The Stress in the Earth *after* Drilling a Borehole:

Before a wellbore is drilled the rock is in a state of equilibrium. This state is called the "Initial State".

The stresses in the earth under this condition are known as the **Far Field Stresses** (σ_h, σ_H, σ_v) or **in-situ stresses**.

When a well is drilled it introduces a perturbation in the initial stress field. The perturbation causes a 'new' set of stresses known as **wellbore stresses** that act on the formation at the mud-formation interface.

The far field stresses have therefore been altered near the wellbore due to the removal of rock and substitution of drilling fluid during the creation of the borehole.

The wellbore stresses are described as follows (Figure 1):

σ_r = Radial Stress

σ_t = Tangential Stress (or Hoop Stress)
σ_a = Axial Stress

Figure 2 shows how these wellbore stresses vary with distance away from the borehole wall.

This shows that the wellbore stresses diminish ***rapidly*** from the borehole wall ***converting to far field stresses.*** This makes sense because away from the wellbore the rock is in an unperturbed state (with stresses σ_h, σ_H, σ_v).

σ_r is seen to converge to σ_h and σ_t is seen to converge to σ_H at very large distances from the wellbore.

At the wellbore wall itself:

$$\sigma_r = p_w \tag{1}$$

$$\sigma_t = (\sigma_H + \sigma_h) - 2(\sigma_H - \sigma_h)\cos 2\theta - p_w \tag{2}$$

$$\sigma_a = \sigma_v - 2(\sigma_H - \sigma_h)\,\mathrm{v}\cos 2\theta \tag{3}$$

Where:

σ_r = radial stress

σ_t = tangential stress

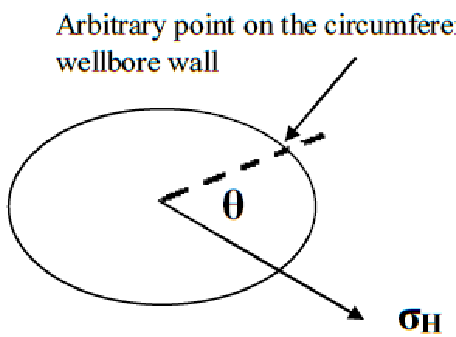
σ_a = axial stress

σ_v = vertical stress

σ_h = minimum horizontal stress

σ_H = maximum horizontal stress

p_w = mud weight



θ = Angle between a point on the circumference of the wellbore and the direction of maximum Horizontal Stress (σ_H)

