt}{\phi\mu c_t A} = \frac{(0.000264)(kh)(24)(p_i - \bar{p})}{(5.615)(qB\mu)}$$

$$= \frac{0.00708 kh (p_i - \bar{p})}{2\pi qB\mu},$$

or

$$2\pi t_{DA} = \frac{0.00708 kh (p_i - \bar{p})}{qB\mu} \quad \dots \dots \dots (E.10)$$

Then, using Eqs. E.8 and E.10,

$$-\frac{0.00708 kh (p_i - p^*)}{qB\mu} + \frac{0.00708 kh (p_i - \bar{p})}{qB\mu} = -p_D(t_D) + \frac{1}{2} \ln(2.246 t_D) + 2\pi t_{DA}.$$

Simplifying,

$$\frac{(p^* - \bar{p})}{70.6 qB\mu} = 4\pi t_{DA} + \ln(2.246 t_D) - 2p_D(t_D) = p_{DMBH}(t_{DA}) \quad \dots \dots \dots (E.11)$$

Note that the left side of Eq. E.11 is the ordinate of the MBH plots (thus, we give it the name p_{DMBH}). Note also that $t_{DA} = 0.000264 kt/\phi\mu c_l A$ is the abscissa of the MBH plots.

Eq. E.11 allows us to use the MBH charts to determine $p_D(t_D)$ for the drainage-area configurations considered in constructing the charts. To understand why this is so, note that Eq. E.11 can be put in the form

$$p_D(t_D) = 2\pi t_{DA} + \frac{1}{2} \ln(2.246 t_D) - \frac{1}{2} p_{DMBH}(t_{DA}) \quad \dots \dots \dots (E.12)$$

For time, t_D , sufficiently small that no boundary effects have appeared (transient flow),

$$p_D(t_D) = \frac{1}{2} \ln(2.246 t_D), \quad t_D < t_{DB}.$$

For $t_D < t_{DB}$, then,

$$\begin{aligned} p_D(t_D) &= \frac{1}{2} \ln(2.246 t_D) \\ &= 2\pi t_{DA} + \frac{1}{2} \ln(2.246 t_D) \\ &\quad - \frac{1}{2} p_{DMBH}(t_{DA}), \end{aligned}$$

or

$$\begin{aligned} p_{DMBH}(t_{DA}) &= 4\pi t_{DA} \\ &= \frac{(p^* - \bar{p})}{70.6 qB\mu}, \quad t_D < t_{DB} \quad \dots \dots (E.13) \end{aligned}$$

Eq. E.13 shows that in transient flow (infinite-acting reservoir), p_{DMBH} is a linear function of t_{DA} . Thus, in principle, a plot of p_{DMBH} vs. t_{DA} could be used to determine the time t_{DA} up to which a reservoir of general shape is infinite acting (by observing the time at which a deviation from linearity occurs). In principle, then, this technique could have

been used to determine the values in Table 1.1 in the column "Use Infinite System Solution With Less than 1% Error for $t_{DA} <$," thus providing us with a means of determining the upper limit of applicability of the relationship $p_D(t_D) = 1/2 (\ln t_D + 0.809)$ in reservoirs of *general drainage-area configuration*.

Now we turn our attention to developing a method for determining $p_D(t_D)$ after pseudosteady-state conditions have been established. Brons and Miller⁵ pointed out that, at pseudosteady state,

$$\begin{aligned} p_{DMBH}(t_{DA}) &= \ln(C_A t_{DA}) \\ &= \ln C_A + \ln t_{DA}, \quad \dots \dots \dots (E.14) \end{aligned}$$

where C_A is a shape factor whose value depends on the specific drainage area configuration. This equation implies a linear relationship between p_{DMBH} and $\ln t_{DA}$, with intercept $\ln C_A$ varying from case to case. The MBH charts (Fig. 2.12) show that this linear relationship does exist and that the intercept does depend on the specific drainage-area configuration. Further, the time at which p_{DMBH} becomes a linear function of $\ln t_{DA}$ establishes the beginning of pseudosteady-state flow.

The task at hand is to show that Eq. E.14 leads to a method for determining $p_D(t_D)$ for general drainage-area shape for pseudosteady-state flow. If we substitute the value of $p_{DMBH}(t_{DA})$ from Eq. E.14 into Eq. E.12, the result is

$$\begin{aligned} p_D(t_D) &= 2\pi t_{DA} + 1/2 \ln(2.246 t_D) \\ &\quad - 1/2 \ln(C_A t_{DA}) \\ &= 2\pi t_{DA} + 1/2 \ln\left(\frac{2.246 t_D}{C_A t_{DA}}\right) \end{aligned}$$

or

$$p_D(t_D) = 2\pi t_{DA} + 1/2 \ln\left(\frac{2.246 A}{C_A r_w^2}\right) \quad \dots \dots (E.15)$$

Thus, we have established a rule by which $p_D(t_D)$ can be established for general drainage shape in pseudosteady-state flow. Table 1.1 gives the times ("Exact for $t_{DA} <$ " and "Less Than 1% Error for $t_{DA} <$ ") at which Eq. E.15 can be applied.

Finally, there is the problem of how to establish $p_D(t_D)$ for general drainage-area configuration when there is a gap between the upper limit of applicability of the transient solution and the lower limit of applicability of the pseudosteady-state solution. Eq. E.12, which applies at *all times*, can be used to fill this gap:

$$\begin{aligned} p_D(t_D) &= 2\pi t_{DA} + 1/2 \ln(2.246 t_D) \\ &\quad - 1/2 p_{DMBH}(t_{DA}). \end{aligned}$$

To avoid both t_{DA} and t_D on the right side of the same working equation, we can rewrite this result as

$$\begin{aligned} p_D(t_D) &= 2\pi t_{DA} + 1/2 \ln(2.246 t_{DA} A/r_w^2) \\ &\quad - 1/2 p_{DMBH}(t_{DA}) \quad \dots \dots \dots (E.16) \end{aligned}$$

Thus, values of p_{DMBH} could be read from their charts at a desired value of t_{DA} , and $p_D(t_D)$ could

be calculated for use in subsequent reservoir analysis.

Earlougher provides values of $p_D(t_D)$ for general drainage-area shapes in Ref. 6 Appendix C. For applications, the reader should find the desired p_D values in that reference.

In summary, this appendix has shown that well test analysis techniques are not limited to wells centered in cylindrical reservoirs or to infinite-acting reservoirs. One can derive or find in the literature dimensionless-pressure values for general drainage-area shapes; one can combine these values (using superposition) to take into account any arbitrary rate history before and during the test. Thus, the title of the appendix: A General Theory of Well Testing.

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Appendix F

Use of SI Units in Well-Testing Equations

This Appendix summarizes the changes required to solve the equations stated in the text by using International System (SI) metric units. To show the necessary changes and to allow the interested reader to apply the SI unit system to typical well-testing problems, this Appendix has four major parts: (1) conversion factors from “oilfield units” to SI units are tabulated in Table F-1 for the units used in the text; (2) a summary of preferred SI units for major variables is given in Table F-2; (3) major equations in the text are restated in Table F-3, with constants given in both oilfield and SI units; and (4) answers to all examples in the text are given in SI units. In addition, in the Nomenclature, preferred SI units are given (in parentheses) for each quantity used in the text.

