Parametric equations and polar coordinates

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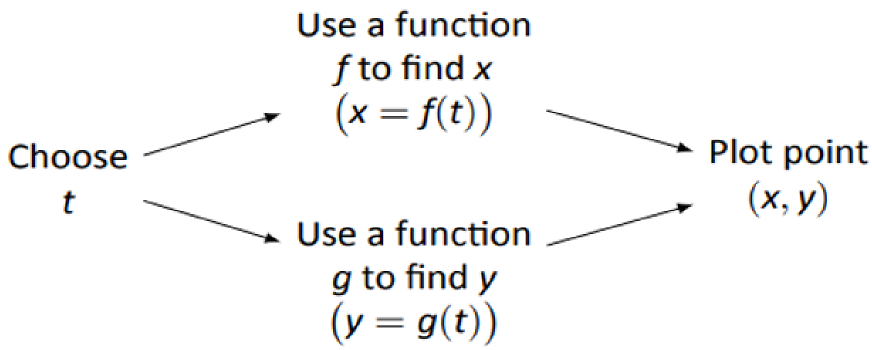
Parametric equations: Parametric equations defined that equations involving the two variables **x** and **y** describe a curve by expressing both coordinates as a function of a third parameter, often called **t**. One way to interpret this is to think of t as \time" where x (t) is the x-coordinate of a moving point at time t and y (t) is the y-coordinate of the point at the same time. In each of the following examples, draw the start/end points and the direction of motion.

$$x = f(t)$$

$$y = g(t)$$



1- **Sketch and identified the parametric functions:**

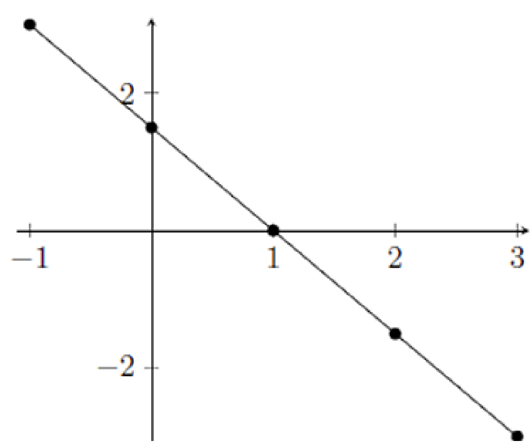
Example1: sketch the graph of the parametric equations and identify

$x = 1 + 2t$

$y = -3t$ for $-1 < t < 1$

Solution

t	x(t)	y(t)
-1	-1	3
-1/2	0	3/2
0	1	0
1/2	2	-3/2
1	3	-3



For identified

$t = \frac{x-1}{2}$ Sub in y

Y= - 3 $(\frac{x-1}{2}) = -\frac{3}{2}x + \frac{1}{2}$ (line equation, slope $= -\frac{3}{2}$)

- **Line equation**

Y=ax + b slope = a

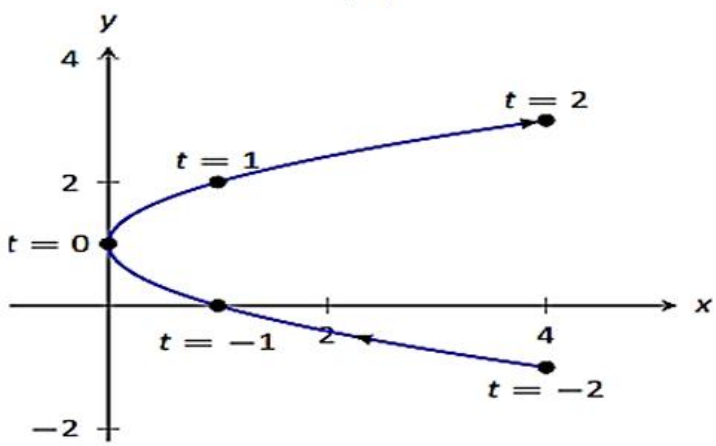
Example2: sketch and identify the graph of the parametric equations

x= t^2, y= t+1 for -2< t < 2

Solution

t	x	y
−2	4	−1
−1	1	0
0	0	1
1	1	2
2	4	3

(a)



