Petroleum and Mining Engineering College
Department of Petroleum & Refining Engineering

Third stage

Petroleum Product Engineering

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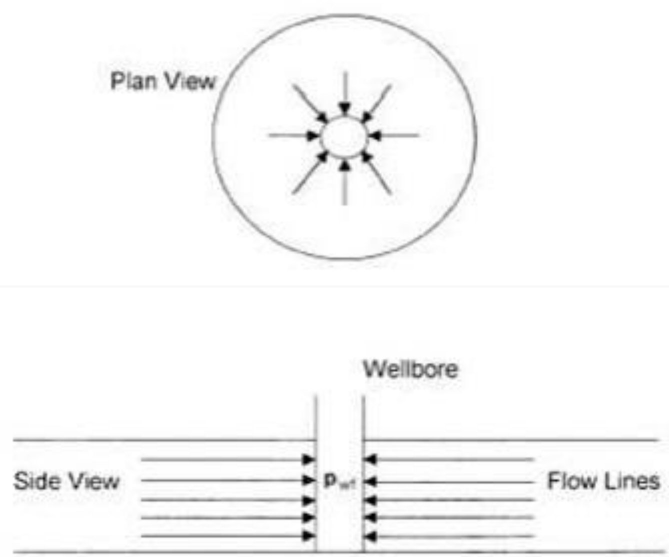
Radial Flow in the Reservoir:

Description of geometry is often possible only with the use of numerical simulators. For many engineering purposes, however, the actual flow geometry may be represented by one of the following flow geometries:

I. Radial flow
II. Linear flow
III. Spherical and hemispherical flow

I. Radial Flow:

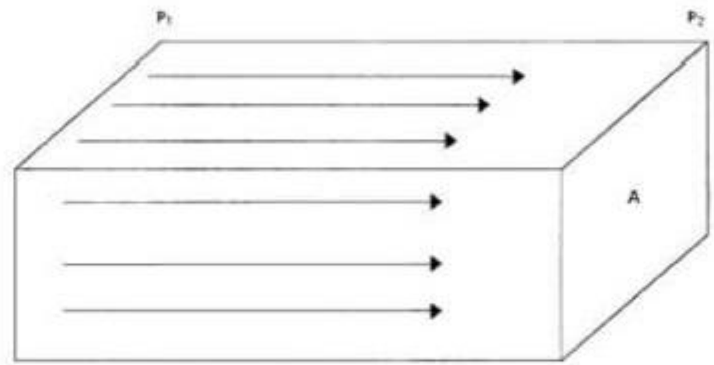
In the absence of severe reservoir heterogeneities, flow into or away from a wellbore will follow radial flow lines from a substantial distance from the wellbore. Because fluids move toward the well from all directions and coverage at the wellbore, the term radial flow is given to characterize the flow of fluid into the wellbore. shows idealized flow lines and iso potential - lines for a radial flow system.



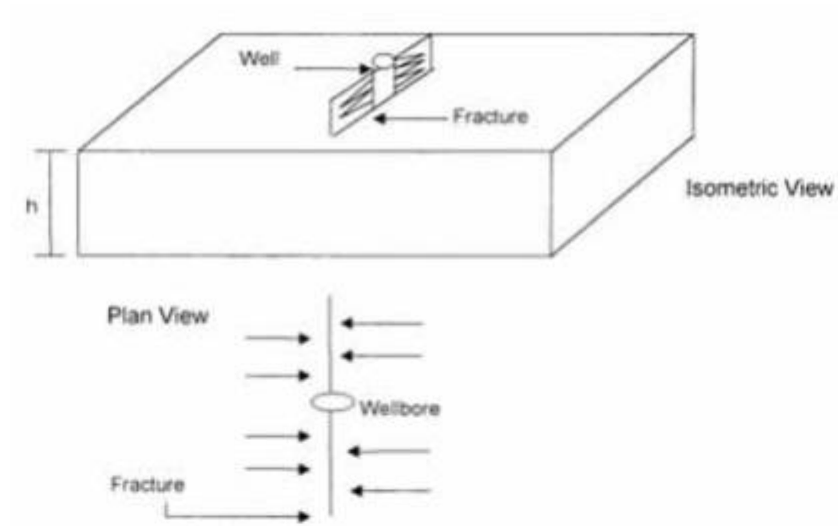
Ideal Radial Flow into the Reservoir

II. Linear Flow

Linear flow occurs when flow paths are parallel and the fluid flows in a single direction. In addition, the cross sectional area to flow must be constant.   shows an idealized linear flow system. A common application of linear flow equations is the fluid flow into vertical hydraulic fractures