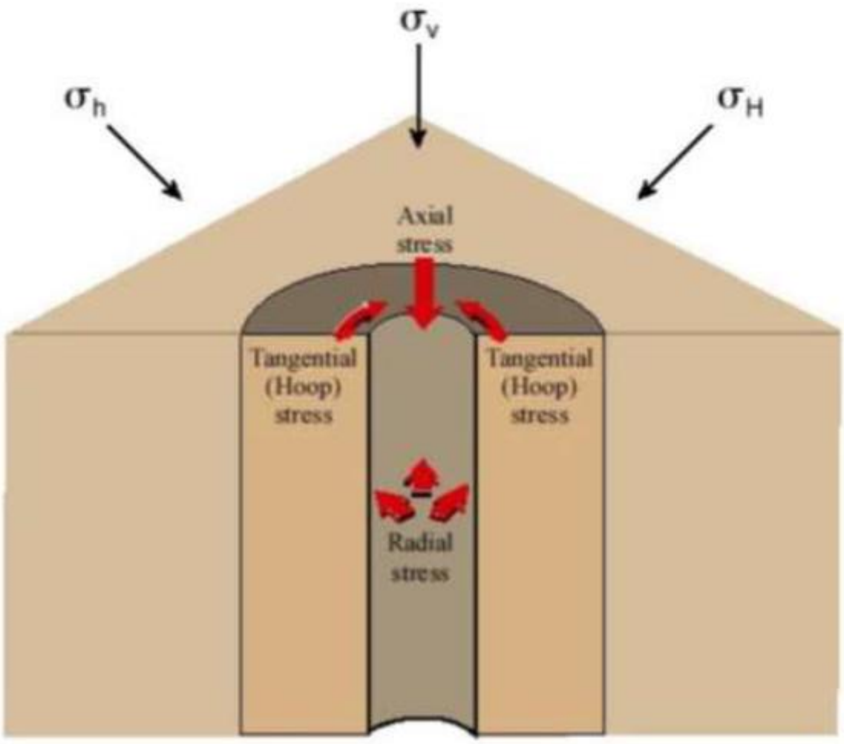


Figure 1: The 3 Wellbore Stresses

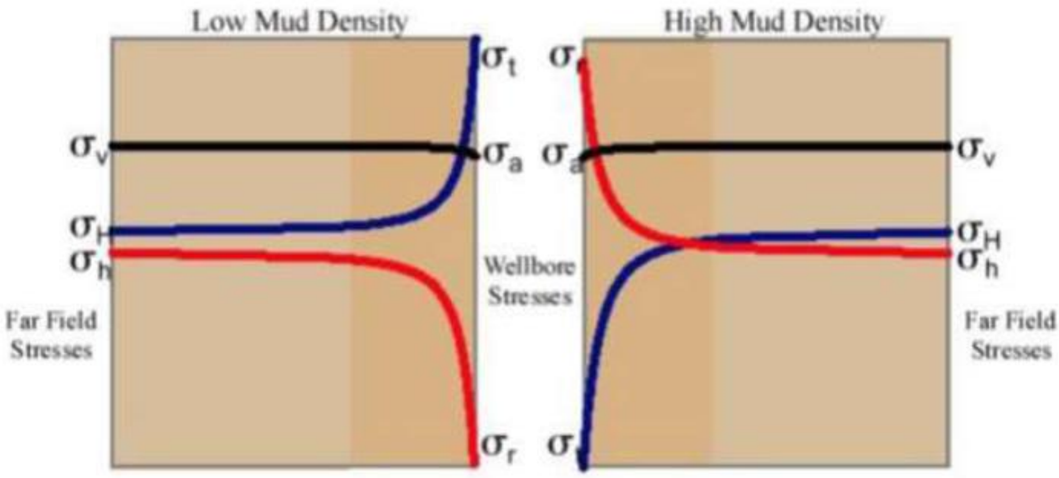


Figure 2: Variation of Wellbore Stresses away from the wellbore

Thus:

- The radial stress σ_r depends on the mud weight (p_w). It acts in all directions perpendicular to the wellbore wall as shown in figure 1.
- When the mud weight is high, then at the wellbore wall σ_r is large and σ_t is small and vice versa. This can be seen from examination of figure 2 and equations 1and 2.
- The tangential (hoop) stress σ_t circles the borehole. Its magnitude depends on:

-The far field stresses (σ_h and σ_H)

-The mud weight (p_w)

-The azimuthal position around the wellbore (θ)

Hence σ_t is a direction dependent variable.

When θ = 0°:

$$(\sigma_t)_{\theta=0} = 3\sigma_h - \sigma_H - p_w \tag{4}$$

When θ = 90°:

$$(\sigma_t)_{\theta=90°} = 3\sigma_H - \sigma_h - p_w \tag{5}$$

The maximum tangential stress occurs in a position that is 90° from the maximum horizontal stress σ_H (equation 5), i.e. in the orientation of the minimum horizontal stress σ_h

e.g.

Wellbore pressure = 3000 psi (in equilibrium with the pore pressure of the reservoir) = p_w

σ_h = 3500 psi, σ_H = 5000 psi

The equations lead to values for the effective tangential stress (σ_t – p) of:

σ_t = (3x3500) – 5000 – 3000 – p = -500 psi (θ = 0°)

σ_t = (3x5000) – 3500 – 3000 – p = 5500 psi (θ = 90°)

(assuming pore pressure p = 3000 psi)

The tangential stress is in compression at θ = 90° and in tension at θ = 0.