TABLE F-1 – CONVERSION FACTORS

To Convert From	To	Multiply By	Inverse
acres	m ²	4.047 E + 03	2.471 E – 04
bbl	m ³	1.590 E – 01	6.290
cp	mPa · s	1.0	1.0
cp	μPa · s	1.0 E + 03	1.0 E – 03
cu ft	m ³	2.832 E – 02	3.532 E + 01
ft	m	3.048 E – 01	3.281
md	m · m ²	9.869 E – 01	1.013
psi	kPa	6.895	1.450 E – 01
°R	K	5.555 E – 01	1.80
sq ft	m ²	9.290 E – 02	1.076 E + 01

A more complete table of conversion factors, emphasizing application to well-testing problems, is given by Earlougher.¹

Table F-2 summarizes oilfield (customary) and preferred SI units (practical) for single variables and groups of variables of major importance in well test analysis.

TABLE F-2 – CUSTOMARY AND METRIC UNITS FOR MAJOR VARIABLES IN EQUATIONS

	Customary Unit	Practical Metric Unit
Compressibility, c_f	psi ⁻¹	kPa ⁻¹
Density, ρ	lbm/cu ft	kg/m ³
Gas flow rate, q_g	Mscf/D	m ³ /d
Gas viscosity, μ_g	cp	μPa · s
Liquid flow rate, q_l , and q_w	B/D	m ³ /d
Liquid viscosity, $\mu_{o,w}$	cp	mPa · s
Permeability, k	md	md
Pressure, p	psi	kPa
Pseudopressure, $\psi(p)$	psi ² /cp	MPa ² /Pa · s
Radius, r	ft	m
Slope, m	psi/cycle	kPa/cycle
Temperature, T	°R	K
Thickness, h	ft	m
Time, t	hr	h
Volume, V	bbl	m ³
Wellbore storage constant, C_w	bbl/psi	m ³ /kPa

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
1.1	$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi \mu c}{c_1 k} \frac{\partial p}{\partial t}$	0.000264	3.557×10^{-6}
1.6	$p_{wf} = p_i - c_1 \frac{qB\mu}{kh} \left\{ \frac{2t_D}{r_e^2 D} + \ln r_{eD} - \frac{3}{4} + 2 \sum_{n=1}^{\infty} \frac{e^{-\alpha_n^2 t_D} J_1^2(\alpha_n r_{eD})}{\alpha_n^2 [J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)]} \right\}$	141.2	1.866×10^3
1.7	$p = p_i + c_1 \frac{qB\mu}{kh} Ei\left(\frac{-c_2 \phi \mu c_l r^2}{kt}\right)$	70.6 948	9.33×10^2 7.036×10^4
1.9	$\Delta p_s = c_1 \frac{qB\mu}{kh} \left(\frac{k}{k_s} - 1 \right) \ln\left(\frac{r_s}{r_w}\right)$	141.2	1.866×10^3
1.11	$p_i - p_{wf} = -c_1 \frac{qB\mu}{kh} \left \ln\left(\frac{c_2 \phi \mu c_l r_w^2}{kt}\right) - 2s \right $	70.6 1,688	9.33×10^2 1.253×10^5
1.12	$p_{wf} = p_i - c_1 \frac{qB\mu}{kh} \left \frac{c_2 kt}{\phi \mu c_l r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right $	141.2 0.000527	1.866×10^3 7.1×10^{-6}
1.13	$\frac{\partial p_{wf}}{\partial t} = -\frac{c_1 qB}{c_l V_p}$	0.234	4.168×10^{-2}
1.16	$\bar{p} - p_{wf} = c_1 \frac{qB\mu}{kh} \left \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s \right $	141.2	1.866×10^3
1.17	$p_i - p_{wf} = c_1 \frac{qB\mu}{kh} \left \frac{c_2 kt}{\phi \mu c_l r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s \right $	141.2 0.000527	1.866×10^3 7.1×10^{-6}
1.19	$J \equiv \frac{q}{\bar{p} - p_{wf}} = \frac{k_j h}{c_1 B\mu \left \ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} \right }$	141.2	1.866×10^3
1.20	$\bar{p} - p_{wf} = c_1 \frac{qB\mu}{kh} \left \frac{1}{2} \ln\left(\frac{10.06 A}{C_A r_w^2}\right) - \frac{3}{4} + s \right $	141.2	1.866×10^3
1.21	$J = \frac{c_1 kh}{B\mu \left \frac{1}{2} \ln\left(\frac{10.06 A}{C_A r_w^2}\right) - \frac{3}{4} + s \right }$	0.00708	5.356×10^{-4}
1.26	$C_s = c_1 \frac{A_{wb}}{\rho} \frac{g_c}{g}$	25.65	101.98
1.29	$p_D = \frac{c_1 kh(p_i - p_w)}{q_i B\mu}$	0.00708	5.356×10^{-4}
1.46	$p_i - p_{wf} = c_1 \frac{qB}{hL_f} \left(\frac{\mu t}{k\phi c_l} \right)^{1/2}$	4.064	6.236
1.47	$r_i = \left(\frac{kt}{c_1 \phi \mu c_l} \right)^{1/2}$	948	7.036×10^4

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
1.48	$t_s = c_1 \phi \mu c_t r_e^2 / k$	948	7.036×10^4
1.52	$t_p = c_1 N_p / q_{\text{last}}$	24	24
2.1	$p_{ws} = p_i - c_1 \frac{qB\mu}{kh} \log[(t_p + \Delta t) / \Delta t]$	162.6	2.149×10^3
2.2	$m = c_1 \frac{qB\mu}{kh}$	162.6	2.149×10^3
2.4	$s = 1.151 \left \frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log\left(\frac{k}{\phi \mu c_t r_w^2}\right) + c_1 \right $	+ 3.23	5.10
2.7	$p_D = \frac{c_1 kh(p_{ws} - p_{wf})}{qB\mu}$	0.00708	5.356×10^{-4}
2.8	$t_D = \frac{c_1 k \Delta t_e}{\phi \mu c_t r_w^2}$	0.000264	3.557×10^{-6}
2.9	$C_{sD} = \frac{c_1 C_s}{\phi c_t h r_w^2}$	0.894	0.159
2.10	$\Delta t_e = \Delta t / (1 + \Delta t / t_p)$	—	—
2.11	$C_s = \frac{qB}{c_1} \frac{\Delta t}{\Delta p}$	24	24
2.12	$t_D \cong 50 C_{sD} e^{0.14 s}$	—	—
2.13	$t_{wbs} \cong c_1 \frac{C_s e^{0.14 s}}{(kh/\mu)}$	170,000	2.247×10^6
2.14	$r_{wa} = r_w e^{-s}$	—	—
2.15	$L_f \cong 2 r_{wa}$	—	—
2.16	$(\Delta p)_s = 0.869 m(s)$	—	—
2.17	$E = \frac{J_{\text{actual}}}{J_{\text{ideal}}} = \frac{\bar{p} - p_{wf} - (\Delta p)_s}{\bar{p} - p_{wf}}$	—	—
2.18	$E \cong \frac{p^* - p_{wf} - (\Delta p)_s}{p^* - p_{wf}}$	—	—
2.19	$s = \frac{h_t}{h_p} s_d + s_p$	—	—
2.20	$s_p = \left(\frac{h_t}{h_p} - 1 \right) \left[\ln \left(\frac{h_t}{r_w} \sqrt{\frac{k_H}{k_V}} \right) - 2 \right]$	—	—
2.21	$m_L = c_1 \frac{qB}{hL_f} \left(\frac{\mu}{k\phi c_t} \right)^{1/2}$	4.064	6.236
2.22	$\log(L_f) = \frac{1}{2} \left[\left(\frac{p_{wf} - p_{1 \text{ hr}}}{m} \right) + \log \frac{k}{\phi \mu c_t} - c_1 \right]$	2.63	4.497

