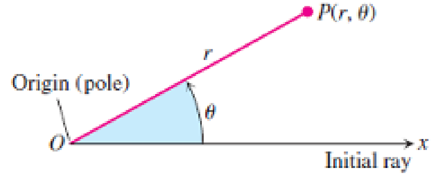
Polar Coordinates and Graphs

Polar Coordinate system



Each point P can be assigned polar Coordinates (r, θ) where:

1) r is the distance from the pole (origin) 0 to the point P. r is positive if measured from the pole along the terminal side of θ and negative if measured along the terminal side extended through the pole.

2) θ is the angle from the Initial ray to (op). The angle θ is positive if the rotation is counterclockwise and negative if the rotation is clockwise.

Review in trigonometric functions:

$$\left.\begin{aligned} \sin(-\theta) &= -\sin\theta \\ \csc(-\theta) &= -\csc\theta \\ \tan(-\theta) &= -\tan\theta \\ \cot(-\theta) &= -\cot\theta \end{aligned}\right\} \text{ odd functions}$$

$$\left.\begin{aligned} \cos(-\theta) &= \cos\theta \\ \sec(-\theta) &= \sec\theta \end{aligned}\right\} \text{ even functions}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y \quad \text{if} \quad x = y \Rightarrow \sin(2x) = 2\sin x\cos x$$

$$\cos(x \mp y) = \cos x\cos y \pm \sin x\sin y \quad \text{if} \quad x = y \Rightarrow \cos(2x) = \cos^2 x - \sin^2 x$$

$$\tan(x \mp y) = \frac{\tan x \mp \tan y}{1 \pm \tan x\tan y} \quad \text{if} \quad x = y \Rightarrow \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

Converting from polar to rectangular form and vice versa

We have the following relationship between rectangular Coordinates (Cartesian) (x, y) and polar Coordinates (r, θ) :

$$x^2 + y^2 = r^2$$

$$\cos\theta = \frac{x}{r} \qquad \text{or} \qquad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \qquad \text{or} \qquad y = r\sin\theta$$

$$\tan\theta = \frac{y}{x} \qquad \text{or} \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$dA = \left.\begin{matrix} dydx \\ dxdy \end{matrix}\right\} \Rightarrow r\ \mathrm{d}r\,\mathrm{d}\theta$$

Cartesian Coordinates

$y = f(x)$

Polar Coordinates

$r = f(\theta)$

Graphing polar equations

Sketch

i) symmetric about x-axis if replacing θ by $(-\theta)$ does not change the function.

ii) Symmetric about y-axis if replacing θ by $(\pi - \theta)$ does not change the function.

iii) Symmetric about the origin if replacing r by $(-r)$ does not change the function.

iv)

$$\theta = 0$$

$$\frac{\pi}{2}$$

$$\pi$$

$$\vdots$$

Ex.1: Converting an equation from Cartesian form to polar form

$x^2 + y^2 - 4y = 0$

Since $x^2 + y^2 = r^2$ and $y = r\sin\theta$

$\Rightarrow \quad x^2 + y^2 - 4y = 0$

$r^2 - 4r\sin\theta = 0$

$r(r - 4\sin\theta) = 0$

$r = 0$ or $r = 4\sin\theta$

the graph of $r = 0$ is the pole. because the pole is included in the graph of $r - 4\sin\theta = 0$, we can discared $r = 0$ and keep only $r = 4\sin\theta$

Ex 2: Converting an equation from polar form to Cartesian form

$r = -3\cos\theta$

$r^2 = -3r\cos\theta$ Multiply both sides by r

$\Rightarrow \quad x^2 + y^2 = -3x$

$\Rightarrow \quad x^2 + y^2 + 3x = 0$

Ex 3: Converting an equation from polar form to Cartesian form

$r\cos(\theta - \pi/3) = 3$

$r(\cos\theta\cos(\pi/3) + \sin\theta\sin(\pi/3)) = 3$

$\frac{1}{2} r\cos\theta + \frac{\sqrt{3}}{2} r\sin\theta = 3$

$\frac{1}{2} x + \frac{\sqrt{3}}{2} y = 3 \Rightarrow x + \sqrt{3}y = 6$

Ex 4: Converting an equation from polar form to Cartesian form

$r = 4\cos\theta$

$r^2 = 4r\cos\theta \Rightarrow x^2 + y^2 = 4x$