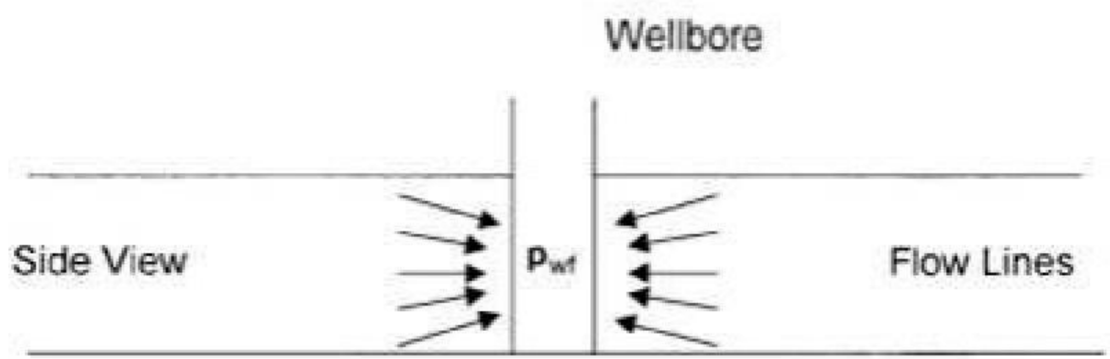
Linear Flow



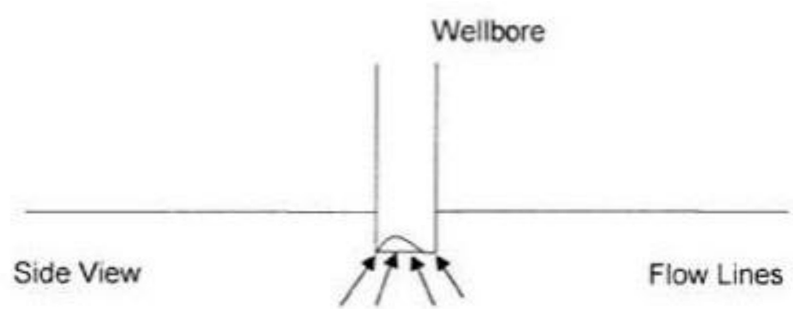
Ideal Linear Flow into Vertical Fracture

III. Spherical and Hemispherical Flow

Depending upon the type of wellbore completion configuration, it is possible to have a spherical or hemispherical flow near the wellbore. A well with a limited perforated interval could result in spherical flow in the vicinity of the perforations as illustrated in Figure **below.** A well that only partially penetrates the pay zone, **also it** could result in hemispherical flow.
The condition could arise where coning of bottom water is important.



Spherical Flow due to Limited Entry



Hemispherical Flow

FLUID FLOW EQUATIONS

Darcy's Law

The fundamental law of fluid motion in porous media is Darcy's Law. The mathematical expression developed by Henry Darcy in 1856 states the velocity of a homogeneous fluid in a porous medium is proportional to the pressure gradient and inversely proportional to the fluid viscosity. For a horizontal linear system, this relationship is:

$$v = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx}$$

v is the apparent velocity in centimeters per second and is equal to q/A, where q is the volumetric flow rate in cubic centimeters per second and A is total cross-sectional area of the rock in square centimeters. In other words, A includes the area of the rock material as well as the area of the pore channels. The fluid viscosity, μ, is expressed in centipoise units, and the

Pressure gradient, dp/dx, is in atmospheres per centimeter, taken in the same direction as v and q. The proportionality constant, k, is the permeability of the rock expressed in Darcy units.

The negative sign in the equation is added because the pressure gradient is negative in the direction of flow.

For a horizontal-radial system, the pressure gradient is positive and Darcy's equation can be expressed in the following generalized radial form:

$$v = \frac{q_r}{A_r} = \frac{k}{\mu}\left(\frac{\partial p}{\partial r}\right)_r$$

Where

q= volumetric flow rate at radius r

A = cross-sectional area to flow at radius r

$(\partial p/\partial r)$= pressure gradient at radius r

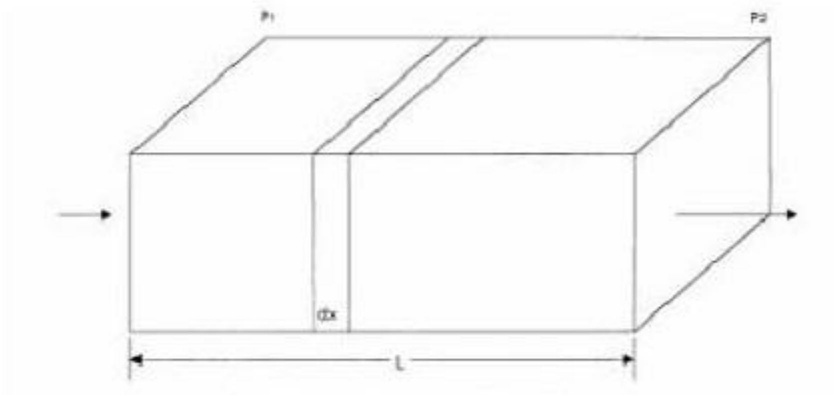
v=apparent velocity at radius r

The cross-sectional area at radius r is essentially the surface area of a cylinder. For a fully penetrated well with a net thickness of h, the cross-sectional area A is given by:

$$A_r = 2\pi rh$$

Linear Flow of Incompressible Fluids

In the linear system, it is assumed the flow occurs through a constant cross-sectional area A, where both ends are entirely open to flow. It is also assumed that no flow crosses the sides, top, or bottom as shown in Figure below:'