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
2.24	$\log(\bar{p} - p_{ws}) = \log\left(c_1 \frac{qB\mu}{kh}\right) - \frac{c_2 k \Delta t}{\phi \mu c_t r_e^2}$	118.6 0.00168	227.37 - 0.8385
2.25	$p_i - p_{ws} = -c_1 \frac{qB\mu}{kh} \left\{ \ln \left \frac{c_2 \phi \mu c_t r_w^2}{k(t_p + \Delta t)} \right - 2s \right\}$ $- c_1 (-q) \frac{B\mu}{kh} \left \ln \left(\frac{c_2 \phi \mu c_t r_w^2}{k \Delta t} \right) - 2s \right $ $- c_1 \frac{qB\mu}{kh} Ei \left \frac{-c_3 \phi \mu c_t L^2}{k(t_p + \Delta t)} \right $ $- c_1 (-q) \frac{B\mu}{kh} Ei \left(\frac{-c_3 \phi \mu c_t L^2}{k \Delta t} \right)$	70.6 1,688 3,792	9.33×10^2 1.253×10^5 2.814×10^5
2.28	$\Delta p_{ws}^* = -c_1 \frac{qB\mu}{kh} \left -Ei \left(\frac{-c_2 \phi \mu c_t L^2}{k \Delta t} \right) \right $	70.6 3,792	9.33×10^2 2.814×10^5
2.29	$L = \sqrt{\frac{c_1 k \Delta t x}{\phi \mu c_t}}$	0.000148	1.944×10^{-6}
2.30	$V_R = \frac{(\Delta N_p)(B_o)}{(\bar{p}_2 - \bar{p}_1)c_t \phi}$	—	—
2.31	$p_{wf} = p_i + c_1 \frac{q_g B_{gi} \mu_i}{kh} \left \log \left(\frac{c_2 \phi \mu c_{ti}}{kt_p} \right) - \frac{(s + Dq_g)}{1.151} \right $	162.6 1,688	2.149 11.638
2.32	$p_{wf}^2 = p_i^2 + \frac{c_1 q_g \mu_i z_i T}{kh} \left \log \left(\frac{c_2 \phi \mu c_{ti}}{kt_p} \right) - \frac{(s + Dq_g)}{1.151} \right $	1,637 1,688	1.508 11.638
2.33	$p_{ws} = p_i - c_1 \frac{q_g B_{gi} \mu_i}{kh} \left \log \left(\frac{t_p + \Delta t}{\Delta t} \right) \right $	162.6	2.149
2.34	$s' = s + Dq_g = 1.151 \left \frac{(p_{1 \text{ hr}} - p_{wf})}{m} - \log \left(\frac{k}{\phi \mu_i c_{ti} r_w^2} \right) + c_1 \right $	3.23	2.10
2.35	$p_{ws}^2 = p_i^2 - c_1 \frac{q_g \mu_i z_i T}{kh} \log \left(\frac{t_p + \Delta t}{\Delta t} \right)$	1,637	1.508
2.36	$s' = s + Dq_g = 1.151 \left[\frac{(p_{1 \text{ hr}}^2 - p_{wf}^2)}{m''} - \log \left(\frac{k}{\phi \mu_i c_{ti} r_w^2} \right) + c_1 \right]$	3.23	2.10
2.37	$p_{wf} = p_i + c_1 \frac{q R_t}{\lambda_t h} \left \log \left(\frac{c_2 \phi c_t r_w^2}{\lambda_t t} \right) - \frac{s}{1.151} \right $	162.6 1,688	2.149×10^3 1.253×10^5
2.38	$p_{ws} = p_i - c_1 \frac{q R_t}{\lambda_t h} \log \left(\frac{t_p + \Delta t}{\Delta t} \right)$	162.6	2.149×10^3
2.39	$q_{Rt} = q_o B_o + \left(q_g - \frac{q_o R_s}{1,000} \right) B_g + q_w R_w$	—	—
2.40	$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} + \frac{k_g}{\mu_g}$	—	—

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
2.41	$\lambda_t = c_1 \frac{q R_t}{mh}$	162.6	2.149×10^3
2.42	$k_o = c_1 \frac{q_o B_o \mu_o}{mh}$	162.6	2.149×10^3
2.43	$k_g = c_1 \frac{\left(q_g - \frac{q_o R_s}{c_2}\right) B_g \mu_g}{mh}$	162.6 1,000	2.149 1.000
2.44	$k_w = c_1 \frac{q_w B_w \mu_w}{mh}$	162.6	2.149×10^3
2.45	$s = 1.151 \left[\frac{p_{1 \text{ hr}} - p_{wf}}{m} - \log \left(\frac{\lambda_t}{\phi c_t r_w^2} \right) + c_1 \right]$	3.23	5.10
3.1	$p_{wf} = p_i + c_1 \frac{q B \mu}{kh} \left[\log \left(\frac{c_2 \phi \mu c_t r_w^2}{kt} \right) - 0.869s \right]$	162.6 1,688	2.149×10^3 1.253×10^5
3.2	$t_{wbs} = \frac{(c_1 + c_2 s) C_s}{kh/\mu}$	200,000 12,000	2.644×10^6 1.586×10^5
3.4	$k = c_1 \frac{q B \mu}{mh}$	162.6	2.149×10^3
3.5	$s = 1.151 \left[\frac{(p_i - p_{1 \text{ hr}})}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + c_1 \right]$	3.23	5.10
3.7	$t_{e,t} = \frac{c_1 \phi \mu c_t A t_{DA}}{k}$	3,800	2.82×10^5
3.8	$\frac{p_i - p_{wf}}{q} = c_1 \frac{\mu B}{kh} \left[\log \left(\frac{c_2 \phi \mu c_t r_w^2}{kt} \right) + 0.869s \right]$	162.6 1,688	2.149×10^3 1.253×10^5
3.9	$s = 1.151 \left[\left(\frac{p_i - p_{wf}}{q} \right)_{1 \text{ hr}} \frac{1}{m'} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + c_1 \right]$	3.23	5.10
3.11	$\frac{p_i - p_{wf}}{q_n} = c_1 \frac{\mu B}{kh} \left\{ \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t - t_{j-1}) + \left[\log \left(\frac{k}{\phi \mu c_t r_w^2} \right) - c_2 + 0.869s \right] \right\}$	162.6 3.23	2.149×10^3 5.10
3.12	$p_i - p_{ws} = c_1 \frac{\mu B}{kh} \sum_{j=1}^n (q_j - q_{j-1}) \log(t - t_{j-1})$	162.6	2.149×10^3
3.15	$p_i - p_{ws} = c_1 \frac{q_2 B \mu}{kh} \left[\frac{q_1}{q_2} \log \left(\frac{t_{p1} + t_{p2} + \Delta t}{t_{p2} + \Delta t} \right) + \log \left(\frac{t_{p2} + \Delta t}{\Delta t} \right) \right]$	162.6	2.149×10^3
3.18	$p_{wf} = p_i - c_1 \frac{q_2 B \mu}{kh} \left[\log \left(\frac{k}{\phi \mu c_t r_w^2} \right) - c_2 + 0.869s \right] - c_1 \frac{q_1 B \mu}{kh} \left[\log \left(\frac{t_{p1} + \Delta t'}{\Delta t'} \right) + \frac{q_2}{q_1} \log(\Delta t') \right]$	162.6 3.23	2.149×10^3 5.10