The latter result indicates the possibility for the occurrence of tensile failure in a direction perpendicular to the minimum stress, solely as a result of drilling the borehole.

The wellbore stresses (given by equations 1, 2, 3) are extremely important because it is at the borehole wall where problems occur that affect the drilling, not far from the wellbore. These wellbore stresses form the basis for wellbore shear failure.

The mud weight (p_w) is the major parameter that the Drilling Engineer has in order to influence these wellbore stresses.

Objective: Describe the various wellbore stresses acting at the wellbore wall.

With reference to figure (3). A coordinate system is used to so that:

x' is parallel with σ_H
y' is parallel with σ_h
z' is parallel with σ_v
x is parallel to the low side of the hole
y is perpendicular to x (and perpendicular to the axis of the borehole)
z is parallel to the axis of the borehole

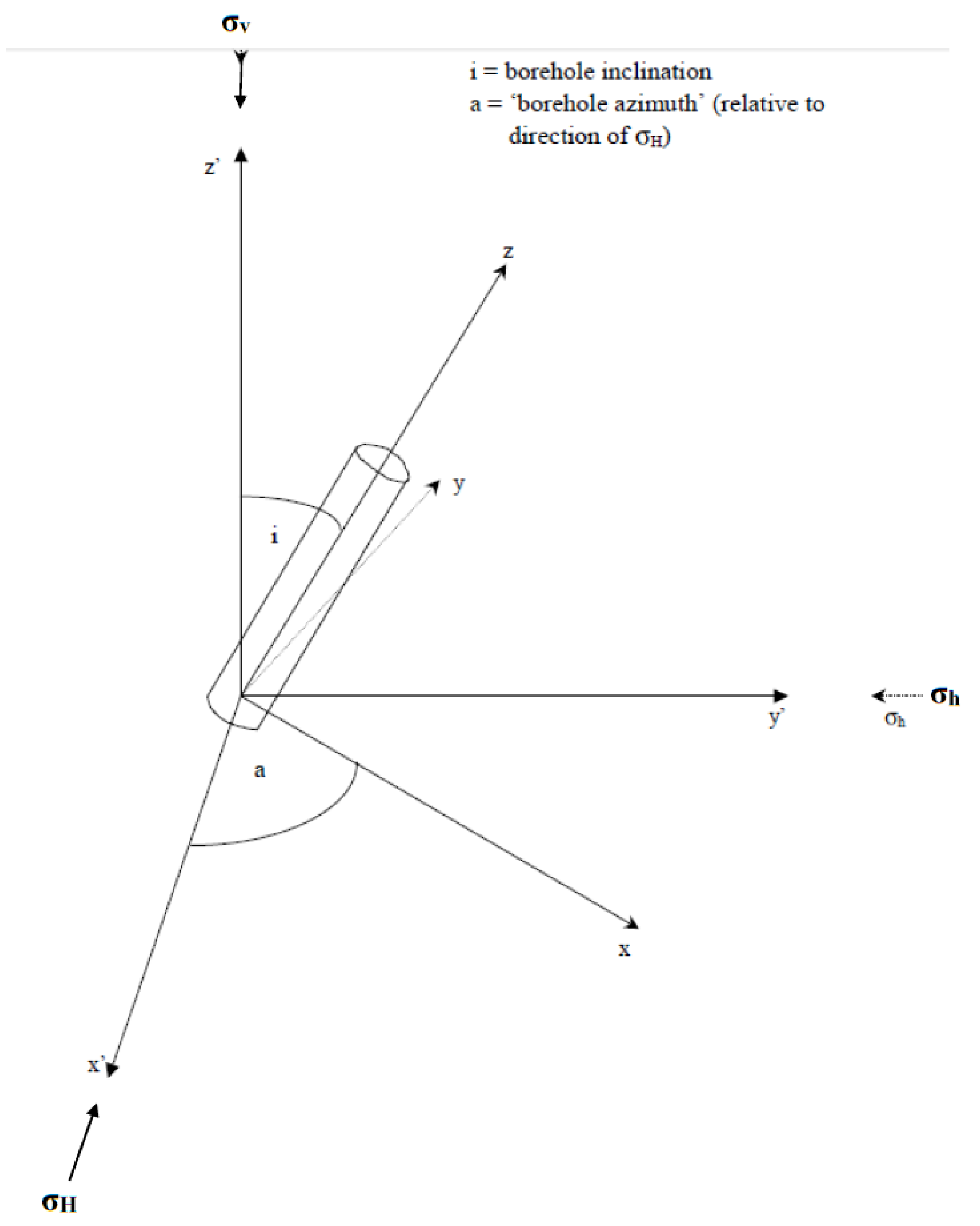


Figure 3: Deviated borehole in an anisotropic stress field (i.e. where $\boldsymbol{\sigma_H \neq \sigma_h}$)

A transform from (x' y' z') to (x, y, z) can be performed by a rotation "a" around the z'-axis and a rotation "i" around the y-axis.

The 6 complete stress solutions are given below. It is assumed that the rock remains linear elastic, and the wellbore fluid pressure pw does not penetrate the rock (here we are assuming the presence of mud cake).