(b)

To eliminate the parameter t=$\sqrt{x} = x^{1/2}$

$$y = x^{1/2} + 1$$

Identify:

- **Circle**

  (x-a) ^{2} **+ (y-b)** ^{2} **=** r^2 **circle center (a,b) and radius** $=\sqrt{r^2} = r$

Example 3: sketch and identify the graph of the parametric equations

X= cos(t) y= sin(t)

$cos^2 t + sin^2 t = 1$

$x^2 + y^2 = 1$ circle center (0,0) r = 1

- **Ellipse**

  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Example 4: sketch and identify the graph of the parametric equations

X= 3 cos2t y = 2sin2t

$\cos 2t = \frac{x}{3}$ $\sin 2t = \frac{y}{2}$

$cos^2 2t + sin^2 2t = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ **(Ellipse a = 3, b = 2)**

Home Work1/ sketch and identify the graph of the parametric equations

X-1=3sint y = 3cost-2

Slope of tangent line:

The slope of the tangent line is still $\frac{dy}{dx}$, and the Chain Rule allows us to calculate this in the context of parametric equations. If x=f(t) and y=g(t), the Chain Rule states that:

$$\frac{dy}{dt} = \frac{dy}{dx} . \frac{dx}{dt}$$

Solving for $\frac{dy}{dx}$, we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Example 5: if $x= 5t^2 -6t +4$ and $y= t^2+ 6t -1$ and let C be the curve defined by these equations.

Find the equation of the tangent line to curve at t=3.

Solution

$\frac{dy}{dt} = 2\,t + 6$, $\quad \frac{dx}{dt} = 10\,t - 6$

$$\frac{dy}{dx} = \frac{2t + 6}{10\,t - 6}$$

At (t = 3) $\frac{dy}{dx} = \frac{2*3+6}{10*3-6} = \frac{12}{24} = \frac{1}{2}$ (slope of tangent line $m = \frac{1}{2}$)

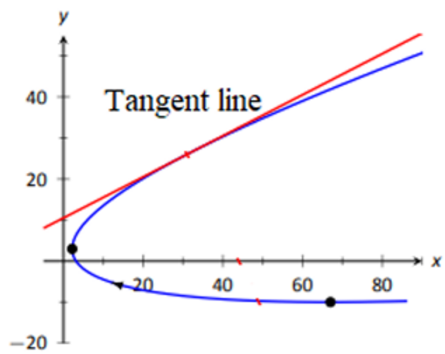
$x_o= 5* (3)^2 - 6(3) +4 = 31$, $\quad y_o= 3^2 +6 *3-1= 26$

Equation of tangent lines

$y - y_o = m(x- x_o)$

$y- 26= \frac{1}{2}(x - 31)$

$y= \frac{1}{2}(x - 31) +26$



$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}(\bar{y})$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(\bar{y})}{\frac{dx}{dt}}$$

Example 6: find $\frac{d^2y}{dx^2}$ as a function of **t** if $x = t - t^2$, $y = t - t^3$

Express $\bar{y} = \frac{dy}{dx}$

$$\bar{y} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-3t^2}{1-2t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\bar{y})}{\frac{dx}{dt}}$$

$$\frac{d}{dt}(\bar{y}) = \frac{d}{dt}\left(\frac{1-3t^2}{1-2t}\right) = \frac{2-6t+6t^2}{(1-2t)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{2-6t+6t^2}{(1-2t)^2}}{1-2t} = \frac{2-6t+6t^2}{(1-2t)^3}$$

<u>**Home Work2**</u>: If x= sec t, y = tan t, $-\frac{\pi}{2} < t < \frac{\pi}{2}$

1- Find the equation of the tangent line $(\frac{dy}{dx})$ to curve at **t**=$\frac{\pi}{4}$.

2- Find $\frac{d^2y}{dx^2}$