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
3.19	$s = 1.151 \left[\frac{q_1}{(q_1 - q_2)} \left(\frac{p_{1 \text{ hr}} - p_{wf1}}{m} \right) - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + c_1 \right]$	3.23	5.10
3.20	$p_i = p_{wf1} + m \left[\log \left(\frac{k t p_1}{\phi \mu c_t r_w^2} \right) - c_1 + 0.869s \right]$	3.23	5.10
3.21	$s = 1.151 \left[\frac{b'}{m'} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + c_1 \right]$	3.23	5.10
4.1	$C_s = c_1 \frac{A_{wb}}{\rho}$	25.65	101.98
4.2	$C_s = c_{wb} V_{wb}$	—	—
4.3	$C_{sD} = c_1 C_s / \phi c_t h r_w^2$	0.894	0.159
4.4	$k = c_1 \frac{q B \mu}{h} \left(\frac{p_D}{p_i - p_{wf}} \right)_{\text{MP}}$	141.2	1.866×10^3
4.5	$\phi c_t = \frac{c_1 k}{\mu r_w^2} \left(\frac{t}{t_D} \right)_{\text{MP}}$	0.000264	3.557×10^{-6}
4.6	$t_D = \frac{c_1 k t}{\phi \mu_i c_{gi} r_w^2}$	0.000264	3.557×10^{-3}
4.7	$\psi_D = \frac{k h T_{sc} [\psi(p_i) - \psi(p_{wf})]}{c_1 p_{sc} q_g T}$	50,300	3.733
4.8	$s' = s + D q_g $	—	—
4.9	$\psi(p) = 2 \int_{p_\beta}^p \frac{p}{\mu(p) z(p)} dp$	—	—
4.10	$p_D = \frac{k h (p_i - p_{wf})}{c_1 q_g \mu_i B_{gi}}$	141.2	1.866
4.11	$B_{gi} = c_1 \frac{z_i T}{p_i}$	5.04	0.351
4.12	$k = c_1 \frac{q_g \mu_i B_{gi}}{h} \left(\frac{p_D}{p_i - p_{wf}} \right)_{\text{MP}}$	141.2	1.866
4.13	$\phi c_{ti} = \frac{c_1 k}{\mu_i r_w^2} \left(\frac{t}{t_D} \right)_{\text{MP}}$	0.000264	3.557×10^{-3}
4.14	$p_D = \frac{k h (p_i^2 - p_{wf}^2)}{c_1 q_g \mu_i z_i T}$	1,422	1.309
4.15	$k = c_1 \frac{q_g \mu_i z_i T}{h} \left(\frac{p_D}{p_i^2 - p_{wf}^2} \right)_{\text{MP}}$	1,422	1.309
4.16	$\phi c_{ti} = \frac{c_1 k}{\mu_i r_w^2} \left(\frac{t}{t_D} \right)_{\text{MP}}$	0.000264	3.557×10^{-3}

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
4.17	$t_{DL_f} = \frac{c_1 kt}{\phi \mu c_t L_f^2}$	0.000264	3.557×10^{-6}
4.18	$L_f = \left[\frac{c_1 k}{\phi \mu c_t} \frac{t_{MP}}{(t_{DL_f})_{MP}} \right]^{1/2}$	0.000264	3.557×10^{-6}
5.1	$\psi(p_{wf}) = \psi(p_i) + c_1 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[1.151 \log \left(\frac{c_2 \phi \mu_i c_{ti} r_w^2}{kt} \right) - (s + D q_g) \right]$	50,300 1,688	3.733 125.3
5.2	$\psi(p) = 2 \int_{p_b}^p \frac{p}{\mu z} dp$	—	—
5.3	$\psi(p_{wf}) = \psi(\bar{p}) - c_1 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s + D q_g \right]$	50,300	3.733
5.4	$\psi(p_{wf}) = \psi(\bar{p}) + c_1 \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[1.151 \log \left(\frac{c_2 \phi \mu_{\bar{p}} c_{i\bar{p}} r_w^2}{kt} \right) - (s + D q_g) \right]$	50,300 1,688	3.733 125.3
5.5	$p_{wf}^2 = \bar{p}^2 + c_1 \frac{q_g \mu_{\bar{p}} z_{\bar{p}} T}{kh} \left[\log \left(\frac{c_2 \phi \mu_{\bar{p}} c_{i\bar{p}}}{kt_p} \right) - \left(\frac{s + D q_g }{1.151} \right) \right]$	1,637 1,688	1.508 11.638
5.6	$p_{wf}^2 = \bar{p}^2 - c_1 \frac{q_g \mu_{\bar{p}} z_{\bar{p}} T}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s + D q_g \right]$	1,422	1.309
5.7	$\bar{p}^2 - p_{wf}^2 = a q_g + b q_g^2$	—	—
5.8	$a = c_1 \frac{\mu_{\bar{p}} z_{\bar{p}} T}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s \right]$	1,422	1.309
5.9	$b = c_1 \frac{\mu_{\bar{p}} z_{\bar{p}} T}{kh} D$	1,422	1.309
5.10	$q_g = C(\bar{p}^2 - p_{wf}^2)^n$	—	—
5.11	$\bar{p}^2 - p_{wf}^2 = a_t q_g + b q_g^2$	—	—
5.12	$a_t = c_1 \frac{\mu_{\bar{p}} z_{\bar{p}} T}{kh} \left[\frac{1}{2} \ln \left(\frac{kt}{c_2 \phi \mu_{\bar{p}} c_{i\bar{p}} r_w^2} \right) + s \right]$	1,422 1,688	1.309 125.3
6.1	$p_i - p_r = -c_1 \frac{q B \mu}{kh} Ei \left(-\frac{c_2 \phi \mu c_t r^2}{kt} \right)$	70.6 948	9.33×10^2 7.036×10^4
6.2	$p_D = -\frac{1}{2} Ei \left(\frac{-r_D^2}{4t_D} \right)$	—	—
A.1	$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = -\frac{\partial}{\partial t}(\rho \phi)$	—	—

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
A.2	$\frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r) = - \frac{\partial}{\partial t} (\rho \phi)$	—	—
A.4	$\frac{\partial}{\partial x} \left(\frac{k_x \rho}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k_y \rho}{\mu} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial z} \left[\frac{k_z \rho}{\mu} \left(\frac{\partial p}{\partial z} + 0.00694 \rho \right) \right]$ $= \frac{1}{c_1} \frac{\partial}{\partial t} (\rho \phi)$	0.000264	3.553×10^{-6}
A.5	$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r \rho k_r}{\mu} \frac{\partial p}{\partial r} \right) = \frac{1}{c_1} \frac{\partial}{\partial t} (\rho \phi)$	0.000264	3.553×10^{-6}
A.8	$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\phi \mu c}{c_1 k} \frac{\partial p}{\partial t}$	0.000264	3.553×10^{-6}
A.9	$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{c_1 k} \frac{\partial p}{\partial t}$	0.000264	3.553×10^{-6}
A.13	$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) = \frac{\phi \mu c_g}{c_1 k} \frac{\partial \psi}{\partial t}$	0.000264	3.553×10^{-6}
A.14	$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi c_t}{c_1 \lambda_t} \frac{\partial p}{\partial t}$	0.000264	3.553×10^{-6}
A.15	$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w}$	—	—
A.16	$c_t = S_o c_o + S_w c_w + S_g c_g + c_f$	—	—
A.17	$c_o = - \frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_s}{dp}$	—	—
A.18	$c_w = - \frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp}$	—	—
D.5	$B_g = c_1 \frac{Tz}{p}$	0.00504	0.351
E.1	$\frac{c_1 kh(p_i - p_{wf})}{qB\mu} = \frac{1}{2} (\ln t_D + c_2) + s$	0.00708 0.809	5.356×10^{-4} -1.062
E.2	$\frac{c_1 kh(p_i - p_{wf})}{qB\mu} = \left(\frac{2t_D}{r_{De}^2} + \ln r_{De} - 0.75 \right) + s$ $= p_D + s, t_D > 0.25 r_{De}^2$	0.00708	5.356×10^{-4}
E.3	$\frac{c_1 kh(p_i - p_{wf})}{qB\mu} = p_D(t_D) + s$	0.00708	5.356×10^{-4}
E.4	$\frac{c_1 kh[p_i - p_{wf}(t_D)]}{B\mu} = \left[\sum_{j=1}^n \Delta q_j p_D(t_D - t_{Dj}) \right] + q_n s$	0.00708	5.356×10^{-4}
E.5	$\frac{c_1 kh(p_i - p_{ws})}{qB\mu} = p_D(t_D + \Delta t_D) - p_D(\Delta t_D)$	0.00708	5.356×10^{-4}

TABLE F-3 – MAJOR EQUATIONS WITH CONSTANT VALUES IN CUSTOMARY AND SI METRIC (Contd.)