$$\sigma_r = \frac{1}{2}(\sigma_x + \sigma_y)\left\{1 - \frac{r_w^2}{r^2}\right\} + \frac{1}{2}(\sigma_x - \sigma_y)\left\{1 + \frac{3r_w^4}{r^4} - 4\frac{r_w^2}{r^2}\right\}\cos 2\theta + \tau_{xy}\left(1 + 3\frac{r_w^4}{r^4} - 4\frac{r_w^2}{r^2}\right)\sin 2\theta + p_w \frac{r_w^2}{r^2}$$

$$\sigma_t = \frac{1}{2}(\sigma_x + \sigma_y)\left\{1 + \frac{r_w^2}{r^2}\right\} - \frac{1}{2}(\sigma_x - \sigma_y)\left\{1 + \frac{3r_w^4}{r^4}\right\}\cos 2\theta - \tau_{xy}\left(1 + 3\frac{r_w^4}{r^4}\right)\sin 2\theta - p_w \frac{r_w^2}{r^2}$$

$$\sigma_a = \sigma_z - \nu\left[2(\sigma_x - \sigma_y)\frac{r_w^2}{r^2}\cos 2\theta + 4\tau_{xy}\frac{r_w^2}{r^2}\sin 2\theta\right]$$

$$\tau_{rt} = \frac{1}{2}(\sigma_x - \sigma_y)\left(1 - 3\frac{r_w^4}{r^4} + 2\frac{r_w^2}{r^2}\right)\sin 2\theta + \tau_{xy}\left(1 - 3\frac{r_w^4}{r^4} + 2\frac{r_w^2}{r^2}\right)\cos 2\theta$$

$$\tau_{ta} = (-\tau_{xz}\sin\theta + \tau_{yz}\cos\theta)\left(1 + \frac{r_w^2}{r^2}\right)$$

$$\tau_{ra} = (\tau_{xz}\cos\theta + \tau_{yz}\sin\theta)\left(1 - \frac{r_w^2}{r^2}\right)$$

σ_r = Radial Stress

σ_t = Tangential Stress (or Hoop Stress)

σ_a = Axial Stress

σ_h = Minimum Horizontal Stress

σ_H = Maximum Horizontal Stress

σ_v = Vertical Stress

r_w = Wellbore Radius

r = Distance from the wellbore

θ = Angle between a point on the circumference of the wellbore and the direction of Maximum Horizontal Stress (σ_H)

p_w = Wellbore fluid pressure

τ_{rt} / τ_{ta} / τ_{ra} = Shear stress in radial/tangential, tangential/axial, radial/axial planes respectively

In the case of a vertical well in an anisotropic stress field (i.e. $\sigma_h \neq \sigma_H$) the problem becomes far simpler.