Linear Flow Model

If an incompressible fluid is flowing across the element dx, then the fluid velocity v and the flow rate q are constants at all points. The flow behavior in this system can be expressed by the differential form of Darcy's equation and integrating over the length of the linear system gives

$$\frac{q}{A}\int_{0}^{L} dx = -\frac{k}{\mu}\int_{p_1}^{p_2} dp$$

$$q = \frac{kA(p_1 - p_2)}{\mu L}$$

OR

It is desirable to express the above relationship in customary field units

$$q = \frac{0.001127\ kA(p_1 - p_2)}{\mu L}$$

Where:

q = flow rate, bbl/day

k = absolute permeability, md

p = pressure, psia

μ=viscosity, cp

L = distance, ft

A = cross-sectional area, ft2

Example: An incompressible fluid flows in a linear porous media with the following properties:

$L = 2000$ ft $h = 20'$ width $= 300'$
$k = 100$ md $\phi = 15\%$ $\mu = 2$ cp
$p_1 = 2000$ psi $p_2 = 1990$ psi

Calculate:

a. Flow rate in bbl/day

b. Apparent fluid velocity in ft/day

c. Actual fluid velocity in ft/day

Solution:

Calculate the cross-sectional area A:

$$A = (h)\ (\text{width}) = (20)\ (300) = 6000\ \text{ft}^2$$

a. Calculate the flow rate from Equation 6-14:

$$q = \frac{(0.001127)(100)\ (6000)\ (2000 - 1990)}{(2)\ (2000)} = 1.6905\ \text{bbl/day}$$

b. Calculate the apparent velocity:

$$v = \frac{q}{A} = \frac{(1.6905)(5.615)}{6000} = 0.0016\ \text{ft/day}$$

c. Calculate the actual fluid velocity:

$$v = \frac{q}{\phi A} = \frac{(1.6905)(5.615)}{(0.15)(6000)} = 0.0105\ \text{ft/day}$$

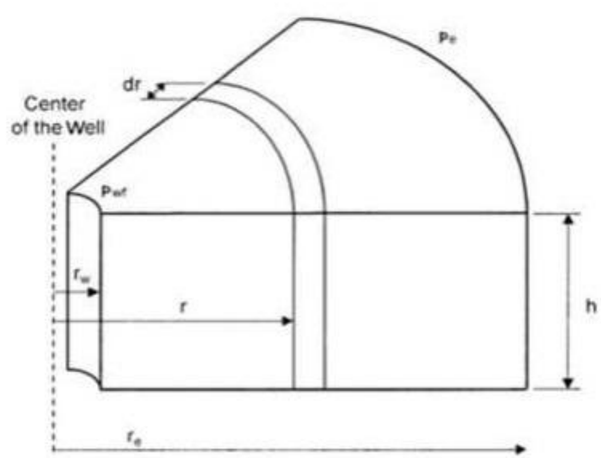
Radial Flow of Incompressible Fluids

In a radial flow system, all fluids move toward the producing well from all directions. Before flow can take place, however, a pressure differential must exist. Thus, if a well is to produce oil, which implies a flow of fluids through the formation to the wellbore, the pressure in the formation at the wellbore must be less than the pressure in the formation at some distance from the well.

The pressure in the formation at the wellbore of a producing well is know as the bottom-hole flowing pressure (flowing BHP, Pwf). Consider Figer below which schematically illustrates the radial flow of

an incompressible fluid toward a vertical well. The formation is considered to a uniform thickness h and a constant permeability k. Because the fluid is incompressible, the flow rate q must be constant at all radii. Due to the steady-state flowing condition, the pressure profile around the wellbore is maintained constant with time. Let Pwf represent the maintained bottom-hole flowing pressure at the wellbore radius rw and pe denote the external pressure at the external or drainage radius. Darcy's equation as described earlier can be used to determine the flow rate at any radius r:

$$v = \frac{q}{A_r} = 0.001127 \frac{k}{\mu} \frac{dp}{dr}$$



Radial Flow Model

Frequently the two radii of interest are the wellbore radius r_w and the *external* or *drainage* radius r_e. Then:

$$Q_o = \frac{0.00708\,kh\,(p_e - p_w)}{\mu_o\,B_o\,\ln(r_e/r_w)}$$

where Q_o = oil, flow rate, STB/day
p_e = external pressure, psi
p_{wf} = bottom-hole flowing pressure, psi
k = permeability, md
μ_o = oil viscosity, cp
B_o = oil formation volume factor, bbl/STB
h = thickness, ft
r_e = external or drainage radius, ft
r_w = wellbore radius, ft

To account for the convergence effects of flow, a simplified model based upon the assumption of radial flow to a central well located in the middle of a cylindrical reservoir unit is assumed as shown in Figure below:

The model assumes:

- The reservoir is horizontal and of constant thickness h.
- The reservoir has constant rock properties of φ and K.
- Single phase flow occurs to the well bore.
- The reservoir is circular of radius re.
- The well is located at the center of the reservoir and is of radius r_w.
- The fluid is of constant viscosity μ.
- The well is vertical and completed open hole, i.e. fluid enters the wellbore through the total height h.