Equation Number in Text	Equation	Numerical Value of Constants in Order of Appearance ($c_1, c_2, c_3 \dots$)	
		Customary	SI
E.6	$\frac{c_1 kh(p_i - p_{ws})}{qB\mu} = \frac{1}{2} \ln\left(\frac{t + \Delta t}{\Delta t}\right) - \frac{1}{2} [\ln(t_D + \Delta t_D) + 0.809] + p_D(t_D + \Delta t_D)$	0.00708	5.356×10^{-4}
E.7	$\frac{c_1 kh(p_i - p_{ws})}{qB\mu} = \frac{1}{2} \ln\left(\frac{t + \Delta t}{\Delta t}\right) + p_D(t_D) - \frac{1}{2} [\ln(2.246 t_D)]$ $= \frac{1}{2} \ln\left(\frac{t + \Delta t}{\Delta t}\right) + \text{constant}$	0.00708	5.356×10^{-4}
E.8	$\frac{c_1 kh(p_i - p^*)}{qB\mu} = p_D(t_D) - \frac{1}{2} \ln(2.246 t_D)$	0.00708	5.356×10^{-4}
E.9	$c_1 V_p(p_i - \bar{p}) = c_1 qBt = c_1 Ah\phi(p_i - \bar{p})$	0.2339	4.167×10^{-2}
E.10	$2\pi t_{DA} = \frac{c_1 kh(p_i - \bar{p})}{qB\mu}$	0.00708	3.975×10^{-2}
E.11	$\frac{(p^* - \bar{p})}{c_1 qB\mu} = 4\pi t_{DA} + \ln(2.246 t_D) - 2p_D(t_D)$ $= p_{DMBH}(t_{DA})$	70.6	9.33×10^2
E.12	$p_D(t_D) = 2\pi t_{DA} + 0.5 \ln(2.246 t_D) - 0.5 p_{DMBH}(t_{DA})$	—	—
E.13	$p_{DMBH}(t_{DA}) = 4\pi t_{DA} = \frac{(p^* - \bar{p})}{c_1 qB\mu}, t_D < t_{DB}$	70.6	41.25
E.14	$p_{DMBH}(t_{DA}) = \ln(C_A t_{DA}) = \ln C_A + \ln t_{DA} - c_3$	—	—
E.15	$p_D(t_D) = 2\pi t_{DA} + 0.05 \ln\left(\frac{2.246 A}{C_A r_w^2}\right)$	—	—
E.16	$p_D(t_D) = 2\pi t_{DA} + 0.5 \ln(2.246 t_{DA} A / r_w^2) - 0.5 p_{DMBH}(t_{DA})$	—	—

Answers to Examples Expressed in SI Units

Chap. 1

Example 1.1. $p_1' = 17\,740$ kPa, $p_{10}' = 20\,464$ kPa, $p_{100}' = 20\,684$ kPa.

Example 1.2. $J = 4.612 \times 10^{-3}$ m³/kPa·d, $k_J = 16$ md, $s = 16$.

Example 1.3.

Geometry	Infinite Solution		Pseudosteadystate (Approximate)		Pseudosteadystate (Exact)	
	t_{DA}	$t(\text{hours})$	t_{DA}	$t(\text{hours})$	t_{DA}	$t(\text{hours})$
Circular	0.10	132	0.06	79.2	0.1	132
Square-centered	0.09	119	0.05	66.0	0.1	132
Square-quadrant	0.025	33	0.30	396.0	0.6	792

Geometry	J		$\frac{q}{(m^3/d)}$
	C_A	(m ³ /d·kPa)	
Circular	31.62	1.213×10^{-2}	41.81
Square-centered	30.88	1.213×10^{-2}	41.81
Square-quadrant	4.513	1.116×10^{-2}	38.47

$t = 30$ hours: $p_i - p_{wf}$

$$= -933 \frac{qB\mu}{kh} \left[\ln\left(\frac{1.253 \times 10^5 \phi \mu c_i r_w^2}{kt}\right) - 2s \right]$$

$t = 200$ hours: no simple equation can be written.

$t = 400$ hours: $\bar{p} - p_{wf}$

$$= 1.866 \times 10^3 \frac{qB\mu}{kh} \left[\frac{1}{2} \ln\left(\frac{10.06 A}{C_A r_w^2}\right) - \frac{3}{4} + s \right]$$

Example 1.4. $t = 75.8$ hours.

Example 1.5. $\Delta p = 112.73$ kPa.

Example 1.6. $t_p = 176$ hours.

Chap. 2**Example 2.1.** $k = 48$ md, $p_i = 13\,445$ kPa, $s = 1.43$.**Example 2.2.** $\Delta t = 6$ hours, $\Delta t = 50$ hours.**Example 2.3.** $k = 7.65$ md.**Example 2.4.** $s = 6.37$, $r_{wa} = 1.036 \times 10^{-4}$ m, $(\Delta p)_s = 2668$ kPa, $E = 0.629$, $t_{ws} = 7.42$ hours.**Example 2.5.** $s = 12.3$, $s_p = 13.2$, $s_d = -0.18$.**Example 2.6.** $\bar{p} = 30\,413$ kPa.**Example 2.7.** $\bar{p} = 30\,420$ kPa.**Example 2.8.** $L = 72.8$ m.**Example 2.9.** $A_r = 5.707 \times 10^6$ m².**Example 2.10.** $k = 9.96$ md, $s' = 4.84$; $k = 9.77$ md, $s' = 4.27$.**Example 2.11.** $\lambda_i = 0.0457$ md/Pa·s, $k_o = 26.2$ md, $k_w = 1.49$ md, $k_g = 0.782$ md, $s = 1.50$.**Chap. 3****Example 3.1.** $k = 7.65$ md, $s = 6.37$.**Example 3.2.** $V_p = 4.992 \times 10^5$ res m³.**Example 3.3.** $k = 7.44$ md, $s = 6.02$.**Example 3.4.** $k = 7.65$ md, $s = 6.32$, $p^* = 30\,385$ kPa.**Example 3.5.** $kh = 31.7$ md·m.**Chap. 4****Example 4.1.** $k = 10.3$ md, $s = 5.0$, $C_s = 2.675 \times 10^{-4}$ m³/kPa.**Example 4.2.** $k_{wb} = 4.01$ md, $k_f = 8.03$ md, $E = 0.607$.**Example 4.3.** $L_f = 18.2$ m, $k = 4.5$ md.**Chap. 5****Example 5.1.** $q_g = \text{AOF} = 1.47 \times 10^6$ m³/d.**Example 5.2.** $q_g = \text{AOF} = 2.373 \times 10^5$ m³/d.**Example 5.3.** $q_g = \text{AOF} = 3.115 \times 10^5$ m³/d.**Example 5.4.** Results are tabulated in Table 5.10 and are plotted in Fig. 5.13.**Example 5.5.** $k = 9.66$ md, $s' = -0.21$.**Chap. 6****Example 6.1.** $k = 1433$ md, $c_i = 3.974 \times 10^{-4}$ kPa⁻¹.**Example 6.2.** $k = 817$ md, $\phi c_i = 1.973 \times 10^{-7}$ kPa⁻¹, $c_i = 2.465 \times 10^{-6}$ kPa⁻¹.**Appendix C****Example C.1.**

t (hour)	p (kPa)
0.001	98.46
0.01	94.04
0.1	65.71

Example C.2.