In a vertical well, the shear stresses are zero and therefore the normal stresses become principal stresses and are σ_H and σ_h.

The above equations simplify to the following 3 equations:

$$\sigma_r = \frac{1}{2}(\sigma_H + \sigma_h)\left(1 - \frac{r_w^{\ 2}}{r^2}\right) + \frac{1}{2}(\sigma_H - \sigma_h)\left(1 + 3\frac{r_w^{\ 4}}{r^4} - 4\frac{r_w^{\ 2}}{r^2}\right)\cos 2\theta + p_w\frac{r_w^{\ 2}}{r^2}$$

$$\sigma_t = \frac{1}{2}(\sigma_H + \sigma_h)\left(1 + \frac{r_w^{\ 2}}{r^2}\right) - \frac{1}{2}(\sigma_H - \sigma_h)\left(1 + 3\frac{r_w^{\ 4}}{r^4}\right)\cos 2\theta - pw\frac{r_w^{\ 2}}{r^2}$$

$$\sigma_a = \sigma_v - \frac{1}{2}(\sigma_H - \sigma_h)\nu\left(\frac{4r_w^{\ 2}}{r^2}\right)\cos 2\theta$$

These equations merely define the value of the wellbore stress (σ_r, σ_t or σ_a) as a function of the distance (r) from the wellbore and the azimuthal position of the wellbore stress θ.

The variables are $\boldsymbol{\sigma_r}$, $\boldsymbol{\sigma_t}$, $\boldsymbol{\sigma_a}$, $\boldsymbol{\sigma_h}$, $\boldsymbol{\sigma_H}$, **r**, $\boldsymbol{\theta}$

The given knowns (constants) are r_w, p_w

(The mud weight p_w and the wellbore radius r_w can be varied before the rock is drilled to see the effect).

a) At the wellbore wall itself: $r = r_w$ and $\frac{r_w}{r} = \mathbf{1}$

$$\sigma_r = p_w \tag{6}$$

$$\sigma_t = (\sigma_H + \sigma_h) - 2(\sigma_H - \sigma_h)\cos 2\theta - p_w \tag{7}$$

$$\sigma_a = \sigma_v - 2(\sigma_H - \sigma_h)\,\mathrm{v}\cos 2\theta \tag{8}$$

These wellbore stresses are used in rock failure analysis.

The radial stress depends only on the mud weight while the tangential stress depends on the azimuthal position around the wellbore θ as well as the mud weight.

Note that from equation 8 the axial stress σ_a also depends on the value of θ. The reason is that the axial stress is affected by the horizontal stresses (which themselves vary with θ), just as the horizontal stresses can be increased (forced laterally) by applying force axially.

b) Far from the wellbore wall $r \to \infty$ and $r_w / r \to 0$

The stresses simplify to:

$$\sigma_r = \frac{1}{2}(\sigma_H + \sigma_h) + \frac{1}{2}(\sigma_H - \sigma_h)\cos 2\theta \tag{9}$$

$$\sigma_t = \frac{1}{2}(\sigma_H + \sigma_h) - \frac{1}{2}(\sigma_H - \sigma_h)\cos 2\theta \tag{10}$$

$$\sigma_a = \sigma_v \tag{11}$$

This shows that the wellbore stresses diminish ***rapidly*** from the borehole wall ***converting to far field stresses.*** This makes sense because at large distances from the wellbore the rock is in an unperturbed state.

For example, when θ = 0 (cos2θ = 1):

$$\sigma_r = \sigma_H$$

$$\sigma_t = \sigma_h$$

$$\sigma_a = \sigma_v$$

When θ = 90° (cos2θ = -1):

$$\sigma_r = \sigma_h$$

$$\sigma_t = \sigma_H$$