t (days)	p_{wf} (kPa)
0.1	18 857
1.0	18 506
100.0	18 375

Example C.3. $Q_p = 0.0370$ m³.**Example C.4.** $Q_p = 3.975 \times 10^4$ m³.**Appendix D****Example D.1.** $T_{pc} = 644$ K, $p_{pc} = 1965$ kPa.**Example D.2.** $p_{bwb} = 13\,307$ kPa.**Example D.3.** $\text{GOR} = 62.34$ m³/m³.**Example D.4.** $B_o = 1.22$ m³/m³.**Example D.5.** $c_o = 5.09 \times 10^{-7}$ kPa⁻¹.**Example D.6.** $c_o = 2 \times 10^{-5}$ kPa⁻¹.**Example D.7.** $\mu_o = 1.21$ mPa·s.**Example D.8.** $R_{sw} = 3.313$ m³/m³.**Example D.9.** $B_w = 1.029$ m³/m³.**Example D.10.** $c_w = 5.076 \times 10^{-7}$ kPa⁻¹.**Example D.11.** $c_w = 0.957 \times 10^{-6}$ kPa⁻¹.**Example D.12.** $\mu_w = 0.33$ mPa·s.**Example D.13.** $T_{pc} = 216.6$ K, $p_{pc} = 4585$ kPa.**Example D.14.** $B_g = 6.355 \times 10^{-3}$ m³/m³.**Example D.15.** $c_g = 5.656 \times 10^{-4}$ kPa⁻¹.**Example D.16.** $\mu_g = 17.8$ μPa·s.**Example D.17.** $c_f = 5.22 \times 10^{-7}$ kPa⁻¹.**Reference**

- Earlougher, R.C. Jr.: *Advances in Well Test Analysis*, Monograph Series, SPE, Dallas (1977) 5.

Appendix G

Answers to Selected Exercises

Chap. 1.

Exercise 1.1.

x	$Ei(x)$	$\ln(1.781x)$
0.01	-4.038	-4.028
0.02	-3.355	-3.335
0.1	-1.823	-1.725
1.0	-0.219	0.5772

Exercise 1.2.

r (ft)	p (psi)
0.333	2,812
1.0	2,837
10	2,888
100	2,940
1,000	2,988
3,160	3,000

$$r_e = 1,989 \text{ ft.}$$

Exercise 1.3. $r_e > 1,989 \text{ ft.}$

Exercise 1.4. (a) $t > 9.68$ seconds; (b) $t \geq 1.21$ seconds; (c) $t \geq 18$ weeks.

Exercise 1.5. $r_i = 1,989 \text{ ft; } r = 1,490 \text{ ft; } \Delta p = 2.45 \text{ psi.}$

Exercise 1.6. $t_s = 25.2$ days.

Exercise 1.7. r_i unchanged, double drawdown at each r .

Exercise 1.8. $(p_i - p_{wf})_{\text{total at A}}$

$$= -70.6 \frac{q_A \mu B}{kh} \left\{ \ln \left[\frac{1.688 \phi \mu c_i r_{wa}^2}{k(t-t_A)} \right] - 2S_A \right\} \\ - 70.6 \frac{q_B \mu B}{kh} Ei \left[\frac{-948 \phi \mu c_i r_{AB}^2}{k(t-t_B)} \right] \\ - 70.6 \frac{q_C \mu B}{kh} Ei \left[\frac{-948 \phi \mu c_i r_{AC}^2}{k(t-t_C)} \right]$$

Assumptions: Sufficiently far from *each* well that *Ei*-function solution can be used for each well.

Exercise 1.9. (a) $p_{wf} = 2,680 \text{ psi}$; (b) $p_{\text{shut-in}} = 2,961 \text{ psi.}$

Exercise 1.10. $p = 2,486 \text{ psi.}$

Exercise 1.11. No influence.

Ignore production before long shut-in in calculating t_p .

Exercise 1.12. (a) $t_p = 126$ hours; $t = 192$ hours (actual total producing time); (b)

r (ft)	p (psi)	
	Super-position	Homer Approximation
1	2,239	2,224
10	2,499	2,484
100	2,760	2,744
1,000	2,972	2,974
2,000	2,996	2,993

Exercise 1.3.

Shape	Infinite-Acting (hours)	PSS (hours)	J (STB/D/psi)	q (STB/D)
	13.2	7.92	0.5262	263.1
	13.2	7.92	0.5262	263.1
	11.9	9.24	0.5230	261.5
	11.9	9.24	0.5225	261.3
	10.6	15.84	0.5177	258.9
	1.98	79.2	0.4184	209.2
	11.9	6.6	0.5256	262.8
	3.96	33.0	0.5061	253.0

Chap. 2

Exercise 2.1. (a) error=0.00689; (b) no difference, $p_{ws}=1,150$ psi; (c) $r_e > 1,233$ ft.

Exercise 2.3. $t_{wbs} \approx 7.85$ hours.

Exercise 2.4. MTR begins at ≈ 7.85 hours; $k=24.5$ md.

Exercise 2.5. $s=0.064$; $(\Delta p)_s=32$ psi; $E=0.992$; $r_{wa}=0.469$ ft.

Exercise 2.7. $\bar{p}=4,308$ psi (p^* method); $\bar{p} \approx 4,405$ psi (modified Muskat method).

Exercise 2.9. $k=9.20$ md; $s+Dq_g \approx -0.952$.

Exercise 2.10. $k_o=32.5$ md; $k_w=3.48$ md; $k_g=1.18$ md; $\lambda_i=80.52$ md/cp; $s=-2.15$.

Exercise 2.11. $L=64$ ft.

Exercise 2.12. (a) in psi:

r (ft)	Δt (days)			
	0	0.1	1.0	10
1.0	2.837	2.948	2.973	2.992
10	2.862	2.948	2.973	2.992
10^2	2.888	2.948	2.973	2.992
10^3	2.914	2.949	2.973	2.992
10^4	2.939	2.951	2.973	2.992
10^5	2.965	2.965	2.976	2.993
10^6	2.988	2.988	2.988	2.994
10^7	3.000	3.000	3.000	3.000

(b)	Δt (days)	r_i (ft)
	0	0
	0.1	200
	1.0	629
	10	1,989

Exercise 2.13. $\bar{p}=4,418$ psi (using t_{pss}); $\bar{p}=4,411$ psi (using t_p).

Exercise 2.14. $t_{wbs} \approx 5$ hours; $k=48$ md; $r_i=371$ ft (at beginning), $r_i=813$ ft (at end); $s=10.93$, $\Delta p_s=950$ psi, $E=0.430$; $\bar{p}=4,325$ psi (MBH p^* method); $\bar{p}=4,325$ psi (modified Muskat method).

Chap. 3

Exercise 3.1. $k=9.55$ md; $s=4.45$; $A=67.9$ acres.

Exercise 3.2. $k=11.1$ md; $s=4.14$.

Exercise 3.3. $k=12.52$ md; $s=4.71$; $p^* \approx 4,380$ psi.

(a)	t (hours)	Plotting Function
	0.5	-0.301
	1.5	0.428
	2.5	1.331

(b) $p_{wf}=2,105$ psi.

Chap. 4

Exercise 4.1. $k=9.68$ md; $s=4.59$; $E=0.636$; $t_{wbs}=7.3$ hours; $V_p=34.2 \times 10^6$ cu ft; $r_e=986$ ft calculated; $r_i=226$ ft (beginning of MTR); $r_i=1,010$ ft (end of MTR).

Exercise 4.2. $s=5.0$ (for $C_{SD} \approx 10^3$); $t_{wbs} \approx 5$ hours; $k=10.3$ md; $C_s=0.0116$ RB/psi.

Exercise 4.3. $C_s=0.01$ RB/psi; $k_{wb}=4.01$ md; $k=8.03$ md; $E=0.606$.

Exercise 4.4. (1) $k=10.1$ md, $s=5.05$, $E=0.594$, $t_{wbs}=6.9$ hours; (2) $\bar{p}=2,854$ psia (p^* method), $\bar{p}=2,864$ psia (modified Muskat method); (3) $V_R=3.01 \times 10^7$ res bbl.

Exercise 4.5. $C_{SD}=10^3$; $s=5$; $k=9.92$ md; $V_R=3.01 \times 10^7$ res bbl.

Exercise 4.6. $C_s=0.0103$; $k_{wb}=4.13$ md; $k=8.26$ md; $E=0.607$.

Exercise 4.8. Type-curve analysis: $L_f=200$ ft; $k=15.2$ md. Conventional analysis: $k=15.4$ md; $L_f=144$ ft. Square root analysis: $L_f=232$ ft.

Chap. 5

Exercise 5.1. (a) AOF=107.0 MMscf/D; (b) AOF=100.0 MMscf/D.

Exercise 5.2. (a) AOF=6.6 MMscf/D (empirical method); (b) AOF=5.6 MMscf/D (theoretical method).

Exercise 5.4. $k=10.3$ md; $s'=0.533$.

Chap. 6

Exercise 6.1. $t=851$ hours; $r_i=12,510$ ft; $\Delta p=68.6$ psi.

Exercise 6.3. $k=101$ md; $s=-2.10$; $E=1.46$; $r_i=290$ ft.

Exercise 6.4. $k=0.52$ md.

Appendix A

Exercise A.1. $\frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r) = - \frac{\partial}{\partial t} (\rho \phi).$

Exercise A.2. $\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r \rho k_r}{\mu} \frac{\partial p}{\partial r} \right) = \frac{1}{0.000264} \frac{\partial}{\partial t} (\rho \phi).$

Exercise A.3. $\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{0.000264 k} \frac{\partial p}{\partial t}.$

Exercise A.4. $\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) = \frac{\phi \mu c_g}{0.000264 k} \frac{\partial \psi}{\partial t}.$

Appendix B

Exercise B.1. Dimensional form:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) = \frac{\phi \mu c_g}{0.000264k} \frac{\partial \psi}{\partial t};$$

Dimensionless form:

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial \psi_D}{\partial r_D} \right) = \frac{\partial \psi_D}{\partial t_D}.$$

Appendix C

Exercise C.1. $q_o = 979$ STB/D.

$$\Delta N_p = 1,056 \text{ STB.}$$

Exercise C.2. $p_{wf} = 3,150$ psia; $N_p = 11,900$ STB.

Exercise C.3.

t (days)	Cumulative Water Influx (res bbl)
100	3.278×10^4
200	5.377×10^4
400	8.959×10^4
800	14.68×10^4

Exercise C.4. Cumulative water influx = 2.342×10^5 res bbl.

Appendix D

Exercise D.1. $q_{Rf} = 145$ RB/D, $\lambda_f = 35.0$ md/cp, $c_f = 9.55 \times 10^{-6}$ psi⁻¹

Exercise D.2. $q_{Rf} = 414$ RB/D, $\lambda_f = 655$ md/cp, $c_f = 1.74 \times 10^{-4}$ psi⁻¹.

Nomenclature

$a = 1,422 \frac{\mu \bar{p} z \bar{p}_g T}{kh} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s \right]$	$g =$ acceleration of gravity, ft/sec ² (m/s ²)
$A =$ drainage area of well, sq ft (m ²)	$g_c =$ gravitational units conversion factor, 32.17 (lbm/ft)/(lbf-s ²), dimensionless
$A_f =$ fracture area, sq ft (m ²)	$h =$ net formation thickness, ft (m)
$A_R =$ reservoir area, acres (m ²)	$J =$ productivity index, STB/D-psi (m ³ /d·kPa)
$A_{wb} =$ wellbore area, sq ft (m ²)	$J_{\text{actual}} =$ actual or observed well productivity index, STB/D-psi (m ³ /d·kPa)
$b = 1,422 \frac{\mu \bar{p} z \bar{p}_g TD}{kh}$	$J_{\text{ideal}} =$ productivity index with permeability unaltered to sandface, STB/D-psi (m ³ /d·kPa)
$b' =$ intercept of $(p_i - p_{wf})/q_n$ plot, psi/STB-D (kPa/m ³ /d)	$J_g =$ gas-well productivity index, Mcf/D-psi (m ³ /d·kPa)
$B =$ formation volume factor, res vol/surface vol	$J_1 =$ Bessel function
$B_g =$ gas formation volume factor, RB/Mscf (m ³ /m ³)	$k =$ reservoir rock permeability, md
$B_{gi} =$ gas formation volume factor evaluated at p_i , RB/Mscf (m ³ /m ³)	$k_f =$ formation permeability (McKinley method), md
$B_o =$ oil formation volume factor, RB/STB (m ³ /m ³)	$k_g =$ permeability to gas, md
$B_w =$ water formation volume factor, RB/STB (m ³ /m ³)	$k_H =$ horizontal permeability, md
$c =$ compressibility, psi ⁻¹ (kPa ⁻¹)	$k_J =$ reservoir rock permeability (based on PI test), md
$c_f =$ formation compressibility, psi ⁻¹ (kPa ⁻¹)	$k_o =$ permeability to oil, md
$c_g =$ gas compressibility, psi ⁻¹ (kPa ⁻¹)	$k_s =$ permeability of altered zone (skin effect), md
$c_{gi} =$ gas compressibility evaluated at original reservoir pressure, psi ⁻¹ (kPa ⁻¹)	$k_v =$ vertical permeability, md
$c_{gw} =$ compressibility of gas in wellbore, psi ⁻¹ (kPa ⁻¹)	$k_w =$ permeability to water, md
$c_o =$ oil compressibility, psi ⁻¹ (kPa ⁻¹)	$k_{wb} =$ near-well effective permeability (McKinley method), md
$c_{pr} =$ pseudoreduced compressibility	$L =$ distance from well to no-flow boundary, ft (m)
$c_t = S_o c_o + S_w c_w + S_g c_g + c_f$ =total compressibility, psi ⁻¹ (kPa ⁻¹)	$L_f =$ length of one wing of vertical fracture, ft (m)
$c_{ti} =$ total compressibility evaluated at p_i , psi ⁻¹ (kPa ⁻¹)	$m = 162.2 qB\mu/kh$ =absolute value of slope of middle-time line, psi/cycle (kPa·cycle)
$c_{tp} =$ total compressibility evaluated at $\bar{p}$, psi ⁻¹ (kPa ⁻¹)	$m' = 162.6 B\mu/kh$ =slope of drawdown curve with $(p_i - p_{wf})/q$ as abscissa, psi/STB-D-cycle (kPa/m ³ /d·cycle)
$c_w =$ water compressibility, psi ⁻¹ (kPa ⁻¹)	$m'' =$ slope of p_{ws}^2 or p_{wf}^2 plot for gas well, psia ² /cycle (kPa·cycle)
$c_{wb} =$ compressibility of liquid in wellbore, psi ⁻¹ (kPa ⁻¹)	$m_L =$ slope of linear flow graph, psi/hr ^{1/2} (kPa·h ^{1/2})
$c_{wp} =$ compressibility of pure (gas-free) water, psi ⁻¹ (kPa ⁻¹)	$m_{\text{max}} =$ maximum slope on buildup curve of fractured well, psi/cycle (kPa·cycle)
$C =$ performance coefficient in gas-well deliverability equation	$m_{\text{true}} =$ true slope on buildup curve uninfluenced by fracture, psi/cycle (kPa·cycle)
$C_A =$ shape constant or factor	$M =$ molecular weight of gas
$C_s =$ wellbore storage constant, bbl/psi (m ³ /kPa)	$n =$ inverse slope of empirical gas-well deliverability curve
$C_{sD} = 0.894 C_s / \phi c_i h r_w^2$ =dimensionless wellbore storage constant	$p =$ pressure, psi (kPa)
$D =$ non-Darcy flow constant, D/Mscf (d/m ³)	$\bar{p} =$ volumetric average or static drainage-area pressure, psi (kPa)
$E =$ flow efficiency, dimensionless	$p^* =$ MTR pressure trend extrapolated to infinite shut-in time, psi (kPa)
$Ei(-x) = -\int_x^\infty (e^{-u}/u) du$ =the exponential integral	$p_D = 0.00708 kh(p_i - p)/qB\mu$ =dimensionless pressure as defined for constant-rate problems
$F' = \Delta t_p / \Delta t_c$ =ratio of pulse length to cycle length	

$p_{DMBH} = 2.303(p^* - \bar{p})/m$, dimensionless

p_i = original reservoir pressure, psi (kPa)

p_{MT} = pressure on extrapolated MTR, psi (kPa)

p_o = arbitrary reference pressure, psia (kPa)

p_{pc} = pseudocritical pressure, psia (kPa)

p_{pr} = pseudoreduced pressure

p_r = pressure at radius r , psi (kPa)

p_{sc} = standard-condition pressure, psia (kPa)
(frequently, 14.7 psia)

p_{wf} = flowing BHP, psi (kPa)

p_{ws} = shut-in BHP, psi (kPa)

$p_{1\text{ hr}}$ = pressure at 1-hour shut-in (or flow)
time on middle-time line (or its
extrapolation), psi (kPa)

q = flow rate, STB/D (m^3/d)

q_D = dimensionless instantaneous flow rate at
constant BHP

q_g = gas flow rate, Mscf/D (m^3/d)

q_{gt} = total gas flow rate from oil well, Mscf/D
(m^3/d)

Q_p = cumulative production at constant BHP,
STB (m^3)

$$Q_{pD} = \frac{BQ_p}{1.119 \phi c_i h r_w^2 (p_i - p_{wf})}$$

= dimensionless cumulative production

R = universal gas constant

R_s = dissolved GOR, scf gas/STB oil (m^3/m^3)

R_{sw} = dissolved gas/water ratio,
scf gas/STB water (m^3/m^3)

R_{swp} = solubility of gas in pure (gas-free) water,
scf gas/STB water (m^3/m^3)

r = distance from center of wellbore, ft (m)

r_{dt} = transient drainage radius, ft (m)

r'_d = radius of drainage, ft (m)

r_e = external drainage radius, ft (m)

$r_{eD} = r_e/r_w$

r_i = radius of investigation, ft (m)

r_s = radius of altered zone (skin effect), ft (m)

r_w = wellbore radius, ft (m)

r_{wa} = effective wellbore radius, ft (m)

s = skin factor, dimensionless

$s' = s + Dq_g$ = apparent skin factor from
gas-well buildup test, dimensionless

$s^* = \log(k/\phi\mu c_i r_w^2) - 3.23 + 0.869s$

$\bar{s} = \log\left(\frac{k}{\phi\mu c_i r_w^2}\right) - 3.23 + 0.869s$

S_g = gas saturation, fraction of pore volume

S_o = oil saturation, fraction of pore volume

S_w = water saturation, fraction of pore volume

t = elapsed time, hours

$t_D = 0.000264 kt/\phi\mu c_i r_w^2$
= dimensionless time

$t_{DA} = 0.000264 kt/\phi\mu c_i A$
= dimensionless time based on drainage
area, A

$t_{DL_f} = 0.000264 kt/\phi\mu c_i L_f^2$
= dimensionless time based on fracture

half-length

t_{end} = end of MTR in drawdown test, hours

t_{lt} = time at which late-time region begins,
hours

= lag time in pulse test, hours

t_p = cumulative production/most recent
production rate = pseudoproducing time,
hours

t_{pss} = time required to achieve pseudosteady
state, hours

t_s = time for well to stabilize, hours

t_{wbs} = wellbore storage duration, hours

T = reservoir temperature, °R (°K)

T_{pc} = pseudocritical temperature, °R (°K)

T_{pr} = pseudoreduced temperature

T_{sc} = standard condition temperature, °R (°K)
(usually 520°R)

u = flow rate per unit area (volumetric
velocity), RB/D-sq ft ($\text{m}^3/\text{d}\cdot\text{m}^2$)

V_p = reservoir pore volume, cu ft (m^3)

V_R = reservoir volume, bbl (m^3)

V_w = wellbore volume, bbl (m^3)

x = distance coordinate used in linear flow
analysis, ft (m)

Y_1 = Bessel function

z = gas-law deviation factor, dimensionless

z_i = gas-law deviation factor evaluated at
pressure p_i , dimensionless

$z_{\bar{p}g}$ = gas-law deviation factor evaluated at $\bar{p}$,
dimensionless

α_n = roots of equation $J_1(\alpha_n r_{eD})Y_1(\alpha_n)$
 $-J_1(\alpha_n)Y_1(\alpha_n r_{eD})=0$

γ_g = gas gravity (air=1.0)

γ_o = oil gravity (water=1.0)

ΔN_p = oil production during a time interval, STB
(m^3)

$\Delta p^* = p^* - p_w$, psi (kPa)

$(\Delta p)_d$ = pressure change at departure (McKinley
method), psi (kPa)

$(\Delta p)_s = 141.2 qB\mu(s)/kh = 0.869 ms$ = additional
pressure drop across altered zone, psi
(kPa)

$\Delta p_{ws}^* = p_{ws} - p_{MT}$ = difference between pressure
on buildup curve and extrapolated
MTR, psi (kPa)

Δt = time elapsed since shut-in, hours

$\Delta t'$ = time elapsed since rate change in two-rate
flow test, hours

Δt_c = cycle length (flow plus shut-in) in pulse
test, hours

Δt_d = time at departure (McKinley method),
hours

Δt_{end} = time MTR ends, hours

Δt_p = pulse-period length, hours

Δt_x = time at which middle- and late-time
straight lines intersect, hours

$\eta = 0.000264 k/\phi\mu c_i$ = hydraulic diffusivity,
sq ft/hr (m^2/h)

$\lambda_t = (k_o/\mu_o + k_g/\mu_g + k_w/\mu_w)$
 =total mobility, md/cp (md/Pa·s)
 μ = viscosity, cp (Pa·s)
 μ_g = gas viscosity, cp (Pa·s)
 μ_i = gas viscosity evaluated at p_i , cp (Pa·s)
 μ_o = oil viscosity, cp (Pa·s)
 $\mu_{\bar{p}}$ = gas viscosity evaluated at $\bar{p}$, cp (Pa·s)
 μ_w = water viscosity, cp (Pa·s)

ρ = density of liquid in wellbore, lbm/cu ft
 (kg/m³)
 ϕ = porosity of reservoir rock, dimensionless
 $\psi(p) = 2 \int_{p_o}^p \frac{p}{\mu z} dp$
 =gas pseudopressure, psia²/cp
 (kPa²/Pa·s)

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